THE SUPER- $W_\infty(\lambda)$ ALGEBRA

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We present the super- $W_\infty(\lambda)$ algebra, an extension of the Virasoro algebra that contains operators of all spins $s \geq \frac{1}{2}$ and depends on an arbitrary parameter λ . It encompasses all previously known versions of W_∞ -type algebras as special cases. We discuss various properties and truncations of the algebra and present a realization in terms of the currents of a supersymmetric bc system.

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Recently extensions of the Virasoro algebra that contain generators of higher spin have been studied intensely from a variety of viewpoints [1-6]. One class of such algebras consists of the W_N algebras [1,2], which involve generators of spin $s \leq N$. Another class contains algebras with an infinite number of higher-spin generators, which will be generically denoted as W_∞ algebras [3,4]. These W_∞ algebras arise as $N \rightarrow \infty$ limits of W_N . Such limits were studied in [3] and [5]. Rather than taking explicit limits one can also directly study the structure of the W_∞ -type algebras. This was the approach followed in [4], where two particular W_∞ -type algebras were constructed. The contraction of these algebras leads to the so-called w_∞ algebra found in [3], which is related to the area-preserving diffeomorphisms of a cylinder.

Algebras containing an infinite number of higher-spin generators have also been studied in the context of higher-spin gauge theories in 3+1 dimensions [7]. Similar algebras have been used in [8] to construct higher-spin theories in 2+1 dimensions. Furthermore, higher-spin algebras occur in the description of relativistic (super)membranes where they are related to the residual symmetry in the light-cone gauge [9]. Although the algebras of [8,9] are not W_∞ algebras (they do not contain the Virasoro algebra as a subalgebra), they do occur as subalgebras of W_∞ algebras.

In this letter we present a one-parameter family $W_\infty(\lambda)$ of W_∞ algebras. It encompasses the W_∞ -type algebras of [4] as special cases. We extend this result to the supersymmetric case and define a one-parameter family of superalgebras, called super- $W_\infty(\lambda)$, extensions of the super-Virasoro algebra that involve operators of spin $s \geq \frac{1}{2}$. Each super- $W_\infty(\lambda)$ algebra contains $W_\infty(\lambda) \oplus W_\infty(\lambda + \frac{1}{2})$ as the bosonic subalgebra. The super- $W_\infty(0)$ algebra is related to the algebra constructed in [10].

We now proceed and give our main results for the super- $W_\infty(\lambda)$ algebras. They include results for the bosonic $W_\infty(\lambda)$ algebras as a subcase. We intend to further explore the properties of the super- $W_\infty(\lambda)$ algebras in a future communication [11]. Our starting point is the expression for the generators of the super-Virasoro algebra acting on superfields, functions of a commuting and an anticommuting coordinate, z and θ , with conformal weight λ . They read

$$\begin{aligned} L_n &= z^{-n+1} \partial - \lambda(n-1)z^{-n} - \frac{1}{2}(n-1)z^{-n}\theta \frac{\partial}{\partial \theta}, \\ G_r &= \left[z^{-r+\frac{1}{2}} \partial - \lambda(2r-1)z^{-r-\frac{1}{2}} \right] \theta + z^{-r+\frac{1}{2}} \frac{\partial}{\partial \theta}, \end{aligned} \quad (1)$$

where $\partial \equiv d/dz$. These generators satisfy the (anti)commutation relations

$$[L_m, L_n] = (m-n)L_{m+n}, \quad [L_m, G_r] = (\frac{1}{2}m-r)G_{m+r}, \quad \{G_r, G_s\} = 2L_{r+s}. \quad (2)$$

In the Neveu-Schwarz sector where the indices r, s take half-integer values there is a finite $osp(1,2)$ subalgebra generated by $L_0, L_\pm$ and $G_{\pm\frac{1}{2}}$.

