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Anomalous Electroweak Processes in a Strong Magnetic Field

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ABSTRACT

We estimate within the standard electroweak theory the magnitude of baryon number violating processes in a strong external magnetic field $H > H_c = m_W^2/e$.

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Based on ideas dating back to 't Hooft[1], there has recently been renewed interest in the study of anomaly-induced baryon and lepton number violating electroweak processes. Compared with the exponentially-suppressed cross sections derived by 't Hooft[1] (see also ref[2]), it has been argued that, effectively, such cross sections may become large at either high temperatures [3-6] or high center-of-mass energies [7-13]. While the possibility of large baryon and lepton number violation at high temperature now appears a likely possibility, whether this is really also the case in zero-temperature high energy collisions has been the subject of much debate [14-19].

In the standard $SU(2)_L \times U(1)_Y$ electroweak theory baryon number (B) and lepton number (L) is strictly conserved only in the combination $B - L$. On account of the anomaly in the baryon and lepton number currents neither B nor L is exactly conserved. But in the dilute instanton gas formalism ($B + L$)-violating electroweak processes occur only through tunneling between topologically-inequivalent vacua[1]. The amplitude for such a process is suppressed by (at least) a factor of $\exp(-8\pi^2/g^2)$, where g is the $SU(2)_L$ gauge coupling constant. This would under normal circumstances make ($B + L$)-violating processes in the standard electroweak theory uninteresting from a phenomenological point of view.

The energy barrier separating topologically distinct vacua has a minimum height of approximately $E_{min} \sim m_W/\alpha_W$. An unstable static saddle point solution to the $SU(2)_L \times U(1)_Y$ theory separating topologically-distinct vacua has been dubbed the sphaleron[8]. It carries half-integer winding number. At sufficiently high temperature or sufficiently large center-of-mass energy ($T, E_{cm} \sim m_W/\alpha_W$) one might hope to overcome the exponential tunneling suppression factor by in effect "passing over" the sphaleron energy barrier. In the case of high-energy scattering processes this naive picture has become somewhat substantiated by a detailed one-instanton computation of Ringwald [11]. Unfortunately, doubts as to the validity of this calculation arise precisely at the point where indeed the cross sections appear to be unsuppressed by the exponential tunneling factor.

It would be of interest to find a different physical regime in which the same suggestion of being able to pass over the sphaleron energy barrier could be tested in a calculable way. One such regime is that of very strong external magnetic fields. The reason is as follows. As the strength of the magnetic field is raised to $H_{c1} \sim m_W^2/e$, where e is the electromagnetic charge, a transition occurs in the $SU(2)_L \times U(1)_Y$ theory from a broken vacuum to an unbroken vacuum [21]. In a semiclassical approximation one finds that for $H < H_{c1} = m_W^2/e$ the vacuum is essentially unchanged, with the expectation value $\langle \phi \rangle \sim v$ being practically constant. The case which can be treated in greatest detail corresponds to a Higgs mass of $m_H = m_Z$. There, in an intermediate regime $m_W^2/e < H < m_W^2/e \cos \theta_W$, where θ_W is the weak mixing angle, $\langle \phi \rangle$ decreases smoothly to zero, which it reaches at $H_{c2} = m_W^2/e \cos \theta_W$. For fields $H > H_{c2}$ the magnetic field appears entirely in the weak hypercharge direction [21]. (In general the upper critical field should be replaced by $H_{c2} = m_W^2/e$, but the effect is presumably of exactly the same kind as long as $m_H > m_W$.) It should be obvious from the above discussion that we are talking about incredibly large $U(1)_Y$ magnetic fields, of at least 10^{24} Gauss, the existence of which for a long period of time can only be entertained within a cosmological context, perhaps in the early universe

or around a cosmic string (see later).

Above the critical field H_{c1} the Higgs vacuum expectation value, and hence the energy barrier between topologically different vacua, is lowered. One might then naively expect that even low energy ($B + L$)-violating processes could be subject to a dramatic enhancement. This would be also implied if one blindly scaled $\langle \phi \rangle = v \rightarrow 0$ in the simplest ($B + L$)-violating cross section[14,11]¹:

$$\sigma_{tot} \sim \text{const} \cdot e^{-16\pi^2/g^2} \left(\frac{s}{v^2} \right)^{13} v^{-2} \quad (1)$$

However, this argument completely ignores all other physical consequences of the external field. In addition, the reasoning used to derive (1) relies on the existence of only one length scale (other than $\langle v \rangle$), namely the instanton core size. When a $U(1)$ magnetic field is present there are, at the very least, two other length scales: $1/\sqrt{H}$ and R , the size of the region where the field is present. These two arguments show that just setting $\langle v \rangle \rightarrow 0$ in (1) is far too naive. A more accurate evaluation of ($B + L$)-violating processes in the presence of large magnetic fields is therefore called for. In this paper we shall derive what we believe is an estimate for the ($B + L$)-violating amplitude in a finite region of strong external magnetic field. Our method relies on the dilute instanton gas calculation of 't Hooft [1] and Ringwald[11,12], with the appropriate modifications due to the external field.

