CRUCIAL EXPERIMENTS AT 200 TeV *

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ABSTRACT

We report on the recent development in high-energy electroweak interactions concerning baryon and lepton number violating processes in association with multi-particle production of W, Z, and Higgs bosons. The present theoretical status is reviewed and the phenomenological consequences at future colliders are discussed.

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1. Introduction

Recently there has been growing interest in non-perturbative effects in the standard electroweak theory. It has been suggested that electroweak interactions become strong at high energies, showing new phenomena like high-multiplicity events involving "weakly" interacting particles and baryon (B) and lepton (L) number violation [1, 3-5]. The purpose of this lecture is to present arguments in favour of this possibility and to discuss the implications for future collider experiments.

![Diagram of (B+L)-violating amplitude](image)

Fig. 1: (B+L)-violating amplitude in the standard model induced by an instanton.

G. 't Hooft was the first to point out B and L are not strictly conserved in the electroweak theory [4]. He showed that topologically non-trivial gauge fields (instantons) induce (B+L)-violating vertices (see Fig. 1). These vertices are proportional to \( \exp(-S_{\text{inst}}) = \exp(-2\pi/\alpha_\text{w}) \approx 10^{-78} \), where \( S_{\text{inst}} \) is the instanton action, and it was commonly believed, due to the smallness of this number, that the (B+L)-violating processes would be unobservable. However, it has been argued recently that such events might occur frequently in high-energy collisions [5, 1-3].

Intuitively one can understand this behaviour through the following reasoning: the factor \( \exp(-S_{\text{inst}}) \) is associated with a tunneling process [6, 7], which occurs between topologically inequivalent vacua [6] in the electroweak theory, characterized, in the temporal gauge, by the winding numbers \( n \) of the vacuum SU(2) gauge fields (see Fig. 2). The transitions between inequivalent vacua have drastic effects due to an anomaly [8] in the B+L current. Whenever the fields make a transition between vacua with different winding numbers, \( \Delta n \neq 0 \), there occur (B+L)-violating processes according to the selection rule

\[
\Delta B = \Delta L = -n \Delta n,
\]

where \( n \) is the number of generations.

![Figure 2: Schematic plot of the vacuum structure in a SU(2) gauge theory](image)

Fig. 2: Schematic plot of the vacuum structure in a SU(2) gauge theory.

However, the transitions with \( \Delta n \neq 0 \) cost energy: the topologically inequivalent vacua are separated by an energy barrier [6]. In the electroweak theory the minimum barrier height between the different vacua is of order \( M_p \approx \pi m_w/\alpha_\text{w} \approx 10 \text{ TeV} \) [9]. The subscript "sp" stands for the sphalerons, which is a static saddle-point solution of the bosonic sector of the electroweak theory and corresponds to the configuration which sits at the top of the barrier between topologically inequivalent vacua [9]. Its energy sets the scale above which the nontrivial structure of the vacuum could possibly play a role and where the anomalous (B+L)-violating processes could occur frequently. Below this energy the processes are assumed to be associated with tunneling and are therefore exponentially suppressed, \( \sim \exp(-2\pi/\alpha_\text{w}) \), with the instanton describing the tunneling event [6, 7]. At energies above \( \sim 10 \text{ TeV} \) one may expect from these simple energy considerations that the large suppression disappears because then transitions are possible on the classical level. It should be noted that the usual perturbation theory corresponds to the expansion around one of the vacua; it ignores \( \Delta n \neq 0 \) transitions. This is appropriate at low energies, but could be misleading at high energies.

There is by now a consensus that (B+L)-violating effects are important in a hot plasma, in particular in the early universe [10]. Hints of the possible importance of (B+L)-violating processes in high-energy, two-particle collisions have been observed in [1] (and...
later also in ref. [2]; for an earlier attempt see ref. [6]). It was found that the relevant 
(B+L)-violating processes are those with the associated production of many, O(1/αw), W, 
Z, and Higgs bosons; this has been conjectured earlier in refs. [9, 11].

The organization of this lecture is as follows. In sect. 2 we review the naive instanton 
calculation of (B+L)-violating amplitudes in the electroweak theory. It is shown that the 
amplitudes, in the leading-order semiclassical approximation, are point-like in the sense 
that there are no form factors suppressing momentum transfers between external legs. 
Due to this feature the leading-order matrix elements will violate partial-wave unitarity 
at high energies (∼ M_{WZ} for associated production of n_w ∼ π/α_w W bosons). We discuss 
the higher-order corrections to the naive instanton result in sect. 3. With the presently 
available techniques we can only investigate the total cross-section for B+L violation at 
energies ≪ M_{WZ}, where it is exponentially growing, but still exponentially small. In sect. 
4 we represent speculations about the behaviour of the total cross section near and above 
the sphaleron scale. If the 't Hooft suppression factor can be completely overcome at the 
sphaleron scale, one expects weak interactions above this scale to resemble closely strong 
interactions, with very interesting phenomenology. Section 5 is devoted to the discussion 
of this phenomenology. In sect. 6 we present our conclusions.

