HADRONIZATION AND CONFINEMENT IN QCD

Nguyen Suan Han^{1,2}

TH Division, CERN, CH-1211 Geneva 23, Switzerland

V.N. Pervushin

Laboratory of Theoretical Physics,
Joint Institute for Nuclear Research

Head Post Office, P.O. Box 79,
101000, Moscow, USSR

Abstract

We discuss the consistency of the standard ideas of confinement with the recent phenomenological procedure of measurement of colour quantum numbers.

We show that the scheme of quantization of gauge fields, most adequate to the covariant description of hadrons, contains also a confinement mechanism as a destructive interference of phase factors of the topological degeneracy.

Geneva-11 October 1990

-
1. Supported by funds from ICSC-World Laboratory and TH Division, CERN.
 2. Permanent address: Department of Theoretical Physics, Hanoi State University, Thuong Dinh, Dong Da, Hanoi, Vietnam.

INTRODUCTION

Traditionally, confinement is connected with the dynamics of the quark-quark interaction and in the long run with hadronization.

Recently in theory, as in phenomenology, these have been arguments pointing out that confinement may be separated from the dynamics of hadronization (or, in any case, for the description of hadronization one should not consider effects of confinement).

In this paper we would like to draw attention to the mechanism of 'topological' confinement (Refs. [1], [2]), which is not connected with the potential of the quark-antiquark interaction, and can be realized in the scheme of quantization of gauge theories where the relativistic transformation properties of the classical and quantum fields coincide (Refs. [3], [4]).

The paper is organized as follows. In section 1, a modern statement of the problem of confinement is given. In section 2 we consider the problem of the gauge-invariant and relativistic covariant description of physical states. Section 3 is devoted to the quantization of gauge fields and to topological confinement.

1 STATEMENT OF THE PROBLEM OF CONFINEMENT

a) Phenomenology

The confinement of quarks in phenomenology means their nonobservation, or the fact that the cross-section for the creation of quarks in a free state is equal to zero.

When we want to prove the nonobservation of any physical object, we first of all need to describe constructively the experiments testifying to the existence of this object. In recent physics the proof of 'the existence' of a physical object means the experimental measurement of all its quantum numbers.

It is worth recalling that the quark idea follows from the classification group of hadrons (Ref. [5]). But even at this step of 'classification' quarks have been considered rather as an auxiliary mathematical construction than as a reality. Quarks became 'real objects' only when the first experiments appeared on the deep-inelastic measurement of the parton quantum numbers which coincided with the ones of the Gell-Mann-Zweig hypothesis quarks hypothesized by Gell-Mann and Zweig.

The parton interpretation of the deep-inelastic experiments on the phenomenological level has been developed by Feynman and Bjorken and its essence is that the sum over all hadron final states of probabilities of processes $e^+e^- \rightarrow \text{hadron}, e^+e^- p \rightarrow \text{hadrons etc.}$, is described as an imaginary part of the corresponding elastic amplitude:

$$\sum_{(\text{hadrons})} \text{phys} \langle f | T | h \rangle_{\text{phys}} \text{phys} \langle h | T^* | i \rangle_{\text{phys}} = 2\text{Im} \text{phys} \langle f | T | i \rangle_{\text{phys}} \quad (1)$$

constructed from quark-gluon diagrams of the QCD perturbation theory

$$2\text{Im} \text{phys} \langle f | T | i \rangle_{\text{phys}} \simeq \sum_{(\text{partons})} \text{phys} \langle f | T | p \rangle \langle p | T^* | i \rangle_{\text{phys}} \quad (2)$$

The resulting relation

$$\sum_{(h)} \text{phys} \langle f | T | h \rangle_{\text{phys}} \text{phys} \langle h | T^* | i \rangle_{\text{phys}} \simeq \sum_{(p)} \text{phys} \langle f | T | p \rangle \langle p | T^* | i \rangle_{\text{phys}} \quad (3)$$

is called quark-hadron duality (QHD), and is used in phenomenology as the energy averaging (global QHD) and without averaging, in the energy region far from resonances (local QHD) (Ref. [7]). For example, the cross-section of the process $e^+e^- \rightarrow \text{hadrons}$ in the nonresonance energy region coincides not only on the average but also at points with the imaginary part of quark loops. The Local QHD means that the perturbation theory real y is used in Minkowski space.