Motivated by these expressions we then construct the complete set of operators that are expressible in terms of positive powers of derivatives ∂ , arbitrary powers of z and the fermionic coordinate and its derivative. A suitable basis is found by classifying these generators according to their maximal power $s-1$ of ∂ , and requiring that they transform

according to appropriate nondecomposable representations of $sl(2)$. Hence s takes positive integer values ($s \geq 1$), so that the bosonic operators carry spin s and the fermionic operators spin $s - \frac{1}{2}$. As there is a two-fold degeneracy for each spin (except for spin $\frac{1}{2}$) we decompose the operators into supermultiplets. The resulting expressions are

$$\begin{aligned}
L_n^{(s)+} &= \sum_{i=0}^{s-1} a_n^i(s, \lambda) z^{-n+i} \partial^i \\
&\quad + \theta \frac{\partial}{\partial \theta} \sum_{i=0}^{s-1} [a_n^i(s, \lambda + \tfrac{1}{2}) - a_n^i(s, \lambda)] z^{-n+i} \partial^i, \\
L_n^{(s)-} &= -\frac{s-1+2\lambda}{2s-1} \sum_{i=0}^{s-1} a_n^i(s, \lambda) z^{-n+i} \partial^i \\
&\quad + \theta \frac{\partial}{\partial \theta} \sum_{i=0}^{s-1} \left[\frac{s-2\lambda}{2s-1} a_n^i(s, \lambda + \tfrac{1}{2}) + \frac{s-1+2\lambda}{2s-1} a_n^i(s, \lambda) \right] z^{-n+i} \partial^i, \\
G_r^{(s)\pm} &= \theta \sum_{i=0}^{s-1} \alpha_r^i(s, \lambda) z^{-r+i-\frac{1}{2}} \partial^i \pm \frac{\partial}{\partial \theta} \sum_{i=0}^{s-2} \beta_r^i(s, \lambda) z^{-r+i+\frac{1}{2}} \partial^i,
\end{aligned} \tag{3}$$

where the coefficients are defined by

$$\begin{aligned}
a_n^i(s, \lambda) &= a^i(s, \lambda) (n-s+1)_{s-i-1}, \\
\alpha_r^i(s, \lambda) &= \alpha^i(s, \lambda) (r-s+\tfrac{3}{2})_{s-i-1}, \\
\beta_r^i(s, \lambda) &= \beta^i(s, \lambda) (r-s+\tfrac{3}{2})_{s-i-2},
\end{aligned} \tag{4}$$

with

$$\begin{aligned}
a^i(s, \lambda) &= \binom{s-1}{i} \frac{(-2\lambda-s+2)_{s-i-1}}{(s+i)_{s-i-1}}, \quad (0 \leq i \leq s-1) \\
\alpha^i(s, \lambda) &= \binom{s-1}{i} \frac{(-2\lambda-s+2)_{s-i-1}}{(s+i-1)_{s-i-1}}, \quad (0 \leq i \leq s-1) \\
\beta^i(s, \lambda) &= \binom{s-2}{i} \frac{(-2\lambda-s+2)_{s-i-2}}{(s+i)_{s-i-2}}, \quad (0 \leq i \leq s-2)
\end{aligned} \tag{5}$$

Here we made use of the definition

$$(a)_n \equiv \frac{\Gamma(a+n)}{\Gamma(a)} = a(a+1)(a+2) \cdots (a+n-1), \quad \text{with } (a)_0 = 1. \tag{6}$$

One easily verifies that $L_n^{(2)+}$ and $G_r^{(2)+}$ coincide with the super-Virasoro generators defined above (cf. (1)).

The $sl(2)$ transformations of the operators (3) are governed by their commutators with L_0 and $L_{\pm 1}$. The indices n and r denote the weight with respect to L_0 (i.e., $[L_0, X_n] = -n X_n$), while the remaining commutation relations are

$$\begin{aligned}
[L_1, L_n^{(s)\pm}] &= (-n+s-1) L_{n+1}^{(s)\pm}, & [L_1, G_r^{(s)\pm}] &= (-r+s-\tfrac{3}{2}) G_{r+1}^{(s)\pm}, \\
[L_{-1}, L_n^{(s)\pm}] &= (-n-s+1) L_{n-1}^{(s)\pm}, & [L_{-1}, G_r^{(s)\pm}] &= (-r-s+\tfrac{3}{2}) G_{r-1}^{(s)\pm}.
\end{aligned} \tag{7}$$

