Signets of CP Violation

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ABSTRACT

In all known realistic theories the CP-odd operators in the effective Lagrangian are the “signets” of CP violation: they summarily but uniquely describe both theory and observable effects. The archetype flavour-diagonal signets are the electric dipole moments (EDM) of fundamental fermions and their neutral-weak and coloured generalizations, as well as the analogous quantities describing CP-odd $\gamma W^+ W^-$ and $Z W^+ W^-$ vertices. The gauge symmetry of the standard model must be respected above the weak scale by any putative theory of the origin of CP violation. This constraint relates EDMs and their generalizations to a blooming ensemble of other higher dimensional signets, some of which involve extra scalar fields and/or gauge bosons and imply novel processes. To gauge what can be done at present and future facilities (e.g. LEP-I and LEP-II) in the CP-violation domain, we give limits on the most relevant of all CP-odd signets, sometimes directly from experiment, often via observable effects induced by electroweak and QCD quantum corrections. These bounds allow us to pinpoint the most promising areas of future development.

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1. Introduction, Motivation and Nomenclature

Back in 1966, Nicola Cabibbo summarized a talk [1] by showing a cartoon of a nuclear mushroom labelled “CP violation” and two ostriches with their proverbially buried heads. In the caption they tell each other “We fully understand the weak interactions”. The situation has improved but not changed.

In the three-generation Standard Model, CP violation originates from the single phase naturally occurring in the Cabibbo-Kobayashi-Maskawa matrix. The observations in the neutral kaon system, as well as the limits gathered elsewhere, agree with the expectations, so... why bother? For one reason, we do not understand why nature should have chosen the number and properties of fundamental fields *just so* that CP violation may be possible, but only large enough to be observable in the kaon sector and, perhaps, in its beauty analog. Furthermore, the CP-odd dynamics allegedly underlying a matter-antimatter asymmetric universe [2] remain totally obscure. Finally, the Peccei-Quinn [3] solution to the QCD CP-conservation problem is no longer very satisfying. A suspiciously evasive axion [4] must be invoked, lest disaster strike in the form of a behemoth θ -term contribution [5] to the neutron’s EDM. But axions have not been found at their natural weak scale, nor should invisible axions exist at the grand-unified level [6]. Astronomical observations constrain the scale of axion dynamics to narrow ranges that do not coincide with any other known physical scale [7].

The conclusions that the axion scale is “unnatural” and that perhaps the whole standard CP machinery is misconstrued, seem to be inescapable. The questions of CP violation and/or conservation continue to tease us from the twilight zone. All of this, as well as the paucity of the observational input, provides a strong motivation to continue to explore the subject of CP non-invariance both theoretically and experimentally.

We have no affirmative empirical information on CP violation other than that stemming from the kaon system or, perhaps, from our own existence [2]. Elsewhere, the most stringent limits concern the electric dipole moments of the neutron and the electron, though bounds for Λ ’s, μ ’s and ν ’s also exist. LEP-I will contribute an extension of these measurements to CP-violating couplings of the Z_0 , and to fermions of the third generation; LEP-II will extend these studies to W ’s. In this paper we systematize the information from the various experimental sources in the form of bounds on CP-violating vertices involving a photon, a gluon or a Z_0 ; plus a quark pair, a lepton pair, or a pair of W ’s. In other words, in the Table of Properties of today’s elementary particles, we attempt to

complete the column pertaining to the CP-violating three-particle vertices, sometimes directly, often via effects induced by electroweak and QCD radiative corrections. Our aim is not numerical, the limits we gather or freshly obtain constitute tight and ready-to-use constraints on past and future theories [8] or experiments. Theoretical consistency will force us to consider not only the above mentioned CP-odd three-particle couplings, but also more involved ones, which we also discuss and constrain.

The electric dipole moment d_f of an on-shell spinor f is the value at vanishing momentum transfer of the coefficient of an effective dimension-five interaction of the form:

$$\mathcal{L}_f^1 = \frac{1}{2} d_f^1 \bar{\psi}_f \{i\sigma_{\mu\nu} \gamma_5\} \psi_f F^{\mu\nu}. \quad (1.1)$$

This conventional expression is not unique. An electric dipole moment (EDM) can equally well be expressed in the form of a “convective axial current”:

$$\hat{\mathcal{L}}_f^1 = d_f^1 \bar{\psi}_f \{ \vec{\partial}_\mu \gamma_5 \} \psi_f A^\mu, \quad (1.2)$$

identical to (1.1) for on-shell fermions. We shall have to discuss in detail the extent to which relations such as $\mathcal{L}_f^1 = \hat{\mathcal{L}}_f^1$ can be extended off-shell. The electric dipole vertex, or its bimodal off-shell generalization, is the lowest-dimension CP-violating operator describing the coupling of a photon to a spinor and not removable within the standard model by an innocuous field redefinition, or a Peccei-Quinn symmetry. It is therefore generally concluded that, if in a process that involves the f - f - A coupling, a measurement is performed sensitive (directly or via convergent quantum corrections) to a momentum scale inferior to that of the weak interactions (or some other unknown CP-violating dynamics), the effects of CP violation manifest themselves to leading order via the corresponding EDM. Thus the central role attributed to the EDMs of the fundamental fermions as means to express limits on CP violation in processes involving electromagnetic interactions. Because of the existence of certain operators closely related to EDMs the quoted customary conclusions, as we shall see, turn out to be much less obvious and somewhat less true than they sound.

The generalization of a spinor electric dipole moment as in (1.1) to its neutral-weak or chromodynamic counterparts, d_f^Z and d_f^g , is obvious:

$$\mathcal{L}_f^Z = \frac{1}{2} d_f^Z \bar{\psi}_f \{i\sigma^{\mu\nu} \gamma_5\} \psi_f F_{\mu\nu}^Z \quad (1.3)$$

$$\mathcal{L}_f^g = \frac{1}{2} d_f^g \bar{\psi}_f \left\{ i\sigma^{\mu\nu} \gamma_5 \frac{\lambda_a}{2} \right\} \psi_f G_{\mu\nu}^a, \quad (1.4)$$

with $F_{\alpha\beta}^Z = \partial_\alpha Z_\beta - \partial_\beta Z_\alpha$, as for the photon's field; and G_α^a its QCD non-abelian counterpart. We often refer to these CP-violating weak- and chromo-“electric” dipole moments¹ by their initials: WEDMs and CEDMs, for short. We shall see that quite a bit can be extracted from experiment in the way of limits on the WEDMs and/or CEDMs of quarks and leptons, even though none of them has been directly measured.

In what follows we shall refer collectively to EDMs, WEDMs and CEDMs simply as “odd” dipole moments: ODMs.

Habit has established the use of the bastardized units “e-cm” in quantifying electric dipole moments, as opposed to some dimensionless characterization. The “e” is cosmetic, but there is no universal way to trade off the “cm” for something less gross, since different models of CP violation suggest different powers of mass of external and virtual particles in the expression for an ODM. But all sensible theories coincide in introducing these dipole moments at the weak scale or beyond. As a particle-demagogical compromise to get rid of dimensions, and to characterize the different types of ODMs in a more natural scale, we have introduced [8] the notion of “weak magneton” and defined dimensionless quantities, δ_f , as follows:

$$d_f^Z \equiv \frac{e}{M_Z} \delta_f^Z, \quad (1.5)$$

$$d_f^Z \equiv \frac{e}{\sin\theta_W \cos\theta_W M_Z} \delta_f^Z, \quad (1.6)$$

$$d_f^g \equiv \frac{g_s}{M_Z} \delta_f^g, \quad (1.7)$$

where the meaning of the undefined symbols is conventional. Notice that we have extracted the coupling strength of the vector bosons in the definition of the different δ_f 's. This departs from “magneton” tradition, but allows for a fairer comparison between the various CP-violating dipole moments. An advantage of this nomenclature is that a value of “ δ ” directly indicates the suppression of the corresponding CP-odd amplitude, relative to its natural or maximal scale, astutely eliminating once and for all the recurring effort of converting “e-cm”, or Bohr magnetons, into something less artificial.

Electromagnetic and weak CP-violating dipole moments can also be defined for the charged intermediate vector boson $W^\pm$. They are conventionally written in the form:

$$\mathcal{L}_W^Z = ie \delta_W^Z \tilde{F}_{\alpha\beta}^Z W^\alpha W^{\beta\dagger} + ie \frac{\delta_W^Z}{M_Z^2} \tilde{F}_{\alpha\beta}^Z W^{\beta\dagger} W^{\mu\alpha}, \quad (1.8)$$

¹ In the coloured case we abuse language by letting the generalization of charge conjugation to a QCD G-parity remain unmentioned.

$$\mathcal{L}_W^Z = ie \frac{\delta_W^Z}{\sin\theta_W \cos\theta_W} \tilde{F}_{\alpha\beta}^Z W^\alpha W^{\beta\dagger} + ie \frac{\delta_W^Z}{\sin\theta_W \cos\theta_W M_Z^2} \tilde{F}_{\alpha\beta}^Z W^{\beta\dagger} W^{\mu\alpha}, \quad (1.9)$$

where $W_\mu^{(\dagger)} = W_\mu^{+(-)}$, the two-index tensors are field strengths, with $\tilde{F}_{\alpha\beta}^Z \equiv \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} F^{\mu\nu}$ their duals, and $\epsilon_{0123} = 1$. The conventional electric dipole and magnetic quadrupole moments of W 's are related to the values at zero momentum transfer of the above δ_f 's by:

$$d_W = \frac{e}{2M_W} [\delta_W^Z - \delta_W^g], \quad (1.10)$$

$$Q_W = \frac{e}{M_W} \delta_W^g. \quad (1.11)$$

The WEDM of a fermion, defined in (1.3), the electric dipole and magnetic quadrupole moments of W 's defined in (1.8), and their weak generalizations of (1.9) have all been written in dressings that do not reflect the $SU(2) \times U(1)$ symmetry of the unbroken standard model. We shall see that use of these non-invariant objects is unwarranted and often leads to results that range from the incomplete to the erroneous. In the next section we convince ourselves that any dynamics “beyond” the standard model must respect the local gauge symmetry of the latter above the weak symmetry-breaking scale. We also complete in that section the construction and analysis of the gauge-invariant operators from which all of the CP-odd couplings we have introduced should “descend”. We encounter various consequences of this underlying gauge-invariant construction throughout the paper. These include the existence of non-vanishing amplitudes for transitions such as $Z \rightarrow \gamma H_0$ or $Z \rightarrow \tau\bar{\tau} H_0$ necessarily related by gauge invariance to non-zero ODMs (with H_0 a Higgs particle, lighter than Z in this example). Some of these processes may be no more difficult to observe than the CP-odd quantities themselves. Gauge invariance imposes relations that are sometimes crucial to the extraction of correct phenomenological conclusions, an example concerns the CP-odd ZWW vertices, that cannot be directly bound at present, but are related to γWW couplings that are severely constrained by limits on the neutron's EDM.

Gauge invariance will also remind us that EDMs, WEDMs and CEDMs are $d=6$ operators in a $d=5$ disguise. Like all $d>4$ operators, these CP-odd effective vertices are induced in renormalizable theories by quantum effects. But there are many other $d=6$ operators related to EDMs and their pals. All these operators have to be considered hand in hand in a systematic analysis of CP violation. Some of them have a potential phenomenological role in high energy e^+e^- annihilation into hadrons, a field in which data

are being gathered at an unprecedented rate. Two CP-odd $d=6$ operators that have been discussed in this context [9] are:

$$\mathcal{L}_q^{Z\gamma} = \frac{e g_s}{M_Z^2 \sin\theta_W \cos\theta_W} G_{\mu\nu}^a \bar{\psi}_q \frac{\lambda_a}{2} [\delta_\gamma^g \gamma^\mu + \delta_a^g \gamma^\mu \gamma_5] \psi_q Z^\nu \quad (1.12)$$

and

$$\mathcal{L}_l^{Z\gamma} = \frac{e^2}{M_Z^2 \sin\theta_W \cos\theta_W} F_{\mu\nu}^I \bar{\psi}_l [\delta_\gamma^g \gamma^\mu + \delta_a^g \gamma^\mu \gamma_5] \psi_l Z^\nu, \quad (1.13)$$

where, once again, we followed our inclination for dimensionless quantities, $\delta_{g,\gamma}^g$. We shall see that the imposition of gauge invariance above the weak symmetry-breaking scale forces the above operators to belong to a rich set that interrelates the couplings of fermion pairs to one, two and three gauge bosons. This ability to grow variable numbers of wiggly legs, as well as other properties of these operators that we shall encounter in due time, strongly suggests that we baptize them as “Polyps”. Polyps are CP-odd, but occur in “vector” and “axial” varieties. Axial polyps turn out to be intimately related to ODMs. Vector polyps are rather novel operators, both from the phenomenological and the purely theoretical points of view.

No experiment in the near future has the remotest hope of being precise enough to measure a nonvanishing δ , of any of the kinds we have defined, should the minimal standard model correctly describe the origin of CP violation. We positively subscribe to the view that not to be plagued by a direct “standard-model background” is an advantage to experiments guided by the suspicion that the standard lore may not constitute the ultimate truth.

Our CP-odd coupling strengths, δ , are all form factors, and enjoy a nontrivial momentum-dependence which in the standard model is characterized by the scale M_Z . No hint of non-standard dynamics has been unearthed, up to energies of $O(M_Z)$. A surprise in the form of a nonvanishing δ would most probably signal new dynamics at a scale Λ comparable to or beyond M_Z . In discussing experiments in existing facilities, one is therefore justified in approximating all δ 's by constants, which, up to QCD renormalization-group effects, we do hereinafter without further ado.

Throughout the phenomenological chapters of this paper our technique will consist in the use of experimental data, either directly or via quantum-loop effects, to extract upper bounds on the absolute values of the EDMs of the fundamental fields, or on their generalizations, also collectively dubbed δ , as in (1.1) to (1.13). These δ 's are then our “signets” of CP violation. Their direct and indirect bounds are directly relevant to many a

future experiment, and to non-standard models of CP violation. The method of comparing experiment and theory at the half-way signet point turns out to be quite general, and very practical. Needless to say, signets and their bounds are directly relevant to the analysis of experiments aimed at the detection of new or novel CP-violating effects.

It is important to begin by acquiring a perspective on the precise expressions of the CP-odd operators that play a role in the description of possible CP-violating effects in flavour-diagonal processes, both in the standard model and in those of its conceivable extensions considered (e.g. by us) to be sensible. Thus, we open the body of the paper with a Section on the role played by electroweak gauge invariance in this respect. In Section 3 we gather those limits on the different CP-odd trilinear couplings that can be currently extracted from experiment without invoking the effects of loop corrections. Sections 4 and 5 are devoted to the study of QCD and electroweak loop effects, respectively, for operators of the ODM variety. The CP-odd trilinear couplings of vector bosons are discussed in their own chapter, Section 6. Polyp operators also deserve separate treatment, which they receive in Section 7. The different bounds we have reviewed or freshly obtained are gathered in Tables and given brief commentary in Section 8. We have divided our conclusions into two chapters, Section 9 and 10, to treat the theoretical and experimental issues separately. Grisly technical details concerning the relations between different types of operators crowd the Appendix.