2. The Naive Instanton Calculation

We will be interested in the total (B+L)-violating cross-section. To this end we consider 
first the exclusive processes (see Fig. 3)

\[ q_1 + q_1 \rightarrow \bar{t}_1 + 3 t_3 + 3 \bar{t}_3 + \bar{t}_1 + \bar{t}_3 + n_w W(Z) + n_h H, \]  

(2.1)

with an arbitrary number of W, Z, and Higgs bosons. The subscripts label the different 
generations (n_f = 3). According to the anomaly selection rule (1.1) these are, in fermion 
content, the simplest anomalous processes (Δn = 1). We are considering processes which 
involves also a number of bosons because the sphaleron picture suggests that the relevant 
processes are those with the associated production of many, O(1/α_w), W, Z, and Higgs 
bosons in addition to the anti-quarks and anti-leptons. The argument goes as follows [9,11]: 
in order to pass over the barrier the fields have to configure themselves into “physical” 
sphaleron-like configurations with masses of order m_w/α_w and radii of order 1/m_w. The 
sphaleron configuration then typically decays into 1/α_w W(Z) and Higgs bosons, producing 
the anti-quarks and anti-leptons as a side effect due to the anomaly.

![Fig. 3: The fermion-number violating vertex (2.1).](image)

In order to compute the S-matrix elements for the exclusive processes (2.1) we use 
the LSZ approach: calculate the appropriate (connected) Green function, amputate it and 
and obtain the on-shell vertex of the processes. In the following we will neglect fermion 
masses and set sin^2 θ_w = 0. The Green function associated with the process (2.1) is

\[ G^{\text{ext}}(\xi_1, \xi_2, \xi_3) = \{ T \left[ \prod_{i=1}^{12} \overline{\psi}_L(\xi_i) \right] W_{\psi}(\xi_i) \prod_{k=1}^{n_h} \psi(\xi_k) \} \}^{\text{ext}}. \]  

(2.2)

The superscripts on the fermion fields label different doublets. The field ϕ(z) appearing 
in eq. (2.2) denotes the interpolating field corresponding to the neutral Higgs boson, H.

Since the Green function involves only φ's it vanishes, in every finite order of conven 
tional perturbation theory in α_w (expansion around the topologically trivial vacuum 
solution). It can be shown [4,7] that the Euclidean path integral expression corresponding 
to (2.2) receives contributions only from the sector of gauge fields which have one unit of 
Euclidean topological (Pontryagin) number

\[ q = \frac{g^2}{16\pi^2} \int d^4 x \text{tr}(\tilde{F}^2) = 1, \]  

(2.3)

where \( \tilde{F} \) denotes the dual SU(2) field strength. This is plausible since Euclidean gauge 
fields with Pontryagin number q correspond to Minkowskian gauge fields which interpolate 
between topologically inequivalent vacua with Δn = q [6,7]. In the presence of q = 1
gauge fields there exists an asymmetry between the number of normalizable right-handed and left-handed zero modes of the Euclidean Dirac operator [12], which is necessary for a non-vanishing fermion-number violating Green function [4, 7].

The path integral over the gauge and Higgs fields can be done in a semiclassical way by expanding around classical gauge and Higgs field configurations which dominate the path integral. It has been shown by 't Hooft [4] and Affleck [13] that the dominant classical gauge fields with \( q = 1 \) can be represented by a family of configurations, labeled by the position \( R, \rho \), a scale size \( \rho \), and a global SU(2)-matrix \( U \), which represents the orientation. The configurations have the following asymptotic form (for \( \rho m \ll 1 \)),

\[
W^\rho_{\alpha}(x - R; \rho, U) = \sum_{\alpha} \frac{U_{\alpha\mu} U_{\gamma\nu}^*(x - R)}{\sqrt{2} (x - R)^2 + \rho^2} \delta_{\alpha \gamma} G_{\mu\nu}(z - R), \quad \text{for } |x - R| < m_x^{-1}, \\
\Phi^\alpha(x - R; \rho) = \frac{v}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix} \begin{pmatrix} (x - R)^2 + \rho^2 \\ 1 - 2\pi^2 \rho^2 G_{\mu\nu}(x - R) \end{pmatrix}^{1/2}, \quad \text{for } |x - R| > m_x^{-1}.
\]

Here \( \Phi_{\mu\nu} = \Phi_{\mu}^\gamma \Phi_{\gamma\nu}^* \) (the \( \Phi \)'s are 't Hooft's symbols [4]), \( v = 246 \text{ GeV} \) is the Higgs field vacuum expectation value, and \( G_{\mu\nu}(x) = 1/(4\pi^2)(m_r/m_r)K_1(m_r \rho) \). This configuration has been called a constrained instanton [13], since it was obtained by introducing a scale constraint into the path integral. Near the center of the configuration the classical gauge field looks like the pure SU(2) instanton [14]. Far away from the center it decays exponentially to a pure gauge.

We obtain, in the leading-order approximation around the constrained instanton, the following form for the Fourier transform (denoted by tildes) of the Euclidean Green function corresponding to (2.2) [1, 2] (see Fig. 4)

\[
G_{\mu\nu}^\rho(p_i, k_j; q_k) = (2\pi)^d \delta^{d}(\sum_{i=1}^{12} p_i + \sum_{j=1}^{n_1} k_j + \sum_{k=1}^{n_2} q_k) \\
\times \int dU \int_{0}^{\infty} dp \rho \prod_{i=1}^{12} S^\rho_{ij}(p_i; \rho, U) \prod_{j=1}^{n_1} S^\rho_{j}(k_j; \rho, U) \prod_{k=1}^{n_2} S^\rho_{k}(q_k; \rho).
\]

Here the "instanton density" \( n(\rho) \) is proportional to (in lowest order in \( \rho(\rho) \))

\[
n(\rho) \propto \exp\left(-S_E(W^\rho, \Phi^\rho)\right) \sim \exp\left(-\frac{2\pi}{\alpha_\rho} - \pi^2 v^2 \rho^2\right),
\]

Fig. 4: The semiclassical expansion of the (B+L)-violating Green function in the single-instanton sector. The first term on the right hand side is the leading-order term (2.6). The second term is an \( \mathcal{O}(\alpha_\rho) \) correction: two external lines are connected by a propagator in the instanton background.