Furthermore the operators (3) were defined such that $(L_m^{(s)+}, G_r^{(s)+})$ and $(L_m^{(s)-}, G_r^{(s+1)-})$ correspond to spin $(s, s - \frac{1}{2})$ and $(s, s + \frac{1}{2})$ supermultiplets, respectively. This can be seen from

$$\begin{aligned} [G_{+\frac{1}{2}}, L_m^{(s)+}] &= -\frac{1}{2}(m - s + 1)G_{m+\frac{1}{2}}^{(s)+}, & [G_{+\frac{1}{2}}, L_m^{(s)-}] &= -G_{m+\frac{1}{2}}^{(s+1)-}, \\ \{G_{+\frac{1}{2}}, G_r^{(s)+}\} &= 2L_{r+\frac{1}{2}}^{(s)+}, & \{G_{+\frac{1}{2}}, G_r^{(s+1)-}\} &= (r - s + \frac{1}{2})L_{r+\frac{1}{2}}^{(s)-}. \end{aligned} \quad (8)$$

In the Neveu-Schwarz sector the operators thus transform according to nondecomposable representations of $osp(1, 2)$.

The differential operators given in (3) form an associative algebra provided we impose no further restrictions on the range of the various indices. On the basis of the commutator one thus finds an explicit representation of the super- $W_\infty(\lambda)$ algebra. Special cases of this infinite-dimensional Lie algebra have been considered in the literature. For instance, the bosonic algebra (generated by the θ -independent terms in $L^{(s)+}$) with $\lambda = \frac{1}{2}$ coincides with the $W_{1+\infty}$ algebra proposed in [4]. For $\lambda = 0$ the algebra is related to the W_∞ algebra of [4] after suppressing the spin-1 operator. This reduction is possible because the coefficients $a^0(s, \lambda)$ for all $s \neq 1$ are proportional to λ . Therefore the bosonic algebra with $\lambda = 0$ can be consistently restricted to generators of spin $s \geq 2$. For general λ such a decoupling of the spin-1 operator is not possible. A similar phenomenon takes place for the super- $W_\infty(0)$ algebra. Because both $a^0(s, \lambda)$ and $\alpha^0(s, \lambda)$ are proportional to λ for $s \neq 1$, the spin-1 operators $L_n^{(1)+}$ decouple from the algebra for $\lambda = 0$, and so do the spin- $\frac{1}{2}$ operators $G_r^{(1)}$. One is then left with the super- W_∞ algebra constructed in [10]. Furthermore, as mentioned in the introduction, the w_∞ algebra can be obtained by a contraction of $W_\infty(\lambda)$. Thus the super- $W_\infty(\lambda)$ algebra encompasses all known cases.

We should point out that super- $W_\infty(\lambda)$ and super- $W_\infty(\frac{1}{2} - \lambda)$ are isomorphic. This follows from the existence of an anti-automorphism which is closely related to hermitian conjugation for superspace differential operators. The anti-automorphism is implemented by changing the order of the various operators and by replacing

$$\partial \rightarrow -\partial, \quad \theta \rightarrow -i\theta, \quad \frac{\partial}{\partial\theta} \rightarrow i\frac{\partial}{\partial\theta}. \quad (9)$$

such that the basic (anti)commutators between z and ∂ and θ and $\partial/\partial\theta$ remain unchanged. Note that the value $\lambda = \frac{1}{4}$ is special in this respect. On the basis of the anti-automorphism one can show that many of the structure constants will vanish in that case. This will enable a consistent reduction of the algebra where one retains only the generators $L^{(s)+}$ with even s , $L^{(s)-}$ with odd s and either $G^{(s)+}$ or $G^{(s)-}$ with even s . The bosonic algebra with $\lambda = \frac{1}{4}$ corresponds to the symplectic higher-spin superalgebra [7,12] (more details can be found in [11]). In the purely bosonic case one finds that the anti-automorphism $\partial \rightarrow -\partial$ relates $W_\infty(\lambda)$ to $W_\infty(1 - \lambda)$.