The ordinary unitarity for the S-matrix:

$$S = 1 + iT; SS^* = 1 \Rightarrow T \cdot T^* = 2\text{Im}T \quad (4)$$

differs from Eq. (1) by the Feynman supposition that the hadrons form the complete set of physical states

$$T \cdot T^* = \sum_{(h)} T | h \rangle_{\text{phys}} \text{phys} \langle h | T^* \quad (5)$$

The last equation means that the probability of hadronization is equal to unity, and, correspondingly, the probability of creation of colour particles is equal to zero, i.e. Equation (5) means the confinement of colour particles (c)

$$\text{phys} \langle i | T | c \rangle_{\text{phys}} = 0; | c \rangle_{\text{phys}} \neq | p \rangle \quad (6)$$

Thus, for the measurement of quantum numbers of colour particles by QHD [Eq. (3)] one uses the confinement hypothesis [Eq. (6)] and two different types of states: physical hadron states (in the left-hand side of [Eq. (3)] which are detected directly by experiments and parton states (in the right-hand side) which reflect particular analytical properties of the 'elastic' hadron amplitude reproducing the imaginary parts of quark diagrams.

In usual field theory, for instance in QED, physical and 'parton' states coincide in the framework of perturbation theory, thus indicating that the theoretical 'observation' of quarks as their parton images in local QHD [Eq. (3)] and their experimental nonobservation [Eq. (6)] take place in one and the same energy region of Minkowski space.

So, the very physical procedure of the measurement of the colour particle quantum number is contradictory, if it does not imply the existence of some mechanism of 'dressing' partons as a result of which they transform into nonobservable physical quarks $| c \rangle_{\text{phys}}$, not losing their quantum numbers.

One of the formulations of the confinement problem is as follows: Why does the coincidence of physical and parton states not occur in QCD? Another formulation: Why is the probability of colour particle production equal to zero [Eq. (6)] while the probability of hadronization is equal to unity?

Up to now the answer to these questions is the confinement hypothesis. However, the parton QCD high-energy phenomenology (Refs. [6], [7]) does not practically depend on the realization of this hypothesis. The same situation is now observed in QCD low-energy phenomenology. For example, chiral Lagrangians are obtained from one-loop quark diagrams (Refs. [8], [9]) and at the same time quarks are removed from the unitary condition corresponding to an implicit use of the hypothesis [Eq. (6)] in the absence of any

dynamics substantiating this hypothesis. The quark hadronization process in low-energy phenomenology is not connected with the confinement process. The same conclusion is made in the phenomenology of the sum rule method (Ref. [10]), where confinement effects do not in any way influence the description of experimental data.

Thus, in the recent quark high and low-energy phenomenology there is a tendency to decouple completely the hadronization process from confinement, and the term of 'confinement' means the specific procedure of measurement of colour particle quantum numbers which uses two sorts of colour states: physical and parton ones in one and the same energy region [Eq. (6)].

b) Theory

QCD arose as the substantiation of parton phenomenology with the help of the asymptotic freedom phenomenon (Ref. [11])

$$d(Q^2) = \left[\beta \ln \frac{Q^2}{\Lambda^2} \right]^{-1} \quad (7)$$

(i.e. of an effective decrease of the coupling constant in the deep Euclidean region).

There is a set of peculiarities of the conventional parton QCD that prevents us from recognizing the phenomenological statement of confinement problem.

Their main peculiarities are the conviction that the formula [Eq. (7)] explains the parton phenomenology and ordinary proof of unitarity relation in perturbative theory.

In agreement with asymptotic freedom, perturbative theory in QCD is valid only in Euclidean space. The correspondence of theoretical quantities with the realistic experimentally measurable cross-sections in Minkowski space is established with the help of dispersion relations.

However, in this way one can explain only the global quark-hadron duality but cannot explain the local QHD, i.e. the fact that the cross-section for the process e^+e^- into hadrons in the energy region far from resonances coincides with the imaginary part of a quark loop at every point. To explain the local QHD and perturbation theory in the Minkowski space, we must understand why in the same energy region quarks play roles simultaneously of nonobservable physical states [the left-hand side of Eq. (3)] and of observable free partons [the right-hand side of Eq. (3)].

This fact is also difficult to understand in the potential version of confinement (Ref. [12]), where argumentation is based on different regimes of the quark behaviour in different energy regions (long and short distances). On the other hand, all confinement-potential attempts to explain the nonobservability of individual quarks by solving the Dyson-Schwinger equation for the quark propagator have led not to confinement, but rather to the spontaneous breaking of chiral symmetry. At present, confinement potentials are successfully used as potentials of hadronization (Refs. [1],[13],[14],[15]).

