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CP violation using non CP eigenstate decays of neutral B mesons

R.Aleksan*, I.Dunietz†, B.Kayser‡, F.Le Diberder†§

Abstract

Neutral B meson decays into non-CP eigenstates are considered as a means for observing CP non conservation. We show that sizeable CP violating asymmetries and clean measurements of the phases of the CKM matrix elements can be obtained using these decays provided they satisfy some necessary conditions that can be experimentally verified. It is also argued that two meson final states, in which the quark content of one meson is the CP conjugate of the quark content of the second one, do fulfill these requirements and are a particularly fertile and promising ground for studying CP violation. Two interesting examples, $B \rightarrow \rho^\pm \pi^\mp$ and $B \rightarrow a_1^\pm \pi^\mp$, are studied in detail. Their reconstruction efficiencies and their backgrounds are discussed for a generic asymmetric B factory detector. It is found that, within this framework, those decay modes can give a better opportunity for observing CP violation and measuring the CKM phases than the $B \rightarrow \pi^+ \pi^-$ decays.

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1 Introduction

The observation of CP violation in the B sector and confronting it with our present understanding of this phenomenon^[1] is going to be one of the most challenging issues in particle physics during this decade. Unfortunately, the expected effects are small for the decay modes that have relatively large branching ratio. On the other hand, interesting channels, in which potentially large effects are to be seen, have small branching ratio^[2]. Among this second class of channels, CP eigenstate final states such as $B \rightarrow \psi K_s^0$ ^[3] or $B \rightarrow \pi^+ \pi^-$ ^[4] are the most promising but the expected number of such events at the future B factories running at the $\Upsilon(4S)$ is not very large. It is therefore very important to study as many B decay modes as possible in order to get the best chances to observe CP violation.

In this paper we shall concentrate on specific neutral B decays into final states that are not CP eigenstates. Section two describes how CP violation can be observed using these decay modes. In section three, we shall discuss what are the particular conditions that these final states should fulfill in order to be useful. The discussion will be illustrated with a typical and interesting example, $B^0 \rightarrow \rho^\pm \pi^\mp$. Finally in section four, the reconstruction efficiencies of this particular final state as well as for $B^0 \rightarrow a_1^\pm \pi^\mp$ and their backgrounds will be studied for a generic B factory detector.

2 CP violation using B^0 decays into non CP eigenstates

In the standard model, every CP-violating effect is the result of an interference between different amplitudes for the same process. When a final state f can come both from a pure B^0 and a pure $\bar{B}^0$, the amplitudes for the direct decay $B^0 \rightarrow f$ and the mixing-induced sequence $B^0 \rightarrow \bar{B}^0 \rightarrow f$ interfere. A time-dependent CP-violating effect can thus appear^[5]. We shall focus in the following on B meson decay modes that can be reached from both a B^0 and $\bar{B}^0$ meson and that are not CP eigenstates. Examples of decays to such final states are given in the next section where they are studied in more detail. Let us for the moment write any such states as f , and suppose that n amplitudes contribute in the decay of a B^0 ($\bar{B}^0$) into f . The overall transition amplitude can be written as :

$$\langle f | T | \bar{B}^0 \rangle = \sum_{j=1}^n e^{i\Phi_j} e^{i\alpha_j} M_j \equiv e^{i\beta} M \quad (1.a)$$

$$\langle f | T | B^0 \rangle = \sum_{j=1}^n e^{-i\Phi_j'} e^{i\alpha_j'} M_j' \equiv e^{-i\beta'} M' \quad (1.b)$$

Here, Φ_j and Φ'_j are the phases coming from the Cabibbo-Kobayashi-Maskawa (CKM) matrix and are at the origin of CP violation in the Standard Model, and α_j and α'_j are the phases due to the strong final state interactions. Finally, M_j and M'_j are the magnitudes of the decay amplitudes. In equations (1), the terms with different j are meant to denote amplitudes with **different** CKM phases. Thus, distinct Feynman diagrams involving the same CKM phase are summed into a single amplitude. It should be noted that one could easily recover the case where f is a CP eigenstate by writing $\Phi'_j = \Phi_j$, $M'_j = M_j$ and $\alpha'_j = \alpha_j$ or $\alpha'_j = \alpha_j + \pi$ according to the nature of f (CP-even or CP-odd). We have used the convention $\text{CP}|B^0\rangle = +|\bar{B}^0\rangle$.

The time-dependent amplitudes for the decay of a neutral B meson created at the time $t_0 = 0$ as a pure $\bar{B}^0$ or as a pure B^0 are, respectively,

$$\begin{aligned} \mathcal{A}(\bar{B}^0 \rightarrow f) &= e^{-\frac{\Gamma}{2}t} e^{-imt} \left[\cos \frac{\Delta m t}{2} \langle f|T|\bar{B}^0\rangle + i \sin \frac{\Delta m t}{2} e^{-2i\Phi_M} \langle f|T|B^0\rangle \right] \\ \text{and} & \\ \mathcal{A}(B^0 \rightarrow f) &= e^{-\frac{\Gamma}{2}t} e^{-imt} \left[\cos \frac{\Delta m t}{2} \langle f|T|B^0\rangle + i \sin \frac{\Delta m t}{2} e^{2i\Phi_M} \langle f|T|\bar{B}^0\rangle \right] \end{aligned} \quad (2)$$

We have defined :

$$\Gamma \equiv \frac{\Gamma_1 + \Gamma_2}{2}, \quad m \equiv \frac{m_1 + m_2}{2}, \quad \Delta m \equiv m_1 - m_2$$

where Γ_i and m_i are the width and the mass of the neutral B mass eigenstates, B_i . Φ_M is the CKM phase that appears in the mixing mechanism. We have assumed $\Delta\Gamma = \Gamma_1 - \Gamma_2 \ll \Gamma$ and $\Delta\Gamma \ll \Delta m$.

It is straightforward to derive the time-dependent decay probabilities using formulae (1) and (2) :

$$\begin{aligned} \text{Pr} \left(\bar{B}^0 \rightarrow f \right) &\propto \left(\frac{A^2}{2} \right) e^{-\Gamma t} \\ &\times \left\{ 1 - R \times \cos \Delta m t - D \sin (2\Phi_M + \beta + \beta') \times \sin \Delta m t \right\} \end{aligned} \quad (3)$$

where $R = \frac{\rho^2 - 1}{\rho^2 + 1}$, $D = \frac{2\rho}{\rho^2 + 1} = \sqrt{1 - R^2}$, $A^2 = M^2 + M'^2$ and $\rho = \frac{M}{M'}$.

Let us now focus on the CP conjugate final state, $\bar{f}$. The decay amplitudes for $B^0 \rightarrow \bar{f}$ and $\bar{B}^0 \rightarrow \bar{f}$ are :

$$\langle \bar{f} | T | B^0 \rangle = \sum_{j=1}^n e^{-i\Phi_j} e^{i\alpha_j} M_j \equiv e^{-i\bar{\beta}} \bar{M} \quad (1'.a)$$

$$\langle \bar{f} | T | \bar{B}^0 \rangle = \sum_{j=1}^n e^{i\Phi_j'} e^{i\alpha_j'} M_j' \equiv e^{i\bar{\beta}'} \bar{M}' \quad (1'.b)$$

These are obtained from the amplitudes for $\bar{B}^0 \rightarrow f$ and $B^0 \rightarrow f$ by applying a CP transformation, under which only the signs in front of Φ_j and Φ_j' change, since CP is conserved in the strong interactions. The decay probabilities are :

$$Pr \left(\bar{B}^0 \rightarrow \bar{f} \right) \propto \left(\frac{\bar{A}^2}{2} \right) e^{-\Gamma t} \times \left\{ 1 + \bar{R} \times \cos \Delta m t - \bar{D} \sin (2\Phi_M + \bar{\beta} + \bar{\beta}') \times \sin \Delta m t \right\} \quad (3')$$

where $\bar{R} = \frac{\bar{\rho}^2 - 1}{\bar{\rho}^2 + 1}$, $\bar{D} = \frac{2\bar{\rho}}{\bar{\rho}^2 + 1} = \sqrt{1 - \bar{R}^2}$, $\bar{A}^2 = \bar{M}^2 + \bar{M}'^2$ and $\bar{\rho} = \frac{\bar{M}}{\bar{M}'}$.