It is easily seen that eq. (2.6) has poles only near the mass shells of the individual particles which, in turn, depend solely on the large distance behaviour of the classical fields (see also ref. [15]): the infrared behavior of the classical fields determines the behaviour of the leading-order on-shell scattering amplitudes. Near the mass-shell the Fourier transforms of the classical fields look like free propagators with non-trivial residue.

Observe that in the leading-order approximation there is no correlation (besides the collective coordinate integrals) between different external legs in the Green function (2.6). The Green function has form factors in the individual virtualities \( p_i^2, k_j^2 \), but not in the pair invariants like \( q_i \cdot q_j \). An immediate consequence of this feature is that off-shell, deeply in the Euclidean region \( (p_i^2, k_j^2, q_k^2 \to \infty) \), the Green function (2.6) scales, due to the finite size of the constrained instanton, with the naive scaling dimension. This behaviour is consistent with renormalizability and asymptotic freedom. On-shell, however, there are no form factors remaining, even if one does not neglect the finite size of the instanton as 't Hooft did [4].

After amputation of the Green function (2.6), i.e. after multiplication by the inverse
free propagators for the external particles, we obtain the on-shell vertex function [1, 2]
\[ \tilde{\Gamma}_{\text{on-shell}} = (2\pi)^4 \delta^{(4)} \left( \sum_{i=1}^{12} p_i + \sum_{j=1}^{n_\omega} k_j + \sum_{k=1}^{n_a} q_k \right) \int \frac{dU}{U} \left( \frac{U(UU^\dagger)^{1/2}}{n_{\omega} n_{a}} \right) \left( -2\pi^2 \rho^2 \right)^{n_{a}}. \] (2.8)

Here \( \xi \) is a constant matrix acting in spin and weak isospin space. For the momenta \( k^\sigma \) and the analytical continuation \( k^0 = i \sqrt{k^2 + m_k^2} = \omega_k \) is implied. The \( \rho \)-integral leads to a factorial growth, \( \sim (n_{\omega} + n_{a})! \), of the amplitudes. It should be noted however that this is strictly true only as long as the dominating \( \rho \) in (2.8) is smaller than \( m^{-1} \). Otherwise the large distance normalizations of the classical fields (2.4) and (2.5) as well as the classical action in (2.7) will change. Since the dominating \( \rho \) according to (2.8) is given by \( \rho_0 \approx \sqrt{n_{\omega} / n_{a}} \), our results are only valid for \( n_{\omega} \ll n_{a} \).

The amplitude (2.8) is very similar to a point-like interaction: only the gauge fields lead to some momentum dependence. On-shell there is no form factor suppressing momentum transfers. In the context of the semiclassical expansion around non-trivial classical background fields there is no way around this paradox. Due to the point-like nature the amplitudes will violate partial-wave unitarity at high energies. The produced particles are concentrated in the lowest partial waves, i.e. they will come out quasi-isotropically from the vertex.

In order to obtain the S-matrix element for the process (2.1) we have to multiply the amplitude (2.8) by the wave functions for the external particles*.

* In ref. [1] the cross-sections for the production of vector particles were underestimated. This was due to the fact that we were not careful enough with the analytic continuation. Since the wave functions of vector particles carry a space-time index they transform non-trivially under the Wick rotation: the zero component of the wave function acquires the factor i. The Euclidean counterparts of the polarization vectors read therefore [10] \( \epsilon^a_\mu(\vec{k}) = (0, \vec{e}^a_{\mu}(\vec{k})) \), \( \epsilon^{*a}_\mu(\vec{k}) = \frac{1}{m^a} (i \mid \vec{k} \mid \vec{e}^{*a}_\mu(\vec{k})) \). In the squared amplitudes one needs the following quantity [10] \( \sum_{a} \epsilon^a_\mu(\vec{k})^* \epsilon^a_{\nu}(\vec{k}) = \delta_{\mu\nu} \omega_k^2 - 2k_{\mu} k_{\nu} + 2\epsilon_{\rho\sigma} k_{\rho} k_{\sigma} \). This formula differs from the corresponding one in ref. [1], eq. (83), in the three-momentum dependence. The origin lies in the factor of i, as explained above.

averaging over the instanton orientation, which unnecessarily complicates the analysis*, we obtain for the squared amplitudes [1, 2, 18]
\[ S \left| \mathcal{M}(qq \rightarrow \bar{q}q, \mathcal{W}, \mathcal{H}) \right|^2 \approx \]
\[ \left( \frac{\Lambda_W}{v} \right)^{24} \left( \frac{2n_{\omega} + n_{a}}{2n_{\omega} - n_{a}} \right)^{2n_{a}} \left( \frac{n_{\omega} + n_{a}}{n_{\omega} n_{a}} \right)^{12} \frac{1}{\left( \frac{\delta}{16\pi^2 v^2} \right)^{n_{a}} \left( \frac{3\pi^2}{8\pi^2 v^2} \right)^{n_{a}}} \] (2.9)

Here \( \Lambda_W \) is the energy scale where the SU(2) gauge coupling would become strong if there were no spontaneous symmetry breaking,
\[ \Lambda_W = \text{const} \cdot v \exp \left( -\frac{2\pi}{b_0} \right) \approx 10^{-22} \text{ GeV}, \] (2.10)
and \( b \) is the coefficient of the \( \beta \)-function, \( b = \frac{11}{3} - \frac{1}{3} n_{\omega} = \frac{11}{3} \).