We see that in QCD theory there is a tendency to consider the dynamics which is traditionally connected with confinement only as the dynamics of hadronization.

The main problem of how the Minkowski space perturbation theory (with a nonzero imaginary part of the quark-gluon diagrams) can be made consistent with the confinement hypothesis [Eq. (6)] remains still open; its answer is given neither by asymptotic freedom, nor by the confinement potential.

One of the first attempts to explain the phenomenological confinement with two sorts of quarks [Eq. (6)] was the version of the two-dimensional chromodynamics suggested

by 't Hooft (Refs. [16],[17]). In this model, physical (dressed) quark and parton (bare) quarks are distinguished. As a consequence of the infrared divergences, all physical quarks have infinite masses whereas colourless amplitudes are expressed in terms of bare quark propagators with finite masses without infrared divergences.

The absence of the amplitude for colour particle production does not contradict the unitarity relation. The point is that when bound states are present, the unitarity relation should not be understood as an identity, but rather as one of the self-consistency conditions of the theory used for normalizing the bound states and their interaction constants (Ref. [18]). (It is worth recalling that in quantum mechanics one and the same Hamiltonian allows different laws of conservation of probability which are considered as additional suppositions). If for some reason the probability of the colour channels disappears, the probability of other channels increases so that the total probability is equal to unity. From this point of view, the perturbation theory for bare quarks in the Minkowski space does not contradict the confinement of physical quarks [Eq. (3)], which is explained not by the interaction potential but by the process of dressing bare quarks.

The similar procedure of dressing bare quarks explaining the phenomenological confinement has been found in the minimal version of quantization of conventional chromodynamics (Refs. [9], [2], [3]).

2 THE RELATIVISTIC-COVARIANT DEFINITION OF PHYSICAL STATES IN QCD

The observable physical states in QCD are hadrons a bound states of quarks and gluons.

To gain some insight into the description of bound states in gauge theory, let us first consider the well-known example of an atom in its rest frame (with the momentum $\mathcal{P}_\mu = (M_A; 0; 0; 0)$). In the lowest order in the radiative corrections the atom spectrum is described by the action

$$W = \int d^4x \bar{\psi}(x)(i\not{\partial} - m^0)\psi(x) + \frac{1}{2} \int d^4x d^4y \bar{\psi}(y)\bar{\psi}(x)K(x,y)\psi(x)\bar{\psi}(y) \quad (8)$$

where K is the Coulomb kernel

$$K(x,y) = (\gamma_0) \cdot (\gamma_0) V_0(\vec{z})\delta(z_0) \quad (9)$$

$z_\mu = x_\mu - y_\mu$ is relative space-time. The action [Eq. (8)] leads to the Salpeter equation and eventually to the Schrödinger equation for an atom wave function $\chi(\vec{z})$ (Ref. [19]) that can be used for the construction of the bilocal atom field

$$\phi(x,y) = \phi(z|X) = e^{iM_A X_0} \chi(\vec{z})\delta(z_0) \quad (10)$$

where $X_\mu = x_\mu + y_\mu/2$ is the total coordinate.

The instantaneous bilocal field [Eq. (10)] is the consequence of the instantaneous interaction [Eq. (9)].

S. Love has proved an important theorem: radiative corrections in any order in the coupling constant, breaking the potential simultaneity, do not break the atom bilocal field simultaneity [Eq. (10)] (Ref. [20]).

The main question in the statement of the problem discussed here is 'What is the action [Eq. (8)] that describes a moving atom?'. The wave function of a relativistic atom (used for the description of the creation of atoms and of their decay [Ref. [21]] is constructed by the usual boost operation

$$\begin{aligned}\phi(x, y) &\rightarrow \phi'(x, y) = e^{i\vec{p}' \cdot \vec{x}} \chi(z^\perp) \cdot \delta(z \cdot \eta') \\ \{Z_\mu^\perp = Z_\mu - \eta'_\mu(z \cdot \eta')\}\end{aligned}\quad (11)$$

where $\vec{p}'_\mu$ is the total momentum $\vec{p}'_\mu = (\sqrt{\vec{p}^2 + M_A^2}, \vec{p}) \neq 0 = M_A \cdot \eta'_\mu$.