All generators (3) are specified by their spin and L_0 eigenvalue. Let us now focus our attention to those generators that belong to a “wedge” in the parameter plane by restricting

the L_0 eigenvalues according to $|n| \leq s-1$ and $|r| \leq s - \frac{3}{2}$ (a super-wedge algebra can only be defined for the Neveu-Schwarz sector). In this way the generators of given spin are restricted to a finite-dimensional (irreducible) representation of $sl(2)$. It turns out that these generators constitute an infinite-dimensional subalgebra, related to a class of algebras denoted by $\mathcal{B}_\lambda$. Recently these algebras were analyzed from a different point of view [13,14]. It was found that they are inequivalent for different values of λ , whereas, for certain values of λ , they correspond to the algebra that is used for the construction of interacting higher-spin gauge theories or to the algebra of area-preserving diffeomorphisms of the two-dimensional hyperboloid. To elucidate the structure of the wedge subalgebra, we show that it is related to the algebra spanned by arbitrary functions of the generators $G_\alpha^{(2)\pm}$ (where $\alpha = \pm\frac{1}{2}$). These generators read

$$G_{\frac{1}{2}}^{(2)\pm} = \theta \partial \pm \frac{\partial}{\partial \theta}, \quad G_{-\frac{1}{2}}^{(2)\pm} = \theta(z \partial + 2\lambda) \pm z \frac{\partial}{\partial \theta}. \quad (10)$$

Denoting $G_\alpha^{(2)+}$ as before by G_α , we establish the following relations [14]

$$\begin{aligned} [G_\alpha, G_\beta] &= \frac{1}{2} \{1 - (1 - 4\lambda) K\} \varepsilon_{\alpha\beta}, \\ G_\alpha^{(2)-} &= G_\alpha K = -K G_\alpha, \end{aligned} \quad (11)$$

where K is the Klein operator defined by $K = 1 - 2\theta \partial / \partial \theta$, which satisfies $K^2 = 1$ and anticommutes with θ and $\partial / \partial \theta$. Following [14] we may thus conclude that functions of $G_\alpha^{(2)\pm}$ can always be decomposed into symmetrized polynomials of G_α multiplied by the Klein operator or the unit operator. From the $sl(2)$ assignments we may then conclude immediately that these functions coincide with (finite linear combinations of) the generators in the wedge.

On the other hand, there are four supersymmetry generators in the wedge algebra, corresponding to $G_\alpha^{(2)\pm}$, so that one expects an extended supersymmetry algebra. Indeed, one is dealing with $N = 2$ supersymmetry, as was already observed in [10] for the special case of $\lambda = 0$ and in [15] for the symplecton case of $\lambda = \frac{1}{4}$. The wedge algebra may thus be viewed as the factor of the enveloping algebra of $so(2, 2)$ over the ideal generated by the Casimir operator of $so(2, 2)$. The latter is proportional to $\lambda(\lambda - \frac{1}{2})$. In fact it turns out that one has to factor over more ideals, as we intend to discuss in [11].

One may view the super- $W_\infty(\lambda)$ algebra as an extension of the $\mathcal{B}_\lambda$ algebra. However, in that context the above results may seem somewhat surprising in view of the results of [4] where for the bosonic case it was shown that, for general λ , it is not possible to extend the wedge algebra without introducing generators of arbitrary *negative* spin; generators of negative spin can only be avoided for $\lambda = 0, \frac{1}{2}$. To appreciate how the $W_\infty(\lambda)$ algebra evades this obstacle, we evaluate the first few terms in the commutator of a spin-2 operator with a spin- s operator. To express the result, it is convenient to sum over all the modes and to define the corresponding differential operators in terms of a function $\Lambda^{(s)}(z)$,

$$L^{(s)}(z) = \sum_{i=0}^{s-1} a^i(s, \lambda) \left((-\partial)^{s-i-1} \Lambda^{(s)}(z) \right) \partial^i, \quad (12)$$

where $\Lambda^{(s)}(z) = \sum_n \Lambda_n z^{n+s-1}$. It turns out that the commutator of $L^{(2)}(z)$ and $L^{(s)}(z)$, characterized by functions $\Lambda^{(2)}(z)$ and $\Lambda^{(s)}(z)$, has the general form

$$[L^{(2)}(z), L^{(s)}(z)] = L^{(s)}(z) + L^{(s-2)}(z) + L^{(s-3)}(z) + \dots, \quad (13)$$

where the generators on the right-hand side are characterized by functions $\xi^{(s)}(z)$ which are expressed in terms of $\Lambda^{(2)}(z)$, $\Lambda^{(s)}(z)$ and derivatives thereof. The most conspicuous feature of this result is that the commutator of two even spins gives rise to both odd and even spins. This is in contradistinction with the W_∞ -type algebras that have been constructed so far, as well as with the wedge subalgebra. Indeed, the W_∞ -type algebras considered in [4] have the structure

$$[L^{(s)}, L^{(s')}] = L^{(s+s'-2)} + L^{(s+s'-4)} + L^{(s+s'-6)} + \dots + (\text{central terms}) \delta^{ss'}. \quad (14)$$