CP invariance demands that the probabilities of CP conjugate processes be identical. Thus, if CP invariance holds, we must have $Pr(\bar{B}^0 \rightarrow \bar{f}) = Pr(B^0 \rightarrow f)$ and $Pr(\bar{B}^0 \rightarrow f) = Pr(B^0 \rightarrow \bar{f})$. These relations must hold for the time-dependent rates and thus for the number of events corresponding to the time-integrated rates (where, for simplicity, equal production of B^0 and $\bar{B}^0$ is assumed) : $N(\bar{B}^0 \rightarrow \bar{f}) = N(B^0 \rightarrow f)$ and $N(\bar{B}^0 \rightarrow f) = N(B^0 \rightarrow \bar{f})$. Thus, CP conservation requires :

$$\bar{M} = M \quad (4.a)$$

$$\bar{M}' = M' \quad (4.b)$$

$$S \equiv \sin(2\Phi_M + \beta + \beta') = -\bar{S} \equiv -\sin(2\Phi_M + \bar{\beta} + \bar{\beta}') \quad (4.c)$$

CP violation occurs if **any** of these three equations is not satisfied. Violation of either of the first two equations is known in the literature as direct CP violation^[6]. The third equation tests CP violation generated by the interference of the direct decay $B^0 \rightarrow f$ and the mixing-induced decay $B^0 \rightarrow \bar{B}^0 \rightarrow f$. From the ratios $N(\bar{B}^0 \rightarrow f)/N(B^0 \rightarrow f)$ and $N(\bar{B}^0 \rightarrow \bar{f})/N(B^0 \rightarrow \bar{f})$, and from the shapes of the four time-dependent decay rates given in equations (3) and (3'), one fits for the four unknowns, ρ , $\bar{\rho}$, S and $\bar{S}$. In addition, the ratio $A^2/\bar{A}^2$ can be extracted from the time-integrated measurements. This ratio is nothing else but $[N(\bar{B}^0 \rightarrow f) + N(B^0 \rightarrow f)]/[N(\bar{B}^0 \rightarrow \bar{f}) + N(B^0 \rightarrow \bar{f})]$. Since $M'^2/\bar{M}'^2 = A^2(1 + \bar{\rho}^2)/\bar{A}^2(1 + \rho^2)$ and $M^2/\bar{M}^2 =$

$M'^2/\bar{M}'^2 \times \rho^2/\bar{\rho}^2$, both equations (4.a) and (4.b) can be verified experimentally. Different values for ρ and $\bar{\rho}$ and/or different values for A^2 and $\bar{A}^2$ would show direct CP violation^[7]. It would also mean that several amplitudes with different CKM phases and with different strong interaction phases have comparable magnitudes in the sums over j in equations (1) and (1'). This would complicate the measurement of the CKM phases and thereby obscure the origin of CP violation. It should be noted that finding that $\rho = \bar{\rho}$ and $A^2 = \bar{A}^2$ does not exclude the possibility (which, however, is unlikely) of having several competing amplitudes with different CKM phases but with almost equal strong interaction phases (see Appendix A).

However, let us now suppose that **one amplitude dominates** in the unmixed $\bar{B}^0 \rightarrow f$ decay, and one in $B^0 \rightarrow f$ decay. In equations (1), we may then keep, say, the term with $j = 1$. By applying a CP transformation, we conclude that the same amplitudes will dominate in the CP conjugate decays. In such a scenario, one gets the following equalities :

$$\bar{M} = M, \quad \bar{M}' = M' \quad (5.a)$$

$$\beta = \Phi_f + \alpha, \quad \beta' = \Phi'_f - \alpha' \quad (5.b)$$

$$\bar{\beta} = \Phi_f - \alpha, \quad \bar{\beta}' = \Phi'_f + \alpha' \quad (5.c)$$

Here, we have defined $\Phi_f \equiv \Phi_1$, $\alpha \equiv \alpha_1$, $\Phi'_f \equiv \Phi'_1$, and $\alpha' \equiv \alpha'_1$. The four previous unknowns have now become only three, $\rho = \bar{\rho} = M/M'$, $S = \sin(2\Phi_M + \Phi_f + \Phi'_f + \Delta\alpha)$ and $\bar{S} = \sin(2\Phi_M + \Phi_f + \Phi'_f - \Delta\alpha)$, where $\Delta\alpha = \alpha - \alpha'$. The sine of the CP violating phase $2\Phi \equiv 2\Phi_M + \Phi_f + \Phi'_f$ can be extracted from the measurements of S and $\bar{S}$, up to a four-fold ambiguity, using :

$$\sin^2 2\Phi = \frac{1}{2} \left[1 + S\bar{S} \pm \sqrt{(1 - S^2)(1 - \bar{S}^2)} \right] \quad (6)$$

Here, one of the signs on the right hand side gives the true $\sin^2 2\Phi$, while the other gives $\cos^2 \Delta\alpha$. How do we resolve this ambiguity ? Consider modes where a theoretical bias for $\Delta\alpha = 0$ or π exists, and where one solution is measured to be compatible with 1. Then, one could attempt to resolve the ambiguity by identifying $\cos^2 \Delta\alpha$ with that solution. Furthermore, one can always use other decay modes (such as CP eigenstates) which yield perhaps a less precise but unambiguous value for $\sin^2 2\Phi$.

Different time-dependent distributions for the $B^0 \rightarrow f$ decay and its CP conjugate $\bar{B}^0 \rightarrow \bar{f}$ decay, or for the $B^0 \rightarrow \bar{f}$ decay and its CP conjugate $\bar{B}^0 \rightarrow f$ decay, would demonstrate in a **model independent** way that CP is not conserved in the B meson decays. CP violation can be simply

exhibited experimentally by adding both final states, f and $\bar{f}$. From (3) and (3') one gets (when one amplitude dominates) :

$$Pr \left(\overset{(-)}{B^0} \rightarrow f \right) + Pr \left(\overset{(-)}{B^0} \rightarrow \bar{f} \right) \propto A^2 e^{-\Gamma t} \quad (7)$$

$$\times \left\{ 1 - \overset{(+)}{D} \cos \Delta \alpha \sin (2\Phi_M + \Phi_f + \Phi'_f) \sin \Delta m t \right\}$$

Since the $\sin \Delta m t$ term in these formulae distinguishes between B^0 and $\bar{B}^0$ decays, this term violates CP conservation. We show in figure 1 this CP violating asymmetry for a particular set of values for ρ , $\sin(2\Phi_M + \Phi_f + \Phi'_f)$ and $\Delta \alpha$.

At this stage, one can make the following remarks.

1. The asymmetry between B^0 and $\bar{B}^0$ decay probabilities will be larger as M and M' tend to be equal. However the asymmetry is not diluted dramatically as the ratio M/M' differs sensibly from 1 (see figure 2).
2. If the B mesons are pair-produced at the $\Upsilon(4S)$, t has to be replaced, in $\sin \Delta m t$ and $\cos \Delta m t$, by $\Delta t = t_2 - t_1$ and by $\Sigma t = t_1 + t_2$ in $e^{-\Gamma t}$ (note that, after integration over Σt , the exponential term is simply $e^{-\Gamma |\Delta t|}$). Here, t_1 and t_2 are the times at which the B^0 and the $\bar{B}^0$ are decaying. One of these decays is the final state f or $\bar{f}$ while the other one is used to tag the flavor of its parent B . It is therefore necessary to know, at least, which of the B mesons has decayed first in order to observe the CP violation effect exhibited by equations (7) (i.e. through the $B^0 \bar{B}^0$ mixing). This can be achieved at an asymmetric B factory^[3,8]. Furthermore, after integrating over Σt and ignoring the time order (i.e. using $|\Delta t|$ instead of Δt) equations (3) and (3') yield

$$Pr \left(\overset{(-)}{B^0} \rightarrow f \right) \propto \left(\frac{A^2}{2} \right) e^{-\Gamma |\Delta t|} \times \left\{ 1 - \overset{(+)}{R} \times \cos \Delta m |\Delta t| \right\} \quad (8)$$

$$Pr \left(\overset{(-)}{B^0} \rightarrow \bar{f} \right) \propto \left(\frac{\bar{A}^2}{2} \right) e^{-\Gamma |\Delta t|} \times \left\{ 1 + \overset{(-)}{\bar{R}} \times \cos \Delta m |\Delta t| \right\} \quad (8')$$

Thus, without the assumption that one amplitude dominates, one can probe direct CP violation in a simple way by measuring independently ρ and $\bar{\rho}$ either from the shapes and/or the relative normalisations of equations (8) and (8') respectively (figure 3).