2. The Role of Electroweak Gauge Symmetry

The impeccable phenomenological success of the standard model has the spontaneous breakdown of its electroweak symmetry, or some extraordinarily good imitation thereof, as an inescapable building block. The standard model *is*, and new hypothetical dynamics *must be*, $SU(2) \times U(1)$ gauge-invariant above the symmetry breaking scale. Else, it is difficult to imagine how the theory would maintain its sacred renormalizability or how symmetry breaking tell-tale signatures would not percolate down to observable levels, particularly in those observables that are understood up to (finite, calculable) electroweak radiative corrections.

In this Section we construct the invariant couplings of which the various CP-odd vertices defined in the Introduction are low energy bits and pieces, and we face the first consequences of this exercise. The various CP-odd operators previously introduced have dimensions ranging from $d=4$ to $d=6$. Gauge invariance puts them all on the same footing:

they all “descend” from $d=6$ operators [10], sometimes inter-related by “Ward” identities². Incidentally, the fact that the $d=4$ operators in (1.8) and (1.9) are after all $d=6$ operators in disguise explains why they do not describe renormalizable couplings. An advantage of the gauge-invariant formalism concerns the way in which it embodies the chiral properties of the standard model and its extensions. The formalism limpidly allows one to gauge the minimum number of mass insertions necessary to generate each type of CP-odd operator (their chiral suppression, if the insertions involve light fermions). Finally, we shall see in subsequent Sections that gauge invariance simply cannot be forgotten in most considerations beyond tree-level.

Let any of the weak- $SU(2)$ quark or lepton doublets be denoted $(U, D)_L$ and let U_R and D_R denote the singlets. Let $\Phi = (\phi^+, \phi^0)$ and v be the customary scalar doublet and vacuum expectation value, with $v = 2M_W \sin\theta_W / e \sim 240$ GeV. (We characterize the trigger of the weak symmetry breaking, even in models beyond the standard one, by an effective Higgs field with the “official” quantum numbers. But on those rare occasions in which gauge invariance forces us to do otherwise, we discuss in this paper only flavour-neutral couplings and we systematically disregard effects explicitly involving new charged scalars). Let $W_{\mu\nu}^i$ and $B_{\mu\nu}$ be the usual $SU(2)$ and $U(1)$ field tensors. The $SU(2)$ -invariant forms of the $d=5$ effective interactions in (1.1), (1.3) and (1.4) involve three independent types of $d=6$ operators³, to wit:

$$\mathcal{L}_1^D = \frac{1}{\sqrt{2}} d_1^D (\overline{U}, \overline{D})_L \{i\sigma_{\mu\nu} B^{\mu\nu} (\Phi/v)\} D_R + h.c., \quad (2.1)$$

$$\mathcal{L}_2^D = \frac{1}{\sqrt{2}} d_2^D (\overline{U}, \overline{D})_L \left\{ i\sigma^{\mu\nu} \frac{\tau_i}{2} W_{\mu\nu}^i (\Phi/v) \right\} D_R + h.c., \quad (2.2)$$

² Operators of dimension $d > 6$ are suppressed at tree level by powers of the large scale Λ characterizing the new dynamics. Their contributions via quantum loops to low-energy observables are suppressed relative to those of operators of lower dimension by at least a $\ln \Lambda$ factor.

³ The expressions for $\mathcal{L}_i$ immediately invoke the threat of flavour-changing neutral currents, since gauge invariance does not preclude the occurrence of a different flavour eigenstate in D_L and D_R or U_L and U_R . In all the examples of dynamics beyond the standard model that we shall bring to the fore the cancellation of this threatening possibility is automatically guaranteed either by a proper extension of the GIM mechanism, or by a judicious choice of Higgs couplings [11]. In other models, such as technicolour, one ought to worry.

$$\mathcal{L}_3^D = \frac{1}{\sqrt{2}} d_3^D (\overline{U}, \overline{D})_L \left\{ i\sigma^{\mu\nu} \frac{\lambda_a}{2} G_{\mu\nu}^a (\Phi/v) \right\} D_R + h.c., \quad (2.3)$$

and similarly for $\mathcal{L}_i^U$, with the substitutions of D_R for U_R and of Φ for $i\sigma_2 \Phi^\dagger$.

Let H_0 be the field of a Higgs particle, so that after spontaneous symmetry breakdown $\Phi/v \rightarrow (1 + H_0/v)/\sqrt{2}$. No longer forgetting the scalar field, we find that the coefficients introduced in (1.1) to (1.4) are not mere c-numbers, they contain a term linear in H_0 and are related to the various d_i 's in (2.1) to (2.3) as follows:

$$d_f^T = (\cos\theta_W d_1^f + \sin\theta_W T_f^3 d_2^f)(1 + H_0/v), \quad (2.4)$$

$$d_f^Z = (-\sin\theta_W d_1^f + \cos\theta_W T_f^3 d_2^f)(1 + H_0/v), \quad (2.5)$$

$$d_f^0 = d_3^f (1 + H_0/v), \quad (2.6)$$

where, with no loss of generality, d_1 , d_2 , and d_3 have been taken to be real, and T^3 is the third component of weak isospin.

To state the obvious, the expressions (2.1), (2.2) and (2.3) are in every way reminiscent of a standard Yukawa coupling, with the Φ field providing the $\Delta T=1/2$ necessary to jump from a left-handed doublet to a right-handed singlet. The non-trivial point, to which we shall come back, is that in contradistinction to the mass of a particle and its related coupling to a Higgs field, there are cases in which, depending on the H_0 mass, it may be simpler to observe the coupling to a Higgs field implied by (2.1) and (2.2) than to measure the corresponding CP-odd operator not involving a scalar field. Examples are the decay $Z \rightarrow \tau\bar{\tau}H_0$ versus the WEDM of the τ , or the decay $Z \rightarrow \nu_\tau\bar{\nu}_\tau H_0$ versus a discrepancy between the observed invisible (!) width of the Z and the corresponding standard expectation.

In the above expressions, and for every type of “ d ”, $d \propto m$ (or, equivalently, a Yukawa coupling) with m the field of “some” particle. To understand what this “ m ” may be, it is useful to refer to specific models of the origin of ODMs, as we parenthetically proceed to do.

In “left-right” models [12] the diagrams generating the effective couplings in (2.1) to (2.3) are of the type depicted in fig. 1a. The CP-odd phase combines in this case the phases of the right and left Cabibbo-Kobayashi-Maskawa matrices and that of the W -mixing matrix, and “ m ” is simply the mass of the internal fermion. In the minimal supersymmetric extension of the standard model the situation is slightly more convoluted [13]. The relevant diagram for quarks is that of fig. 1b, where the CP-odd phase can be

ascribed to a complex gluino mass. The end result is proportional to the difference of the squares of the masses of the scalar partners of left- and right-handed quarks. This difference can in turn be traded for the product of a supersymmetric mass scale times the mass m of the quark external to the diagram, all in all $d \propto m$. In multi-Higgs models the relevant diagram is that of fig. 1 c. The result is $d \propto m^3$, with the m 's standing for masses of the various fermions involved. Once more the vector-boson to fermion couplings with and without an extra H_0 field appear in the combination $(1 + H_0/v)$, as in (2.4) to (2.6). In a generic unspecified model, such as that of fig. 1 d, the result for the various d 's would again obey (2.4) to (2.6) and be proportional to a mass, that may be as large as the scale characterizing the new dynamics.

The expressions (1.8) and (1.9) for the CP-odd trilinear vertices of intermediate vector bosons are not $SU(2) \times U(1)$ gauge invariant. The lowest-dimension ($d=6$) gauge-invariant operators [10] inducing the couplings in (1.8) and (1.9) in their low energy limits are:

$$O_{\tilde{B}} \equiv \frac{e^4}{4 \sin\theta_W \cos^3\theta_W M_Z^2} (\Phi^\dagger \tau^i \Phi) W_{\mu\nu}^i \tilde{B}^{\mu\nu}, \quad (2.7)$$

$$O_{\tilde{W}} \equiv \frac{e^3}{6 \sin\theta_W M_Z^2} \epsilon_{ijk} \tilde{W}_{\mu}^{\nu i} W_{\nu}^{\kappa j} W_{\kappa}^{\mu k}, \quad (2.8)$$

with Φ the standard scalar doublet. Upon spontaneous symmetry breaking, the above operators acquire the fertile effective low-energy forms:

$$\begin{aligned} \cos\theta_W O_{\tilde{B}} = & \left[-e^2 \sin\theta_W \left(\frac{\sin(2\theta_W)}{4} \{ \tilde{F}^\gamma \cdot \tilde{F}^\gamma - \tilde{F}^Z \cdot \tilde{F}^Z \} + \frac{\cos(2\theta_W)}{2} \tilde{F}^Z \cdot \tilde{F}^\gamma \right) \right. \\ & \left. - i e^3 (\cos\theta_W \tilde{F}_{\mu\nu}^{\gamma\gamma} - \sin\theta_W \tilde{F}_{\mu\nu}^Z) W^\mu W^{\nu\gamma} \right] \times \left\{ 1 + \frac{H_0}{v} \right\}^2, \end{aligned} \quad (2.9)$$

$$\sin\theta_W O_{\tilde{W}} = i \frac{M_Z^2}{M_W^2} W^{\nu\lambda} W_{\lambda}^{\mu\gamma} (\sin\theta_W \tilde{F}_{\mu\nu}^{\gamma\gamma} + \cos\theta_W \tilde{F}_{\mu\nu}^Z + i \frac{e}{\sin\theta_W} \epsilon_{\mu\nu\rho\sigma} W^\rho W^{\sigma\gamma}), \quad (2.10)$$

where W^μ stands for the $W^\pm$ field, and

$$W^{\mu\nu} \equiv \partial^\mu W^\nu - \partial^\nu W^\mu + i \frac{e}{\sin\theta_W} (W_3^\mu W^\nu - W^\mu W_3^\nu), \quad (2.11)$$

$$W_3^\mu \equiv \cos\theta_W A^\mu - \sin\theta_W Z^\mu. \quad (2.12)$$

In (2.9) the terms $\tilde{F}^\gamma \cdot \tilde{F}^\gamma$, $\tilde{F}^Z \cdot \tilde{F}^\gamma$ and $\tilde{F}^Z \cdot \tilde{F}^Z$ with no extra H_0 are total derivatives, and may be erased. For an effective Lagrangian of the form

$$\mathcal{L} = C_{\tilde{B}} O_{\tilde{B}} + C_{\tilde{W}} O_{\tilde{W}}, \quad (2.13)$$

the translation to the usual low energy couplings of (1.8) and (1.9) reads:

$$\delta_W^\gamma = -e^2 C_{\tilde{B}}, \quad \delta_W^Z = e^2 \sin^2\theta_W C_{\tilde{B}}, \quad (2.14)$$

$$\delta_W^\gamma = e^2 C_{\tilde{W}}, \quad \delta_W^Z = e^2 \cos^2\theta_W C_{\tilde{W}}. \quad (2.15)$$

In other words, gauge invariance entails the relations:

$$\delta_W^Z = -\sin^2\theta_W \delta_W^\gamma, \quad (2.16)$$

$$\delta_W^Z = \cos^2\theta_W \delta_W^\gamma, \quad (2.17)$$

The above expressions, the first of which was recently noted in [14], teach us an important fact: if, as we shall see, the photonic CP-odd couplings of W 's are severely constrained by limits on the neutron's EDM, there should be no hope to measure their neutral-weak analogs at LEP-II; a non-vanishing result would signal a non-standard breakdown of gauge-invariance and would be particularly surprising. By recalling (2.9) and (2.14) we also learn the curious fact that CP violation in trilinear vector-boson couplings implies the existence of nonvanishing rare vertices, such as $Z H_0 \gamma$.

The "polyp" operators in (1.12) and (1.13) are not $SU(2) \times U(1)$ gauge invariant and, once again, we must proceed to repair their birth defect. To introduce the invariant ancestry of polyps, let

$$\mathcal{F}_{\mu\nu}^j \equiv \left(\frac{e}{\cos\theta_W} B_{\mu\nu}, \frac{e}{\sin\theta_W} W_{\mu\nu}^i, \frac{\tau_i}{2}, g_s G_{\mu\nu}^a \frac{\lambda_a}{2} \right) \quad (2.18)$$

gather in a single vector the ensemble of standard gauge field tensors. Let $\Psi_L \equiv (U, D)_L$ with U, D denoting again any quarks or leptons, once more required to be mass eigenstates, with the usual warning concerning flavour-changing neutral currents³. The polyps of (1.12) and (1.13) are pieces of the luxuriant set:

$$\mathcal{P}_{\Psi}^j \equiv i \bar{\Psi}_L \gamma^\mu \left\{ \mathcal{F}_{\mu\nu}^j, \tilde{D}^\nu \right\}_+ \Psi_L, \quad (2.19)$$

$$\mathcal{P}_{U_R}^j \equiv i \bar{U}_R \gamma^\mu \left\{ \mathcal{F}_{\mu\nu}^j, \tilde{D}^\nu \right\}_+ U_R, \quad (2.20)$$

$$\mathcal{P}_{D_R}^j \equiv i \bar{D}_R \gamma^\mu \left\{ \mathcal{F}_{\mu\nu}^j, \tilde{D}^\nu \right\}_+ D_R. \quad (2.21)$$

Here D_ν denotes the generalized $SU(3) \times SU(2) \times U(1)$ covariant derivative, and the curly brackets denote anticommutators. Notice that the operators $\mathcal{P}^j$ describe the couplings of a fermion pair to one, two or three gauge fields of various types, they are indeed polyp-like. Those bits of polyps that contain charged gauge bosons either describe flavour-non diagonal processes that we do not discuss in this paper, or result in higher order corrections to the effects that we do discuss: we remain oblivious to these operators in what follows. Parity will soon play an important role and it is convenient, to discuss the neutral-current content of the above expressions for a particular flavour f , to introduce the effective Lagrangians⁴:

$$\mathcal{L}_f^j = \frac{1}{M_Z^2} \left(\Sigma_f^j \mathcal{P}_{f+}^j + \Delta_f^j \mathcal{P}_{f-}^j \right), \quad (2.22)$$

$$\mathcal{P}_{f\pm}^j \equiv i \bar{\psi}_f \gamma^\mu \left\{ \mathcal{F}_{\mu\nu}^j \left[\vec{\partial}^\nu + ie Q_f A^\nu + ie(v_f - a_f \gamma_5) Z^\nu + i g_s A_b^\nu \frac{\lambda^b}{2} \right] \right\} \pm \left(\frac{1}{\gamma_5} \right) \psi_f, \quad (2.23)$$

where $\mathcal{F}_{\mu\nu}^j$ for $j = 2$ hereinafter stands for only the neutral part of the $\mathcal{F}_{\mu\nu}^2$ of (2.18). We have used the again the conventional notation for anticommutators, the $\pm$ sign on the left hand side of (2.23) corresponds to 1 or γ_5 in the right hand side. We have also used the standard notation

$$v_f = \frac{(T_f^3 - 2Q_f \sin^2 \theta_w)}{2 \sin \theta_w \cos \theta_w}, \quad (2.24)$$

$$a_f = \frac{T_f^3}{2 \sin \theta_w \cos \theta_w}. \quad (2.25)$$

In (2.23) one can already discern the presence of (1.12) and (1.13), but its gauge-invariant pedigree affords several interesting conclusions, besides the obvious relations between the four terms within the square brackets. Consider the “kinetic” piece linear in $\vec{\partial}^\nu$. We shall call $\mathcal{P}_+$ and $\mathcal{P}_-$ “vector” and “axial” polyps, respectively, in reference to the kinetic (and photonic and gluonic) terms of the covariant derivative within the square brackets in (2.23) (because of the mixed parity of the accompanying Z -coupling this is a slight yet useful abuse of language). The kinetic term of the axial polyp $\mathcal{P}_-$, reduces for on-shell fermions to an ODM:

$$i \bar{\psi}_f \left\{ \gamma^\mu \vec{\partial}^\nu, \mathcal{F}_{\mu\nu}^j \right\} \gamma_5 \psi_f = 2m \times \frac{1}{2} \bar{\psi}_f \{ i \sigma^{\mu\nu} \mathcal{F}_{\mu\nu}^j \gamma_5 \} \psi_f. \quad (2.26)$$

There is therefore an intimate relation between the axial polyps and the (P-odd) dipole moments, and bounds on the latter will often imply limits on the former. The vector polyps $\mathcal{P}_+$ cannot be as simply related to dipole moments, that are P-odd. Their kinetic term vanishes on-shell. We shall find the vector polyps, $\mathcal{P}_+$, to be particularly sneaky.