Since except for a narrow interval $H_{c1} < H < H_{c2}$ we either have (roughly) $\langle \phi \rangle = v$ ($H < H_{c1}$) or $\langle \phi \rangle = 0$ ($H > H_{c2}$), we choose to simplify the calculation by assuming that $H > H_{c2}$. The intermediate region $H_{c1} < H < H_{c2}$ could, in principle, be treated by analogous methods. For $H > H_{c2}$ the $SU(2)_L \times U(1)_Y$ symmetry is fully restored [21], and we shall accordingly treat all fermions as being strictly massless. The magnetic field is for $H > H_{c2}$ aligned along the $U(1)_Y$ direction, so we effectively have $\theta_W = 0$.

We take the electroweak Lagrangian in the background of an external magnetic field to be

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_H + \mathcal{L}_F + \mathcal{L}_{ext} \quad (2)$$

where

$$\begin{aligned} \mathcal{L}_G &= -\frac{1}{4} W_{\mu\nu}^a W_{\mu\nu}^a - \frac{1}{4} F_{\mu\nu} F_{\mu\nu} + \text{ghosts} + \text{gauge fixing} \\ \mathcal{L}_H &= (D_\mu \Phi)^\dagger D_\mu \Phi + \lambda (\Phi^\dagger \Phi - \frac{1}{2} v^2)^2 \\ \mathcal{L}_F &= i \bar{\Psi}_L^{(i)} \not{D} \Psi_L^{(i)} + i \bar{\Psi}_R^{(i)} \not{D} \Psi_R^{(i)} \\ \mathcal{L}_{ext} &= -A_{\mu\nu} J_{\mu\nu}^{ext} \end{aligned} \quad (3)$$

Here Φ represents the standard $SU(2)_L$ Higgs doublet; ψ_L and ψ_R are the left handed (doublet) and right handed (singlet) fermions, respectively. $W_{\mu\nu}^a \equiv \partial_\mu W_\nu^a - \partial_\nu W_\mu^a +$

¹Clearly the regime in center-of-mass energy $\sqrt{s}$ where this expression would be applicable must decrease as well. So this certainly cannot be viewed as a serious argument for an enhanced cross section.

$g\epsilon^{abc}W_\mu^a W_\nu^b W_\nu^c$ is the $SU(2)_L$ field strength tensor, and $F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu$ is the one corresponding to $U(1)_Y$. $\vec{D}$ and $\vec{\partial}$ are the covariant derivatives acting on $SU(2)$ doublets and singlets, respectively. Since we will perform a semiclassical calculation, all effects due to the Yukawa couplings between bosons and fermions can be neglected. However, it is quite obvious that, in contrast to earlier calculations of instanton-induced $(B+L)$ -violating effects, we now cannot ignore the $U(1)_Y$ sector of the theory. The $U(1)_Y$ gauge field is coupled to the external field through the current $j_\mu^{ext} = \partial_\nu F_{\mu\nu}^{ext}$, which is equivalent to replacing $A_\mu \rightarrow A_\mu + A_\mu^{ext}$. For A_μ^{ext} we take

$$\begin{aligned} A_i^{ext} &= g' \frac{H}{2} \epsilon_{ij} x_j, & i, j = 1, 2 \\ A_0 &= A_3 = 0 \end{aligned} \quad (4)$$

which corresponds to a constant magnetic field in the positive x_3 direction, here represented in the symmetric gauge.

We now wish to argue that inside the region of strong magnetic field we have, in addition, approximate "singular gauge" instanton solutions of the equations of motion

$$W_\mu^a(x) = \frac{2}{g} \frac{\rho^2}{(x-z)^2} \frac{\eta_{a\mu\nu}(x-z)_\nu}{(x-z)^2 + \rho^2}; \quad \Phi(x) = 0 \quad (5)$$