3 Decays to CP-Self-Conjugate Collections of Quarks

Suppose that the final state f is a pair of mesons a_1 and $\bar{a}_2$ in which the two mesons differ in J^{PC} or mass, so that $a_1\bar{a}_2$ is not a CP eigenstate. Suppose, however, that at the quark level, $a_1\bar{a}_2 = (q_x\bar{q}_y)(\bar{q}_xq_y)$, so that f is made up of a CP-self-conjugate collection of quarks and antiquarks (for example $f = D^{*+}D^-, \rho^+\pi^-, a_1^+\pi^-, K^{*+}K^- \dots$). Then, as we shall argue, f is sufficiently close to being a CP eigenstate that CP violation in $\bar{B}^0 \rightarrow f$ is expected to be large.

As has been said in section two, CP violation is the result of several interfering amplitudes. If the interference is to be a large effect, then the magnitudes M and M' of the interfering $\bar{B}^0$ and B^0 decay amplitudes in equations (2) must be of comparable size. The quantity D in the decay rates will then be near its maximum possible value. Now as pointed out some time ago^[5,9], when f is a CP eigenstate and $|\langle f|T|B^0\rangle|$ is dominated by one CKM factor, M and M' are of equal size. This is one of the reasons why decays to CP eigenstates are of particular interest.

When f is not a CP eigenstate, but is a final state of the type $a_1\bar{a}_2 = (q_x\bar{q}_y)(\bar{q}_xq_y)$, the CP-self-conjugacy of the quark content has the consequence that for each diagram for $B^0 \rightarrow a_1\bar{a}_2$, there is a related one for $\bar{B}^0 \rightarrow a_1\bar{a}_2$ which is just the CP conjugate of the $B^0 \rightarrow a_1\bar{a}_2$ diagram except for the way in which the final quarks are joined up to make hadrons. This is illustrated in figures 4 and 5. Because the $B^0 \rightarrow a_1\bar{a}_2$ diagram and its $\bar{B}^0 \rightarrow a_1\bar{a}_2$ partner are almost CP conjugates, one expects that in general these two diagrams are of comparable size. If some single diagram dominates $B^0 \rightarrow a_1\bar{a}_2$, then its "almost-CP-conjugate" partner is expected to dominate $\bar{B}^0 \rightarrow a_1\bar{a}_2$, and the amplitudes for these two decays should be of comparable size, as desired*. As discussed earlier, when one diagram dominates $B^0 \rightarrow f$ and one dominates $\bar{B}^0 \rightarrow f$, equations (5) apply, and the weak phase $2\Phi = 2\Phi_M + \Phi_f + \Phi'_f$ can be extracted from the data using equation (6). Since the "almost-CP-conjugate" diagrams dominating $B^0 \rightarrow a_1\bar{a}_2$ and $\bar{B}^0 \rightarrow a_1\bar{a}_2$ have equal and opposite weak phases (see, for example, figure 5) $\Phi'_f = \Phi_f$.

A promising example of the final states of the type $(q_x\bar{q}_y)(\bar{q}_xq_y)$ is $\rho^\pm\pi^\mp$. The CKM phase involved in this final state is the same as the one involved in the $\pi^+\pi^-$ mode. This latter mode was considered, up to now, as the

Using the heavy quark techniques^[10,11] in addition to factorization, it has been shown^[12,13] that the diagram which dominates $B^0 \rightarrow D^{-}D^+$ and the "almost-CP-conjugate" one which dominates $\bar{B}^0 \rightarrow D^{*-}D^+$ are of equal size. Furthermore, since only one universal form factor governs the $\bar{B}_d \rightarrow D^*$ and $\bar{B}_d \rightarrow D$ transitions and since the D and D^* decay constants are related, no difference in final state phases is expected, i.e. $\cos^2 \Delta\alpha = 1$. Those modes, then, would measure the same CKM phase as D^+D^- and ψK_s , and complement them.

practical candidate for measuring this particular phase. Since the branching ratio for $B^0 \rightarrow \pi^+\pi^-$ is expected to be small ($\sim 2 \times 10^{-5}$), it is important to find other competitive channels. This is one of the reasons why we are focussing our study on $\rho^\pm\pi^\mp$ (and $a_1^\pm\pi^\mp$, which is a similar mode) in the following. The diagrams expected to dominate the decays into $\rho^\pm\pi^\mp$ are the $(\bar{b})$ quark decay diagrams shown in the top row of figure 5 (comments on the complications that arise from other diagrams will be made in Appendix A). Henceforth, we assume that the $(\bar{b})$ decay diagram dominates and that the rescattering processes such as $B_d^0 \rightarrow D^{*-}D^+ \rightarrow \rho^-\pi^+$ are negligible. Let us discuss the ρ parameter. Under the previous assumptions, the factorization approximation^[14] ought to hold for the $\rho^\pm\pi^\mp$ and $\pi^+\pi^-$ decay modes. One predicts that

$$\frac{\Gamma(\bar{B}_d^0 \rightarrow \rho^-\pi^+)}{\Gamma(\bar{B}_d^0 \rightarrow \pi^+\pi^-)} \approx \left| \frac{F_\rho}{f_\pi} \right|^2 \times \left| \frac{f_+(m_\rho^2)}{f_+(m_\pi^2)} \right|^2 = \left| \frac{F_\rho}{f_\pi} \right|^2 = \left(\frac{190 \text{ MeV}}{132 \text{ MeV}} \right)^2 \simeq 2 \quad (9)$$

Here, the ρ^- meson is made from the virtual W boson, F_ρ and f_π are the decay constants of the ρ and the pion, and the relevant vector form factor for the $\bar{B}_d^0 \rightarrow \pi^+$ transition is denoted by f_+ . Only factorization is required to obtain the quoted numerical value for this ratio, since the decay constant F_ρ is measured from $\tau \rightarrow \rho\nu_\tau$ decay, and the vector form factor f_+ ought to be the same for the two light scales $q^2 = m_\rho^2$ and $q^2 = m_\pi^2$.

In contrast, at present we cannot predict reliably the ratio $\frac{\Gamma(\bar{B}_d^0 \rightarrow \rho^+\pi^-)}{\Gamma(\bar{B}_d^0 \rightarrow \pi^+\pi^-)}$

and therefore $\frac{\Gamma(\bar{B}_d^0 \rightarrow \rho^+\pi^-)}{\Gamma(\bar{B}_d^0 \rightarrow \rho^-\pi^+)} = \frac{M^2}{M'^2} = \rho^2$ will be kept as unknown in the following. However, using many additional assumptions, we estimate the ratio $\Gamma(\bar{B}_d^0 \rightarrow \rho^+\pi^-)/\Gamma(\bar{B}_d^0 \rightarrow \pi^+\pi^-)$ as being either 10 ± 4 or 31 ± 7 in Appendix B.

While we have allowed for final state phases, we believe that there are no such phases involved in the decays of a heavy neutral B meson into two charged light particles. In the factorization approximation, the $q\bar{q}$ system issued from the virtual W is created at the same space-time point. Because of its pointlike creation, the $q\bar{q}$ pair starts out as a color singlet without any color dipole moment. It is a light system which flies off with a large Lorentz boost factor γ ^[15]. Only after having flown a distance of a few Fermis from its creation point does the $q\bar{q}$ system develop a sizeable color dipole moment through which strong interaction could have been felt. But by that time the $q\bar{q}$ system is too far away from the other quark pair to interact appreciably with this other light particle. Furthermore, this latter particle has also travelled a sizeable distance in the opposite direction. In short, no

final state interactions are expected and, thus, **no final state interaction phases**.