For $s' = 2$ this general form implies that there is no $L^{(s-3)}$ generator at the right hand side of the commutator between a spin-2 and a spin- s operator. On the other hand an explicit calculation reveals that the function $\xi^{(s-3)}(z)$ characterizing the $L^{(s-3)}$ generator in (13) is given by ($s \geq 4$)

$$\xi^{(s-3)}(z) = -\frac{1}{6}(2\lambda - 1)\lambda(\lambda - 1) \left[2(\partial\Lambda^{(s)}) (\partial^3\Lambda^{(2)}) + (s-1)\Lambda^{(s)} (\partial^4\Lambda^{(2)}) \right]. \quad (15)$$

We see that $\xi^{(s-3)}(z) = 0$ only for $\lambda = 0, 1$ and $\frac{1}{2}$. Therefore, only the $W_\infty(0) \sim W_\infty(1)$ and $W_\infty(\frac{1}{2})$ algebras exhibit the structure shown in (14). This thus explains the apparent discrepancy with the results of [4]. We note that the possibility for extending the $\mathcal{B}_\lambda$ algebras for all values of λ without introducing negative-spin generators was already considered in [6].

We now present a field-theoretic representation of the super- $W_\infty(\lambda)$ algebra in terms of an operator product expansion for quasi-primary field operators*. This requires the introduction of two conformal superfields, a commuting field $B = \beta + \theta b$ and an anticommuting field $C = c + \theta \gamma$, with conformal weights λ and $\frac{1}{2} - \lambda$, respectively. Since θ has weight $-\frac{1}{2}$, we find that b, c, β and γ have conformal weights $\lambda + \frac{1}{2}$, $-\lambda + \frac{1}{2}$, λ and $-\lambda + 1$. The supersymmetric action equals [18]

$$S = \frac{1}{\pi} \int d^2z d^2\theta B \bar{D}C = \frac{1}{\pi} \int d^2z \{ \beta \bar{\partial}\gamma + b \bar{\partial}c \}. \quad (16)$$

The generators of the super- $W_\infty(\lambda)$ algebra are then related to the following conserved currents,

$$V^{(s)+}(z) = \sum_{i=0}^{s-1} a^i(s, \lambda) \partial^{s-i-1} \{ (\partial^i \beta) \gamma \} + \sum_{i=0}^{s-1} a^i(s, \lambda + \frac{1}{2}) \partial^{s-i-1} \{ (\partial^i b) c \},$$

* Free-field realisations of special W_∞ -type algebras have been given in [16,17,10].

$$\begin{aligned}
V^{(s)-}(z) &= -\frac{s-1+2\lambda}{2s-1} \sum_{i=0}^{s-1} a^i(s, \lambda) \partial^{s-i-1} \{(\partial^i \beta) \gamma\} \\
&\quad + \frac{s-2\lambda}{2s-1} \sum_{i=0}^{s-1} a^i(s, \lambda + \frac{1}{2}) \partial^{s-i-1} \{(\partial^i b) c\}, \\
Q^{(s)\pm}(z) &= \sum_{i=0}^{s-1} \alpha^i(s, \lambda) \partial^{s-i-1} \{(\partial^i \beta) c\} \pm \sum_{i=0}^{s-2} \beta^i(s, \lambda) \partial^{s-i-2} \{(\partial^i b) \gamma\}.
\end{aligned} \tag{17}$$

The correspondence with the previous representation of the algebra is established by evaluating the operator products of the currents (17) with the conformal fields and comparing the relevant term to the superspace differential operators (3).