where ρ is the instanton scale parameter, z is the instanton center location, and the $\eta_{\mu\nu}$ are the usual 't Hooft symbols [1]. First of all, the fact that $\Phi = 0$ is not surprising: for large magnetic fields all fluctuations from the $\Phi = 0$ value have large (proportional to H) and positive energies; the symmetry is restored [21], and the $\Phi = 0$ solution is energetically favoured. The presence of the instanton background will not modify this result for large H . Secondly, we have to find the saddle point solution for W_μ^a . In order to do that we need to know the effect of the magnetic field on the effective action for the $SU(2)_L$ gauge sector. The classical action for the $SU(2)_L$ field being unchanged, the only way the $U(1)_Y$ background can affect the $SU(2)_L$ field effective action and, hence, its saddle point, is through scalar and fermion loops. (In addition to scalar and fermion loops we also have pure $SU(2)_L$ gauge fluctuations, described by the appropriate determinant. This is known exactly in the background of a pure $SU(2)_L$ instanton [1], but we first have to make sure that instantons are saddle-point solutions when a large magnetic field is considered, so we will return to it later.) We therefore need to know scalar and fermion determinants in the mixed background of an arbitrary $SU(2)_L$ field and a $U(1)_Y$ magnetic field. The one-loop effective action of fermions and scalars coupled to a constant external $U(1)$ magnetic field is known exactly [22, 23]. We are, however, interested in the case when both backgrounds are present. Exact expressions are not available then, but we can carry out a $1/H$ expansion. This is just what we need, as we are assuming that the external magnetic field H is indeed very large.

We do this in the following manner. Knowing it exactly, we use the exact propagator of a fermion (or scalar) in the magnetic field to derive the effective action for the $SU(2)_L$ part. This yields an expansion in gauge-invariant operators of increasing scaling dimension.

multiplying by appropriate powers of $(g'H)^{-1}$. In an obvious notation we write $D_\mu = \partial_\mu + W_\mu + A_\mu$. Then

$$\begin{aligned} Det(\vec{D}) &= \exp\{Tr \ln(\vec{\partial} + W + A)\} \\ &= \exp\{Tr[\ln(\vec{\partial} + A) + \ln(1 + \frac{1}{\vec{\partial} + A} W)]\} \\ &= Det(\vec{\partial} + A) \exp\{Tr[\frac{1}{\vec{\partial} + A} W + \dots]\} \end{aligned} \quad (6)$$

Here $Det(\vec{\partial} + A)$ is known for a constant external magnetic field. The exact expression is [22, 23]:

$$\ln Det(\vec{\partial} + A) = H^2 \frac{g'^2 Y^2}{24\pi^2} \left\{ \ln \left(\frac{g' Y H}{\mu^2} \right) + \ln 2 - 1 + 12\zeta'(-1) \right\} \quad (7)$$

as obtained from ζ -function regularization. This result is valid for one Dirac fermion of weak hypercharge Y . Note that in the Standard Model lagrangian (3) A_μ couples both to the left-handed and right-handed part. The analogous expression for a complex scalar field is [22, 23]

$$\ln Det(\partial_\mu + A_\mu)(\partial^\mu + A^\mu) = -H^2 \frac{g'^2 Y^2}{96\pi^2} \left\{ \ln \left(\frac{g' Y H}{\mu^2} \right) + \ln 2 - 1 - 24\zeta'(-1, \frac{1}{2}) \right\} \quad (8)$$

Thus, in total, adding the classical piece $\Gamma_{U(1)}^{(0)} = -\frac{1}{2}H^2$, we get the one-loop effective action of the pure $U(1)_Y$ part:

$$\begin{aligned} \Gamma_{U(1)}^{(0)} + \Gamma_{U(1)}^{(2)} &= -\frac{1}{2}H^2 + N_g \sum_f H^2 \frac{g'^2 Y_f^2}{24\pi^2} \ln \left(\frac{H}{\mu^2} \right) \\ &\quad + N_d \sum_b H^2 \frac{g'^2 Y_b^2}{96\pi^2} \ln \left(\frac{H}{\mu^2} \right) \end{aligned} \quad (9)$$

where we have already neglected all constant parts, which will not be important for our discussion. N_g stands for the number of generations and the sum $\sum_f$ refers to the counting of fermions within each generation. Likewise N_d is the number of complex doublets (just one in the minimal Standard Model) and $\sum_b$ refers to the fields within each doublet. Eq. (9) corresponds to the factor $Det(\vec{\partial} + A)$ in (6).

