World Summary of $\alpha_s$ (2015)

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Abstract: This is a preliminary update of the measurements of $\alpha_s$ and the determination of the world average value of $\alpha_s(M_Z^2)$ presented in the 2013/2014 edition of the Review of Particle Properties [1]. A number of studies which became available since late 2013 provide new results for each of the (previously 5, now) 6 subclasses of measurements for which pre-average values of $\alpha_s(M_Z^2)$ are determined.

In the following, we list those new results which are used to determine the new average values of $\alpha_s$, i.e. which are based on at least complete NNLO perturbation theory and are published in peer-reviewed journals, as well as those which are used for demonstrating asymptotic freedom (although being based on NLO only)$^\dagger$:

- updated results from $\tau$-decays [2,3,4], based on a new release of the ALEPH data and on complete N$^3$LO perturbation theory,
- more results from unquenched lattice calculations, [5,6],
- more results from world data on structure functions, in NNLO QCD [7],
- results from $e^+e^-$ hadronic event shape (C-parameter) in soft collinear effective field theory (NNLO) [8],
- $\alpha_s$ determinations at LHC, from CMS data on the ratio of inclusive 3-jet to 2-jet cross sections [9], from inclusive jet production [10], from the 3-jet differential cross section [11], and from energy-correlations [12], all in NLO QCD, plus one determination in complete NNLO, from a measurement of the $t\bar{t}$ cross section at $\sqrt{s} = 7$ TeV [13];
- and finally, an update of $\alpha_s$ from a global fit to electro-weak precision data [14].

All measurements available in subclasses of $\tau$-decays, lattice results, structure functions, and $e^+e^-$ annihilation are summarized in Fig. 1. With the exception of lattice results, most results within their subclass are strongly correlated, however to an unknown degree, as they largely use the same data sets. The large scatter between many of these measurements, sometimes with only marginal or no agreement within the given errors, indicate the presence of additional systematic uncertainties from theory or caused by details of the analyses. In such cases, a pre-average value is determined with a symmetric overall uncertainty that encompasses the central values of all individual determinations (‘range averaging’). For the subclass of lattice results, the average value determined in Ref. [5] is taken over. For the subclasses of hadron collider results and electroweak (ewk) precision fits, only one result each is available in full NNLO, so that no pre-averaging can

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†Note that this does not fully account for all studies and results presented at this workshop, but rather reflects the restricted summary currently intended for the new edition of Ref. [1].
be applied. Note, however, that more measurements of top-quark pair production at the LHC are meanwhile available, indicating that on average, a larger value of $\alpha_s(M_Z^2)$ is likely to emerge in the future [15]. The emerging subclass averages are plotted in Fig. 1, and summarized in Table 1.

![Image](summarized.png)

**Figure 1:** Summary of determinations of $\alpha_s$ from: (a) hadronic $\tau$-decays (full circles obtained using CIPT, open circles FOPT expansions, see text), (b) lattice calculations, (c) DIS structure functions, and (d) $e^+e^-$ annihilation. The shaded bands indicate the pre-average values explained in the text, to be included in the determination of the final world average of $\alpha_s$.

<table>
<thead>
<tr>
<th>Subclass</th>
<th>$\alpha_s(M_Z^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau$-decays</td>
<td>$0.1187 \pm 0.0023$</td>
</tr>
<tr>
<td>lattice QCD</td>
<td>$0.1184 \pm 0.0012$</td>
</tr>
<tr>
<td>structure functions</td>
<td>$0.1154 \pm 0.0020$</td>
</tr>
<tr>
<td>$e^+e^-$ jets &amp; shapes</td>
<td>$0.1174 \pm 0.0051$</td>
</tr>
<tr>
<td>hadron collider</td>
<td>$0.1151^{+0.0028}_{-0.0027}$</td>
</tr>
<tr>
<td>ewk precision fits</td>
<td>$0.1196 \pm 0.0030$</td>
</tr>
</tbody>
</table>

**Table 1:** Pre-average values of subclasses of measurements of $\alpha_s(M_Z^2)$. The value from $\tau$-decays was converted from $\alpha_s(M_T^2) = 0.322 \pm 0.019$, using the QCD 4-loop $\beta$-function plus 3-loop matching at the charm- and bottom-quark pole masses.
Assuming that the resulting pre-averages are largely independent of each other, the final world average value is determined as the weighted average of the different input values. An initial uncertainty of the central value is calculated treating the uncertainties of all measurements as being uncorrelated and of Gaussian nature, and the overall $\chi^2$ to the central value is determined. If the initial value of $\chi^2$ is smaller than the number of degrees of freedom, an overall, a-priori unknown correlation coefficient is introduced and determined by requiring that the total $\chi^2$/d.o.f. equals unity. Applying this procedure to the values listed in Table 1 results in a preliminary new world average of

$$\alpha_s(M_Z^2) = 0.1177 \pm 0.0013.$$ 

This value is in reasonable agreement with that from 2013/2014, which was $\alpha_s(M_Z^2) = 0.1185 \pm 0.0006$ [1], however at a somewhat decreased central value and with an overall uncertainty that has doubled. These changes are mainly due to the following reasons:

- the uncertainty of the lattice result, now taken from the estimate made by the FLAG group, is more conservative than that used in the previous review, leading to a larger final uncertainty of the new world average, and to a reduced fixing power towards the central average value;
- the decreased pre-average value of $\alpha_s(M_Z^2)$ from $\tau$-decays, due to the most recent re-evaluations and their unexplained, increased inconsistency with respect to each other;
- the relatively low value of $\alpha_s$ from the new sub-class of hadron collider results, which currently consists of only one measurement of the $t\bar{t}$ cross section at $\sqrt{s} = 7$ TeV, and which appears to be “lowish” if compared to further measurements at higher $\sqrt{s}$ [15].

Note that pending discussions about inclusion or exclusion of some of the most recent results, as well as refinements of the procedures applied may still change the final value of $\alpha_s(M_Z^2)$ and its assigned overall uncertainty. While there is still room for improved measurements and treatments of systematic uncertainties, the data and results, especially when including measurements which are available at NLO only, consistently demonstrate and prove asymptotic freedom and the running of $\alpha_s$, as predicted by QCD, up to energies beyond 1 TeV, see Fig. 2.

References

Figure 2: Summary of measurements of $\alpha_s$ as a function of the energy scale $Q$. The respective degree of QCD perturbation theory used in the extraction of $\alpha_s$ is indicated in brackets (NLO: next-to-leading order; NNLO: next-to-next-to leading order; res. NNLO: NNLO matched with resummed next-to-leading logs; $N^3$LO: next-to-NNLO).