⁴ The various $\mathcal{L}_f^j$ are not totally independent, since (2.19) imposes the following relations: $\Sigma_f^0 - \Delta_f^0 = \Sigma_f^D - \Delta_f^D$ and $\Sigma_f^{U,D} = -\Delta_f^{U,D}$.

3. Direct Experimental Bounds

Direct bounds from dedicated experiments on electric dipole moments exist only for neutrons and atoms; for Λ 's and μ 's the best direct limits are by-products of measurements of magnetic moments; for ν 's, τ 's, c 's and b 's the existing bounds follow from the agreement of calculated and observed rates, and are therefore quadratic in the EDMs. We extract similar quadratic bounds on quark and lepton WEDMs from the recent LEP data (these data turn out not to be competitive as a constraint on photon-mediated EDMs). We also discuss in this section quasi-direct bounds on the chromoelectric dipole moments of u and d quarks, dug out from limits on the neutron's EDM in a way reminiscent of the extraction of bounds on the electron's EDM from atomic data. We shall see in subsequent sections that the direct limits discussed here are, in general, not the tightest for a given observable.

The most stringent and recent results on the neutron's EDM from the Leningrad [15] and Grenoble [16] groups are:

$$d_n^L = (-1.4 \pm 0.6) \cdot 10^{-25} \text{ e cm} \quad (\text{Leningrad}), \quad (3.1)$$

$$d_n^G = (-0.3 \pm 0.5) \cdot 10^{-25} \text{ e cm} \quad (\text{Grenoble}). \quad (3.2)$$

The tightest of these bounds, to the 95% ($\sim 2\sigma$) confidence level to which we shall subscribe throughout, entails the constraint $|\delta_n^L| < 6 \cdot 10^{-10}$, in the weak magneton units introduced in (1.5). Use of the naive quark model or $SU(4)$ relation $\delta_n = (4/3)\delta_d - (1/3)\delta_u$ allows one to conclude that, barring cancellations,

$$|\delta_u^L| < 4.4 \cdot 10^{-10}, \quad |\delta_d^L| < 1.8 \cdot 10^{-9}. \quad (3.3)$$

The bounds on δ_u and δ_d from the proton's EDM are not competitive with their neutron counterparts. A similar “valence-quark” bound for the strange quark's EDM can be derived, with use of $SU(6)$ quark-model wave functions, from the experimental result on the Λ [17]:

$$d_\Lambda^L = (-3.0 \pm 7.4) \cdot 10^{-17} \text{ e cm}, \quad (3.4)$$

$$|\delta_s^L| < 1.2. \quad (3.5)$$

The tautest bound on the electron's EDM, painfully extracted from a measurement in ^{250}Tl [18], is $|d_e^L| = (-2.7 \pm 8.3) \cdot 10^{-27} \text{ e cm}$. This corresponds to

$$|\delta_e^L| < 8.9 \cdot 10^{-11}. \quad (3.6)$$

The limit on the muon's EDM, a by-product of the $(g-2)$ experiment [19], is $|d_\mu^T| < 7.3 \cdot 10^{-19} e \text{ cm}$, or, in our convention,

$$|\delta_\mu^T| < 3.4 \cdot 10^{-3}. \quad (3.7)$$

Laboratory limits $|d_\mu^T| < 8 \cdot 10^{-21} e \text{ cm}$, $|d_\mu^T| < 2 \cdot 10^{-20} e \text{ cm}$, have been extracted [20] from the agreement between observations and standard-model expectations in the process $\nu e \rightarrow \nu e$. In weak magnetons,

$$|\delta_\mu^T| < 3.7 \cdot 10^{-5}, \quad |\delta_\mu^T| < 9.2 \cdot 10^{-8}. \quad (3.8)$$

Stiffer limits have been derived from cosmology, SN1987A, and the evolution of red giants. The latter [21] do not involve dubious assumptions, and follow from the constraint that the emission of $\nu\bar{\nu}$ pairs by plasmon decay in the inner regions of the star not excessively delay its He-core ignition. For neutrinos sufficiently less massive than the core's temperature ($T \sim 10 \text{ keV}$) the 2σ result is $|d_\mu^T| < 2.1 \cdot 10^{-26} e \text{ cm}$, or

$$|\delta_\mu^T| < 10^{-10}, \quad (3.9)$$

an amazingly tight constraint, that we have simply paraphrased from the generally quoted [21] limits on neutrino magnetic moments.

In terms of the conventionally defined ratio, R , of photon-mediated e^+e^- annihilation cross sections, the EDM of a spinor f would induce a step

$$\Delta R_f \equiv \frac{\Delta\sigma(e^+e^- \rightarrow \gamma^* \rightarrow f\bar{f})}{\sigma(e^+e^- \rightarrow \gamma^* \rightarrow \mu\bar{\mu})} = (\delta_f^T)^2 \frac{s}{2M_Z^2} \left[\frac{2\beta_f^3}{\beta_\mu(3-\beta_\mu^2)} \right] D_e, \quad (3.10)$$

$$\beta \equiv \sqrt{1 - \frac{4m^2}{s}}, \quad (3.11)$$

with $D_e = 3, 1$ for quarks or leptons, respectively. For $f = \tau$, Barr and Marciano [22], demand less than a 3% excess cross section at typical Petra energies $\sqrt{s} \sim 35 \text{ GeV}$, to obtain $|d_\tau^T| < 10^{-19} e \text{ cm}$, that is

$$|\delta_\tau^T| < 0.46. \quad (3.12)$$

A similar analysis can be used to limit the EDMs of charmed and bottom quarks. We impose that the yield of hadrons and leptons as the charmed threshold is crossed and the charmonium resonances are left behind, agree with theory to 3% (this $\sim 2\sigma$ constraint

includes a 0.6% theoretical error on the total cross-section from the indetermination in α_s). From data centered at c.m.s. energies $\sqrt{s} \sim 8 \text{ GeV}$, we obtain

$$|\delta_c^T| < 4.7. \quad (3.13)$$

In writing the limit (3.13) we are justified in ignoring the EDMs of the lighter quarks, better constrained by (3.3) and (3.5). In the region around 35 GeV the bottomium resonances are no longer prominent and the electroweak interference effects are negligible within errors. The QCD e^+e^- annihilation cross-section and observations do not deviate by more than 2% at our usual $\sim 95\%$ confidence level. This precision is better than the one we just discussed for charmed quarks, and the sensitivity to an EDM is improved by the higher energy. We may therefore add to our cumulating knowledge the bounds

$$|\delta_c^T| < 0.8, \quad |\delta_s^T| < 0.8. \quad (3.14)$$

At the Z resonance, limits on δ^Z can be extracted for all leptons and known quarks. At present the data analysis least biased by theoretical prejudice proceeds as follows. The resonant peak is fitted to a Breit-Wigner expression with a linearly energy-dependent width and a multiplicative correction describing initial-state radiation. The peak position and breadth are M_Z and Γ_{tot} (in practice the determination of these quantities is dominated by the copious hadronic channel). For a particular final state $f\bar{f}$, add to the previous information the peak value of $\sigma(e^+e^- \rightarrow f\bar{f})$ to get $\Gamma_e \cdot \Gamma_f$. Even for the $e^+e^- \rightarrow e^+e^-$ process, where Ehabha scattering is important, the width Γ_e is best determined so far by the procedure just outlined, even though overall multiparameter fits are also employed in the analysis of LEP experiments. As a consequence of all this, it is very simple to extract constraints on the WEDMs of quarks and leptons. Suffice it to compare the maximum (2σ) allowed difference between observation and standard-model expectation with the contribution, $\Delta\Gamma_f$, of a particular WEDM to the corresponding partial width:

$$\Delta\Gamma_f = \frac{\alpha M_Z}{6 \sin^2\theta_W \cos^2\theta_W} \beta_f^3 D_e |\delta_f^Z|^2. \quad (3.15)$$

The Standard Model prediction [23] for the total width of the Z into hadrons is $\Gamma_H = 1747 \pm 34 \text{ MeV}$, while the LEP result [24], averaged over all experiments, is $\Gamma_H = 1764 \pm 25 \text{ MeV}$. The 2σ mismatch is at most $\sim 84 \text{ MeV}$. The corresponding limit on the WEDM of each single accessible quark flavour, $q = u, d, s, c, b$, is:

$$|\delta_q^Z| < 0.21. \quad (3.16)$$

The Standard Model prediction for a charged-lepton width of the Z is $\Gamma_l = 83.8 \pm 0.9$ MeV. The average LEP results are $\Gamma_e = 83.6 \pm 1.8$ MeV, $\Gamma_\mu = 84.1 \pm 1.8$ MeV, $\Gamma_\tau = 83.2 \pm 1.9$ MeV. Thus, at the 2σ level, and for $l = e, \mu, \tau$,

$$|\delta_f^Z| < 8.0 \cdot 10^{-2}. \quad (3.17)$$

Finally, the LEP experiments assume the standard model to be correct, and quote $N_\nu = 2.90 \pm 0.10$ neutrino generations from the measurement of the “invisible” width of the Z , $\Gamma_{inv} = 482 \pm 26$ MeV. The standard prediction for an assumed $N_\nu = 3$ is $\Gamma_{inv} = 502 \pm 5$ MeV, and we deduce, for each individual neutrino type:

$$|\delta_{\nu_l}^Z| < 0.26. \quad (3.18)$$

It has been pointed out [25] that a ν_τ magnetic moment can produce anomalous single photon events via the process $Z^0 \rightarrow \bar{\nu}_\tau \nu_\tau \gamma$, illustrated in fig. 2. A neutrino EDM has the same effect and results in a partial width:

$$\Gamma(Z^0 \rightarrow \bar{\nu} \nu \gamma) = \frac{\alpha^2}{128 \pi} \frac{M_Z}{\sin^2 \theta_w \cos^2 \theta_w} |\delta_\nu^Z|^2, \quad (3.19)$$

which corresponds to a branching ratio:

$$BR(Z^0 \rightarrow \bar{\nu} \nu \gamma) \simeq 2.7 \cdot 10^{-5} |\delta_\nu^Z|^2. \quad (3.20)$$

At present there is no confessed evidence for anomalous single photons. With no events observed, Poisson statistics allows for a maximal expectation of three events, at a 95% confidence level (tantamount to our usual “ 2σ ” limits). Since some 10^5 Z_0 -decays have been fully analyzed so far, the branching ratio (3.20) is limited to $\sim 3 \cdot 10^{-5}$, yielding the result:

$$\delta_\nu^Z < 1.0. \quad (3.21)$$

This limit cannot compete with (3.8), nor with (3.9) unless $m(\nu) > 10$ keV, in which case the stellar bounds do not apply. Such a value of the neutrino mass is excluded for electron neutrinos, barely acceptable for muon neutrinos and well below the present bound on tau neutrinos.

SLC and LEP are unlikely to measure or constrain the *photonic* EDMs of τ 's, c 's and b 's to a better precision than that reflected in (3.12) and (3.14). Let a fermion f have an EDM and a WEDM. The ratio of photon-mediated to Z -mediated modifications of the $f\bar{f}$

cross section at the peak of the Z is $\sim (\alpha \delta_f^Z / 12 B_e \delta_f^Z)^2$, with B_e the Z -decay branching ratio into electron pairs. Thus the sensitivity to a non-vanishing $|\delta_f^Z|$ is some three to four orders of magnitude less than the sensitivity to $|\delta_f^Z|$, and there is little hope to improve on our lower-energy limits on $|\delta_f^Z|$ from data at the Z peak. Measurements way down the tail of the resonance will not easily provide significant constraints, since the Z - γ interference of dipole moments is of order $\alpha(1 - 4 \sin^2 \theta_w) (\delta_f^Z / \delta_f^Z) (s - M_Z^2) / (6 M_Z \Gamma_e)$, relative to the purely weak effect: the interference is reduced by the fortuitously small $Z\bar{e}e$ vector coupling. Eventually one would like to measure observables linear in the CP-odd moments, but this requires flavour tagging and, for most final states, subsequent research.

In the case of the WEDM of the τ , a theoretical analysis exists [26] of T-odd observables⁵ in Z -decay linear in δ_τ^Z . The expected sensitivity is at the level $\delta_\tau^Z \sim 0.48$ for 10^5 Z -decays, somewhat weaker than the quadratic bound (3.17). Even for 10^7 Z -decays, the quadratic and linear tests would have a similar sensitivity, though the latter has the obvious advantage of being based on an honest to goodness T-odd observable.

Incidentally, all quoted limits from (3.8) to (3.18) inclusive are also bounds on CP-even anomalous magnetic dipole moments (expressed in weak magnetons), since they were extracted from rates, to which the contributions of the two types of dipole couplings are identical.