The currents $(Q^{(2)+}, V^{(2)+})$ reproduce the $N = 1$ super-Virasoro algebra (cf. (1)). It is well known that the supersymmetric bc system actually has a $N = 2$ superconformal invariance [18]. Indeed the currents $(V^{(1)-}, Q^{(2)\pm}, V^{(2)+})$ define an $N = 2$ super-Virasoro algebra. As explained previously when discussing the wedge algebra there exists a finite $osp(2, 2)$ subalgebra in the Neveu-Schwarz sector corresponding to the generators $(L_0^{(1)-}, G_{\pm 1/2}^{(2)\pm}, L_0^{(2)+}, L_{\pm}^{(2)+})$. All currents fit into $N = 2$ supermultiplets with respect to this $osp(2, 2)$ subalgebra. The resulting combinations are

$$\begin{aligned}
&(V^{(s)-}, Q^{(s+1)\pm}, V^{(s+1)+}) \quad s = 1, 2, 3, \dots \\
&(Q^{(1)}, V^{(1)+}),
\end{aligned} \tag{18}$$

where $(Q^{(1)}, V^{(1)+})$ constitutes a so-called $N = 2$ scalar multiplet.

One can show that, in addition to the truncation noted below (9), a truncated version of the operator algebra must also exist for $\lambda = 0, \frac{1}{2}$. For instance, for $\lambda = 0$ the conformal weights of b and c are equal to $\frac{1}{2}$ while the conformal weights of β and γ are equal to 0 and 1, respectively. In the operator expansion one can then replace γ by $\partial\phi$, where ϕ has conformal weight 0. The truncation can now be implemented by identifying b with c and β with ϕ . This identification has the effect that the currents (17) will no longer be independent: $Q^{(s)+} = 0$ for s odd, $Q^{(s)-} = 0$ for s even, and $Q^{(1)\pm} = V^{(1)\pm} = 0$, while all operators $V^{(s)\pm}$ with odd s can be expressed as derivatives of operators $V^{(s)\pm}$ with s even. The resulting algebra then coincides with the super- $W_{\infty/2}$ algebra of [10]. It is generated by $V^{(s)\pm}, Q^{(s)+}$, with $s = 2, 4, \dots$ and $Q^{(s)-}$ with $s = 3, 5, \dots$. The bosonic subalgebra of the super- $W_{\infty/2}$ algebra is given by $W_{\infty/2} \oplus W_{(1+\infty)/2}$ where $W_{\infty/2}$ and $W_{(1+\infty)/2}$ are the truncated versions of W_{∞} and $W_{1+\infty}$ discussed in [4].

The realization given above can be used to calculate an expression for the central extension of the algebra. To keep matters simple let us confine ourselves to the bosonic case. The central charge $c(s, s'; \lambda)$ is defined by the coefficient of the leading term in the operator product expansion of two currents,

$$V^{(s)}(z) V^{(s')}(w) \sim \frac{c(s, s'; \lambda)}{2(z-w)^{s+s'}} + \dots \tag{19}$$

It satisfies the condition

$$c(s, s'; \lambda) = (-)^{s+s'} c(s', s; \lambda) = c(s', s; 1 - \lambda). \quad (20)$$

It is easy to check that in the spin-2 sector one obtains the standard result $c(2, 2; \lambda) = 2(6\lambda^2 - 6\lambda + 1)$. We find that, in general, $c(s, s'; \lambda) \neq 0$ for $s \neq s'$. For instance,

$$c(s+1, s; \lambda) = 2 \frac{s!(s-1)!}{(2s-2)!} (2\lambda)_{s-1} (1-2\lambda)_s. \quad (s \geq 1) \quad (21)$$

This shows once again that, for general λ , the $W_\infty(\lambda)$ algebras do not have the structure indicated in (14).

We should finally discuss the relationship between two super- $W_\infty(\lambda)$ algebras with different values of λ . Again let us confine ourselves to the purely bosonic case. One can regard the generators $L_n^{(s)}(\lambda)$ as λ -dependent linear combinations of the basis elements $z^m \partial^n$ (cf. (3)), so that all algebras $W_\infty(\lambda)$ are isomorphic (of course, this observation does not apply to truncations or reductions of the algebra which presuppose a fixed value of λ). Note, however, that this is not quite the point of view that one adopts for extensions of the Virasoro algebra with $s \leq 2$. In that case the classical algebra remains independent of λ (the structure constants are λ independent), while the central extension depends on λ . However, representations with different central charges are regarded as inequivalent. In the case at hand the situation is even more complicated because one is dealing with an algebra that involves an infinite range of spin values. Although for fixed s one can express the generators $L_n^{(s)}(\lambda)$ as linear combinations of a finite set of generators $L_n^{(s')}(\lambda')$, this is not so for the dual space which can be identified with the space of transformation parameters. To see this one may compare the same Lie-algebra valued expression in terms of two bases corresponding to different values of λ . For the transformation parameters a change of λ leads to redefinitions that always involve sums over infinite numbers of terms. It is well known that such infinite summations tend to make these redefinitions meaningless. Therefore we prefer to interpret algebras super- $W_\infty(\lambda)$ with different λ as inequivalent. We intend to return to this topic in [11].