Using eq. (2.9) we obtain [1 - 3, 18] for the (parton) cross-sections of the exclusive (B+L)-violating processes (2.1) (here we use the formula for relativistic phase space)
\[ \delta_{n_{\omega}, n_{a}} \approx \frac{1}{v^2} \left( \frac{\Lambda_W}{v} \right)^{24} \frac{1}{n_{\omega}! (n_{a}!)^3} \left( \frac{\delta}{16\pi^2 v^2} \right)^{13} \frac{\delta}{4\pi^2 v^2} \left( \frac{3\pi^2}{8\pi^2 v^2} \right)^{n_{a}}. \] (2.11)

From (2.11) we find that for (parton) cm energies \( \sqrt{s} \) larger a few times \( m_{\omega} \) the total (B+L)-violating cross-section rises exponentially
\[ \delta_{\Delta(B+L)} = \sum_{n_{\omega}, n_{a}} \delta_{n_{\omega}, n_{a}} \approx \frac{1}{v^2} \left( \frac{\delta}{16\pi^2 v^2} \right)^{13} \exp \left( -\frac{4\pi}{\alpha_{\omega}} F \left( \frac{\sqrt{s}}{M_{\omega}} \right) \right), \] (2.12)
where the function \( F \) is given by [15, 18]
\[ F(x) = 1 - \left( \frac{9}{16\pi^2} \right)^{3/4} x^{3/2}. \] (2.13)

Here we have defined **
\[ M_{\omega} \equiv \frac{m_{\omega}}{\alpha_{\omega}} = 7.2 \text{ TeV}. \] (2.14)

** In refs. [16, 17] it was found that averaging over the orientation will not affect the final formula for the inclusive (B+L)-violating cross-section, (2.12)-(2.14).

** The mass (2.14) is of the order but not identical to the sphaleron mass [9], which depends slightly on the Higgs mass and varies between 7 and 14 TeV if one varies the Higgs mass between 0 and \( \infty \).
The first term in eq. (2.13) gives rise to the familiar 't Hooft suppression factor, \( \sim \exp(-4\pi/\alpha_w) \) [4]. The second term in eq. (2.13), which leads to an exponential growth of the cross-section, comes from W and Z production: for energies below the sphaleron, (B+L)-violating processes are dominated by the associated production of W's and Z's [16–18]. The average multiplicity increases according to a power law,

\[
\langle n_w \rangle \approx \frac{4\pi}{3\alpha_w} \left( \frac{9}{16\sqrt{8}} \right)^{2/3} \left( \frac{\sqrt{3}}{M_{\text{eq}}} \right)^{4/3}. \tag{2.15}
\]

Using eqs. (2.12)-(2.14) we find that the cross-section blows up exponentially, thereby violating S-wave unitarity, at the scale \( \sqrt{3} \approx M_{\text{eq}} \),

\[
M_0 = 2^{-1/4} \frac{8}{3} M_{\text{eq}} \approx 16 \text{ TeV}, \tag{2.16}
\]

where the mean multiplicity is \( \langle n_w \rangle \approx (4\pi/3)\alpha_w^{-1} \approx 120^* \). The results seem to fit nicely onto the suggestions from the sphaleron picture and related energy considerations: above a threshold of the order of 10 TeV anomalous (B+L)-violating processes are unassuppressed and associated with the multi-particle production of gauge bosons. However, the leading-order calculation has a very bad disease: partial-wave unitarity will be violated at high energies. Therefore it is necessary to consider higher-order corrections to the amplitudes.

3. Corrections to the Naive Instanton Approximation

The leading-order calculation of the total cross-section for B+L violation certainly breaks down at the energy scale \( M_0 \), eq. (2.16). The key question is whether the breakdown does not occur already much earlier, say at \( m_\text{H} \)? There are many indications that perturbative corrections in the single-instanton sector become important already at \( \sqrt{3} \sim m_\text{w} \) for the exclusive cross-sections (2.11) at large \( n \) [20–24]. However, there are also arguments that the corrections to the leading-order total cross-section exponentiate in such a way that their effects are irrelevant up to energies of order \( M_{\text{eq}} \): it was argued [16, 21–24] that the total cross-section for the processes (2.1) has the generic form, for \( \alpha_w \rightarrow 0 \), \( \sqrt{3}/M_{\text{eq}} \) fixed,

\[
\delta_{\Delta(B+L)} \sim \exp \left( -\frac{4\pi}{\alpha_w} F_0 \left( \frac{\sqrt{3}}{M_{\text{eq}}} \right) (1 + O(\alpha_w)) \right). \tag{3.1}
\]

* Using the leading order matrix elements, eq. (2.9), but taking into account exact phase space changes \( M_0 \) to 17 TeV and \( n_w \) at this energy to 86 [19].