We have seen in the previous section that gauge invariance forces ODMs to appear hand in hand with couplings to neutral Higgs scalars, as in (2.4) to (2.6). A non-vanishing WEDM of a (light) fermion f implies the existence of the process $Z \rightarrow H_0 f \bar{f}$, or $H_0 \rightarrow Z f \bar{f}$, depending on what the value of M_H turns out to be. At the Z peak of the e^+e^- cross section, the limit from the failure to discover “the Higgs” ($M_H > 40$ GeV, for a single standard scalar) may be used to constrain WEDMs, an exercise whose result competes with other constraints for $f = \tau$ and, if $m(\nu_\tau) > 10$ keV, for $f = \nu_\tau$. With input from (1.3), (1.6) and (2.5), we compute the width of the Z into one of these channels to be:

$$\Gamma(Z^0 \rightarrow H_0 f \bar{f}) = |\delta_f^Z|^2 \left(\frac{\alpha}{12 \sin^2 \theta_w \cos^2 \theta_w} \right)^2 \frac{M_Z}{8 \pi} (3 + 10x - 18x^2 + 6x^3 - x^4 + 12x \ln x). \quad (3.22)$$

⁵ A T-odd observable is one that changes sign under a flip of direction of three-momenta and spins in a given process. Nonvanishing T-odd effects are induced either by CP-violating interactions, or by CP-conserving perturbative effects (“final-state interactions”) involving the absorptive part of the scattering amplitude. In the case at hand these perturbative effects are $O(\alpha)$ and not a threatening standard-model background to potential new CP-violating effects, at LEP’s expected level of sensitivity.

where $x = M_H^2/M_Z^2$. For $M_H > 40$ GeV, this corresponds to a branching ratio $BR < 1.0 \cdot 10^{-5} |\delta_l^Z|^2$, to be compared with the standard model prediction for $l = \mu$: $BR < 5 \cdot 10^{-6}$, the prediction on which the limit on M_H is based. We conclude that:

$$|\delta_l^Z| < 0.71, \quad (3.23)$$

with a somewhat weaker result on $|\delta_{\nu}^Z|$ from the non-observation of Z decays into hadrons plus missing energy.

We have also seen in the previous section that CP violation in trilinear vector-boson vertices implies nonvanishing couplings for $ZH_0 \gamma$ and other rare combinations, recall (2.9) and (2.14). For $M_Z > M_H$ we compute a partial width:

$$\Gamma(Z \rightarrow H \gamma) = \frac{\alpha}{3} |\delta_W^\gamma|^2 \left(\frac{\cos 2\theta_w}{\cos^2 \theta_w} \right)^2 \frac{(M^2 - M_H^2)^3}{8 M_Z^5}, \quad (3.24)$$

leading to a branching ratio of $3.1 \cdot 10^{-3} |\delta_W^\gamma|^2$ for $M_H = 40$ GeV. The limit we could extract from the non-observation of this rare decay would not compete with the indirect bounds to be derived in Section 6.

“Direct” limits on the chromo-electric dipole moments of u , d and s can be extricated from bounds on the EDMs of the baryons that incorporate these quarks as “valence” constituents. The technique [27] is not unlike the one used to extract limits on the electron’s EDM from experiments on atoms, only simpler. For the sake of completeness, we briefly recall the rules. The aim is to estimate the magnitude of the coefficient of an “external” operator constructed exclusively with “external” fields, such as the neutron’s EDM d_n^i in the effective interaction term $(d_n^i/2) \bar{n} \gamma_5 \sigma^{\mu\nu} n F_{\mu\nu}$. The operator can be generated by the cooperative effect of various “internal” operators involving “constituent” fields (gluons and valence quarks). In the example at hand these internal operators will be the quark’s minimal coupling $Q_q \bar{q} \gamma_\mu q A^\mu$ and the quark’s CEDM $(d_q^a/2) \bar{q} \gamma_5 \sigma_{\mu\nu} (\lambda_a/2) q G_{\mu\nu}^a$. Let an operator of dimension D , $\mathcal{O}_N^D$, be built of a total of N fields, and let g be its coupling strength in the relevant effective Lagrangian. For the neutron’s EDM and the quarks’ CEDM $N = 3$, $D = 5$ and $g = d_n^i/2$ or $d_q^a/2$, respectively; while for the minimal quark-photon interaction $N = 3$, $D = 4$ and $g \sim e$. Let $M = 2\pi f_\pi \simeq 1190$ MeV be the scale of chiral symmetry breaking. Define the reduced coupling of $\mathcal{O}_N^D$ as $\bar{g} = (4\pi)^{2-N} M^{D-4} g$. The rule [27] is that the reduced coupling of the external operator can be estimated as the product of the reduced couplings of the internal operators that suffice to generate it. In the case of a CEDM of a *constituent* quark and its minimal photonic coupling conspiring

to generate a baryon’s EDM, the rule⁶ leads to $d_B^\gamma \sim (e/4\pi) d_q^a$, or, in our weak-magneton parlance:

$$\delta_B^\gamma = \frac{g_s(M)}{4\pi} \delta_q^a. \quad (3.25)$$

The size of $g_s(M)$ may be estimated [28] by demanding that the first and second order terms in the perturbative expression for its scale-dependence be of equal magnitude: $g_s(M) \sim 4\pi/\sqrt{6}$. Given the limits on the neutron’s and Λ ’s EDM, this entails:

$$|\delta_u^a|, |\delta_d^a| < 1.5 \cdot 10^{-9}, \quad (3.26)$$

$$|\delta_s^a| < 2, \quad (3.27)$$

to be taken *cum grano salis*: an up or down uncertainty of a factor of “a few”.

4. Indirect Bounds: QCD Loops.

In this Section we review, update and offer new commentary on those bounds on quark EDMs and CEDMs whose derivation [8] involves the use of QCD radiative corrections. As defined in the Introduction, these (C)EDMs appear to be the lowest-dimension ($d=5$) CP-odd operators that may reflect non-standard CP-violating dynamics originating at a scale Λ beyond the weak scale. Given their record-low dimension, one would naively conclude that (C)EDMs are the dominant effective operators to be considered in calculations involving convergent quantum corrections, and that limits on possible non-standard models of CP violation can be obtained by retaining only the effects of these $d=5$ couplings. This naive conclusion is threatened by three facts. First, the equality between the “conventional” CP-odd dipoles of (1.1) and their “convective axial current” counterparts of (1.2) is an on-shell result: the existence of both forms cannot be ignored off-shell. Second, the (C)EDMs are, as we have seen, $d=6$ operators in disguise, and there are polyp CP-odd operators of the

⁶ There is no water-tight argument why we should have chosen $g = d_n^i/2$ and $d_q^a/2$ and not some reasonable multiple of the same operators as the objects that play a role in the estimates of neutron matrix elements. Our choice is based on an analogy with the best crude estimate of the magnetic moment of the proton (μ_p) one may obtain following similar rules. Choosing in this case $g = \mu_p/2$ as the relevant operator leads to $\mu_p = 2e/M = 3.12e/(2m_p)$, in fair agreement with the experimental result $2.73e/(2m_p)$. One could easily in this instance “improve” the rules to get a result $-2/3$ times as large for the neutron, an exercise of doubtful value for most of the operators we shall deal with.

same dimension: neither can the polyps of (2.19) to (2.21) be simply ignored. Finally, some of the quantum corrections we are led to consider are plagued by anomalies reminiscent of the Steinberger-Adler-Bell-Jackiw one. In spite of all this, the end result emerging from a marmagnum of technicalities that we relegate to a large extent to the Appendix, is that, had we ignored all of the complications, we would have obtained results that are, in practice, the correct ones. A bonus to our efforts is that we establish bounds on the two types of CP-odd dipoles, as well as on some of the polyp operators.

Consider a specific theory of CP violation that generates quark EDMs or CEDMs at a scale Λ close to or above M_Z . Focus now on a quark q that is *not* a valence constituent of the neutron. An EDM or a CEDM of one of these medium-weight or heavy quarks⁷ can induce [8] a non-zero neutron electric dipole moment via quantum effects whose contributions are not chirally suppressed by light-quark masses.

Examples of loop diagrams that may transmit the CP-odd infection are given in fig. 3 and fig. 4, in which the original CP-violating vertices are depicted as heavy dots. Since the (C)EDM is the lowest-dimension CP-odd vertex operator, it is adequate, in the limit $\Lambda \gg m_q$, to “integrate out” the heavier degrees of freedom and to embody all CP-violating effects in these single operator(s). For a t -quark this approximation is dubious and complete satisfaction may require a two-loop calculation. If the quark in the loop is much heavier than the inverse of the neutron’s radius R_n , it can in turn be integrated out to generate new “local” effective interactions involving only the photon and gluon fields that propagate with a longer range. This time, the procedure is dubious [30] for an s quark, whose inverse constituent mass is $O(R_n)$. But the insertions of the effective local operators into the neutron cannot be estimated in any case to better precision than (half?) an order of magnitude. It would take a surprisingly scale-dependent “form factor”, or cancellation with operators of higher dimensionality, to modify the insertion of the s -quark-induced “local” operator by more than this amount, a point to which we return in more detail once we have made our way to the final results (4.6) and (4.16). Finally, once the quark degrees of freedom have been integrated “in”, the neutron matrix elements of the resultant effective operators are to be evaluated. The end result is an estimate of the neutron EDM in terms of the original quark (C)EDMs or the physics that underlies them. The details of the plot are thickened by QCD scale-dependent effects. The quark CP-odd dipole moments

⁷ In [29] the limits of [28] on multi-Higgs models were also reanalyzed in terms of the CEDM of the top quark as a “signet” of CP violation.

and the quark mass itself are, by convention, defined at a scale m_q . The effective operator resulting from the integration of the quark loop must be QCD-evolved from m_q down to the quark-confining scale, prior to its insertion into the neutron. Moreover, the (C)EDMs themselves must be evolved up from m_q to the scale Λ , if contact is to be made with the theory that generates them.

The result of integrating out the quark in the loop of fig. 3 is an effective Lagrangian [28] involving a $d=6$ CP-odd three-gluon operator:

$$\mathcal{L}_w = K_w(\mu) g_s^3(\mu) \mathcal{O}_w(\mu) \quad (4.1)$$

$$\mathcal{O}_w = -\frac{1}{3!} f_{abc} \tilde{G}_\beta^{a\alpha} G_\gamma^{b\beta} G_\alpha^{c\gamma}, \quad (4.2)$$

with μ the renormalization scale, $\tilde{G}_{\mu\nu} = (1/2)\epsilon_{\mu\nu\rho\sigma} G^{\rho\sigma}$ the dual to the gluon field G , and $\epsilon_{0123} = 1$.

The explicit value of the coefficient in (4.1) is:

$$K_w(m_q) = \frac{1}{16\pi^2 m_q} \frac{\delta_q^g(m_q)}{M_Z} \quad (4.3)$$

We computed [8] in background gauge the anomalous dimension of the operator (4.2) and found that the scale-dependence of (4.1) is governed by

$$\frac{K_w(\mu')}{K_w(\mu)} = \left[\frac{g_s(\mu')}{g_s(\mu)} \right]^{-\frac{108}{33-2N_f}}, \quad (4.4)$$

with N_f the active number of quark flavours. Our exponent in (4.4) is equal in magnitude but of opposite sign to that [31] quoted by S. Weinberg [28] in his influential paper, and agrees with earlier [32] and more recent [33] calculations. The correct sign considerably weakens the potential constraints on heavy quark CEDMs and on the theories that may generate them.

These results for the diagram of fig. 3, and the rules [27] to estimate matrix elements that we have reviewed, yield an expected EDM for the neutron, of magnitude:

$$\begin{aligned} \delta_n^7 &= \frac{1}{32\pi^3} \frac{M}{m_q} g_s^3(M) \left[\frac{g_s(M)}{g_s(m_q)} \right]^{-\frac{108}{33-2N_f}} \delta_q^g(m_q) \\ &\sim \frac{1}{3\sqrt{6}} \frac{M}{m_q} \left[\frac{33-2N_f}{18} \ln \left(\frac{m_q^2}{\lambda^2} \right) \right]^{-\frac{54}{33-2N_f}} \delta_q^g(m_q), \end{aligned} \quad (4.5)$$

where we have used the conventional expression for the QCD running coupling constant, with $\lambda \sim 150$ MeV. The limits obtained from the inversion of (4.5), for s, c, b, t quarks of mass 0.45, 1.3, 4.9, 100 GeV, are:

$$|\delta_s^g| < 1.7 \cdot 10^{-8}, |\delta_c^g| < 2.0 \cdot 10^{-7}, |\delta_b^g| < 2.5 \cdot 10^{-6}, |\delta_t^g| < 2.7 \cdot 10^{-4}. \quad (4.6)$$

We re-emphasize that the limits for $q = s, t$ are not as defensible as the others, thus the “caveat” dot over the inequality sign, a notation that we shall again use when needed. One may consider the possibility of computing the effects of higher-dimensional multi-gluon operators, beginning the list with the four-gluon analogs to (4.2). This would result in a substitution of (4.5) by a power series expansion in $M^2/(4\pi m_q^2)$, no doubt converging fast for all quarks but s . For the strange quark, it behaves one in this connection to use, as we have done, a “constituent” mass, for which the expansion parameter is ~ 0.5 . To reach a level of satisfaction [30] one must appeal to the fact that the final estimates of neutron matrix elements are based on considerations whose precise numerical credibility is affected by uncertainties [27] of the very same type and putative size.

Limits on quark EDMs and CEDMs can also be extracted from the diagrams of fig. 4, whose analysis parallels that of fig. 3. The result of integrating out the quark loops is an effective Lagrangian involving two d=8 CP-odd, one-photon-three-gluon operators:

$$\mathcal{L}_4 = e g_s^3(\mu) [K_F(\mu) \mathcal{O}_F(\mu) + K_G(\mu) \mathcal{O}_G(\mu)], \quad (4.7)$$

$$\mathcal{O}_F = \frac{1}{3!} d_{abc} \tilde{F}_{\mu\nu} G^{a\mu\nu} G_{\rho\sigma}^b G^{c\rho\sigma}, \quad (4.8)$$

$$\mathcal{O}_G = \frac{1}{3!} d_{abc} \tilde{G}_{\mu\nu}^a G^{b\mu\nu} G_{\rho\sigma}^c F^{\rho\sigma}. \quad (4.9)$$

The explicit values of the coefficients in (4.7) in terms of the EDM of fig. 4a and the CEDM of fig. 4b are:

$$K_G = \frac{1}{64 \pi^2 m_q^3 M_Z} (Q_q \delta_q^g + \delta_q^{\gamma}), \quad (4.10)$$

$$K_F = \frac{1}{64 \pi^2 m_q^3 M_Z} (2 Q_q \delta_q^g). \quad (4.11)$$

Notice that the scale factor relating the coefficients of the d=6 operator $\mathcal{O}_w$ to those of the d=8 operators $\mathcal{O}_F$ and $\mathcal{O}_G$ is m_q^{-2} . Since this quark may be a medium weight one,

the contributions of the d=8 operators are not a priori (nor, as we shall see, a posteriori) negligible relative to the contribution of the d=6 operator. The eigenstates of the renormalization group are not $\mathcal{O}_F$ and $\mathcal{O}_G$ of (4.8) and (4.9), but the combinations

$$\mathcal{O}^+ = \mathcal{O}_F + \frac{\sqrt{6}-2}{2} \mathcal{O}_G, \quad (4.12)$$

$$\mathcal{O}^- = \mathcal{O}_G - (\sqrt{6}-2) \mathcal{O}_F, \quad (4.13)$$

so that it befits one to re-express (4.7) in terms of $\mathcal{O}^\pm$, the scale-evolution of whose coefficients we computed [8] in background gauge to be:

$$R^\pm \equiv \frac{K^\pm(\mu')}{K^\pm(\mu)} = \left[\frac{g_s(\mu')}{g_s(\mu)} \right]^{-\frac{75 \pm 5\sqrt{2}}{33-2N_f}}. \quad (4.14)$$

Even for the longest-distance run we undertake (that from m_t to the confining scale) the ratio R^+/R^- is only ~ 2 , certainly within the inescapable uncertainty of the rough estimate of neutron matrix elements. It would therefore be merely pedantic to maintain the difference between R^+ and R^- in our expressions. We simply evolve K_G and K_F with the average of the anomalous dimensions in (4.14). This, and the rules [27] to estimate matrix elements, yield an expected EDM for the neutron, of magnitude:

$$\delta_n^\gamma \sim \frac{1}{12\sqrt{6}} \left(\frac{M}{m_q} \right)^3 \left[\frac{33-2N_f}{18} \ln \left(\frac{m_q^2}{\lambda^2} \right) \right]^{-\frac{75}{2(33-2N_f)}} [3 Q_q \delta_q^g(m_q) + \delta_q^\gamma(m_q)]. \quad (4.15)$$

Assuming no accidental cancellation between EDMs and CEDMs, the limits obtained from the inversion of (4.15) are, for s, c, b, t quarks of mass 0.45, 1.3, 4.9, 100 GeV:

$$\begin{aligned} |\delta_s^\gamma| &< 4.8 \cdot 10^{-9}, |\delta_c^\gamma| < 1.5 \cdot 10^{-7}, |\delta_b^\gamma| < 3.6 \cdot 10^{-5}, |\delta_t^\gamma| < 5.0 \cdot 10^{-1}, \\ |\delta_s^\gamma| &< 4.8 \cdot 10^{-9}, |\delta_c^\gamma| < 3.0 \cdot 10^{-7}, |\delta_b^\gamma| < 3.6 \cdot 10^{-5}, |\delta_t^\gamma| < 1.0. \end{aligned} \quad (4.16)$$

These are tighter than (4.6) on the light-quark side, and much looser on the heavy side, as could be expected from the diverse dimensionalities of the effective operators in (4.1) and (4.7), and the different resulting m_q -dependences of (4.3) versus (4.10) or (4.11). For the strange quark, the same considerations made after (4.6) are to be repeated here.