Next we want to use the exact propagators in the external $U(1)_Y$ magnetic field to determine the effective action for the $SU(2)_L$ part. This corresponds to the expansion in powers of W_μ in (6). Simple considerations show that to evaluate the coefficient in front of $-\frac{1}{4}W_{\mu\nu}^a W^{\mu\nu a}$ coming from the fermion and boson loops it is sufficient to take the second derivative d^2/dH^2 of $\Gamma_{U(1)}^{(2)}$, and then put the appropriate group-theoretical coefficient in by hand. This gives, for a doublet of left-handed fermions, the following quantum correction to the coefficient in front of $-\frac{1}{4}W_{\mu\nu}^a W^{\mu\nu a}$

$$-\frac{g^2}{48\pi^2} \ln \left(\frac{H}{\mu^2} \right) \quad (10)$$

and for a complex scalar doublet

$$-\frac{g^2}{96\pi^2} \ln\left(\frac{H}{\mu^2}\right) \quad (11)$$

The right-handed fermions do not couple to the $SU(2)_L$ field, so they do not contribute to the effective action for W_μ . Finally, to obtain the effective action in the combined background we have to replace $-\frac{1}{4}W_\mu^a W^{\mu a}$ by its value in an instanton field, $-8\pi^2/g^2$.

We should next consider higher-dimensional operators involving the $SU(2)_L$ gauge field. However, these terms will be suppressed by powers of $1/H$. If we substitute the instanton field into the $1/H$ expansion we will find that higher-order terms give contributions of $\mathcal{O}((\rho^2 H)^{-1})$. Hence the instanton (5) is still an approximate saddle-point solution of the $SU(2)_L$ action, at least for $\rho \gg 1/\sqrt{H}$. For very small core instantons this expansion is clearly not a valid one; the classical solution will no longer be that given by eq. (5). Nevertheless, we will in what follows restrict ourselves to instantons fulfilling the condition $\rho\sqrt{H} \gg 1$, since the dominant contribution to the amplitude, as we shall see, comes from large-size instantons. Then we can neglect all higher orders in the $1/H$ expansion, the classical solutions for W_μ^a are those given in (5), and the contribution from the $SU(2)_L$ gauge loop is just the same as in [1]. We now see that when all these effects are included the difference between having the large constant magnetic field or not lies, at this level, in the fermion and scalar determinants leading to (10) and (11). At leading order we have just to replace the “ ρ ” that appears in the usual case [1] by “ $1/\sqrt{H}$ ” in those matter determinants, but not in the pure gauge determinant.

Of course it is illusory to expect exactly constant magnetic fields. In practice we must expect small local deviations δH . Provided $\delta H/H \ll 1$ we can always carry out a double expansion in $\delta H/H$ and $1/\rho\sqrt{H}$ of the determinants and the above results will still hold approximately.

When considering Green functions, the pure $U(1)_Y$ part of the effective action disappears when one normalizes with the no-instanton partition function. We will be left only with the usual exponential prefactor $\exp(-8\pi^2/g^2)$, the contribution from the determinants (eqs. (10) and (11)), and the contribution from the zero modes. The zero modes for the pure $SU(2)_L$ field will not change (for large H), so we can directly borrow the collective coordinate measure of 't Hooft [1]. The one-loop contribution from $SU(2)_L$ gauge fluctuations will not change either, as we just said, and we hence have from the corresponding determinant (ignoring unimportant constants)

$$(\mu\rho)^{22/3} \quad (12)$$

We must now specify precisely which baryon-lepton number violating process we wish to focus on. To avoid complications due to many external quanta, we choose the simplest $(B+L)$ -violating process possible:

$$q_1 + q_1 \rightarrow \bar{q}_1 + 3\bar{q}_2 + 3\bar{q}_3 + \bar{l}_1 + \bar{l}_2 + \bar{l}_3, \quad (13)$$

assuming $N_g = 3$. The notation, which follows ref. 11, should be obvious.

Finally, collecting all terms, we can write down the effective action computation of the euclidean Green function associated with the process (13) [1,11]

$$G(x_1, \dots, x_{12}) = C \int d^4z \int_0^R d\rho g^{-8} \rho^{-5} (\mu\rho)^{22/3} \times \left(\frac{\sqrt{H}}{\mu}\right)^{4N_g/3} \left(\frac{\sqrt{H}}{\mu}\right)^{N_d/6} \exp\left(-\frac{8\pi^2}{g^2(\mu)}\right) \prod_i \Psi_i^0(x_i, -z) \times (1 + \mathcal{O}(1/\rho^2 H)) \quad (14)$$

where Ψ_i^0 denotes the zero mode for fermion i in the combined background of the $SU(2)_L$ instanton and the constant background $U(1)_Y$ magnetic field:

$$p\Psi_i^0 = 0 \quad (15)$$

and C is the following numerical constant

$$C = 2^{10} \pi^6 \exp[-\alpha(1) + \frac{N_g}{12} \sum_f (\ln(g'Y_f) + \ln 2 - 1 + 12\zeta'(-1))] + \frac{N_d}{24} \sum_f (\ln(g'Y_f) + \ln 2 - 1 - 24\zeta'(-1, \frac{1}{2})) \quad (16)$$