Only if \( F \) has a zero is there a chance that electroweak B+L violation in association with multi-W production becomes relevant at future colliders.

For energies much less than the sphaleron energy the function \( F \) may be computed in perturbation theory, which is equivalent to a low-energy expansion in terms of \( z = \sqrt{3}/M_{\text{eq}} \ll 1 \). It is possible to show [16], that at low energies, \( z \ll 1 \), \( F \) depends on energy as in eq. (2.13). Unfortunately, the function \( F \) is not known at \( \sqrt{3} \approx M_{\text{eq}} \). At \( \sqrt{3} \approx M_{\text{eq}} \) all tree graphs in the background of the instanton seem to contribute substantially to \( F \), [16, 21–24], indicating a breakdown of perturbation theory around the instanton at this energy. This may signal that it is inappropriate to expand around the instanton and that a better zeroth-order approximation has to be found (the so-called distorted instanton [3, 25]).

The arguments leading to the expectation that in general \( \delta_{\Delta(B+L)} \) will behave as in eq. (3.1) were mainly combinatorial [16, 21–23]. To calculate explicitly the corrections to the exclusive amplitudes (and afterwards to sum over all channels in order to obtain the correction to \( F \)) is extremely difficult, since the propagators in the background of the constrained instanton - the building blocks of perturbation theory in the single-instanton sector - are not known*. An alternative and perhaps easier approach to the problem of higher-order corrections to \( F \), eq. (2.13), is to consider dispersion relations [17, 18], i.e. to calculate the total cross-section for B+L violation via the imaginary part of the forward elastic scattering amplitude. The main reason why dispersion relations are particularly convenient is that phase space and quantum corrections due to W's and H's are automatically taken into account. Also, unitarity is manifestly built in.

The dispersion relation for the total cross-section for fermion-number violating processes (the sum over all W's and H's in eq. (2.1)) reads

\[
\delta_{\Delta(B+L)} = \frac{1}{4\pi} \cdot \text{Im} \left\{ T(p_1, p_2, p_1, p_2) \right\} \tag{3.2}
\]

\[
= \frac{1}{4\pi} \cdot \text{Im} \left\{ (\text{LSZ}) \left( 0 \mid T[\psi_1(x_1)\psi_2(x_2)\psi_1^*(x_3)\psi_2^*(x_4)] \mid 0 \right) \right\}.
\]

* The propagators in the background of the pure SU(2) instanton are known explicitly [26]. However, the gauge propagator, which gives probably the most important correction, is singular and must be modified [27, 23].
Here $T$ denotes the elastic scattering amplitude and "LSZ" stands for the usual LSZ procedure. One has to select only those intermediate states which contain exactly 10 fermions (see eq. (2.1)) [17] by means of a projection operator $P$.

The Euclidean functional integral for the 4-point Green function entering in eq. (3.2)

$$G_E = \int DWD\Phi D\Phi^* \psi_1\psi_2\psi_3\psi_4 \exp\left(-I[W,\Phi,\psi]\right)$$

(3.3)

can be calculated systematically by semiclassical methods; it receives its dominant contribution [28, 29] from an instanton/anti-instanton valley* in functional space, labeled by the double set of collective coordinates of an instanton/anti-instanton pair. The dominant configuration in Minkowski space is a saddle-point [29] - one point of the valley bottom - with one negative mode, which gives rise to the imaginary part of the forward scattering amplitude (see also ref. [17]). Unfortunately, the valley in functional space is only known for large instanton/anti-instanton separation [28], which in our case means for energies low compared to the sphaleron energy [17, 29]. One finds [29] that the total cross-section of the processes $\sigma_{\mu}$ behaves like in eq. (3.1), where

$$F(x) = 1 - \left(\frac{9}{16\sqrt{2}}\right)^{2/3} x^{4/3} + \frac{3}{32} x^2 + O\left(\frac{1}{x^2}\right)$$

(3.4)

We see that indeed the sphaleron energy (remember, $\sqrt{s}/M_{sp}$) sets the scale for the energy dependence of the total cross-section. We have therefore verified the conjecture [16, 21 - 24] that the total cross-section for B+L violation has the form (3.1). Note, since we have a complete control over the order of the neglected contributions, we can state explicitly that (3.4) gives the correct behaviour of the total fermion-number-violating cross-section in the energy range $m_\tau < \sqrt{s} < M_{sp}$. The prefactors before the exponential in (3.1) can also be calculated [29].

If we namely omit the unknown corrections in eq. (3.4) we find that the cross-section has a maximum at $x = 3\sqrt{2}/3$, where it is still exponentially small, $\sigma_{\mu}(B+L) \sim \exp(-4\pi/\alpha_s)(1/3) \sim 10^{-32}$. It becomes much larger than the 't Hooft factor but is still observably small. This maximum is of course outside the region of validity of our formula. However, it demonstrates that there is a possibility that the total cross-section, even in the single-instanton sector, will not violate the unitarity bound. In case one does not need to invoke multi-instanton induced processes [32] in order to save unitarity.