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References

- [1] A.B. Zamolodchikov, Teo. Mat. Fiz. **65** (1985) 347.
V.A. Fateev and S. Lykhanov, Int. J. Mod. Phys. **A3** (1988) 507.
- [2] F. Bais, P. Bouwknegt, M. Surridge and K. Schoutens, Nucl. Phys. **B304** (1988) 348,371.
A. Bilal and J.-L. Gervais, Nucl. Phys. **B318** (1989) 579.
C.M. Hull, Phys. Lett. **B240** (1989) 110.
K. Schoutens, A. Sevrin and P. van Nieuwenhuizen, Phys. Lett. **B243** (1990) 245.

- E. Bergshoeff, C.N. Pope, L.J. Romans, E. Sezgin, X. Shen and K.S. Stelle, Phys. Lett. **243B** (1990) 350.
- [3] I. Bakas, Phys. Lett. **B228** (1989) 57; "The structure of the W_∞ algebra", Maryland preprint, PP-90-085 (1989).
 - [4] C.N. Pope, L.J. Romans and X. Shen, Phys. Lett. **B236** (1990) 173; Phys. Lett. **B242** (1990) 401; Phys. Lett. **B245** (1990) 72; Nucl. Phys. **B339** (1990) 191; "A brief history of W_∞ ", Texas A&M preprint, CTP TAMU-89/90 (October 1990).
 - [5] A. Bilal, Phys. Lett. **227B** (1989) 406.
 - [6] E.S. Fradkin and V.Ya. Linetsky, Mod. Phys. Lett. **A4** (1989) 2649; "Infinite-dimensional Generalizations of Finite-dimensional Symmetries", ICTP preprint, IC/90/128 (June 1990).
 - [7] E.S. Fradkin and M.A. Vasiliev, Ann. of Phys. (NY) **177**, 63 (1987).
M.A. Vasiliev, Fortschr. Phys. **36**, 33 (1988).
 - [8] M.P. Blencowe, Class. Quantum Grav. **6**, 443 (1989).
 - [9] J. Hoppe, M.I.T. PhD thesis, 1982; Aachen preprint PITHHA 86/24; proceedings *Int. Workshop on Constraints Theory and Relativistic Dynamics*, eds. G. Longhi and L. Lusanna (Singapore, World Scient. 1987).
B. de Wit, J. Hoppe and H. Nicolai, Nucl. Phys. **B305** [FS23], (1988) 545
 - [10] E. Bergshoeff, C.N. Pope, L.J. Romans, E. Sezgin and X. Shen, Phys. Lett. **245B** (1990) 447.
 - [11] E. Bergshoeff, B. de Wit and M. Vasiliev, work in progress.
 - [12] L.C. Biedenharn and J.D. Louck, Ann. Phys. (NY) **63** (1971) 459.
 - [13] E. Bergshoeff, M.P. Blencowe and K.S. Stelle, Commun. Math. Phys. **128** (1990) 213.
M. Bordemann, J. Hoppe and P. Schaller, Phys. Lett. **B232** (1989) 199.
 - [14] M.A. Vasiliev, Pis'ma Eksp. Teor. Fiz. **50** (1989) 344; Lebedev preprint 214 (1989), to appear in Int. J. Mod. Phys. A.
 - [15] E.S. Fradkin and M.A. Vasiliev, Int. J. Mod. Phys. **A3** (1988) 2983.
 - [16] I. Bakas and E. Kiritsis, Nucl. Phys. **B343** (1990) 185.
 - [17] E. Witten, Comm. Math. Phys. **113** (1988) 529.
 - [18] D. Friedan, E. Martinec and S. Shenker, Nucl. Phys. **B271** (1986) 93.