We have seen that the ratio of rates $\overline{B}^0 \rightarrow \rho^- \pi^+ / \overline{B}^0 \rightarrow \pi^+ \pi^- = B^0 \rightarrow \rho^+ \pi^- / B^0 \rightarrow \pi^+ \pi^- \simeq 2$ is theoretically well established and not very model dependent. With this in mind, let us study the error on the measurement of $\sin 2\Phi$ using the decay mode $B \rightarrow \rho^\pm \pi^\mp$ as compared to $B \rightarrow \pi^+ \pi^-$, where B stands for the sum of the B^0 and $\overline{B}^0$ mesons. This error, $\sigma[\sin 2\Phi]$, depends on the value of the CP violating asymmetry (cf. equation 7) and the total number of available events. Therefore, neglecting for the moment the relative selection efficiencies and backgrounds, one gets :

$$\frac{\sigma[\sin 2\Phi]_{\rho^\pm \pi^\mp}}{\sigma[\sin 2\Phi]_{\pi^+ \pi^-}} = \sqrt{\frac{N_{\pi^+ \pi^-}}{N_{\rho^\pm \pi^\mp}}} \times \frac{1}{d} \quad (10)$$

where $N_{\pi^+ \pi^-}$ and $N_{\rho^\pm \pi^\mp}$ are the numbers of observed $B \rightarrow \pi^+ \pi^-$ and $B \rightarrow \rho^\pm \pi^\mp$ decays, respectively, and $d = [2\rho/(\rho^2 + 1)] \times \cos \Delta\alpha$ is the dilution factor for $B \rightarrow \rho^\pm \pi^\mp$ (this factor is 1 for $B \rightarrow \pi^+ \pi^-$). If we suppose, as we have argued previously, that $\Delta\alpha = 0$ and acknowledging that

$$N_{\rho^\pm \pi^\mp} \propto A^2 = M'^2(\rho^2 + 1) \propto \Gamma(\overline{B}^0 \rightarrow \rho^- \pi^+) \left(\frac{\Gamma(\overline{B}^0 \rightarrow \rho^+ \pi^-)}{\Gamma(\overline{B}^0 \rightarrow \rho^- \pi^+)} + 1 \right) \quad (11)$$

we obtain :

$$\frac{\sigma[\sin 2\Phi]_{\rho^\pm \pi^\mp}}{\sigma[\sin 2\Phi]_{\pi^+ \pi^-}} = \sqrt{\frac{\rho^2 + 1}{4\rho^2}} \times \sqrt{\frac{\Gamma(\overline{B}^0 \rightarrow \pi^+ \pi^-)}{\Gamma(\overline{B}^0 \rightarrow \rho^- \pi^+)}} \simeq \sqrt{\frac{\rho^2 + 1}{8\rho^2}} \quad (12)$$

We show this ratio as a function of the value of ρ^2 in figure 6. It can be seen from this figure that the error on $\sin 2\Phi$ as measured in the $B \rightarrow \rho^\pm \pi^\mp$ channel is smaller than the one obtained from the $B \rightarrow \pi^+ \pi^-$ channel as long as $\rho^2 \equiv \Gamma(\overline{B}^0 \rightarrow \rho^+ \pi^-) / \Gamma(\overline{B}^0 \rightarrow \rho^- \pi^+)$ is larger than 1/7.

4 Analysis

We address now the issue of the experimental feasibility of using the $\rho^\pm \pi^\mp$ and the $a_1^\pm \pi^\mp$ decays of the B mesons to study CP violation at an asymmetric e^+e^- collider whose center of mass energy is set to the $\Upsilon(4S)$ mass.

The $\Upsilon(4S)$ decay into a $B_d^0 \overline{B}_d^0$ pair involves a p-wave final state which ensures a definite correlation between the decay times of the two mesons^[2]. Having observed at time t_1 the decay of one of the B mesons into a final state tagging the flavor of a $(\overline{B})^0$, and the decay of the second B into the final state

f at time t_2 , the time difference $\Delta t = t_2 - t_1$ follows the distribution given by equations 3, where t is replaced by Δt in the sine and cosine terms and by $|\Delta t|$ in the exponential term. (We assume that $t_1 + t_2$ is not measured.)

The use of an asymmetric e^+e^- collider is prompted by the need of a moving $\Upsilon(4S)$ system in the laboratory frame in order to facilitate the measurement of Δt . More precisely the determination of $\sin 2\Phi$ is performed on the basis of the event distribution with respect to ΔZ , the distance along the beam axis between the vertex of the $B_d^0(\overline{B}_d^0) \rightarrow f$ disintegration and the decay vertex of the tagged B_d^0 or $\overline{B}_d^0$. The two quantities are related according to

$$\Delta Z = \gamma\gamma^{cm}\beta c\Delta t + \gamma\gamma^{cm}\beta^{cm}c \cos\theta_2^{cm}(t_2 + t_1) \quad (13)$$

where c is the light velocity and

- β and γ define the boost which links the $\Upsilon(4S)$ rest frame to the laboratory frame. For the high energy beam of the machine we assume in the following $E_b(e^-) = 9 \text{ GeV}$, which gives $\beta = 0.49$ and $\gamma = 1.14$.
- $\beta^{cm} = 0.066$ and $\gamma^{cm} \simeq 1$ define the boost which links the B rest frames to the $\Upsilon(4S)$ rest frame.
- θ_2^{cm} is the polar angle of the B decaying at time t_2 in the $\Upsilon(4S)$ rest frame.

Although the second term in equation (13) spoils the ideal one-to-one correspondence between ΔZ and Δt that one would obtain with $\beta^{cm} = 0$, its impact on the $\sin 2\Phi$ measurement turns out to be very small. This results from $\langle \cos\theta_2^{cm} \rangle = 0$ and from the $\sin^2\theta_2^{cm}$ angular distribution which favors small values of $\cos\theta_2^{cm}$. Moreover, the final state f being completely reconstructed, the value of θ_2^{cm} is available and can be used to limit even further the dilution of the information. A detailed analysis shows that the increase on the resolution $\sigma[\sin 2\Phi]$ induced by the non zero value of β^{cm} is given by the factor

$$\Sigma_{\beta^{cm}} = 1 + \left(\frac{\beta^{cm}}{\beta}\right)^2 \frac{2x^2 + 1}{20} \quad (14)$$

for $\beta^{cm}x \ll \beta$, where $x = \Delta m/\Gamma$ is the B_d^0 mixing parameter. For $x = 0.7$ one gets the negligible correction $\Sigma_{\beta^{cm}} = 1.002$. Hence, one can safely ignore the non zero value of β^{cm} and use the approximate relation $\Delta Z = \beta\gamma c\Delta t$.

Using $\tau_d = 1.18 \cdot 10^{-12} \text{ s}$ for the B_d lifetime, the average longitudinal distance between the two B decays is $L_{lab} = \langle \Delta Z \rangle = \beta\gamma c\tau_d \simeq 200 \mu$. For a typical^[3] detector resolution $\sigma[\Delta Z] = 55 \mu$, the inequality $L_{lab} \gg \sigma[\Delta Z]$ is not satisfactorily satisfied. However, the quantity which governs the measurableness of the CP-violating effect is not L_{lab} but the wave length of

the CP violating component of the ΔZ -distribution, $\lambda_{CP} = \frac{2\pi}{\sigma} L_{lab} \simeq 1800 \mu$. For a 9 GeV high energy beam (but also for a smaller machine asymmetry) the relevant inequality $\lambda_{CP} \gg \sigma[\Delta Z]$ is satisfied. Hence, the quality of the vertex resolution of the detector is not critical for an experiment at an asymmetric collider. Accordingly, the algorithm used to measure ΔZ will not be discussed (details can be found in Reference [3]).

For small values of $\sin 2\Phi$ (≤ 0.5) and assuming that $\cos \Delta\alpha \approx 1$, equation (6) simplifies to :

$$\sin^2 2\Phi = \left(\frac{S + \bar{S}}{2 \cos \Delta\alpha} \right)^2 \quad (15)$$

$$\text{with } \cos \Delta\alpha \simeq \sqrt{1 - \left(\frac{S - \bar{S}}{2} \right)^2}.$$

The measurement of $\sin 2\Phi$ is obtained from the shape of the four distributions (3 and 3') using the likelihood method. The overall resolution is :

$$\sigma^2[\sin 2\Phi] \simeq \frac{1 + 4x^2}{2x^2} d^{-2} \frac{1 + r\eta_Z}{N} (1 - 2\omega)^{-2} e^{\sigma_t^2(x^2 + \frac{1}{2})} \times [1 + \mathcal{O}(\sin^2 2\Phi)] \quad (16)$$

where

- $d = \frac{2\rho}{\rho^2 + 1} \cos \Delta\alpha$ is the dilution factor defined in section three.
- N is the total number of selected $B_d^0 \bar{B}_d^0$ events.
- r is the ratio of the number of selected background events over the number of selected signal events.
- η_Z is an effective rejection factor resulting from the shape differences between the ΔZ distributions of background and signal events. The η_Z value depends on the resolution $\sigma[\Delta Z]$. For example, with $\sigma[\Delta Z] = 55 \mu m$, one has $\eta_Z \simeq 0.20$ for the continuum $q\bar{q}$ background. The conservative value $\eta_Z = 0.5$ is used below.
- ω represents the fraction of $B_d^0 \bar{B}_d^0$ events where the flavor of the tagged B is wrongly identified.
- The exponential term accounts for the loss in resolution due to the finite vertex resolution, expressed in unit of L_{lab} : $\sigma_t = \sigma[\Delta Z]/L_{lab} \simeq 0.3$, with our numerical values. For $x = 0.7$, the vertex resolution correction term on $\sigma[\sin 2\Phi]$ is then very small ; 4%.