4. High-Energy Electroweak Interactions

To summarize, we know that the total cross-section for electroweak B+L violation grows exponentially at energies small compared to the sphaleron scale. We know nothing about the behaviour of the cross-section near or above this scale. There are still two possibilities for the function $F(\sigma_{\mu}(B+L) \sim \exp(-4\pi/\alpha_s)F(x))$: at high energies

i) $F$ has a minimum above zero; in this case B+L violation will always be suppressed;

ii) $F$ has a zero or approaches zero, in this case B+L violation will be relatively strong.

McLerran, Vainshtein and Voloshin [3] gave arguments in favour of the second possibility. They suggested that above a sharp threshold at $\sqrt{s} \sim M_{sp}$ weak interactions

* In a different approach [30], based on a Minkowski-space calculation, it was claimed that in theories in which the instanton has a fixed size - unlike the electroweak theory - the cross-section will have the same qualitative behaviour as obtained here: it will first grow exponentially, but will never be unsuppressed because the overlap of the initial states with the sphaleron decreases exponentially with energy. One comment on this approach is in order: The Minkowski-space calculation done in ref. [30] relies on the notion of the "most probable escape path in Minkowski space" which is obtained from the instanton by interpreting its Euclidean time variable as a path parameter, the corresponding Minkowskian classical field interpolates between topologically equivalent vacua and passes the sphaleron. However, it is not clear that for energies approaching the sphaleron it is appropriate to consider just one particular path which exactly passes the sphaleron - there may be many other paths which are equally probable: we do not have to create exactly the sphaleron. That there is indeed the possibility that large coherent fields can be produced at high energies in the electroweak theory has been discussed in several works [31]. In any case, both approaches, the Euclidean and Minkowskian approach, should ultimately converge to give the same answer.
would become "strong" in the sense that \( n_\text{w} \sim \pi/\alpha_\text{w} = O(100) \) weakly-interacting particles (mainly W-bosons) will be produced with a relatively large cross-section, \( O(0.1) \) nb. Close to the threshold these particles come out quasi-isotropically from the vertex. At higher energies, the S-wave amplitude would then become unity, and for other partial waves, as well as (B+L)-conserving amplitudes would become strong, inducing large forward-peaked cross-sections at asymptotic energies \( \sqrt{s} \gg M_\text{w} \).

Possibly the strong interaction above \( M_\text{w} \sim m_\text{w}/\alpha_\text{w} \) is related to the long-expected breakdown of conventional (in the topologically trivial sector) perturbation theory at high orders \( N > 1/\alpha_\text{w} \) (see e.g. [33] and references therein). Indeed, it was argued in ref. [31] that conventional perturbation theory, applied to (B+L)-conserving processes, breaks down at similar energies and multiplicities, at least in the high-energy, fixed-angle regime.

There may even be deeper analogies between the electroweak interactions and QCD at very high energies, as both are non-Abelian gauge theories with similar infrared divergences [34] and topological features. These analogies refer not only to the scattering of a few particles at large angles, which is well described by (parton model) perturbation theory, but also to the large, logarithmically-rising, total inelastic cross-section, where perturbation theory presumably fails.

Based on the above considerations it was suggested [3, 34, 35]:

1) At high energies, the total weak inelastic cross-section becomes approximately energy-independent

\[
\sigma_\text{w} = 4\pi c_\text{w} m_\text{w}^{-2} \approx 0.1 \text{nb} \sim 10 \text{nb},
\]

where \( c_\text{w} \) is a logarithmically-varying coefficient;

2) the asymptotic behaviour sets in abruptly above a certain threshold energy \( E_\text{w}^{\text{crit}} \) of the order of a few TeV to about 20 TeV;

3) the mean constituent multiplicity is high, typically of order \( 1/\alpha_\text{w} \sim 30 \);

4) there is a substantial contribution of (B+L)-violating events to the total weak cross-section;

5) the inclusive differential cross-sections decrease rapidly for transverse momenta exceeding a characteristic value of order \( m_\text{w} \).

In a simple picture [3, 34, 35] these processes arise from the scattering of the clouds of weak gauge bosons and scalars carried along by colliding quarks or leptons. Both the almost constant multi-particle cross-sections and the characteristic \( p_\text{T} \) distribution directly reflect the finite size of these clouds (\( \propto m_\text{w}^{-1} \)) which, of course, related to the finite range of the underlying interactions. We therefore call processes with these characteristics Geometrical Flavour Interactions (GFI) [35]. In collisions with low \( p_\text{T} \), the resolution is "bad", \( \Delta x > m_\text{w}^{-1} \). Therefore, only the gross geometrical features are relevant here, namely, the "weak size" of quarks or leptons corresponding to the size of the W, Z, Higgs clouds, and the "opacity" of the colliding objects, which is measured by the factor \( c_\text{w} \) in (4.1). Within such a geometrical scattering scenario, the most speculative assumptions concern the high multiplicity and the large value of \( c_\text{w} \). (The "opacity" of the colliding objects is actually related to the large multiplicity.) Intuitively, one may envisage that a non-perturbative behaviour of the multiparticle cross-sections arises from the strong "flavourelectric" and "flavourcemagnetic" fields created by the presence of more than \( \alpha_\text{w}^{-1} \) weak charges in a volume with a characteristic linear size \( m_\text{w}^{-1} \) [34, 35].