Integration of the heavy degrees of freedom of our yet-unspecified theory of CP violation yields predictions for $\delta_q^\gamma(\Lambda)$ and $\delta_q^g(\Lambda)$. To transcribe the theoretical predictions

from a scale Λ to the quark-mass scale we need to know the QCD evolution of the various dipole moments. The relevant anomalous dimensions are known [34] and yield the result:

$$\frac{\delta_q^{7,Z}(\mu')}{\delta_q^{7,Z}(\mu)} = \left[\frac{g_s(\mu')}{g_s(\mu)} \right]^{-\frac{8}{33-2N_f}}, \quad (4.17)$$

$$\frac{\delta_g^{8,Z}(\mu')}{\delta_g^{8,Z}(\mu)} = \left[\frac{g_s(\mu')}{g_s(\mu)} \right]^{-\frac{4}{33-2N_f}}. \quad (4.18)$$

The exponents in the above expressions are very small in comparison with those describing the evolution of multi-gluon operators, as in (4.4) and (4.14). Even for the large gap between $\mu' = \Lambda \sim M_Z$ and the confining scale $g(M) = 4\pi/\sqrt{6}$, the numerical values of (4.17) are $\sim 1.6, 1.3$, indistinguishable from unity above the “noise” in the estimates of hadronic matrix elements. Operator mixing is of even lesser numerical relevance⁸. Thus, we are justified in neglecting all QCD-evolution effects but the large ones pertinent to the evolution of multi-gluon vertices.

5. Indirect Bounds: Electroweak Loops.

Bounds on the weak and electromagnetic CP-odd moments of leptons (and quarks) can also be extracted from limits on the EDMs (and CEDMs) they induce via radiative corrections. In various cases in which the latter limits are sufficiently tight, the indirect bounds are stronger than those obtained otherwise.

Consider the Z-exchange diagram of fig. 5. in which the heavy dot is the WEDM of a fermion, whose standard-model vector coupling to the Z, v_f , is recalled in (2.24). Unlike the quantum corrections discussed in the previous chapter, the current one is ultraviolet divergent, and must be regularized with a cut-off Λ , characterizing the CP-odd dynamics (the divergence is logarithmic, indicating that all momentum scales up to Λ are similarly weighed). At these scales, comparable to or larger than M_Z , not only the

⁸ Matching of the coefficients of the various effective Lagrangians (obtained as successive fields are integrated out) mimicks a mixing between operators of different field-dimensionality. The numerical incidence of this effect on the evolution of dipole-moment-like operators in the $\Delta S=1$ weak Hamiltonian [34] is minor. Similar “mixing” effects between CEDMs and the operator (4.2) have been found to be numerically small [33] in Higgs-mediated and supersymmetric [35] models. We expect no surprises in the extension of these results to include the operators (4.8) and (4.9).

fermion’s WEDM, but all the operators in (2.1) and (2.2) (that were forced upon us by the necessarily restored $SU(2) \times U(1)$ gauge invariance of the theory) do play a role. The W -exchange diagrams in fig. 5 must be included in the calculation, whose overall logarithmic dependence, with the help of (2.4), (2.5), (1.5) and (1.6), can be cast in the form:

$$\delta_f^7 \doteq -\left(\frac{\alpha}{\pi}\right) \left\{ \frac{Q_f v_f}{\sin\theta_w \cos\theta_w} \left[\ln \frac{\Lambda^2}{M_Z^2} \right] + \frac{T_f^3 Q_f'}{\sin^2\theta_w} \left[\ln \frac{\Lambda^2}{M_W^2} \right] - \frac{3T_f^3(Q_f' - Q_f)}{2\sin^2\theta_w} \left[\ln \frac{\Lambda^2}{M_W^2} \right] \right\} \delta_f^Z. \quad (5.1)$$

Here Q_f' is the charge of f ’s partner in a weak doublet, and a term $\propto \alpha \delta_f^7/\pi$ in the r.h.s. has been consistently dropped. The caveat dot over the equality sign in (5.1) is a reminder of the fact that, if $\Lambda \gg M_Z$, the result is approximately correct; while if $\Lambda \sim M_Z$, (5.1) is at best an estimate, and the full result calls for a two-loop calculation. The above relation, and similar ones to follow, holds for δ ’s as defined at the scale Λ but, as in the previous Section, this point is numerically moot. With use of the direct EDM limits of Section 2 and the indirect ones (4.16), we obtain, upon inversion of (5.1) with its logs set to unity:

$$\begin{aligned} |\delta_q^Z| &< 2.5 \cdot 10^{-7}, \quad |\delta_q^Z| < 8.1 \cdot 10^{-8}, \quad |\delta_q^Z| < 8.9 \cdot 10^{-7}, \quad |\delta_e^Z| < 4.2 \cdot 10^{-3}, \\ |\delta_e^Z| &< 6.7 \cdot 10^{-3}, \quad |\delta_e^Z| < 1.1 \cdot 10^{-8}, \quad |\delta_e^Z| < 3.7 \cdot 10^{-8}, \quad |\delta_\mu^Z| < 0.41, \end{aligned} \quad (5.2)$$

where we have piled up the caveats on the strange quark result. Because of its reliance on (3.9), the bound for neutrinos is valid if their mass is less than ~ 10 keV. We have not listed limits whose numerical value (in weak magneton units) is significantly bigger than one. Notice that the despairingly tight bound on neutrinos stems from the strictures imposed by gauge invariance. Notice also that the bounds for electrons and quarks are much tighter than any of the direct bounds derived from LEP data in Section 3.

Consider next the diagrams of fig. 6. The loop integration yields the following logarithmic dependence:

$$\begin{aligned} \delta_q^g \doteq -\left(\frac{\alpha}{\pi}\right) &\left\{ \left(\frac{v_q}{\sin\theta_w \cos\theta_w} \left[\ln \frac{\Lambda^2}{M_Z^2} \right] + \frac{T_q^3}{\sin^2\theta_w} \left[\ln \frac{\Lambda^2}{M_W^2} \right] \right) \delta_q^Z \right. \\ &\left. + \left(Q_q \left[\ln \frac{\Lambda^2}{m_q^2} \right] + T_q^3 \left[\ln \frac{\Lambda^2}{M_W^2} \right] \right) \delta_q^7 \right\}. \end{aligned} \quad (5.3)$$

If no accidental cancellation takes place between the two terms in the right hand side, inversion of this expression and use in each case of the strongest of the bounds (4.6) and (4.16) gives strictures sometimes tighter than those in (5.2) :

$$\begin{aligned} |\delta_q^Z| &< 6.1 \cdot 10^{-7}, \quad |\delta_e^Z| < 2.2 \cdot 10^{-5}, \quad |\delta_e^Z| < 3.2 \cdot 10^{-4}, \quad |\delta_e^Z| < 4.0 \cdot 10^{-2}, \\ |\delta_\mu^Z| &< 4.8 \cdot 10^{-7}, \quad |\delta_\mu^Z| < 9.8 \cdot 10^{-6}, \quad |\delta_\mu^Z| < 4.1 \cdot 10^{-4}, \quad |\delta_\mu^Z| < 9.3 \cdot 10^{-2}. \end{aligned} \quad (5.4)$$

In the numerics we have conservatively substituted $\ln(\Lambda^2/m_q^2)$ by $\ln(M_Z^2/m_q^2)$ for all quarks except the top, and set all other logarithms to unity. The double-caveat dot in δ_f^Z reminds us that, in a theory in which CP violation originates at a scale $\sim M_Z$, a full three-loop calculation may be necessary to go beyond a mere estimate. No caveat accompanies the inequalities extracted from the second line of (5.3), but its insertion into the loop diagrams, from which the limits on δ_q^Z were obtained, always does. The point is that the correct calculation of the quark loops of fig. 3 and fig. 4 with an extra internal photon is always a two-loop enterprise. Thus the accumulation of caveat dots in the results of the inversion of (5.3) with input from the tightest of (4.6) and (4.16). For δ_f^Z , δ_c^Z and δ_b^Z the inequalities in (5.4) are weaker than (4.16), not so for δ_f^Z .

It is important to realize that, even though the bounds of (5.1) and (5.4) may be sprinkled with our honest caveat dots to remind the reader that they may have an order of magnitude imprecision, they are so much tighter than the direct bounds from Z -decay that there is no question that they supersede them. An experimental measurement amply contradicting the indirect bounds would be a rather challenging result.

6. Indirect Bounds on the CP-odd couplings of W 's.

A nonvanishing δ_W^Z induces fermionic EDMs via quantum effects. The contribution of the diagram of fig. 7a to the quark EDM has been previously computed [36], but a full calculation involving all the relevant couplings in (2.13), i.e. all the diagrams of fig. 7, is called for by the requirement of gauge invariance. The result for a d -type quark, D , which we have derived in the Feynman and unitary gauges, is:

$$\begin{aligned} \delta_D^Z = & -\frac{\alpha}{16\pi} \frac{m_D}{\sin^2\theta_W \cos^2\theta_W M_Z} \left\{ \delta_W^Z \left[\ln \frac{\Lambda^2}{M_W^2} - 2 + \frac{4}{3} \sin^2\theta_W \ln \frac{\Lambda^2}{M_H^2} \right. \right. \\ & + \left. \left(\frac{1}{4} - \frac{1}{3} \sin^2\theta_W \right) \frac{\cos(2\theta_W)}{\cos^2\theta_W} \left(\ln \frac{\Lambda^2}{M_Z^2} + \ln \frac{\Lambda^2}{M_H^2} + \frac{M_H^2 + M_Z^2}{M_H^2 - M_Z^2} \ln \frac{M_Z^2}{M_H^2} \right) \right] \\ & \left. - \delta_W^Z \left[2 \frac{\cos^2\theta_W}{\sin^2\theta_W} \ln \frac{\Lambda^2}{M_W^2} \right] \right\}, \end{aligned} \quad (6.1)$$

where the “constant” terms, but not the $\ln \Lambda$ ones, depend on the regularization scheme (a momentum cutoff $\Lambda^2 \leq \Lambda^2$, in the above expression). Barring unlikely cancellations between the δ and $\bar{\delta}$ terms, (6.1) can be inverted to set bounds on either. We have refrained from giving the up-quark (U) or lepton (l) analogs of (6.1), but setting to unity the contents of

the square brackets in (6.1) and its U and l counterparts allows us to write an expression valid for all fermions:

$$\delta_f^Z \sim \frac{\alpha}{16\pi \cos^2\theta_W \sin^2\theta_W} \frac{m_f}{M_Z} \left(\delta_W^Z \text{ or } \bar{\delta}_W^Z \right). \quad (6.2)$$

An entirely analogous treatment of ZWW couplings results in:

$$\delta_f^Z \sim \frac{\alpha}{16\pi \cos^3\theta_W \sin^3\theta_W} \frac{m_f}{M_Z} \left(\delta_W^Z \text{ or } \bar{\delta}_W^Z \right). \quad (6.3)$$

We may invert these equations and deduce a long list of constraints on $\delta_W^{f,Z}$. The strongest turn out to be those based on $f = e, u, d, s, c$, that exploit the limits (3.6), (3.3), (4.16), (5.2) and (5.4). To be on the safe side, we first use “current” masses for the quarks in the inversion of (6.2) and (6.3): $m_q = 5, 7, 150, 1300$ MeV, for $q = u, d, s, c$, respectively, and obtain:

$$\begin{aligned} |\delta_W^Z(\Leftarrow e)| & < 1.8 \cdot 10^{-2}, \quad |\delta_W^Z(\Leftarrow u)| < 3.7 \cdot 10^{-2}, \quad |\delta_W^Z(\Leftarrow d)| < 6.5 \cdot 10^{-3}, \\ |\delta_W^Z(\Leftarrow s)| & < 3.3 \cdot 10^{-3}, \quad |\delta_W^Z(\Leftarrow c)| < 2.4 \cdot 10^{-2}, \quad |\delta_W^Z(\Leftarrow b)| < 0.76, \end{aligned} \quad (6.4)$$

$$\begin{aligned} |\delta_W^Z(\Leftarrow e)| & < 0.94, \quad |\delta_W^Z(\Leftarrow d)| < 0.50, \\ |\delta_W^Z(\Leftarrow s)| & < 0.18, \quad |\delta_W^Z(\Leftarrow c)| < 0.74, \end{aligned} \quad (6.5)$$

where “ $\Leftarrow$ ” is a reminder of the origin of the bound, and similar results apply to the corresponding $\bar{\delta}$'s.

The quark-based bounds (6.4) and (6.5) may be unduly ample. Verily, the quark mass occurring in (6.1) is a constituent mass: either a mass insertion in (or the coupling of an H_0 to) an external leg in the diagrams of fig. 7. It can therefore be soundly argued that the limits based on (3.3) should be strengthened to read:

$$|\delta_W^Z(\Leftarrow u)| < 6.2 \cdot 10^{-4}, \quad |\delta_W^Z(\Leftarrow d)| < 1.5 \cdot 10^{-4}, \quad (6.6)$$

where we have used $m_{u,d} = 300$ MeV.