The contribution to $\sigma^2[\sin 2\Phi]$ of the measurement error on ρ enters in the $\mathcal{O}(\sin^2 2\Phi)$ term of equation (16). For $x = 0.7$, $\sigma[\rho]/\rho$ goes from $\sim 4\%$ when $\rho = 1$ to $\sim 12\%$ when $\rho = 5$ (for $N = 10^3$ events).

The study presented below was carried out using the Monte-Carlo program simulating a generic B detector as described in Reference [4]. The most relevant detector characteristics assumed in the simulation are the following. The tracking device consists of a Drift Chamber completed with a silicon vertex detector. The transverse momentum resolution is $\sigma(p_\perp)/p_\perp^2 = (0.23^2 + 0.27^2/(p\beta_c)^2(p_z/p_\perp)^3)^{1/2}$ ($\% \text{ GeV}^{-1}$), where $p_\perp$, p , and p_z are the transverse, total and longitudinal momentum of the charged particle and β_c is its velocity. The vertex resolution in the longitudinal direction is $\sigma = (17^2 + 25^2/(p\beta_c)^2(p_z/p_\perp)^5)^{1/2}$ (μm). The calorimeter is made of 18 radiation length CsI crystals. The angular and energy resolutions for the electromagnetic showers are respectively $\sigma(\theta) = \sigma(\phi) = 10 \text{ mrd}$ and $\sigma(E)/E = ((0.02/E^{1/4})^2 + 0.01^2)^{1/2}$ (E in GeV). Both the tracking device and the calorimeter cover the polar angle region $|\cos\theta| < 0.95$. For the sake of simplicity the particle identification capabilities are assumed to be perfect in this analysis. More details can be found in Reference [4].

Although the analysis implies the use of $\rho^+\pi^-$ together with $\rho^-\pi^+$ events and $a_1^+\pi^-$ together with $a_1^-\pi^+$ events, as a matter of convention we refer to these final states as $\rho^-\pi^+$ and $a_1^-\pi^+$ in the following. In distinction to the notation used in the previous sections, we will denote the generic final states $\rho^\pm\pi^\mp$ and $a_1^\pm\pi^\mp$ by f in this section. In both final states, the observed signal consists of 4 particles; 2 charged pions and 2 photons for the $\rho^-\pi^+$ mode, 4 charged pions for the $a_1^-\pi^+$ mode. In the latter case, we have restricted the analysis to the charged-pions decay of the a_1 ($a_1^- \rightarrow \pi^-\rho^0 \rightarrow \pi^-\pi^+\pi^-$) because events from the single charged pion decay of the a_1 involve two π^0 and are much more difficult to extract experimentally. Both final states involve a π^+ with a well defined high-momentum in the Υ rest frame: $P_{\pi^+} \simeq M_{\Upsilon(4S)}/4$. Because of this particularity, the background coming from B decays is very effectively suppressed. The most important background to these final states is due to continuum $q\bar{q}$ events, which, accordingly, are the only ones considered below.

In addition to the previously quoted values, the following numbers have been used in our analysis :

- $q\bar{q}$ continuum cross-section at the $\Upsilon(4S)$ center of mass energy :
 $\sigma_{q\bar{q}} = 2.7 \text{ nb}$.
- $e^+e^- \rightarrow \Upsilon(4S)$ cross-section : $\sigma_{\Upsilon(4S)} = 1.2 \text{ nb}$.
- Branching ratios

1. $\Upsilon(4S)$ into $B_d^0 \bar{B}_d^0$ pairs : $B_r(B_d^0) = 50\%$.
2. B_d^0 into $\pi^- \pi^+$: $B_r(\pi^- \pi^+) = 2 \cdot 10^{-5}$.
3. B_d^0 into $\rho^- \pi^+$ or $\rho^+ \pi^-$: $B_r(\rho^\mp \pi^\pm) = 6 \cdot 10^{-5}$.
4. B_d^0 into $a_1^- \pi^+$ or $a_1^+ \pi^-$: $B_r(a_1^\mp \pi^\pm) = 6 \cdot 10^{-5}$.
5. $a_1^\mp$ into $\rho^0 \pi^\mp$: 50%.

- Dilution factor : $d = 0.94$ ($\rho = 1/\sqrt{2}$ and $\Delta\alpha = 0$).

The quoted values for $B_r(\rho^\mp \pi^\pm)$ and $B_r(a_1^\mp \pi^\pm)$ are chosen as an example. The dependence of $\sigma[\sin 2\Phi]$ on these branching ratios is presented at the end of this section.

The following quantities are used to select the $\rho^- \pi^+$ and $a_1^- \pi^+$ final states.

- $\delta W_f = |W_f - M_B|$, where W_f is the invariant mass formed by the four particle 4-momenta, and M_B is the B mass. The experimental resolutions on W_f are $\sigma[W_f] = 31 \text{ MeV}$ for the $\rho^\pm \pi^\mp$ events and $\sigma[W_f] = 17 \text{ MeV}$ for the $a_1^\pm \pi^\mp$ events. The larger $\rho^\pm \pi^\mp$ resolution results from the angular and energy resolutions attached to the two photons of the π^0 decay which are significantly poorer than the ones of the charged pions.
- $\delta \bar{W}_f = |\bar{W}_f - M_B|$ with $\bar{W}_f^2 = M_{\Upsilon(4S)}^2/4 - \mathbf{P}_{cm}^2$ and where $\mathbf{P}_{cm}$ is the total 3-momentum of the four particles in the $\Upsilon(4S)$ rest frame. The experimental resolutions on $\bar{W}_f$ are essentially determined by the beam energy spread of the machine (assumed to be $\sigma[E_b(e^\pm)] = 7 \cdot 10^{-4} \times E_b(e^\pm)$). The resolutions are therefore almost the same, $\sigma[\bar{W}_f] \simeq 3 \text{ MeV}$, for both the $\rho^\pm \pi^\mp$ events and $a_1^\pm \pi^\mp$ events.
- $\delta \rho = |W_{\pi\pi} - m_\rho|$, where m_ρ is the ρ mass and where $W_{\pi\pi}$ is the invariant mass of the two pions which should originate from a ρ decay : $\rho^- \rightarrow \pi^- \pi^0$, for the $\rho^\pm \pi^\mp$ events and $\rho^0 \rightarrow \pi^+ \pi^-$, for the $a_1^\pm \pi^\mp$ events.
- $\delta a_1 = |W_{\pi-\rho^0} - m_{a_1}|$ (defined for the $a_1^- \pi^+$ final state), where m_{a_1} is the a_1 mass and where $W_{\pi-\rho^0}$ is the invariant formed by the π^- and a pair of charged pions attributed to a ρ^0 decay.
- $\delta \pi^0 = |W_{\gamma\gamma} - m_{\pi^0}|$ (defined for the $\rho^- \pi^+$ final state), where m_{π^0} is the mass of the π^0 and where $W_{\gamma\gamma}$ is the invariant mass formed by a photon pair, candidate for a π^0 decay. The experimental resolution is $\sigma[W_{\gamma\gamma}] = 7.5 \text{ MeV}$. The typical value of the smallest of the two photon energies is $E_{min}^\gamma \simeq 0.4 \text{ GeV}$ (cf figure 7) and is well within the foreseen capabilities of future B factory detectors.

- P_f^v the χ^2 probability of the fit constraining the charged particles of the final state f to share the same vertex. This quantity is particularly useful in the case of the $a_1^\pm \pi^\mp$ final state, where four charged particles are involved. For example, the cut $P_f^v > 0.05$ rejects 45% of the $q\bar{q}$ background to this final state.
- N_c the number of charged particles detected in addition to the charged particles of the final state f .
- $\cos \theta_+^{cm} = | \vec{u}^+ \cdot \vec{u}_{ph} |$, where $\vec{u}^+$ is the unit vector along the direction of the high-energy π^+ and where $\vec{u}_{ph}$ is a unit vector pointing along the sphericity axis of the final state (the four particles of the final state f are excluded). For $q\bar{q}$ background events, the high energy π^+ direction tends to be close to the sphericity axis ($\cos \theta_+^{cm} \simeq 1$), while the two directions are uncorrelated for the signal events.