with $\alpha(1) = 0.4433\dots$ Equation (14) is, just as the equivalent one in 't Hooft's original calculation, independent of the scale μ up to one loop order. For the sake of definiteness we will take $\mu = m_W$. Eq. (14) however, has striking differences with respect to the analogous equation in [1]. First of all, it shows that the natural scale appearing in the fermion and boson loops is the $U(1)_Y$ magnetic field, but not in the gauge loops. Since $\langle\phi\rangle = 0$ the integral over the instanton size is infrared divergent. Here, however, we have a natural cut-off provided by the size of the region over which the magnetic field has $H > H_c$. We call it R . By analogy with QCD instantons one might think that if R is larger than the electroweak confinement radius r_c (recall that we are in the unbroken phase), the ρ integral should be effectively cut-off by r_c instead of R . On account of the large energy density present this is actually not entirely obvious, but a rough estimate of the electroweak confinement scale is given by

$$r_c = \frac{1}{\mu} \exp\left(-\frac{8\pi^2}{\beta_1 g^2(\mu)}\right) \quad (17)$$

β_1 being the first coefficient of the β function. In our case,

$$\beta_1 = -\frac{22}{3} + 4\frac{N_g}{3} + \frac{N_d}{6} \quad (18)$$

and, with the matter content of the minimal Standard Model, $N_g = 3$, $N_d = 1$, this yields a scale of the order $r_c \simeq 10^{23} \text{ GeV}^{-1}$, many orders of magnitude larger than any length scale we shall be concerned with. Note that in the calculation leading to the Green function (14) we have been implicitly assuming $R = \infty$. In fact, this is a good approximation

provided that $\rho \ll R$. This is automatically accomplished (within the sort of accuracy we are aiming for) when we cut-off the integral over the instanton size by R itself.

To proceed we need to know the zero modes in the combined field of the instanton and the magnetic field. To begin with, how many of them are there? Without the magnetic field it is well known that only one exists [1,11]. In the chiral representation for the γ matrices

$$\psi_{\alpha\alpha}^0(x-z) = i \left(\frac{2\rho^2}{\pi^2} \right)^{1/2} \frac{|x-z|}{((x-z)^2 + \rho^2)^{3/2}} \frac{\epsilon_{\alpha\beta}}{((x-z)^2 + \rho^2)^{3/2}} \quad (19)$$

where $\sigma_\mu = (i, \sigma)$. It satisfies $\gamma_5 \psi^0 = -\psi^0$. It is easy to see that in the combined background of a large constant magnetic field and the instanton there is still one and only one zero mode. To begin with, since there is no electric field $F\tilde{F} = 0$, while $W\tilde{W}$ does not change. The index theorem then ensures that $n_+ - n_- = -1$. So we certainly have zero modes. The next step consists of proving that $n_+ = 0$. Let us first assume that we have a solution with the "wrong chirality" $\gamma^5 \psi = \psi$. Then by standard manipulations[24] we can write

$$\int d^4x (D^\mu \psi^\dagger D_\mu \psi + g' H \psi^\dagger \sigma_3 \psi) = 0 \quad (20)$$

The piece in $W_{\mu\nu}\sigma^{\mu\nu}$ has cancelled because $\gamma^5 \psi = \psi$ and $W_{\mu\nu} = \tilde{W}_{\mu\nu}$. If H is large the spin will be roughly aligned with the magnetic field. Therefore, the second term will be positive and, since the first one is positive definite, the only way out is $\psi = 0$. This proves that in the combined background $n_+ = 0$ and $n_- = 1$.

To see what this zero mode looks like we have to recall some well known facts concerning the spinor eigenstates in a constant magnetic field. For a massless spin 1/2 particle, the solutions of the corresponding Dirac equation are of the form[25]

$$\varphi(x) = \phi_n(x_1, x_2) e^{ip_3 x_3 + i\epsilon_n t} \chi \quad (21)$$

where χ is a constant spinor that is an eigenstate of σ_3 (in our case with $s_3 = +1/2$). The energies are $\epsilon_n^2 = p_3^2 + (2n+1)g'H - 2g'Hs_3$. For large values of H only the $n = 0$, $s_3 = +1/2$ sector corresponding to $\epsilon_n = 0$ (for $p_3 = 0$) will be populated. In fact, the ground state and any others are infinitely degenerate in a constant magnetic field of infinite extent. (On a finite but large area R^2 the degeneracy is of the order $eHR^2/2\pi$). An appropriate basis of the $n = 0$ sector is provided by the functions

$$\phi_0(x_+, x_-) = f(x_+) \exp\left(-\frac{g'H}{4} x_+ x_-\right) \quad (22)$$

f being an arbitrary function of x_+ , with $x_\pm = x_1 \pm ix_2$. Switching on the instanton must break the degeneracy of the ground state, since we know we end up with a unique zero mode. It is convenient to consider also a different basis for the purely magnetic zero modes, namely the set of functions

$$\exp\left[-g'\frac{H}{4}(x_+ - z_+)(x_- - z_-)\right] \quad (23)$$

Note that a gaussian centered at z , such as the one above, can be obtained from (22) by taking $f(x_+) = \exp[-g'\frac{H}{4}(x_+ z_-)]$ and doing a gauge transformation with gauge function

$\Lambda = -g'\frac{H}{4}(x_+ z_- - x_- z_+)$. Conversely, any function of the form (22) can be obtained by a superposition of the gaussians (23) centered at different points.