5. Possible Consequences at Future Colliders

There remain considerable theoretical uncertainties in these non-perturbative estimates, notably whether the rapidly-rising (B+L)-violating cross-section flattens out at some energy below \( M_\text{w} \), and whether it ever reaches a sizeable fraction of the unitarity limit. Nevertheless, the consequences of large non-perturbative electroweak interactions are so far-reaching and dramatic*, that we believe it is worthwhile to study the phenomenology that would be expected at future colliders. As working hypotheses we will adopt points 1)-5) of sect. 4.

We explore the notion that GFI events set in abruptly once the energy is high enough to produce a critical number \( n_\text{w}^{\text{crit}} \) of weakly-interacting particles. As mentioned above, these phenomena are presumably related to the breakdown of perturbation theory for large particle numbers, namely when \( n_\text{w} \alpha_\text{w} \) becomes large. We therefore assume \( n_\text{w}^{\text{crit}} \alpha_\text{w} = \nu_\text{w} \) with \( \nu_\text{w} \) some constant of order one. The threshold energy can then be roughly guessed by requiring that \( \sqrt{s} \) must be large enough to produce \( n_\text{w}^{\text{crit}} \) particles with mass \( m_\text{w} \) without

* For a detailed discussion of the expected phenomenology see ref. [35]. A short version is also contained in ref. [36].
a strong phase space suppression. We therefore parametrize

\[ E_{\text{W}}^{\text{crit}} = f_{\text{w}} n_{\text{w}}^{\text{crit}} m_{\text{w}} = f_{\text{w}} \nu_{\text{w}} m_{\text{w}} / \sigma_{\text{w}}. \] 

(5.1)

Both instanton estimates [1-3] and a comparison with QCD [34, 35] suggest values of \( n_{\text{w}}^{\text{crit}} \) and \( E_{\text{W}}^{\text{crit}} \) in the range \( n_{\text{w}}^{\text{crit}} \approx 30 - 100, E_{\text{W}}^{\text{crit}} \approx (3.5 - 20) \text{ TeV} \). We use a conservative value \( n_{\text{w}} = 30 \) for the phenomenological discussion: all results can be rescaled easily for the experimentally even more advantageous case of higher multiplicity.

![Graph showing number of GFI events per second as a function of the (parton) threshold energy \( E_{\text{W}}^{\text{crit}} \) (taken from ref. [35]). We assume a constant cross-section \( \sigma_{\text{w}} = 1 \text{ nb} \) for weakly-interacting partons with energies above \( E_{\text{W}}^{\text{crit}} \) and a pp luminosity of \( L = 10^{33} \text{ cm}^{-2}\text{sec}^{-1} \).](image)

Let us assume an abrupt onset of geometrical QCD cross-sections for \( \sqrt{s} \geq E_{\text{W}}^{\text{crit}} \), and approximate \( \delta_{\text{w}}(\sqrt{s}) = \sigma_{\text{w}} \delta(\sqrt{s} - E_{\text{W}}^{\text{crit}}) \), with \( \sigma_{\text{w}} \approx 0.1 \text{ nb} - 10 \mu\text{b} \) [3, 34, 35]. After folding \( \delta_{\text{w}} \) with the parton distributions (no gluons) we obtain the event rate for the production of many weakly-interacting particles in proton-proton collisions, see fig. 5. The limit of detectability should be around 100 events per year. We conclude that GFI events can be seen for \( E_{\text{W}}^{\text{crit}} < 11, 28, \) and 130 TeV for the LHC, SSC, and ELOISATRON, respectively, if \( \sigma_{\text{w}} \) is around 1 nb. The possibility for detection at the LHC depends crucially on the precise values of \( E_{\text{W}}^{\text{crit}} \) and \( \sigma_{\text{w}} \). A future collider in the 100 TeV range (ELOISATRON) would be needed for a full exploration of the asymptotic behaviour beyond the threshold energy.

Let us concentrate [35] on the light fermions of mass smaller than the transverse momenta of the produced particles \( \sim 30-50 \text{ GeV} \), which may be produced both promptly and as decay products of W, Z bosons, etc. The associated jets and charged leptons are directly accessible experimentally. One finds [35] the following characteristic features of “geometrical flavour interaction” (GFI) events (for \( n_{\text{w}} = 30 \)):

1) For parton kinematics near the threshold \( (\sqrt{s} \approx E_{\text{W}}^{\text{crit}}) \), the events look rather central, i.e. jets and charged leptons are distributed over the whole angular range. Only partons with energies much above \( E_{\text{W}}^{\text{crit}} \) produce more forward-oriented events. For “threshold” machines with \( \sqrt{s} = 16-40 \text{ TeV} \) most of the activity will be in the central detector \( (10^\circ < \vartheta < 170^\circ) \). Even at \( \sqrt{s} = 200 \text{ TeV} \) one may expect on the average about one half of the produced particles in this central region.

2) The average transverse momentum per light particle is estimated to be \( p_t \approx 35 \text{ GeV} \), and the total transverse energy is large: \( \langle E_{\text{T}} \rangle \approx 1.6 - 2.2 \text{ TeV} \).

3) We expect on average at least 3.5 “isolated” electrons or muons per event.

4) A similar number of neutrinos is responsible for an average missing transverse momentum \( \langle q_{\text{T}} \rangle \approx 100 \text{ GeV} \).

5) More than 20 jets/event, corresponding to \( \sim 200 \) charged hadrons, should be seen in the central detector.