The preselection is obtained by applying the following cuts :

$$\begin{aligned}
 \rho^- \pi^+ &: \delta W_f < 100 \text{ MeV} , \delta \bar{W}_f < 10 \text{ MeV} , \delta_\rho < 300 \text{ MeV} \\
 &\delta_{\pi^0} < 20 \text{ MeV} , P_f^v > 0.05 , N_c \geq 3 \\
 a_1^- \pi^+ &: \delta W_f < 50 \text{ MeV} , \delta \bar{W}_f < 10 \text{ MeV} , \delta_\rho < 300 \text{ MeV} \\
 &\delta_{a_1} < 660 \text{ MeV} , P_f^v > 0.05 , N_c \geq 3
 \end{aligned}$$

The overall efficiencies of these cuts are $\epsilon_{\rho\pi} = (58.5 \pm 0.5)\%$ and $\epsilon_{a_1\pi} = (30 \pm 0.3)\%$, respectively. The latter value includes the 50% reduction factor coming from the $a_1^- \rightarrow \rho^0 \pi^-$ branching ratio. The rejection factors on the $q\bar{q}$ background events are $\eta_{\rho\pi} = (1.7 \pm 0.1) \times 10^{-4}$ and $\eta_{a_1\pi} = (1.8 \pm 0.1) \times 10^{-4}$, respectively. The above quoted errors reflect the limited Monte-Carlo statistics used ($2 \cdot 10^6$ generated $q\bar{q}$ Monte-Carlo events). At this stage of the preselection, the detectable signal events have been almost completely preserved.

In addition, the remaining N_c charged particles not associated with the final state f must allow a $B_d^0/\bar{B}_d^0$ tagging. This tagging is done by requiring the presence of either one high-energy lepton (electron or muon) or one charged Kaon. The minimum lepton energy in the $\Upsilon(4S)$ rest frame is taken as $E_{min}^l = 1.4 \text{ GeV}$. The lepton-tagging efficiency is $\epsilon^l = 11\%$. The fraction of the lepton-tagged B which are wrongly identified is $\omega^l = 4\%$. The rejection factor on the $q\bar{q}$ background associated with a lepton-tagging is taken as $\eta^l = 1\%$, although, in the limit of a perfect particle identification, the Monte-Carlo simulation gives $\eta^l = (.3 \pm .1)\%$. The corresponding tagging efficiency using Kaons, including the losses due to the decays in flight, is $\epsilon^K = 35\%$. The fraction of the Kaon-tagged B which are wrongly identified is $\omega^K = 7\%$. The background rejection factor associated with the kaon-tagging

is $\eta^K = 25\%$. Depending on the lepton or kaon-tagging mode of the B_d^0 , the background/signal ratios are $r_{\rho\pi}^l = 1$, $r_{\rho\pi}^K = 8$, $r_{a_1\pi}^l = 2$ and $r_{a_1\pi}^K = 16$.

To optimize the $\sigma[\sin 2\Phi]$ resolution one must improve the rejection factors and, inevitably, one must accept also a reduction of the selection efficiencies. For a given tagging mode and a given final state, let us define ϵ_* and η_* as the efficiency and the background rejection factor associated with the final cuts of the selection, and r_o as the background/signal ratio obtained at the preselection level, before these final cuts are applied. In order to optimize $\sigma[\sin 2\Phi]$, it can be seen, from equation (16), that the final cuts must minimize the quality factor :

$$Q = \frac{\epsilon_* + r_o \eta_*}{\epsilon_*^2} \quad (17)$$

Five distributions are available for this purpose, the $\cos \theta_+^{cm}$ distribution and the δ -invariant mass distributions. The shapes of the latter distributions and the correlations between the variables have not been used in the preselection. However, for most of the selected $q\bar{q}$ events, at least one of the δ -invariant masses is close to the preselection cut. This is in contrast with the signal events, for which all the δ -invariant masses tend to be close to zero. This information can be used to reduce the rejection factor. In order to perform easily the optimization of Q , we express the 5-dimension distribution in term of a single variable distribution as follows. For a given event i , let us define f^i , the fraction of Monte-Carlo signal events for which **all** the variables $\cos \theta_+^{cm}$, δW_f , $\delta \bar{W}_f$, $\delta \rho$ and $\delta \pi^o$ (or δa_1) have a larger value than the corresponding ones of the i -event. Having calculated f^i , we define for the same i -event, the R_{Rarity}^i variable as being the probability, for a Monte-Carlo signal event, to be associated with a **smaller** f^i . Hence, the smaller the R_{Rarity}^i , the more likely the i -event is to be a background event. In other words, the R_{Rarity} provides a definition of the probability for an event to be a signal event. The R_{Rarity} distribution of the $q\bar{q}$ background events for the $\rho^-\pi^+$ channel is shown in figure 8 (lepton and Kaon tagged events are combined). The dashed line indicates the signal-event distribution; it is flat by definition of the R_{Rarity} variable. Both background and signal distributions correspond to an integrated luminosity of 50 fb^{-1} (970 selected $\rho^\pm\pi^\mp$ signal events). The shape of the R_{Rarity} distribution of the $q\bar{q}$ background for $a_1^\pm\pi^\mp$ events turns out to be very similar.

Depending on the tagging mode of the B_d^0 , the minimal values of Q are found to be : $Q^l(\rho\pi) = 1.2$, $Q^K(\rho\pi) = 1.9$, $Q^l(a_1\pi) = 1.4$ and $Q^K(\rho\pi) = 2.5$. The corresponding $\sin 2\Phi$ resolutions, for an integrated luminosity of 50 fb^{-1} , are $\sigma[\sin 2\Phi] = 0.15, 0.11, 0.23$ and 0.18 , respectively. The various numerical values obtained in our analysis are summarized in tables 1 to 4. We also give in these tables the corresponding values for the $\pi^+\pi^-$ mode for comparison.

	$\rho^\pm\pi^\mp$	$a_1^\pm\pi^\mp$	$\pi^+\pi^-$
ϵ	58%	30%	43%
η	$1.7 \cdot 10^{-4}$	$1.8 \cdot 10^{-4}$	10^{-6}

Table 1: Preselection efficiencies and background rejection factors for the three decay modes.

	Lepton-tagging	Kaon-tagging
ϵ	11%	35%
η	1%	25%
ω	4%	7%

Table 2: Lepton and Kaon tagging efficiencies, background rejection factors and wrong identification probabilities.

	$\rho^\pm\pi^\mp$		$a_1^\pm\pi^\mp$		$\pi^+\pi^-$	
	Lepton	Kaon	Lepton	Kaon	Lepton	Kaon
r_0	1	8	2	16	0.02	0.18
r_{eff}	0.13	0.42	0.27	0.57	0.01	0.09
ϵ_*	0.93	0.76	0.89	0.62	1	1
$\sqrt{Q}-1$	10%	40%	20%	60%	0.6%	5%

Table 3: Background/signal ratios at the preselection level(r_0), effective background/signal ratios ($r_{eff} = r_0\eta_Z\eta_*/\epsilon_*$), selection efficiencies of the final stage of the selection (ϵ_*) and increase on $\sigma[\sin 2\Phi]$ due to the background ($\sqrt{Q}-1$).

	$\rho^\pm\pi^\mp$	$a_1^\pm\pi^\mp$	$\pi^+\pi^-$
ϵ_{tot}	21.5%	9.5%	20%
B_r	$6 \cdot 10^{-5}$	$6 \cdot 10^{-5}$	$2 \cdot 10^{-5}$
N_{tot}	775	340	240
$\sigma[\sin 2\Phi]$	0.09	0.14	0.14

Table 4: Overall selection efficiencies (ϵ_{tot}), assumed branching ratios (B_r), numbers of signal events for a 50 fb^{-1} integrated luminosity (N_{tot}) and the corresponding resolutions $\sigma[\sin 2\Phi]$.