The instanton provides an additional attractive potential, which, in a sense, can be viewed as inducing a weak perturbation on the purely magnetic zero modes. Physically it is clear that of all possible gaussians the one whose center coincides with the instanton location will be the one with lowest action. A reasonable ansatz for the zero mode will then be (taking $z = 0$ for simplicity)

$$\psi^0(x) = N \exp\left[-\frac{g'H}{4}(x_1^2 + x_2^2)\right] \psi(x) \quad (24)$$

where N is a normalization constant. Although ψ will in general depend on all coordinates, for large values of H the wave function will collapse to the origin in the (x_1, x_2) plane. At the price of making an error of $\mathcal{O}(1/H)$, we can take $\psi(x) = \psi(x_3, t)$. Furthermore, we know from the index theorem that $\gamma_5 \psi = -\psi$ and that for large H , $\sigma_3 \psi = \psi$. When we substitute the ansatz (24) into the full wave equation and consistently use the large H approximation we discover that there is a unique solution for ψ . After inserting it in eq. (24) we get

$$\psi^0(x) = N \exp\left[-\frac{g'H}{4} x_+ x_- \right] g(w_+, w_-) \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad (25)$$

where $w_\pm = x_3 \pm it$. The two vectors correspond to Dirac and $SU(2)_L$ indices, respectively. The function $g(w_+, w_-)$ is the solution of the following differential equation

$$\partial_{w_+} g - i w_- f(w_+, w_-) g = 0 \quad (26)$$

with

$$f(w_+, w_-) = \frac{2}{g} \rho^2 \frac{1}{w_+ w_-} \frac{1}{w_+ w_- + \rho^2} \quad (27)$$

Note that for $|w| \rightarrow \infty$, $g \sim 1/|w|^2$. Although a numerical evaluation is probably necessary to get precise results, we are interested in order of magnitude estimates and the above approximation (exact for $H \rightarrow \infty$) is enough to trace potentially large factors depending on the magnetic field.

The anomalous process will take place inside a finite region of space (of extent R) where the strong, constant, external $U(1)_Y$ magnetic field is switched on. We know in fact that the cross section for the process (13) *outside* the region of strong magnetic field is vanishingly small [1,11] at the energies we are interested in. The physical picture of the way $(B+L)$ -violating processes occur is the following. Particles coming from the region outside the magnetic field are trapped into Landau orbits. When the number of particles is large enough the projection of their wave functions in the plane perpendicular to the magnetic field will begin to overlap. (These wave functions are gaussians in the (x_1, x_2) plane, as we have seen.) Then the anomalous process associated with the Green function (14) takes place (we will see later with what probability). The resulting particles will also be bound by the magnetic field, at least until they manage to escape by thermal fluctuations or by having the background field configuration relax, presumably at a later stage of cosmic

evolution. Thus we are led to consider transition elements between *bound* states. This is somewhat unusual in field theory, and quite difficult in general. Fortunately the states are bound by an external field so the usual separation between a “free” hamiltonian, $\mathcal{H}_0$ (to be treated exactly), and an interaction hamiltonian, $\mathcal{H}_I$, still leads to a quadratic operator for $\mathcal{H}_0$, namely $\mathcal{H}_0 = \sum_n \epsilon_n c_n^\dagger c_n$, where $\epsilon_n, c_n^\dagger$ are Landau states’ annihilation and creation operators and the ϵ_n have been defined following eq. (21) (we have lumped the degeneracy label into n to avoid cluttering the notation). The usual manipulations to obtain the transition amplitude from the appropriate Green function remain valid in this case, and one obtains for a generic matrix element

$$M_{n,m,\dots} = \int d^4 x \varphi_n^\dagger(x) (\not{\partial} + \not{A})_x \int d^4 y \varphi_m^\dagger(y) (\not{\partial} + \not{A})_y \dots G(x, y, \dots) \quad (28)$$