This relativistic atom bilocal field is described by the action [Eq. (8)] with the moving Coulomb kernel

$$K(x, y) = K(z|X) = \not{p}' V_0(z^\perp) \not{p}' \delta(z \cdot \eta') \quad (12)$$

This means that we choose the new radiative gauge depending on the arbitrary unit time-like vector η' (that one calls the time-axis of quantization) and this vector has been chosen parallel to the total momentum of an atom ($\eta' \sim \vec{p}'$). According to the Love theorem (Ref. [20]), the old structure of the bilocal field [Eq. (10)] cannot be restored by the radiation corrections. We cannot say here that the atom wave function does not depend on gauge.

So, the usual boost of the matrix elements with the atom wave function corresponds to the Lorentz transformation of field operators accompanied by the gauge change. Just this field transformation law has first been pointed out by Heisenberg and Pauli in 1930 (Ref. [22]). Another question is 'What is the relativistic atom for which we do not change gauge?'

The answer to this question is given by the general theory of bilocal fields (Refs. [23],[24]).

It is easy to see that the bilocal fields [Eq. (10) and (11)] satisfy the Yukawa condition (Ref. [23])

$$Z_\mu \frac{\partial}{\partial X_\mu} \phi(z|X) = 0 \quad (13)$$

which means that the bilocal field is an irreducible representation of the Lorentz group (i.e. it has the mass $p^2 = M_A^2$ and spin). Expression (13) is a generalized condition of irreducibility of vector, tensor and other fields: $\partial_\mu A_\mu = 0$; $\partial_\mu T_{\mu\nu} = 0$; ... (Ref. [24]).

If we shall do not change gauge, then the irreducibility condition is not fulfilled and the relativistic dispersion law breaks down, $p^2 \neq M_A^2$ (see for example Ref. [15]). Thus, not only the wave function but also the atom spectrum depends on gauge.

There are several papers (Refs. [20],[25]) devoted to the proof of gauge independence of an atom spectrum. In these treatment, the Coulomb interaction is used in the rest frame with the choice of the time-axis $\eta_\mu = (1, 0, 0, 0)$. However, all the authors have not taken into account that the vector η_μ (contained in the Coulomb part of interaction) can indeed be arbitrary, and that a transition from one vector η_μ to another η'_μ ($\eta'^2_\mu = 1$) is realized by means of a special change of the gauge.

We have seen that the real action for the relativistic QED atom in the lowest order in coupling constant is given by the action [Eq. (8)] with the kernel [Eq. (12)] where the time-axis η'_μ is the unit eigenvector of the bound state total momentum operator [Eq. (13)]

$$\eta'_\mu \phi(z|X) \sim \frac{\partial}{\partial X_\mu} \phi(z|X) \quad (14)$$

These prescriptions [Eqs. (8), (12), (14)] lead to a new relativistic potential model (Refs. [9] [26]), which unifies the potential model for the heavy-quarkonia spectroscopy and the bilocal generalization of the chiral Lagrangian for light quarkonia.

If we define any hadron bound state by formulae (11,13) as the irreducible nonlocal representation of the Poincaré group it is easy to understand that the covariant instantaneous gluon interaction $\sim \delta(\eta \cdot z)$ at the point $\eta \cdot z = 0$ is greater than the remaining retarded part of this interaction in QCD.

In any gauge theory we are faced with two types of physical problem: i) scattering and ii) bound states.

For a relativistic covariant description of the S-matrix with asymptotic free states of initial particles of the gauge theory it is sufficient to pass to relativistic gauge, and for this problem the potential model is the nonrelativistic approach and retardation corrections are the relativistic ones. Such an S-matrix does not depend on the gauge.

In the other case ii) instantaneous bound state physics not only depends on the gauge but this dependence is necessary for the relativistic covariance of physical observables (in particular the mass spectrum). In this case, the potential model [Eqs. (8), (12)] becomes the relativistic one (Refs. [9],[13],[15]). Moreover, any relativistic gauge does not contain instantaneous singularities [if you do not add it by hand (Refs. [20],[25])] and is important for describing the instantaneous bound states.