Combining the measurements of the lepton and Kaon tagging modes, one gets $\sigma[\sin 2\Phi](\rho\pi) = 0.09$ and $\sigma[\sin 2\Phi](a_1\pi) = 0.14$. These results should be compared with the resolution expected with the $\pi^+\pi^-$ events. The $\pi^+\pi^-$ event selection is similar to the one described in Reference [4], and close to the $\rho^\pm\pi^\mp$ and $a_1^\pm\pi^\mp$ selection. It yields $\epsilon_{\pi\pi} = 0.43$ and $\eta_{\pi\pi} = 10^{-6}$, which give

$\sigma[\sin 2\Phi](\pi\pi) = 0.14$. A much smaller reduction factor $\eta_{\pi\pi}$ can be achieved at the price of a significantly smaller selection efficiency. However, the quality factor, which has not been optimized here, is close to one and would increase with the use of more stringent cuts.

Hence, for the particular values of the branching ratios assumed above, the measurement of $\sin 2\Phi$ using the $\rho^\pm\pi^\mp$ final states is more accurate than the one using the $\pi^+\pi^-$ mode. Combining the above resolutions, one gets

$$\sigma[\sin 2\Phi](\pi^+\pi^- + \rho^\pm\pi^\mp + a_1^\pm\pi^\mp) = 0.07 \quad .$$

Compared to the $\pi^+\pi^-$ resolution only, it represents a factor 4 improvement on the luminosity needed to reach a given level of precision on $\sin 2\Phi$. The expected resolutions on $\sin 2\Phi$ as a function of the branching ratios $B_r(\rho^\pm\pi^\mp)$ and $B_r(a_1^\pm\pi^\mp)$ are shown on figure 9. The dotted lines indicated the resolution that would be achieved in the absence of background ($Q = 1$).

5 Conclusion

We have shown that neutral B mesons decaying into non-CP eigenstates are very interesting reactions for studying CP violation in particular when these final states are made out of two mesons for which the quark content of one meson is the CP conjugate of the quark content of the second one. Provided this condition is fulfilled, sizeable CP violating asymmetries can be expected and the measurement of the CKM phases is possible. Two interesting examples of such decays, $B \rightarrow \rho^\pm\pi^\mp$ and $B \rightarrow a_1^\pm\pi^\mp$, have been studied in detail in terms of their reconstruction efficiencies and their background using a generic asymmetric B factory detector. It is found that these two modes can indeed be efficiently used for probing CP non-conservation and for measuring the CKM phases. Furthermore, these reactions can turn out to be better candidates than the final state $B \rightarrow \pi^+\pi^-$.

Appendix A

In the main body of this paper, we neglected rescattering and penguin diagrams (see the bottom row of figure 5). This appendix discusses some physical effects arising when those diagrams are not negligible. When the magnitudes of these diagrams are comparable to the magnitude of the $\langle \bar{b} \rangle$ quark decay one, we would expect that direct CP violation occurs, i.e. $M \neq \bar{M}$ and $M' \neq \bar{M}'$. The reason is that different CKM phases are involved, and there is no reason to believe that the final state phases of the rescattering and penguin diagrams and the $\langle \bar{b} \rangle$ decay one should be the same. In section three, we have argued that the final state phase of the $\langle \bar{b} \rangle$ quark decay diagram vanishes, but this argument by no means suggests that the same is true of other diagrams. However, if for some reason the final state phases of the rescattering and penguin diagrams do vanish, then $\alpha_j = \alpha'_j = 0$ for all j in equations (1) and (1') and we have $M = \bar{M}$ and $M' = \bar{M}'$. Nevertheless, since the various contributing diagrams have different CKM phases, when one measures the time-dependent asymmetry for, say, $B^0 \rightarrow \rho^+ \pi^-$ versus $\bar{B}^0 \rightarrow \rho^- \pi^+$, one does not measure $\sin 2\alpha^\dagger$ but rather a different, unknown combination of CKM elements. To guard against such a wrong interpretation of the measurements (that is in terms of $\sin 2\alpha$), one must be able to disentangle penguin and rescattering contributions from the $\langle \bar{b} \rangle$ quark decay ones. Such efforts have recently begun to appear in the literature^[17].

Appendix B

In this Appendix, we estimate the decay rate for $\Gamma(\bar{B}_d^0 \rightarrow \rho^+ \pi^-)$ relative to the $\Gamma(\bar{B}_d^0 \rightarrow \pi^+ \pi^-)$ one. Neglecting penguin and rescattering contributions, factorization ought to hold for color-allowed two-body B-decays into light mesons^[14]. With this assumption, the ratio of rates is given in terms of form factors

$$\frac{\Gamma(\bar{B}_d^0 \rightarrow \rho^+ \pi^-)}{\Gamma(\bar{B}_d^0 \rightarrow \pi^+ \pi^-)} \approx \frac{|a + f|^2}{4m_\rho^2 |f_+|^2} \quad (\text{B.1})$$

where the π^- is made from the virtual W boson. Here the form factors are defined by :

$$\begin{aligned} \langle \rho^+ | \mathcal{J}_\mu | \bar{B}^0 \rangle &= ig \epsilon_{\mu\alpha\beta\gamma} \epsilon^{*\alpha} (P_B + P_\rho)^\beta (P_B - P_\rho)^\gamma - f \epsilon_\mu^* \\ &\quad - a_+ \epsilon^* \cdot P_B (P_B + P_\rho)_\mu - a_- \epsilon^* \cdot P_B (P_B - P_\rho)_\mu \end{aligned} \quad (\text{B.2})$$

[†]Where α is one of the angles of the unitarity triangle^[16].

and

$$a \equiv a_+(m_B^2 - m_\rho^2) + a_- m_\pi^2 \approx a_+(m_B^2 - m_\rho^2) \quad (\text{B.3})$$

The 4-momenta of the B and ρ mesons are denoted by P_B and P_ρ respectively. The numerator of equation (B.1) depends upon axial form factors of the transition $\overline{B}_d^0 \rightarrow \rho^+$ whereas the denominator depends on the vector form factor for $\overline{B}_d^0 \rightarrow \pi^+$. All the form factors are evaluated at very low $q^2 = m_\pi^2$. Many assumptions and indirect pieces of information are used to estimate this ratio to be 10 ± 4 or 31 ± 7 . Let us first list them and then discuss how they are used to obtain our estimate.

1. We use the work of Isgur and Wise where they show how the unknown form factors of $B \rightarrow K^*$ can be related to the measured ones of $D \rightarrow K^*$, when the 4-velocity transfers of the two transitions are the same^[18].
2. A universal q^2 -dependence is used for the form factors in all the $B \rightarrow X$ transitions^[14], where X denotes any light meson ρ, π, K^*, K .

$$\text{form factor}^{B \rightarrow X}(q^2) = \frac{\text{form factor}^{B \rightarrow X}(q^2 = 0)}{1 - \frac{q^2}{m_B^2}} \quad (\text{B.4})$$

3. Factorization is assumed to hold for the two decays $B \rightarrow K^* \psi$ and $B \rightarrow K \psi$. That yields information on the form factors for $B \rightarrow K^*$ and $B \rightarrow K$ at $q^2 = m_\psi^2$ ^[14].
4. ARGUS found that, for large ψ momenta, the decay $B \rightarrow \psi X$ proceeds mainly through its helicity zero amplitude^[19].
5. SU(3) invariance holds when relating $B \rightarrow K^*$ to $B \rightarrow \rho$ form factors, if the two transitions have the same 4-velocity transfer :

$$\text{form factor}^{B \rightarrow K^*} = \text{form factor}^{B \rightarrow \rho} \quad (\text{B.5})$$

Whereas the vector form factors of $D \rightarrow K$ and $D \rightarrow K^*$ are in agreement with theory, the axial-vector form factors of $D \rightarrow K^*$ are not. Experiment E691 measures amongst others the axial-vector form factors, f and a_+ , and finds them to be lower than what existing theoretical models predict^[20].

Our objective is to numerically estimate the ratio in equation (B.1). The denominator depends on the vector form factor $f_+^{B \rightarrow \pi}$. As no inconsistencies between theory and experiment have yet been found for the vector form factors, we use the theoretical estimate of Reference [14]. Conditional support for this choice comes from the $B \rightarrow K \psi$ decay. Under assumption (3), only one form factor -that of the vector $f_+^{B \rightarrow K}$ - is involved in the $B \rightarrow K \psi$ decay. The form factor $f_+^{B \rightarrow K}$ can thus be estimated from the $B \rightarrow K \psi$ rate^[21]

and agrees well with the predictions of Reference [14]. What is even more fortunate is that we measure $f_+^{B \rightarrow K}$ at a rather low $q^2 = m_\psi^2$. This $B \rightarrow K$ form factor is closely related to the $f_+^{B \rightarrow \pi}$ at $q^2 = 0$, which must be known in order to estimate the denominator in the ratio of equation (B.1). Thus, we have some confidence in using the result of Reference [14] for this vector form factor and use $f_+^{B \rightarrow \pi}(q^2 = 0) = 0.333$.