where $G(x, y, \dots)$ is the Green function given in (14), the φ_n ’s are the initial and final (bound) states, and A_μ is the $U(1)_Y$ field (4). We choose to normalize $\phi_n(x)$ to unity in the (x_1, x_2) plane (and to momentum conserving delta functions in the (x_3, t) plane). To finish the calculation we now simply need to estimate

$$\int d^4 x \varphi^\dagger(x) (\not{\partial} + \not{A})_x \Psi^0(x - z) = - \int d^4 x \varphi^\dagger(x) \not{W}(x - z) \Psi^0(x - z) \quad (29)$$

where we have used the equation satisfied by Ψ^0 , and where W_μ is the instanton field. Since both φ and Ψ^0 are peaked gaussians in the (x_1, x_2) plane, the (by far) dominant contribution will occur when all initial and final particles are approximately at $(x_1, x_2) = (z_1, z_2)$. Using the expression for the wave functions with and without the instanton (eqs. (21) and (25)), it is trivial to evaluate (29). For large H the integral factorizes and simple considerations show that it is $\mathcal{O}(\sqrt{\rho})$. As we have already discussed, corrections will be $\mathcal{O}(1/\rho^2 H)$, which we systematically neglect. All this considered, and ignoring all constants, the amplitude for the process (13) will, with our normalization convention, approximately be

$$\delta(\sum p_3) \delta(\sum \epsilon) R^2 (R\mu)^{23/3} \left(\frac{\sqrt{H}}{\mu} \right)^{25/6} \exp\left(-\frac{8\pi^2}{g^2(\mu)}\right) F(y_1, \dots, y_{12}) \quad (30)$$

where

$$F(y_1, \dots, y_{12}) \simeq \int dz_1 dz_2 \prod_i \exp\left(-g' \frac{H}{4} (y_i - z)^2\right) \quad (31)$$

and $y_1, \dots, y_{12}$ are the Landau orbit centers of the external fermions in the (x_1, x_2) plane. This is our final result.

The geometry chosen, and the assumption of a uniform magnetic field within this geometry, represents of course a rather idealized situation. A physical scattering process one could conceive is a baryon number violating processes around superconducting cosmic strings [26]. Baryon number violating processes around such strings (and ordinary grand unified cosmic strings) have been considered earlier [27], but the mechanism discussed here is of an entirely different origin. In the previous computations the essential ingredient was the existence of an unbroken grand unified vacuum at the core of the string, with

corresponding massless baryon number nonconserving gauge bosons. In our case the anomaly-induced baryon and lepton number violation arises solely due to a very strong $U(1)$ magnetic field close to the string. Hence probably only *superconducting* cosmic strings are of interest in this context. Although different geometries would affect our results to some extent, we believe that the key ideas would certainly survive. (In [21] it is proven that the main ingredient — symmetry restoration — also holds for magnetic fields created by a cosmic string.) Of course, the total amount of $B + L$ violation will depend not only on the matrix element we have derived, but also on the density of initial and final particles (i.e. how often their wave functions overlap, both for the direct process and its inverse) and the specific string geometry. But all this goes beyond the scope of the present work.

Although in high-energy laboratory collisions one can also achieve (for very short periods of time) magnetic fields of strength larger than H_{c1} [28], the present mechanism will be of no interest for the “minimal” baryon number violating processes associated with such collisions. In that case all scales are set by m_W , and all factors in the amplitude (30) will essentially cancel, leaving $\sim m_W^4$ which is still to be multiplied by the exponential suppression factor. It is thus clear that the only way the present baryon number violating process can be of substantial magnitude is if the region of strong magnetic field is very large.

To get a very rough idea of the magnitude of the magnetic field around the superconducting cosmic string, we follow ref. [26] and estimate an upper bound for the current that creates the magnetic field of $I \simeq M_X \simeq 10^{20}$ A. This gives a magnetic field near the core $H_{\max} \sim IM_X$ (M_X is the grand unified scale.) Given that the field decreases as $1/r$, the radius R inside which the *electroweak* vacuum is unbroken is [21] $R \sim (M_X/m_W^2)\epsilon \sim 10^{11}$ GeV $^{-1}$. Plugging these estimates into eq.(30) would give a huge enhancement factor; embarrassingly large in fact. The reason is of course to be found in the huge disparity in scales between the breaking/restoration of the electroweak vacuum and the grand unified scale M_X . For the cosmic string this gives rise to an extraordinarily large region (of radius $R \sim M_X/m_W^2$), inside which the anomalous electroweak processes certainly are expected to be enhanced. The large amplitude is partly caused by the infrared cutoff v^{-1} in the broken vacuum being replaced by R (a replacement which involves a huge amplification of scales), partly by effects entirely due to the magnetic field (the factors of H).