The gauge invariance of Lagrangian $L(A)$ means that it does not vary under gauge transformations of the fields A

$$\mathcal{L}(A^g) = \mathcal{L}(A) \quad (15)$$

where

$$A^g = g(A + \partial)g^{-1}$$

The choice of the gauge is a specific gauge transformation g' depending on the field A , so that the new field $A'[A] = g'[A](A + \partial)g'[A]^{-1}$ satisfies the additional condition

$$f(A'[A]) = 0 \quad (16)$$

The quantization of the fields and the Feynman rules are always formulated in terms of a certain gauge: $f_1 = 0, f_2 = 0, \dots$. We would like to draw your attention to some not well known consequences of these definitions.

i) The explicit solution of gauge condition [Eq. (16)] gives the physical variables A' as a functional on the initial fields A . In QED this is the axial field

$$A_\mu^{(3)}[A] = (\delta_{\mu\nu} - \partial_\mu \frac{1}{\partial^2} \partial_\nu) A_\nu, \quad A_3^{(3)}[A] = 0$$

or the transversale field

$$A_i^T[A] = (\delta_{ij} - \partial_i \frac{1}{\partial^2} \partial_j) A_j, \quad \partial_i A^T[A] = 0$$

and so on. These functions are invariant under gauge transformations of the initial fields in the sense of Eq. 15. So, any gauge choice is a transition from the initial fields to the gauge invariant physical variables (i.e. 'gauge' [Eq. (16)] is the choice of variables).

ii) The change of the gauge (from $A^{(1)}$ to $A^{(2)}$) is fulfilled by the substitution

$$A^{(2)}[A^{(1)}] = V[A^{(1)}](A^{(1)} + \partial)V[A^{(1)}]^{-1}, \quad \psi^{(2)} = V[A^{(1)}]\psi^{(1)}, \quad (17)$$

All Green functions are invariant under the operation [Eq. (17)]

$$\langle \psi^{(2)} \dots \bar{\psi}^{(2)} \rangle = \langle V[A^{(1)}]\psi^{(1)} \dots \bar{\psi}^{(1)}V[A^{(1)}]^{-1} \rangle \quad (18)$$

(if anomalies are absent). This substitution contains not only the modification of the Feynman rules (i.e. the gauge change) but also the spurious diagrams induced by the factor $V[A^{(1)}]$ (which do not follow from the initial Lagrangian).

On the mass-shell, these additional diagrams do not contribute, and we have invariance under a gauge change. But off the mass-shell dependence on the gauge occurs but this does not mean gauge noninvariance (any variables [Eq. (16)] are gauge invariant as we have seen above).

The experimental value of the Lamb-shift is described by any 'gauge' up to the spurious diagrams. The radiative 'gauge' is the unique gauge which does not demand these spurious diagrams to reproduce the observed Lamb-shift in atomic spectra, the problem is the foundation of the radiative gauge for the description of the bound state spectrum in the rest frame and the Heisenberg-Pauli group transformation.

3 THE MINIMAL QUANTIZATION SCHEME AND CONFINEMENT IN QCD

Let us consider the quantization of two simplest systems of quantum mechanics with lagrangians

$$L_1 = \frac{\dot{q}^2}{2}, \quad L_2 = \frac{q^2}{2} \left(\dot{q} = \frac{d}{dt}q \right)$$

The Lagrangian describes the nontrivial dynamics of a classical point particle with the momentum $p = \partial L_1 / \partial \dot{q} = \dot{q}$ and the Euler equation $\ddot{q} = 0$. The quantization means the transition to operators of momentum and coordinate with the commutation relation $i[\hat{p}, \hat{q}] = \hbar$.

The second Lagrangian describes the static classical point at rest with the zero momentum $p = \frac{\partial L_2}{\partial \dot{q}} = 0$ and zero coordinate, as a solution of the Euler equation $\partial L_2 / \partial q = q = 0$.

The second system L_2 in principle cannot be considered as quantum one in the spirit of the Dirac quantization (Ref. [27]). If we consider an equation $\hat{p} = 0$ in this model as the constraint on a quantum variable, we get also the second constraint $-i[\hat{p}, \hat{q}] = i[\hat{p}, L_2] = \dot{\hat{q}} = 0$, and both these constraints ($\hat{p} = 0, \dot{\hat{q}} = 0$) contradict to the commutation relation $i[\hat{p}, \hat{q}] = \hbar$ and the Heisenberg one $\Delta p \Delta q \geq \hbar/2$.

Just such a 'nonquantum' static system is the Lagrangian of electrodynamics with neglect of the space field component

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}^2 + A_\mu J_\mu|_{A=0} = \frac{1}{2}(\partial_t A_0)^2 + A_0 J_0.$$

that is the foundation of the spectrum and wave function of an atom in the lowest order in coupling constant.