LEP-II would be sensitive [37] to values of $\delta^{f,Z}$ or $\bar{\delta}^{f,Z} \sim \mathcal{O}(10^{-2})$. The limits (6.5) allow one to sustain the momentary hope that the measurement of a nonvanishing δ_W^Z or $\bar{\delta}_W^Z$ is conceivable. The gauge-invariance conditions (2.16) and (2.17), however, relate the bounds on Z couplings to the much stronger photon bounds (6.4) or (6.6), yielding

$$\begin{aligned} |\delta_W^Z(\Leftarrow e)| & < 4.1 \cdot 10^{-3}, \quad |\delta_W^Z(\Leftarrow u)| < 1.4 \cdot 10^{-4}, \quad |\delta_W^Z(\Leftarrow d)| < 3.5 \cdot 10^{-5}, \\ |\bar{\delta}_W^Z(\Leftarrow e)| & < 1.3 \cdot 10^{-2}, \quad |\bar{\delta}_W^Z(\Leftarrow u)| < 4.7 \cdot 10^{-4}, \quad |\bar{\delta}_W^Z(\Leftarrow d)| < 1.2 \cdot 10^{-4}. \end{aligned} \quad (6.7)$$

There appears to be absolutely no hope to observe CP violation at LEP-II in the form of an electric dipole or a magnetic quadrupole moment the W , nor in the form of their weak-interaction generalizations. A non-vanishing result would indicate very serious theoretical misconceptions on our part.

An important difference between the operators $\mathcal{O}_{\tilde{B}}$ and $\mathcal{O}_{\tilde{W}}$ defined in (2.7) and (2.8) concerns their properties under “custodial” [38] $SU_C(2)$: a global symmetry of the gauge couplings of the standard model that survives the spontaneous breakdown of its pristine gauge symmetry. $\mathcal{O}_{\tilde{W}}$ is $SU_C(2)$ -symmetric, while $\mathcal{O}_{\tilde{B}}$ is not. Let Σ be the self-energy insertion of an intermediate vector boson, renormalized at vanishing momentum. Its contribution to Veltman’s “ ρ -parameter” can be defined [39] as a shift in $\Sigma_Z/M_Z^2 - \Sigma_W/M_W^2$ away from the standard model prediction (with its $SU_C(2)$ -breaking mass effects explicitly included). $SU_C(2)$ -invariant modifications of the standard model, such as an effective interaction proportional to $\mathcal{O}_{\tilde{W}}$, preserve the result $\delta\rho = 0$. An extra interaction linear in $\mathcal{O}_{\tilde{B}}$ results in $\delta\rho \propto (C_{\tilde{B}}\Lambda)^2$, a contribution that “decouples” as expected, since dimensional analysis dictates $C_{\tilde{B}} \propto M_Z^2/\Lambda^2$. The current empirical bounds on $\delta\rho$ are insufficient to yield a limit on $C_{\tilde{B}}$ competitive with the ones we have discussed.

Nonvanishing EDMs and WEDMs of W ’s occur in the standard model at the two-loop level, they induce the corresponding quantities for electrons and neutrons via an extra loop. Non-standard contributions to W ’s appear at one loop, and have been analyzed in “left-right” models [40], in models with “mirror” fermions [41], and in supersymmetric theories [41] where they may be parametrized via phases in the wino-higgsino mass matrix [42]. None of the above theoretical expectations exceeds the limits that we have discussed, or even flirts with them.

In conclusion, the CP-odd couplings involving three gauge bosons are bound to levels that make them unlikely to be measurable in the near future. They are also predicted to be small in known extensions of the standard model. Yet, they are novel and fundamental signets of CP violation: direct experimental information on them is mandatory.

7. Limiting Polyps.

The “polyp” operators defined in (2.23) can be directly bound from experimental constraints whenever the on-shell expression (2.26) that relates them to dipole moments is applicable. To proceed in this direction, it is commodious to define:

$$\Delta_f^j = (\Delta_f^I + T_f^3 \Delta_f^J), \quad (7.1)$$

$$\Delta_f^Z = (-\sin^2\theta_W \Delta_f^I + \cos^2\theta_W T_f^3 \Delta_f^J), \quad (7.2)$$

$$\Delta_f^g = \Delta_f^J, \quad (7.3)$$

and entirely similar relations for the vector-polyp coefficients Σ_f .

It follows from (2.26) and (2.4) to (2.6) that any *direct* (on shell) bound on each of the dipole coefficients $\mathcal{O}_f^{Z,g}$ is a bound on the value of the corresponding $2m_f \Delta_f^{Z,g}/M_Z$. It is also legitimate to apply this argument to the quasi-direct bounds on the CEDMs of constituent quarks (3.26) and (3.27). From the limits (3.3), (3.6) and (3.26) we obtain, for the axial polyp coefficients of (7.1), (7.2) and (7.3):

$$|\Delta_f^J| < 6.7 \cdot 10^{-8}, |\Delta_f^I| < 2.7 \cdot 10^{-7}, |\Delta_f^Z| < 7.9 \cdot 10^{-6}, |\Delta_f^g| < 2.3 \cdot 10^{-7}, |\Delta_f^Z| < 2.3 \cdot 10^{-7}, \quad (7.4)$$

where we have omitted all results larger than unity.

Indirect bounds on axial-polyp couplings via their quantum-loop effects can also be laboriously worked out though, in some cases, they can be extricated without recourse to an independent calculation from the indirect bounds on (C)EDMs that we have discussed, a delicate procedure that we cover in detail in the Appendix. We learn there that an anomaly not entirely unlike the Adler-Bell-Jackiw one completely obliterates the contribution of axial polyps to the operator $\mathcal{O}_w$ of (4.2). As a consequence the constraints obtained via the contribution of this operator to the neutron’s EDM cannot be translated into strictures on axial polyps. Contrarywise, and once again as detailed in the Appendix, the bounds (4.16) that we derived with the help of the operators $\mathcal{O}_F$ and $\mathcal{O}_G$ of (4.8) and (4.9) can be converted into limits on axial-polyp couplings exactly as the direct bounds can. This way we obtain:

$$\begin{aligned} |\Delta_f^g| &< 4.9 \cdot 10^{-7}; |\Delta_f^g| < 5.3 \cdot 10^{-6}; |\Delta_f^g| < 3.4 \cdot 10^{-4}; |\Delta_f^g| < 2.3 \cdot 10^{-1}, \\ |\Delta_f^Z| &< 4.9 \cdot 10^{-7}; |\Delta_f^Z| < 1.1 \cdot 10^{-5}; |\Delta_f^Z| < 3.4 \cdot 10^{-4}; |\Delta_f^Z| < 4.6 \cdot 10^{-1}. \end{aligned} \quad (7.5)$$

Drawing nigher to the end of our efforts, we quote some limits on axial and vector polyp coefficients that we have derived the hard way: via an independent calculation of a loop-induced effect. Consider fig. 8, that illustrates how a non-vanishing Σ^Z or Δ^Z induces an EDM or a CEDM. The results of the corresponding computation are:

$$\delta_f^j = \frac{\alpha}{\pi} Q_f \left(\frac{\Sigma_f^Z a_f + \Delta_f^Z v_f}{\sin\theta_W \cos\theta_W} \right) \frac{m_f \ln\left(\frac{\Lambda^2}{M_Z^2}\right)}{M_Z}, \quad (7.6)$$

$$\delta_q^g = \frac{\alpha}{\pi} \left(\frac{\Sigma_q^Z a_q + \Delta_q^Z u_q}{\sin\theta_w \cos\theta_w} \right) \frac{m_q}{M_Z} \ln \left(\frac{\Lambda^2}{M_Z^2} \right). \quad (7.7)$$

If there are no fortuitous cancellations between Δ 's and Σ 's, inversion of the above and use of (3.26), (4.16) and (4.6) results in:

$$\begin{aligned} |\Delta_e^Z| &< 5.7 \cdot 10^{-2}, \quad |\Delta_u^Z| < 2.0 \cdot 10^{-2}, \quad |\Delta_d^Z| < 7.0 \cdot 10^{-3}, \quad |\Delta_s^Z| < 1.2 \cdot 10^{-3}, \\ |\Delta_c^Z| &< 7.7 \cdot 10^{-3}, \quad |\Delta_b^Z| < 1.9 \cdot 10^{-2}, \quad |\Delta_t^Z| < 1.8 \cdot 10^{-1}, \end{aligned} \quad (7.8)$$

for the axial polyp coefficients, and:

$$\begin{aligned} |\Sigma_e^Z| &< 4.6 \cdot 10^{-3}, \quad |\Sigma_u^Z| < 7.7 \cdot 10^{-3}, \quad |\Sigma_d^Z| < 4.9 \cdot 10^{-3}, \quad |\Sigma_s^Z| < 8.2 \cdot 10^{-4}, \\ |\Sigma_c^Z| &< 3.0 \cdot 10^{-3}, \quad |\Sigma_b^Z| < 1.3 \cdot 10^{-2}, \quad |\Sigma_t^Z| < 7.0 \cdot 10^{-2}, \end{aligned} \quad (7.9)$$

for the corresponding vector ones. Here we have used current quark masses in every case, since m_q in (7.6) and (7.7) permeates from inside the loop. Beware that use of non-gauge invariant forms such as (1.12) and (1.13) and the consequent forsaking of the diagrams of fig. 8c,d would have resulted in fake quadratic divergencies in (7.6) and (7.7), that would not even “decouple” as they should in the large Λ limit. Gauge invariance also implies that the vector polyp interactions of (2.22), (2.23) and (7.2) have their coefficients thus trimmed, that Σ^g and Σ^V do not contribute to fermion EDMs when inserted in loops such as those depicted in fig. 8.

8. Compilation and comparison of bounds.

We have summarized in Table I the tightest direct or indirect limits on each of the EDMs, WEDMs and CEDMs of all fundamental fermions, as well as on the coefficients describing the electromagnetic and weak “electric” dipole and “magnetic” quadrupole moments of W 's. The limits on vector and axial polyp coefficients, for those fermions for which we have derived them so far, are gathered in Table II.

The EDMs of leptons are systematically best constrained by observational bounds, with the possible exception of massive ($m > 10$ keV) ν_τ 's, that would evade the very tight constraints from the cooling of red giants. Most WEDMs of leptons, contrarily, are best constrained by limits relating them, via electroweak radiative corrections, to the corresponding EDM. Here the exceptions are the μ , the τ and, with the usual mass proviso, the ν_τ .

Practically all ODMs of quarks are most limited by the neutron's EDM they would induce by quantum corrections. The exceptions are the EDMs and CEDMs of u and d quarks, that would generate a neutron EDM in a more direct manner.

The electric dipole and magnetic quadrupole moments of W 's or, more precisely, the linear combinations thereof: δ_W^V and δ_W^A of (1.10) and (1.11), can at present be only bound indirectly, via their radiatively induced effect on the EDM of the neutron. The tightest limits on the analogous quantities for the ZW^+W^- couplings require the use of the gauge invariance conditions (2.16) and (2.17).

The limits on polyps in Table II are all indirect but for the photonic (and gluonic) couplings of electrons and light quarks, for which we only employed the Dirac equation in relating them to the (C)EDM of e 's u 's and d 's.

In the Tables, figure numbers are indicated in those cases in which a limit follows from a loop-induced effect. These limits are always orders of magnitude tighter than the corresponding direct ones, that often do not exist at all. We have therefore refrained from recalling once again the caveats that accompany some of these limits, which may be relevant in establishing precise constraints on potential theories of CP violation, but are quite irrelevant to the analysis of experiments with a discovery potential.

	δ^γ	δ^g	δ^Z
ν_e	10^{-10}	(RG)	$3.7 \cdot 10^{-8}$ (GI+5+RG)
e	$8.9 \cdot 10^{-11}$	(d_e)	$1.1 \cdot 10^{-8}$ (5)
ν_μ	10^{-10}	(RG)	$3.7 \cdot 10^{-8}$ (GI+5+RG)
μ	$3.4 \cdot 10^{-3}$	($g-2$)	$8.0 \cdot 10^{-2}$ (L)
ν_τ	1.0 ($m > 10$ keV), (L)		0.26 ($m > 10$ keV), (L)
	10^{-10}	(RG)	$3.7 \cdot 10^{-8}$ (GI+5+RG)
τ	0.46	(P)	$8.0 \cdot 10^{-2}$ (L)
u	$1.8 \cdot 10^{-9}$	(d_n)	$1.5 \cdot 10^{-9}$ (d_n)
			$2.5 \cdot 10^{-7}$ ($5+d_n$)
d	$4.4 \cdot 10^{-10}$	(d_n)	$1.5 \cdot 10^{-9}$ (d_n)
			$8.1 \cdot 10^{-8}$ ($5+d_n$)
c	$3.0 \cdot 10^{-7}$	($4a+d_n$)	$1.5 \cdot 10^{-7}$ ($4b+d_n$)
			$2.2 \cdot 10^{-5}$ ($6+4b+d_n$)
s	$4.8 \cdot 10^{-9}$	($4a+d_n$)	$4.8 \cdot 10^{-9}$ ($4b+d_n$)
			$6.1 \cdot 10^{-7}$ ($6+4b+d_n$)
t	$9.3 \cdot 10^{-2}$	($4a+d_n$)	$2.7 \cdot 10^{-4}$ ($3+d_n$)
			$4.0 \cdot 10^{-2}$ ($6+3+d_n$)
b	$3.6 \cdot 10^{-5}$	($4a+d_n$)	$2.5 \cdot 10^{-6}$ ($3+d_n$)
			$3.2 \cdot 10^{-4}$ ($6+3+d_n$)
$W[\delta_W^{\gamma,Z}]$	$1.5 \cdot 10^{-4}$	($7+d_n$)	$3.5 \cdot 10^{-5}$ (GI+7+ d_n)
$W[\delta_W^{\gamma,Z}]$	$1.5 \cdot 10^{-4}$	($7+d_n$)	$1.2 \cdot 10^{-4}$ (GI+7+ d_n)

Table I

Compendium of the tightest bounds on the electric, weak and chromoelectric dipole moments (δ^γ , δ^Z and δ^g , respectively) introduced in equations (1.5) to (1.7). The experimental constraints from which the bounds are extricated are indicated: RG for red giants, L for LEP, P for Petra, d_n and d_e for the neutron's and electron's EDMs. In those cases in which the bounds stem from quantum corrections, the numbers of the corresponding figures are indicated. GI, for gauge invariance, reminds one that this condition plays a crucial role in the derivation of a particular limit from a related one. The bounds quoted for the W refer either to $\delta_W^{\gamma,Z}$ or to $\hat{\delta}_W^{\gamma,Z}$, defined in (1.8) and (1.9).