6) In addition, we expect many events with a high charged hadron multiplicity \( (n_h > 100) \) in the forward and/or backward detectors (\( \eta > 2.5 \)).

7) If B+L violation is relatively strong in comparison to the (B+L)-conserving GFI processes, it may be possible to observe the L violation by measuring average lepton charge asymmetries and/or \( \langle E_{\text{T},e^+} \rangle > \langle E_{\text{T},e^-} \rangle \).

There is essentially no background for these events [35].

Let us enlarge [35] on point 7, since, of course, B+L violation would be the most spectacular consequence of our speculations. If B+L violation becomes strong in the TeV range [1-3], the topology of the (B+L)-violating events will, presumably, resemble roughly the one corresponding to generic GFI events described above. It will be rather central close to the threshold energy and increasingly forward-oriented as the parton energy increases beyond the threshold. A search for B+L violation should therefore directly concentrate on quantum numbers [35] (see also ref. [19]).
A $(B+L)$-violating interaction

\[ q + q \rightarrow 7q + 3l + X \]  \hfill (5.2)

(see eq. (2.1)) produces on average more positrons and $\mu^+$ than electrons and $\mu^-$. Thus one should measure the average lepton charge asymmetries,

\[
\left( \frac{N_{e^+}(\mu^+) - N_{e^-}(\mu^-)}{N_{e^+}(\mu^+) + N_{e^-}(\mu^-)} \right)
\]  \hfill (5.3)

for isolated electrons and muons (with $p_t > 20$ GeV) in the central region ($10^\circ < \theta < 170^\circ$). The asymmetry vanishes, in principle, for L-conserving GFI events and gives, therefore, a direct measure of the relative strength of $(B+L)$-violating interactions.

In addition, one expects the primary $e^+$'s and $\mu^+$'s from a $(B+L)$-violating interaction to be more energetic than the decay products of the associated gauge bosons. This leads to an asymmetry in the mean energy of the fastest anti-lepton as compared to that of the fastest lepton,

\[
\left( E_{e^+,\mu^+} \right) > \left( E_{e^-,\mu^-} \right). \]  \hfill (5.4)

As to the usefulness of the average charge asymmetries (5.3) or the energy asymmetries between leptons/anti-leptons (5.4), some comments [35] are in order. i) Since the charges for electrons and muons are almost uncorrelated for a given event, we expect rather large fluctuations of $N_{e^+}(\mu^+) - N_{e^-}(\mu^-)$ on an event by event basis. For $(B+L)$-conserving interactions the average value of the asymmetry should only vanish for a large number of events, with a statistical error determined by the average size of the fluctuations and the number of events. ii) Lepton number conservation implies $(N_{e^+}(\mu^+) = N_{e^-}(\mu^-)) = 0$ only if the experimental cuts (like e.g. angular cuts, isolation cuts or $p_t$ cuts) are completely symmetric with respect to the lepton charge distribution. This is far from trivial, since an asymmetry in the kinematic distributions of $W^+$ and $W^-$, for example, may reflect itself in an induced effective charge asymmetry of the lepton cuts. To minimize such possible effects it seems advantageous to concentrate on isolated leptons (with $p_t > 30$ GeV) in a central rapidity region. iii) Using special properties of the primordial fermions to filter out the $(B+L)$-violating events, like the asymmetry (5.4), has to compete always with the "tails" of geometrical flavour production.

Therefore, only $L$ violation that is large compared to the $(B+L)$-conserving GFI background can be detected by a measurement of (5.3) and/or (5.4).

6. Conclusions

We have discussed in this lecture the possibility that non-perturbative effects in the electroweak theory, like multi-particle production of weakly interacting particles and $B+L$ violation, may become important at energies $\sim M_{\nu} \sim 4\pi f$ $\sim$ 10 TeV, which will be reachable at colliders in the near future.

It is by now established that the total cross-section for electroweak $B+L$ violation grows exponentially at energies below the sphaleron scale, $m_\nu \ll \sqrt{s} \lesssim M_{\nu}$, and becomes much larger than the 't Hooft factor, $\exp(-4\pi f/\alpha_s)$, would suggest. It is dominated by the associated production of (many) W bosons. Unfortunately, we do not know the behaviour and size of the cross-section near and above the sphaleron scale. The presently-available calculational techniques break down at this scale. Much theoretical work is needed to come to a definitive conclusion whether $B+L$ violation becomes of observable size.

If the rapidly-rising $(B+L)$-violating cross-sections are qualitatively correct near the sphaleron scale, we expect very interesting phenomenology at future colliders. Multi-particle production of $O(\alpha^{-1})$ $\sim$ 30 weakly-interacting particles and $B+L$ violation would occur with relatively large cross-sections, $\delta_{\nu} \sim 0.1$ nb $\sim 10$ pb. Pictorially, these processes could be ascribed to the scattering of the clouds of weak gauge bosons and scalars carried along by colliding quarks or leptons.

We have discussed the phenomenology associated with this "geometrical" flavour production in the multi-TeV regime. We found that there is essentially no QCD background for these processes. At least the multi-particle production of weakly-interacting particles would be observed, if the threshold is below 11, 28, or 130 TeV for the LHC, SSC, or ELOISATRON, respectively. The observation of $B+L$ violation would require a lot of statistics and would be a challenge for experimentalists.
References

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