So all methods of quantization of the gauge fields can roughly be separated into two approaches. In the first one tries to quantize all components to keep manifest relativistic covariance (the Dirac quantization); in the second, one solves all constraints explicitly and quantizes only the minimal set of physical variables (that remains after the substitution of the explicit solution of the constraints into the initial Lagrangian). We have seen that the second approach is more adequate to the task of describing bound states. The 'minimal quantization' has been formulated in Refs. [1]–[4]. At first we consider this quantization using a free electromagnetic field as an example

$$\mathcal{L}(x) = -\frac{1}{4}F_{\mu\nu}^2 = \frac{1}{2}F_{\alpha i}^2 - \frac{1}{2}B_i^2 \quad (19)$$

$$F_{\alpha i} = \partial_\alpha A_i - \partial_i A_\alpha, \quad B_i = \epsilon_{ijk}\partial_j A_k \quad (20)$$

Before quantization we should solve explicitly the classical equation for the time component of the gauge field

$$\bar{\partial}^2 A_0 = \partial_i \partial_0 A_i \Rightarrow A_0(x) = \frac{1}{\bar{\partial}^2}(\partial_j \partial_0 A_j) = - \int \frac{d^3 y \bar{\partial}_j^2 \partial_0 A_j(y)}{4\pi|\vec{x} - \vec{y}|} \quad (21)$$

The substitution of this gauge invariant solution into the Lagrangian and field strength $F_{\alpha i}$ eliminates from these expressions two components

$$F_{\alpha i} = \partial_\alpha A_i - \partial_i \left[\frac{1}{\bar{\partial}^2} \partial_j \partial_0 A_j \right] = \partial_\alpha A_i^T[A] \quad (22)$$

where

$$A_i^T[A] = \left(\delta_{ij} - \partial_i \frac{1}{\bar{\partial}^2} \partial_j \right) A_j = \sum_{a=1,2} e_i^a (e_j^a A_j) = \sum_{a=1,2} e_i^a A^a \quad (23)$$

e_i^a are two transversal unit vectors.

As a result, we passed from the gauge invariant Lagrangian to the gauge invariant variables A^a in terms of which the Lagrangian has the form

$$\mathcal{L}^{ph} = \sum_{a=1,2} \frac{1}{2} \partial_\mu A^a \partial^\mu A^a \equiv \frac{1}{2} \partial_\mu A_i \left(\delta_{ij} - \partial_i \frac{1}{\bar{\partial}^2} \partial_j \right) \partial^\mu A_j \quad (24)$$

The energy-momentum tensor as the symplectic structure on the measure [Eq. (24)] (Ref. [28])

$$T_{\mu\nu}^{ph} = \sum_{a=1,2} \partial_\mu A^a \partial_\nu A^a - \delta_{\mu\nu} \mathcal{L}^{ph} \quad (25)$$

coincides using the Gauss equation (21) with the gauge invariant energy-momentum tensor of the initial theory

$$T_{\mu\nu}^{B\text{ein}/\text{ante}} = F_{\mu\alpha} F_{\nu}^{\alpha} - \delta_{\mu\nu} \mathcal{L}$$

Quantization means the commutation relation only for the minimal set of two independent components

$$i[\partial_0 A^0(\vec{x}, t), A^k(\vec{y}, t)] = \delta^{0k} \delta^3(\vec{x} - \vec{y}) \quad (26)$$

or

$$i[\partial_0 A^T(\vec{x}, t), A^T_j(\vec{y}, t)] = (\delta_{ij} - \partial_i \frac{1}{\partial^2} \partial_j) \delta^3(\vec{x} - \vec{y})$$

The minimal quantization is the unique one of gauge theory in which the quantum Lorentz transformation

$$i\varepsilon_k [M_{0i}, A^T_i] = \delta^0_L A^T_i + \partial_k \Lambda[A^T] \quad (27)$$

$$M_{0*} = \int d^3x [x_k T^{0k} - x_0 T_{0*}]$$

$$\delta^0_L A^T_i = \varepsilon_i (x'_\mu \partial_\mu - x'_0 \partial'_0) A^T_i(x'), \Lambda[A^T] = \varepsilon_k \frac{1}{\partial^2} \partial_0 A^T_k \quad (28)$$

coincides completely with the classical transformations of the physical variables (Ref. [3])

$$A^T_k[A + \delta^0_L A] - A^T_k[A] = \delta^0_L A^T_k + \partial_k \Lambda[A^T],$$

where

$$\delta^0_L A_k(x) = \varepsilon_i (x'_\mu \partial_\mu - x'_0 \partial'_0) A_k(x') + \varepsilon_k A_0[A_j(x)],$$

The conventional Lorentz transformations in terms of minimal quantization A^T means just the Heisenberg-Pauli turning of the time-axis of quantization (Ref. [22]).