	Δ^γ	Δ^g	Δ^Z	Σ^Z
e	$7.9 \cdot 10^{-6}$		$5.7 \cdot 10^{-2}$ (8)	$4.6 \cdot 10^{-3}$ (8)
u	$2.7 \cdot 10^{-7}$	$2.3 \cdot 10^{-7}$	$2.0 \cdot 10^{-2}$ (8)	$7.7 \cdot 10^{-3}$ (8)
d	$6.7 \cdot 10^{-8}$	$2.3 \cdot 10^{-7}$	$7.0 \cdot 10^{-3}$ (8)	$4.9 \cdot 10^{-3}$ (8)
c	$1.1 \cdot 10^{-5}$ (WI+4a)	$5.3 \cdot 10^{-6}$ (WI+4b)	$7.7 \cdot 10^{-3}$ (8+4b)	$3.0 \cdot 10^{-3}$ (8+4b)
s	$4.9 \cdot 10^{-7}$ (WI+4a)	$4.9 \cdot 10^{-7}$ (WI+4b)	$1.2 \cdot 10^{-3}$ (8+4b)	$8.2 \cdot 10^{-4}$ (8+4b)
t	$4.6 \cdot 10^{-1}$ (WI+4a)	$2.3 \cdot 10^{-1}$ (WI+4b)	$1.8 \cdot 10^{-1}$ (8+3)	$7.0 \cdot 10^{-2}$ (8+3)
b	$3.4 \cdot 10^{-4}$ (WI+4a)	$3.4 \cdot 10^{-4}$ (WI+4b)	$1.9 \cdot 10^{-2}$ (8+3)	$1.3 \cdot 10^{-2}$ (8+3)

Table II

Bounds on the axial polyps of (2.22), (2.23) and (7.1) to (7.3). All limits except the electron's are ultimately based on the experimental bound on the neutron's EDM. If a limit involves a loop calculation, the corresponding figure number is indicated. The acronym WI indicates that a bound exploits the Ward identities derived in the Appendix.

9. Theoretical Conclusions.

The only empirical evidence on CP violation in flavour-diagonal processes is negative. Direct limits on electric dipole moments (EDMs) exist only for e 's, μ 's, neutrons and Λ 's. The neutron and Λ bounds can be translated into limits on the EDMs of u , d and s quarks. Quadratic bounds stemming from measurements of rates are also available for all elementary fermions but the t quark. These direct limits are much weaker than the ones we have deduced from indirect effects on the neutron's EDM, and summarized in Table I. For the EDMs of neutrinos lighter than ~ 10 keV the tightest quadratic constraints are astrophysical, based on considerations on the cooling of red-giant stars.

Recent data from LEP add to the above bounds a list of quadratic limits on WEDMs: the neutral-weak generalizations of EDMs. For all fundamental fermions but μ 's, τ 's and $m < 10$ keV ν_τ 's, much tighter constraints can be dug out from the effects on EDMs that WEDMs induce via radiative corrections, see Table I.

CEDMs, the coloured analogs of EDMs, are also quasi-directly bound for u and d quarks by the limit on the neutron's EDM. This same experimental constraint imposes

amazingly tight strictures on the CEDMs of all heavier quarks, that transmit their CP-odd effect to the neutron via quantum loop corrections. These results are also summarized in Table I.

The conventionally defined ODMs (EDMs, WEDMs and CEDMs) are operators of mass dimension $d=5$. This characterization is untenable once the full impact of electroweak gauge invariance is invoked, as we discussed in gory detail in Section 2. ODMs are $d=6$ operators in a $d=5$ clothing, indissolubly spoused to a Higgs field or, in their low-energy incarnation, to its vacuum expectation value, as in (2.1) to (2.6). In a gauge invariant framework, the restriction to ODMs as signets of CP violation, suggested by their record-low dimensionality, is no longer automatically justified. New and interesting CP-odd couplings involving a fermion pair, a gauge boson and a Higgs particle are an inescapable consequence of nonvanishing ODMs.

Once gauge invariance has forced us to move to the $d=6$ realm, the number and types of relevant CP-violating effective operators increases manifold, even in the flavour-conserving subdomain involving no more than two fermion fields, that we have covered in this paper. Rather peculiar are the $d=6$ operators of (2.23), that we have dubbed “polyps”, in an attempt to reflect the profusion of their extremities. The proper handling of polyps calls for their analysis in an $SU(2) \times U(1)$ gauge invariant context. Polyps, unlike ODMs, describe chirality-conserving interactions, and it is useful to recombine them into vector and axial varieties. Polyps of the axial ilk turn out to be rather intimately related to ODMs, an example is the on-shell relation (2.26), exploited in Section 7 to limit axial-polyp coefficients in terms of bounds on ODMs, see (7.4). Off-shell relationships between axial polyps and ODMs are sometimes anomalous and more dangerous to handle, they are discussed in detail in the Appendix, and exploited in Section 7 to constrain axial-polyp coefficients, as in (7.5). Axial polyp interactions can also be bound without recourse to their explicit on-shell or off-shell relations to ODMs by direct computation of the ODMs they induce via quantum corrections. Examples are the bounds (7.8), gathered along the rest of the limits on axial-polyp coefficients in Table II. In the few cases in which the results are not a priori forced to vanish by the local $SU(3) \times U_4(1)$ symmetry, the same technique of direct computation of a loop-induced effect also applies to vector polyps, that are not explicitly related to ODMs either on- or off-shell. Such an example is provided by the limits in (7.9). Direct bounds on vector polyps can also be extracted from their potentially observable effects in various reactions, an enterprise that we relegate to subsequent work.

In the pure $SU(3)$ gauge sector, the $d=6$ three-gluon operator $\mathcal{O}_W$, defined in (4.1), has received multitudinous attention. We have completed the discussion of how ODMs and polyps may generate a non-vanishing $\mathcal{O}_W$ and the related one-photon-three-gluon $d=8$ operators $\mathcal{O}_P$ and $\mathcal{O}_G$ of (4.8) and (4.9). In the $SU(2) \times U(1)$ gauge sector, the $d=6$ CP-odd trilinear couplings of Z 's and γ 's to W pairs, (2.7) and (2.8), have in their full gauge-invariant plumage acquired a new liveliness. Their low-energy forms (1.10) and (1.11) appear to be $d=4$ and $d=6$, and are linear combinations of the conventionally defined electric dipole and magnetic quadrupole moments of the W , or their neutral-weak generalizations. The difference between the $d=4$ operators and their $d=6$ forebears is accounted for by a total of two powers of a Higgs field and its vacuum expectation value. Much as for fermionic ODMs, gauge invariance entails the existence of new couplings between scalars and gauge bosons that cannot be consistently ignored. The very stringent limits (6.1) on the CP-odd $\gamma W^+ W^-$ interactions extracted from their radiative effect on the neutron's EDM are examples of the need to use the full consequences of gauge invariance to obtain theoretically-defensible results. Another example is the relation between the $\gamma W^+ W^-$ and $Z W^+ W^-$ coupling strengths, (2.16) and (2.17).

The standard model amply obeys all bounds on CP-odd couplings listed in I and II. Alternative models of CP violation are often significantly constrained by these limits, examples based on the ODM strictures were discussed in detail in [8] for supersymmetric, left-right and multiple-Higgs models.

Polyps (or their limits) are particularly relevant to models of CP violation that involve charged scalars, they arise at the one-loop level and they are less chirally suppressed, by one power of mass, than the corresponding ODMs. The opposite is the case for the contributions of neutral scalars in the same type of model. In supersymmetric and left-right models the effective polyp vertices also arise with weaker strengths than the corresponding ODMs. Polyps are also special in that they are the lowest-dimension CP-odd operators constructible with the massless left-handed neutrino fields of the standard model.

Even the least wily amongst our devoted readers has surely noticed that most of our considerations not explicitly involving T-odd effects can be translated with modest effort from ODMs and CP-odd couplings of W s to their CP-even counterparts: anomalous magnetic moments and their generalizations. Particularly relevant in this respect are the relations between different processes imposed by the $SU(2) \times U(1)$ gauge invariance of the standard model and its possible extensions. We devote a forthcoming article to this topic.

10. Experimental Consequences.

For years on end, the limits on the electric dipole moments of neutrons and electrons have been the (negative) pillars of our information on CP in the flavour-diagonal domain. The empirical bound on the Λ 's EDM, and the consequent limit on the s-quark's EDM, are orders of magnitude less stringent, and can be substantially improved by the analysis of their indirect $O(\alpha_s^3)$ effects on the neutron's EDM. No similar improvement can be found on the rather loose empirical bound on the muon's EDM, that induces a negligible $O(\alpha^3)$ neutron EDM. The motivation to continue the effort to improve the measurements of the EDMs of neutrons, electrons and muons is stronger than ever: the better the limit, the closer the experiment is to a possibly revolutionary discovery.

LEP-I and SLC offer the possibility to extend our empirical knowledge of EDMs to their neutral-weak generalization, WEDMs. LEP-II will enlarge these studies to the analogous CP-odd couplings of W 's. As it turns out, however, the classical neutron and electron limits, as well as the red giant bounds on neutrino EDMs, are so flabbergastingly stringent that they constrain most WEDMs, that would induce EDMs via radiative corrections, well below the level of conceivable observability, see the results summarized in Table I. The WEDM of the muon, and those of the τ lepton and its neutrino (should it be more massive than ~ 10 keV and thus skirt the constraint from the cooling of red giants) constitute the only exceptions to the conclusion that indirect limits preclude any hope of observability at LEP-I. Consequently, they deserve separate paragraphs.

The WEDM of the muon, δ_μ^Z , is best constrained by the agreement between the standard and the observed values of the muonic width of the Z , as in (3.17). This bound is quadratic in δ_μ^Z and can only improve or collapse as the square root of the accumulated statistics. We know of no realistic T-odd observable linear in this WEDM.

The very same comments apply to the WEDM of the τ , except that this lepton decays within the detectors, and the directions of its decay products are sensitive to its polarization. A T-odd observable linear in the τ 's WEDM can be constructed with the directions of its decay fragments [43]. Even though the T-odd observables have the advantage of their intimate relation to potential sources of CP violation, we find that in this case they are unlikely to compete in sensitivity with our quadratic bounds, based on the success of the standard prediction for $\Gamma(Z \rightarrow \tau\bar{\tau})$.

For a ν , heavier than 10 keV, LEP-I is the winner of the context to limit or measure its EDM or WEDM. Much as for neutrino-counting, this is done by comparing the theoretical

and experimental values for the invisible width of the Z . It follows that one should not be satisfied with the result that the measured N_ν is "close" to three, extra precision will continue to be welcome.

For the particularly interesting case of the P-odd and CP-odd ZW^+W^- coupling, the conclusion that it should not be observable at LEP-II is entirely based on the constraints (2.16) and (2.17) imposed by $SU(2) \times U(1)$ gauge invariance. But there is no entirely satisfactory alternative to a direct measurement. This is the attitude taken in [37], wherein the T-odd observables explorable in W -pair production at LEP-II are thoroughly analyzed.

The imposition of $SU(2) \times U(1)$ gauge invariance made us conclude that fermion ODMs, much as fermion masses, are low-energy manifestations of couplings to neutral scalars. At sufficiently high energy, and for scalars that are not prohibitively heavy, the measurement of nonvanishing ODMs and the search for processes explicitly involving a scalar particle may be competitive ways to unearth an effect directly or indirectly reflecting a novel source of CP violation. As an example, a non-vanishing WEDM of the τ not only induces T-odd effects and a non-standard rate in the process $Z \rightarrow \tau\bar{\tau}$, but must necessarily be related to a non-null $Z\tau\bar{\tau}H_0$ coupling. The latter may be searched for in Z decay or H_0 decay, depending on which of the two particles is more massive.

Considerations of gauge invariance have also inescapably led us to consider d=6 CP-odd "polyp" operators more baroque than EDMs or their weak and coloured generalizations. Axial polyps involving gauge bosons and quark pairs can be bound, as in Table 2, to levels that seriously compromise their observability in collider facilities. Vector polyps involving a quark pair and a Z would be of particular interest for LEP, but we constrain them to unobservable levels. But for electrons, leptonic polyps cannot be bound at present in any significant way and they offer a target of opportunity for experimental studies that we have not analyzed in this paper in terms of actual observables and prospects. We devote a forthcoming article to the analysis of polyp-induced non-standard widths and T-odd effects in the $Z \rightarrow l\bar{l}\gamma$ channel.

Further improvements of the classical measurements of the electric dipole moments of neutrons and electrons may well constitute the best hope for detection of a novel effect. It is intriguing to realize that if a non-zero coupling of the form $\vec{\sigma} \cdot \vec{E}$ were to be found, one would not a priori know which CP-odd operator is dominant in transmitting the CP-violating effect from the underlying theory to the observed effective dipole moment. In the neutron's case, for instance, it may be the dipole moment of a constituent quark or, as in the (unobservable) case of the standard model, a complicated multiloop effect dimly

reflecting the underlying dynamics, and involving more than a single constituent quark. A non-vanishing neutron's EDM may also be dominantly induced by the EDM of a heavy quark, percolating out via quantum effects. More entangled would be the possibility that a $\vec{\sigma} \cdot \vec{E}$ coupling of the neutron or electron reflect the low-energy limit of a polyp operator.

To establish the pedigree of a non-vanishing EDM it would be necessary to observe related processes, such as those involving a Higgs boson in the case of a fundamental coupling of the dipole type, or those involving more than one gauge boson in the polyp case. Since an EDM is a non-renormalizable effective operator, we do not know how it could exist as a fundamental interaction. At a deep level an EDM must be something other than an EDM.

Much remains to be done in the field of CP violation. Our indirect limits imply that, but for a few exceptions, there should be little hope of progress at collider facilities, even particularly clean and luminous ones such as LEP. But the exceptions are what matters most, besides the possibility of finding some effect that bluntly (and inexplicably) contradicts our theoretical expectations.

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Appendix A. Some "Ward identities".

For on-shell fermions the following identities hold between the convective axial current vertex in (1.2), the conventional CP-odd dipole in (1.1) and the kinetic term of the axial polyp operators of (2.23) in their electromagnetic, strong and neutral-weak manifestations:

$$\begin{aligned} i \bar{\psi} \{ \gamma_\mu \vec{\partial}_\nu \gamma_5 \} \psi F^{\mu\nu} &= m \bar{\psi} \{ i \sigma_{\mu\nu} \gamma_5 \} \psi F^{\mu\nu} \\ &= 2m \bar{\psi} \{ \vec{\partial}_\mu \gamma_5 \} \psi A^\mu \end{aligned} \quad (\text{A.1})$$

$$\begin{aligned} i \bar{\psi} \left\{ \gamma_\mu \frac{\lambda^a}{2} \vec{\partial}_\nu \gamma_5 \right\} \psi \hat{G}_a^{\mu\nu} &= m \bar{\psi} \left\{ i \sigma_{\mu\nu} \frac{\lambda^a}{2} \gamma_5 \right\} \psi \hat{G}_a^{\mu\nu} \\ &= 2m \bar{\psi} \left\{ \frac{\lambda^a}{2} \vec{\partial}_\mu \gamma_5 \right\} \psi G_a^\mu \end{aligned} \quad (\text{A.2})$$

$$\begin{aligned} i \bar{\psi} \{ \gamma_\mu \vec{\partial}_\nu \gamma_5 \} \psi \hat{F}_Z^{\mu\nu} &= m \bar{\psi} \{ i \sigma_{\mu\nu} \gamma_5 \} \psi \hat{F}_Z^{\mu\nu} \\ &= 2m \bar{\psi} \{ \vec{\partial}_\mu \gamma_5 \} \psi Z^\mu. \end{aligned} \quad (\text{A.3})$$

where the hats mean that only the abelian terms containing one gauge boson are to be included in the field strengths. For a massive fermion, these relations imply that any "direct" limit (i.e. a limit not involving the fermion propagating off-shell in a loop) on a ODM can be translated into a limit on the corresponding axial-polyp or axial-convective operator. For massless fermions the relation between ODMs and the concomitant axial convective vertices still holds. As we try to generalize the above relations off-shell, we shall find the various operators to be even more intimately related than the on-shell relations dictate, particularly the conventional and axial-convective dipoles, that involve the same number and types of fields.