Of course, as we have mentioned above, the matrix element derived above is not enough to obtain a final answer, because the result will depend crucially of the density of particles around the string. It should also be added that large matrix elements indicate that higher order corrections (multi-instanton or otherwise) are required to restore unitarity, but it seems more than likely that even the most suppressed baryon and lepton number violating processes occur inside extended regions of large magnetic fields with substantially enhanced rates. Processes involving several gauge bosons and Higgs scalars in the final state should be further enhanced.

In view of the interest in cosmic strings from the point of view of cosmological scenarios, galaxy formation, and bounds on their density on account of their contributions

to baryon number violation [27,29], it seems worthwhile to refine the crude estimates for the amplitudes of the present paper. We hope that our work may stimulate more work in this direction.

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References

- [1] G. 't Hooft, Phys. Rev. Lett. **37** (1976) 8; Phys. Rev. D **14** (1976) 3432.
- [2] R. Noble, Phys. Rev. D **25** (1982) 825.
- [3] A. Linde, Phys. Lett. **B70** (1977) 306.
- [4] V. Kuzmin, V. Rubakov and M. Shaposhnikov, Phys. Lett. **155B** (1985) 36.
- [5] P. Arnold and L. McLerran, Phys. Rev. D **36** (1987) 581.
- [6] H. Aoyama, H. Goldberg and Z. Ryzak, Phys. Rev. Lett. **60** (1988) 1902.
- [7] N. Christ, Phys. Rev. D **21** (1980) 1591.
- [8] N. Manton, Phys. Rev. D **28** (1983) 2019; F. Klinkhamer and N. Manton, Phys. Rev. D **30** (1984) 2212.
- [9] H. Aoyama and H. Goldberg, Phys. Lett. **188B** (1987) 506.
- [10] P. Arnold and L. McLerran, Phys. Rev. D **37** (1988) 1020.
- [11] A. Ringwald, Nucl. Phys. **B330** (1990) 1.
- [12] O. Espinosa, Nucl. Phys. **B343** (1990) 310.
- [13] L. McLerran, A. Vainshtein and M. Voloshin, Phys. Rev. D **42** (1990) 171.
- [14] J. Ellis, R. Flores, S. Rudaz and D. Seckel, Phys. Lett. **194B** (1987) 241.
- [15] A.G. Cohen, M. Dugan and A.V. Manohar, Phys. Lett. **222B** (1989) 41.
- [16] A.H. Mueller, Phys. Lett. **240B** (1990) 414.
- [17] K. Aoki and P. Mazur, preprint UCLA/89/TEP/67.
- [18] M. Cornwall, Phys. Lett. **243B** (1990) 271.
- [19] V. Zakharov, preprint TPI-MINN-90/7-T.
- [20] T. Banks, G. Farrar, M. Dine, D. Karabali and B. Sakita, preprint RU-90-20.
- [21] J. Ambjørn, N.K. Nielsen and P. Olesen, Nucl. Phys. **B310** (1988) 625; J. Ambjørn and P. Olesen, Nucl. Phys. **B315** (1989) 606; Phys. Lett. **218B** (1989) 67; Int. J. Mod. Phys. **A5** (1990) 4525.
- [22] W. Dittrich, J. Phys. **A9** (1976) 1171.
- [23] W. Dittrich and M. Reuter: "Effective Lagrangians in Quantum Electrodynamics", Springer (Berlin) 1985.

- [24] S.Coleman in *Aspects of Symmetry*, pp 347 and ff, Cambridge University Press, Cambridge 1985
- [25] M.H.Johnson and B.A.Lippmann, *Phys. Rev.* **76** (1949) 828
- [26] E. Witten, *Nucl. Phys.* **B249** (1985) 557.
- [27] P. Bhattacharjee, T.W.B. Kibble and N. Turok, *Phys. Lett.* **119B** (1982) 95; R.H. Brandenberger, A.C. Davis and A. Matheson, *Nucl. Phys.* **B307** (1988) 909; R.H. Brandenberger and L. Perivolaropoulos, *Phys. Lett.* **208B** (1988) 396; M.G. Alford, J. March-Russell and F. Wilczek, *Nucl. Phys.* **B328** (1989) 140.
- [28] J. Ambjørn and P. Olesen, Niels Bohr Institute preprint NBI-HE-90-73.
- [29] T.W.B. Kibble, *J. Phys.* **A9** (1976) 1387; A. Vilenkin, *Phys. Rev. Lett.* **46** (1981) 1169; N. Turok, *Nucl. Phys.* **B242** (1984) 520; A. Albrecht and N. Turok, *Phys. Rev. Lett.* **54** (1985) 1868; N. Turok and R. H. Brandenberger, *Phys. Rev. D* **33** (1986) 2175.