Electrodynamics

$$\mathcal{L}(x) = -\frac{1}{4} F_{\mu\nu}^2 + \bar{\psi}(i \not{\partial} - m^0) \psi + A_\mu j^\mu, \quad j^\mu = e \bar{\psi} \gamma^\mu \psi.$$

using the solution of the Gauss equation

$$A_0 = \frac{1}{\partial^2} (\partial_i \partial_0 A_i + j_0) \quad (29)$$

also is expressed in terms of gauge-invariant variables A^T, ψ^T (Ref. [3])

$$\mathcal{L}^{ph} = \frac{1}{4} (\partial_\mu A^T)^2 + \frac{1}{2} j_0^T \frac{1}{\partial^2} j_0^T + \bar{\psi}^T (i \not{\partial} - m^0) \psi^T - j_i^T A^T_i \quad (30)$$

where

$$ie A^T_k = V^T [A] (ie A_k + \partial_k) V^T [A]^{-1} \quad (31)$$

$$\psi^T = V^T [A] \psi$$

$$V^T [A] = \exp \left\{ ie \frac{1}{\partial^2} \partial_i A_i \right\}.$$

The variables [Eq. (31)] differ from the conventional 'radiation gauge' by the classical relativistic transformation of the Heisenberg-Pauli type:

$$\psi^T [A + \delta^0_L A, \psi + \delta^0_L \psi] - \psi^T [A, \psi] = \delta^0_L \psi^T + ie \Lambda \psi^T,$$

$$\Lambda = \varepsilon_k \frac{1}{\partial^2} \left(\partial_0 A^T_k + \partial_k \frac{1}{\partial^2} j_0^T \right) \quad (32)$$

that completely coincides with the Belinfante quantum boost (Refs. [3],[29])

$$i\varepsilon [M^B_{0k}, \bar{\psi}^T] = \delta^0_L \bar{\psi}^T + ie \Lambda \bar{\psi}^T, \quad (33)$$

if the nonlocal classical field ψ^T [Eq. (31)] is compared to the quantum field $\bar{\psi}^T$ with the local commutation relation

$$\{\psi^{T+}(\vec{x}, t), \psi^T(\vec{y}, t)\} = \delta^3(\vec{x} - \vec{y})$$

In this case the quantum boost [Eq. (33)] means the relativistic covariance of the Coulomb field: the new diagrams will describe the motion of the Coulomb field (Refs. [3],[4]).

Just in the same way it is possible to solve the problem of restoration of the relativistic covariance of the Bethe-Salpeter equation with the Coulomb-like potential in QCD.

On the level of 'minimal' quantization based on the classical theory with gauge invariant variables [Eq. (31)] the problems of dependence of the one-particle Green function on the choice of gauge, zeroes of the Faddeev-Popov determinant and other problems, connected with the choice of gauge in the Dirac method, have an absolutely different meaning and a different solution than in the Dirac method.

For example, nonlocal physical variables of the type [Eq. (31)], obtained by the explicit solution of the Gauss equation, may provide a new physical information lost in the Dirac quantization. In particular, the classical construction of the covariant variables [Eq. (31)] is defined up to zero-eigenvalues of the Gauss differential operator $\tilde{\partial}^2$ [Eq. (29)]

$$\exp \left\{ ie \frac{1}{\partial^2} \partial_i A_i \right\} \Rightarrow W(x) \exp \left\{ ie \frac{1}{\partial^2} \partial_i A_i \right\} \quad (34)$$

$$W(x) = \exp i \lambda(x), \quad \tilde{\partial}^2 \lambda(x) = 0 \quad (35)$$

The variables

$$A^{ph}_k = W(x) (A^T_k - ie^{-1} \partial_k) W^{-1}(x), \quad \psi^{ph} = W(x) \psi^T \quad (36)$$

satisfy the same gauge constrain that [Eq. (31)]

$$\partial_i A^{ph}_i = \partial_i A^T_i \equiv 0$$

The variables A^{ph} and A^T have physically equivalent Lagrangians (if W is a smooth function) up to the definition of asymptotical states by the classical sources

$$\text{i) } \bar{\psi}^{ph} \eta^{ph} + \bar{\eta}^{ph} \psi^{ph}, \quad \text{ii) } \bar{\psi}^T \eta^T + \bar{\eta}^T \psi^T \quad (37)$$