In section 4 many stringent bounds on EDMs and CEDMs were derived via QCD loop effects. The question is the extent to which similar limits may apply to the axial convective vertices and to polyps, which are independent entities off-shell. As it turns out, and as we prove in this Appendix, for those cases involving only photons and gluons, several of the (C)EDM limits of Section 4 can be directly transmogrified into limits on the other CP-odd couplings, via ordinary or anomalous Ward identities requiring no extra computational effort. The same is not true of loop effects involving intermediate vector bosons and scalars, these would ask for additional calculations not all of which we have performed as of now. Consequently, we restrict ourselves in what follows to the unbroken $SU(3) \times U_\gamma(1)$ subgroup of the standard model.

We discuss first the relation between the axial convective and electric dipole moment operators. Let the $SU(3) \times U_\gamma(1)$ covariant derivative be:

$$D_\mu = \partial_\mu + i g_s A_\mu^a \frac{\lambda^a}{2} + i e Q A_\mu, \quad (\text{A.4})$$

where A_μ^a (A_μ) is the gluon (photon) field, eQ is the fermion charge.

Let $\mathcal{O}^{\gamma,g}$ be the EDM and CEDM operators:

$$\mathcal{O}^\gamma = \frac{ie}{2} \bar{\psi} \{ \sigma_{\mu\nu} \gamma_5 \} \psi F^{\mu\nu}, \quad (\text{A.5})$$

$$\mathcal{O}^g = \frac{ig_s}{2} \bar{\psi} \left\{ \sigma_{\mu\nu} \gamma_5 \frac{\lambda^a}{2} \right\} \psi G_a^{\mu\nu}. \quad (\text{A.6})$$

Introduce two auxiliary operators, the first of which is a gauge invariant ancestor of the convective axial current:

$$\tilde{\mathcal{O}} = -i\bar{\psi} \{D_\mu D^\mu + m^2\} \gamma_5 \psi, \quad (\text{A.7})$$

$$\mathcal{O} = i\bar{\psi} \{(\not{p} + im)\gamma_5(\not{p} + im)\} \psi. \quad (\text{A.8})$$

The above operators are not independent since the following rigorous identity holds:

$$\tilde{\mathcal{O}} - \mathcal{O} = \mathcal{O}^g + Q\mathcal{O}^\gamma. \quad (\text{A.9})$$

Consider the insertion of (A.9) in a Green's function. In regularized Feynman graphs, the use of field equations is allowed. In this situation the insertion of $\mathcal{O}$ vanishes identically (we are neglecting corrections involving scalars, W 's and Z 's). Thus:

$$\tilde{\mathcal{O}} = \mathcal{O}^g + Q\mathcal{O}^\gamma, \quad (\text{A.10})$$

to be understood as an equality between convergent Green's functions with the above operator insertions and with any number of outgoing gluons and photons.

The expression (A.10) can be exploited order by order in perturbation theory in g_s and e . We are interested in the $\mathcal{O}(g_s^3)$ and $\mathcal{O}(e g_s^3)$ results, relevant to the three-gluon and three-gluon-one-photon operators in (4.1) and (4.7).

To $\mathcal{O}(g_s^3)$, (A.10) implies that the sum of all regularized diagrams with a color axial convection vertex-insertion equals that of all regularized diagrams with a CEDM-insertion, the insertion that begat the three-gluon-operator result (4.3). Note that in most regularization schemes some of the diagrams with an $\tilde{\mathcal{O}}$ or $\mathcal{O}^g$ insertion are divergent (an exception is the background gauge scheme, with constant gluon field strengths). Regularize these results à la Pauli-Villars, with a fermionic regulator mass M . Once all diagrams are summed, the Pauli-Villars counterterm vanishes as $1/M$ when $M \rightarrow \infty$, as (4.3) indicates. In a model of CP violation that generates CEDMs and coloured axial convection operators (as independent entities off-shell) in the combination $\delta^g \mathcal{O}^g + \delta \tilde{\mathcal{O}}$, the effective Lagrangian once the quark degrees of freedom are integrated out is:

$$\mathcal{L} = g_s^3(\mu) \tilde{\mathcal{K}}_w(\mu) \mathcal{O}_w(\mu), \quad (\text{A.11})$$

$$\tilde{\mathcal{K}}_w(m) = \frac{1}{16\pi^2 m M_z} (\delta^g + \delta). \quad (\text{A.12})$$

The meaning of the above expressions is the following. A given model of CP violation may generate a CEDM and a coloured axial convection operator, that are not the same

object off-shell. Both should be inserted into the loops that enter into the computation of the effective three-gluon operator of (A.11). But given that their contributions are the same, as in (A.12), it would have sufficed to compute one or the other on-shell, where they are indistinguishable, and to insert the result in (4.3), as we did without excessive ado in Section 4.

To $\mathcal{O}(e g_s^3)$, (A.10) can be used to generalize the results on three-gluon-one-photon operators discussed in Section 4 to the insertion of axial-convective vertices. This time the loop integrations are even more convergent, and once again the regularization procedure yields anomaly-free results. In a model of CP violation generating EDM, CEDM and axial convection operators (as unrelated objects off-shell) in the combination $\delta^\gamma \mathcal{O}^\gamma + \delta \tilde{\mathcal{O}}$, the effective Lagrangian once the quark degrees of freedom are integrated out is:

$$\mathcal{L}_4 = e g_s^3(\mu) \left[\tilde{\mathcal{K}}_F(\mu) \mathcal{O}_F(\mu) + \tilde{\mathcal{K}}_O(\mu) \mathcal{O}_O(\mu) \right]. \quad (\text{A.13})$$

$$\tilde{\mathcal{K}}_O = \frac{1}{64\pi^2 m^3 M_z} (Q\delta^g + \delta^\gamma + 2Q\delta) \quad (\text{A.14})$$

$$\tilde{\mathcal{K}}_F = \frac{1}{64\pi^2 m^3 M_z} (2Q\delta^g + 2Q\delta). \quad (\text{A.15})$$

Consider once more a theory that generates the operators $\mathcal{O}^\gamma$, $\mathcal{O}^g$ and the axial convection $\tilde{\mathcal{O}}$ as independent off-shell effective interactions. They should all be inserted into the loops that mediate the computation of the effective three-gluon-one-photon operator of (A.11). The expressions (A.13) to (A.15) indicate that this lengthy procedure can be shortened by computing the EDM and CEDM on-shell and inserting the result in (4.10) and (4.11), the procedure followed in Section 4.

We still have to discuss the potential effect of the last $d=6$ CP-odd operators: the polyps of (2.23), restricted here to their $SU(3) \times U_\gamma(1)$ pieces, of well defined parity. Consider first the axial polyps

$$\mathcal{P}_-^\gamma = ie\bar{\psi} \left\{ \gamma_\mu \tilde{D}_\nu \gamma_5 \right\} \psi F^{\mu\nu}, \quad (\text{A.16})$$

$$\mathcal{P}_-^g = ig_s \bar{\psi} \left\{ \gamma_\mu \tilde{D}_\nu \frac{\lambda^a}{2} \gamma_5 \right\} \psi G_a^{\mu\nu}, \quad (\text{A.17})$$

which are P-odd. Here the derivative in the covariant derivative acts only on fermion fields. Introduce the auxiliary operator

$$\tilde{\mathcal{P}}_- = \bar{\psi} \left\{ \gamma_\mu [D_\mu, D_\nu] \gamma_5 D^\nu \psi - (D^\nu \bar{\psi}) \gamma_\mu [D_\mu, D_\nu] \gamma_5 \right\} \psi. \quad (\text{A.18})$$

The above constructs are related by the identity:

$$\tilde{\mathcal{P}}_- = \mathcal{P}_-^g + Q\mathcal{P}_-^{\gamma}. \quad (\text{A.19})$$

The consequences of this operator relation are fraught with the presence of an anomaly. Naive use of $SU(3) \times U_1(1)$ field equations would result in the equality $\mathcal{P}_-^g + Q\mathcal{P}_-^{\gamma} = \tilde{\mathcal{P}}_- = 2m(\mathcal{O}^g + Q\mathcal{O}^{\gamma})$. Application of this identity to the $O(g_s^3)$ reading of (A.19) (relevant to the calculation of the CP-odd three-gluon operator in (4.1)) would be incorrect. The Pauli-Villars counterterm regulating the insertion of the gluonic axial polyp $\mathcal{P}_-^g$ in the loops of fig. 9a,b is proportional to M times the counterterm that regulates the insertion of $\mathcal{O}^g$. Since the latter is proportional to $1/M$, as reflected in (4.3), the counterterm survives, as in the Steinberger-Adler-Bell-Jackiw anomaly. The correct relation, to all orders in e and g_s , contains an anomalous term:

$$\mathcal{P}_-^g + Q\mathcal{P}_-^{\gamma} = 2m(\mathcal{O}^g + Q\mathcal{O}^{\gamma}) - \frac{1}{8\pi^2}\mathcal{O}_w, \quad (\text{A.20})$$

where $\mathcal{O}_w$ is the effective three-gluon operator (4.2). In the three-gluon effective vertex, the naive and anomalous terms generated by a polyp operator exactly cancel. As a consequence, the bounds that follow from the insertion of the three-gluon effective vertex in a neutron cannot be exploited to constrain the gluonic axial-polyp operator.

Consider next the insertion of photonic and gluonic axial polyps in the $O(e g_s^3)$ calculation of the effective one-photon-three gluon vertices depicted in fig. 9c to fig. 9f. This time the overall counterterm vanishes as $O(M/M^4)$ and the anomaly is ineffectual. Yet, the relation (A.20) appears to be insufficient to extend the results of Section 4 to the insertion of axial polyps, since a general theory would generate the axial polyps in the arbitrary combination $\Delta^g \mathcal{P}_-^g + \Delta^{\gamma} \mathcal{P}_-^{\gamma}$ and not in the combination occurring in the left hand side of (A.20). But the limits of Section 4, based on mere estimates on neutron matrix elements, exploit the combination $K_F \mathcal{O}_F + K_G \mathcal{O}_G$, with $< \mathcal{O}_F >_n < \mathcal{O}_G >_n$. These operators have very similar anomalous dimensions, a circumstance that allowed us to obtain limits on the combination $K_F + K_G$. Lo and behold, Bose statistics and the fact that the relevant colour combinations are totally symmetric also imply that the polyp contribution to $K_F + K_G$ is proportional to the combination in the left hand side of (A.20). All in all, we obtain:

$$\tilde{\mathcal{K}}_F + \tilde{\mathcal{K}}_G = \frac{1}{64\pi^2 m^3 M_z} \left(3Q\delta^g + \delta^{\gamma} + 4Q\delta + \frac{2m}{M_z}(3Q\Delta^g + \Delta^{\gamma}) \right). \quad (\text{A.21})$$

This expression is all we need to extend the bounds on EDMs and CEDMs obtained via the operators $\mathcal{O}_F$ and $\mathcal{O}_G$ to axial polyps, since as it is apparent in (4.15), we have obtained in practice a bound on $K_F + K_G$.

To dispatch this Appendix, consider the vector polyps of (2.23):

$$\mathcal{P}_+^{\gamma} = ie\bar{\psi} \left\{ \gamma_{\mu} \vec{D}_{\nu} \right\} \psi F^{\mu\nu}, \quad (\text{A.22})$$

$$\mathcal{P}_+^g = ig_s \bar{\psi} \left\{ \gamma_{\mu} \vec{D}_{\nu} \frac{\lambda^a}{2} \right\} \psi G_a^{\mu\nu}, \quad (\text{A.23})$$

which, in the $SU(2) \times U_1(1)$ realm discussed here, are purely P-even. Introduce the auxiliary operator:

$$\tilde{\mathcal{P}}_+ = \bar{\psi} \{ \gamma_{\mu} [D_{\mu}, D_{\nu}] \psi - (D_{\nu} \bar{\psi}) \{ \gamma_{\mu} [D_{\mu}, D_{\nu}] \} \psi. \quad (\text{A.24})$$

The above operators are related by the identity:

$$\tilde{\mathcal{P}}_+ = \mathcal{P}_+^g + Q\mathcal{P}_+^{\gamma}. \quad (\text{A.25})$$

Whenever the use of $SU(2) \times U_1(1)$ field equations is justified the contribution of $\tilde{\mathcal{P}}_+$ to Green's functions is identically zero and:

$$\mathcal{P}_+^g + Q\mathcal{P}_+^{\gamma} = 0. \quad (\text{A.26})$$

In its $O(g_s^3)$ reading, (A.26) implies that $\mathcal{P}_+^g$ vanishes on-shell and contributes not to regularized diagrams with only quarks and gluons. Similarly, $\mathcal{P}_+^{\gamma}$ vanishes on-shell and does not contribute to regularized diagrams involving only fermions and photons. We conclude that vector polyps manifest themselves only in processes involving at least one gluon and one photon or, more generally, at least a Z_0 and a massless gauge boson. Should we have extended our considerations to polyps linear in $Z_{\mu\nu}$, and discussed their off-shell avatars, we would not have dared say so.

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Figure Captions

- Fig. 1: Diagrams generating ODMs in [a] left-right, [b] supersymmetric, [c] multi-Higgs and [d] unspecified models of CP violation.
- Fig. 2: The EDM of a ν , induces the process $Z^0 \rightarrow \bar{\nu}_\tau \nu_\tau \gamma$.
- Fig. 3: A CEDM ($\bullet$) generates a neutron EDM via the three-gluon operator (4.2).
- Fig. 4: [a] A quark EDM generates a neutron EDM via the operators (4.8) and (4.9); [b] Analogous contribution from a CEDM (also a $\bullet$).
- Fig. 5: An EDM is generated from a WEDM via Z and W exchanges; the heavy dot is the WEDM.
- Fig. 6: A quark's CEDM is generated by the operators (2.1) and (2.2) via photon, Z and W exchange.
- Fig. 7: A fermion's (W)EDM is generated by the full gauge invariant operator (2.9) describing the CP-odd couplings of gauge bosons: [a] W exchange contribution; [b,c] Z-Higgs and photon-Higgs exchange contributions.
- Fig. 8: Fermionic (C)EDMs induced by the polyp operators Σ^Z and Δ^Z defined in (2.22), (2.23) and (7.2).
- Fig. 9: Insertions of the axial polyp (2.23) generate, via a quark loop, the three-gluon operator (4.2) and the three-gluon one-photon, operators (4.8) and (4.9).

Fig. 1

Fig. 2

Fig. 3

Fig. 4b

Fig. 4 a

Fig. 5

Fig. 8 a

Fig. 8 b

Fig. 8 c

Fig. 8 d

Fig. 9

Fig. 6

Fig. 7 a

Fig. 7 b

Fig. 7 c