In contrast, the numerator depends on the axial form factors $a + f$, where experiment and theory disagree. Thus, to estimate the form factors in the numerator, we wish to maximize experimental and minimize theoretical input. We start by utilizing assumption (1) and relate f and g for the $B \rightarrow K^*$ transition at $q_0^2 = 16.5 \text{ GeV}^2$ to the $D \rightarrow K^*$ one at $q^2 = 0$. This step is fairly sound^[22]. However, scaling the form factors f and g from q_0^2 down to $q^2 = m_\psi^2$, with hypothesis (2), is riskier, but not absurd as the value $q_0^2/m_B^2 \simeq 0.6$ and is not too close to the poles.

Assuming factorization (assumption (3)) the rate measurement of $B \rightarrow K^*\psi$ involves all the three form factors f, g , and a_+ . Only a_+ is unknown, and is obtained by solving a quadratic equation, and thus two solutions are obtained^[23]. Having a_+, f , and g at $q^2 = m_\psi^2$, hypothesis (2) scales them down to $q^2 = 0$. From assumption (5), i.e. SU(3) invariance, we obtain the form factor $a + f$ for the $B \rightarrow \rho$ transition. Hypothesis (2) yields this form factor at the desired $q^2 = 0$ scale.

This analysis assumes that the errors are gaussians and uses the inputs below:

- The $D \rightarrow K^*$ measurements^[20] :

$$V(0) = 0.9 \pm 0.3 \pm 0.1 \quad , \quad A_1(0) = 0.46 \pm 0.06 \pm 0.03 \quad (\text{B.6})$$

where V and A_1 are proportional to g and f respectively, up to different mass factors.

- The $B \rightarrow K^*\psi$ rate and the B meson lifetime^[24] :

$$B_r(B \rightarrow K^*\psi) = 0.11 \times 10^{-2} \pm 35\% \quad (\text{B.7})$$

$$|V_{cb}|^2 \tau_B = 3.6 \times 10^9 \text{ GeV}^{-1} \pm 40\% \quad (\text{B.8})$$

Without incorporating assumption (4), we get for the ratio of equation (B.1) either 4 ± 4 or 20 ± 9 . This prediction allows tiny values for this ratio and is not much of a prediction at all.

We can do better than that by incorporating the information from ARGUS (assumption (4)). Even though the $B \rightarrow K\psi$ component always has $\lambda = 0$, we still note that the rates for $B \rightarrow K^*\psi$ are larger than $B \rightarrow K\psi$ and take the finding of ARGUS as indicating that $\lambda = 0$ dominates $B \rightarrow K^*\psi$ decays.

Incorporating this fact in a standard way, one gets the numerical estimates 10 ± 4 or 31 ± 7 .

Some of the assumptions involve larger uncertainties than others. We cannot justify the use of hypothesis (2), as many poles ought to be included. However, a recent calculation by J.L.Rosner hints that when one fits the $b \rightarrow c$ transitions to one universal pole, this universal pole is lower than expected^[12]. That is the reason why our pole was taken as m_B^2 and not at a higher mass. As for hypothesis (3), rescattering from colour-allowed processes, which have a much larger branching rate, such as $B_d^0 \rightarrow D_s^+ D^{*-} \rightarrow K^{*0} \psi$ can invalidate it. Assumption (5) probably holds to 20%, that is supposedly the level where SU(3) breaking terms become important^[25].

It should be noted that were the K and K^* treated as heavy mesons, the Isgur-Wise symmetry limit^[11] applied to the $B \rightarrow K^{(*)}$ transition and were assumption (3) made, one would predict :

$$B_r(B_d^0 \rightarrow K^{*0} \psi) / B_r(B_d^0 \rightarrow K^0 \psi) \approx 3 \quad (\text{B.9})$$

and

$$\frac{\Gamma[B_d^0 \rightarrow K^{*0}(\lambda=0) \psi(\lambda=0)]}{\Gamma[B_d^0 \rightarrow K^{*0} \psi]} = 0.43 \quad (\text{B.10})$$

Since equation (B.10) disagrees with the ARGUS measurement (assumption (4)), this approach was not used in estimating the ratio of equation (B.1).

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- [23] This quadratic equation depends, among other things, on $Re(fa_+^*)$. For an exclusive transition, such as the $B \rightarrow K^*$ one considered here, the relative phase between f and a_+ is probably 0 or π .
- [24] We thank S. Stone for providing us the latest CLEO, ARGUS and ALEPH results which were used here.
- [25] We thank Mark Wise for informing us about his educated guess.

$$\sin(2\phi + \phi_f + \phi'_f) = .4, \quad \frac{\Delta m}{\Gamma} = .7, \quad \rho = 2, \quad \Delta\alpha = 0$$

Fig. 1 : Evolution, in proper time units, of the decay probabilities for the sum of the reactions $B^0 \rightarrow f$ and $B^0 \rightarrow \bar{f}$ on the one hand, and reactions $\bar{B}^0 \rightarrow \bar{f}$ and $\bar{B}^0 \rightarrow f$ on the other hand. Here, $f = \rho^+ \pi^-$ and $\bar{f} = \rho^- \pi^+$ are chosen as an example.

Fig. 2 : The CP violating asymmetry dilution factor D as a function of ρ .

Fig. 3 : Evolution, in proper time units, of the decay probabilities for the reactions $\bar{B}^0 \rightarrow \bar{f}$ and $B^0 \rightarrow \bar{f}$. Here, $\bar{f} = \rho^- \pi^+$ is chosen as an example.

Fig. 4 : Related diagrams for $B^0 \rightarrow a_1 \bar{a}_2$ and $\bar{B}^0 \rightarrow a_1 \bar{a}_2$. Except for the way in which the final quarks are joined up to make hadrons, the diagrams are CP conjugates of each other.

Fig. 5 : Diagrams for $B^0 \rightarrow \rho^+ \pi^-$ and $\bar{B}^0 \rightarrow \rho^+ \pi^-$. In the penguin diagrams (bottom row), q runs over all the positively charged quarks, t , c , and u . The product of CKM elements to which each diagram is proportional is written beneath it. All the products in this figure are of order λ^3 , where λ is the Wolfenstein parameter. Replacing the quark $u(\bar{u})$ by the quark $c(\bar{c})$ wherever it is written explicitly, leads to another interesting final state, namely $D^{*+} D^-$.

Fig. 6 : The expected error on $\sin 2\Phi$ using the four reactions $B^0(\bar{B}^0) \rightarrow \rho^\pm \pi^\mp$ relative to the error obtained using the two reactions $B^0(\bar{B}^0) \rightarrow \pi^+ \pi^-$ as a function of $\Gamma(\bar{B}^0 \rightarrow \rho^+ \pi^-)/\Gamma(\bar{B}^0 \rightarrow \rho^- \pi^+)$.

Fig. 7 : Energy spectrum of the softest photon issued from the cascade decay $B^0(\overline{B}^0) \rightarrow \rho^{\pm}\pi^{\mp} \rightarrow \pi^+\pi^-\pi^0 \rightarrow \pi^+\pi^-\gamma\gamma$.

Fig. 8 : The distribution of the $q\bar{q}$ background events contaminating the $\rho^\pm\pi^\mp$ channels as a function of the R_{rarity} variable (see text for definition). Events tagged with leptons and kaons are combined. The dashed line shows the distribution for the signal-events and is flat by definition of the R_{rarity} variable. Both background and signal distributions correspond to an integrated luminosity of 50 fb^{-1} (970 selected $\rho^\pm\pi^\mp$ signal events).

Fig. 9 : The expected experimental resolutions on $\sin 2\Phi$ as a function of the branching ratios $B_r(\rho^\mp \pi^\pm)$ and $B_r(a_1^\mp \pi^\pm)$ for $\rho = 1/\sqrt{2}$ and for an integrated luminosity of 50 fb^{-1} . The dotted lines indicated the resolution that would be achieved in the absence of background.