The first 'physical' case for nontrivial phase factors¹ differs from the second one by the 'spurious' diagrams

$$\bar{\psi}^T (W^{-1} \eta^{ph}) + (\eta^{ph} W) \psi^T \quad (38)$$

which describe the degeneracy of physical states.

¹In Ref. (1) the function is called the Weyl phase factor.

In refs. ([1]–[4]) it is shown that nontrivial phase factors and degeneracy of physical states arise in those gauge theories where phase factors $W(x)$ as mappings of the space $R(3)$ into the gauge internal group G have nontrivial topological properties. (For example, in the Schwinger model, where the homotopy group is $\Pi_1(U(1)) = \mathbb{Z}$ or in QCD:

$$\Pi_3[SU(3)] = \mathbb{Z}$$

As a result of such a topological degeneracy, physical variables are distinguished from ‘bare’ fields A^T, ψ^T used in the diagrams of perturbation theory.

In QED equation (35) $\partial^2 \lambda(r) = 0$, has generally speaking, two solutions

$$\partial^2 \lambda(x) = \frac{1}{r} \frac{d^2}{dr^2} [r \lambda(r)] = 0 \Rightarrow \lambda(r) = \frac{c_1}{r} + c_2, \quad r = |\vec{x}|$$

We should put one of them equal to zero, $c_1 = 0$, because of the singularity and the other, c_2 , which describes the constant phase, has, as is known, nophysical consequences.

In QCD we meet another situation. The same equation for the non-Abelian phase (Ref. [3])

$$\partial^2 \lambda(x) = 0$$

has nontrivial solution in the finite space $|x| \leq R$

$$\begin{aligned} \lambda(x)_{n,\phi} &= i\tau^a \Omega^{ab}(\phi_i) \frac{x^b}{R} \pi n \\ \tau^a \Omega^{ab}(\phi_i) &= U(\phi_i) \tau^a U^{-1}(\phi_i) \\ U(\phi_i) &= \exp\left(\frac{i\tau_1 \phi_1}{2}\right) \exp\left(\frac{i\tau_2 \phi_2}{2}\right) \exp\left(\frac{i\tau_3 \phi_3}{2}\right) \end{aligned} \quad (39)$$

where τ^a ($a = 1, 2, 3$) are the generators of the ‘minimal’ $SU(2)$ subgroup of the $SU(3)$ group (‘Minimality’ means that the fundamental representation of $SU(3)$ is simultaneously the irreducible representation of the $SU(2)$ subgroup; for example, the generators of the minimal subgroup are the Gell-Mann matrices $\lambda_2, \lambda_5, \lambda_7$ which coincide with the adjoint $SU(2)$ representation).

The solutions [Eq. (39)] correspond to smooth mappings of space $R(3)$ into the subgroup $SU(2)$ with the topological integer index of mapping

$$n = \frac{1}{24\pi^2} \int d^3x \varepsilon_{ijk} \text{tr}(\hat{V}_i \hat{V}_j \hat{V}_k) = 0, \pm 1, \dots; \quad \hat{V}_i = W \partial_i W^{-1}$$

The three Euler angles ϕ_i in [Eq. (39)] fix the position of the coordinate system of the colour space with respect to the coordinate system of the ordinary space $R(3)$.

The existence of such solutions indicates that the physical variables [Eq. (36)] are degenerate in a space of finite volume.

However, the degeneracy index n does not disappear even in the limit $R \rightarrow \infty$ and represents an example of topological quantum anomalies of the type of the axial current divergence $\partial_\mu j_5^\mu \sim F_{\mu\nu} \tilde{F}^{\mu\nu}$. Both these quantities, the index and divergence $\partial_\mu j_5^\mu$, after removing the regularization are not equal to zero despite the disappearance of the initial elements of their construction: the field V_i or the Pauli–Villars propagators.

As degeneracy phase factors are nonsingular, the Lagrangian and Belinfante tensor do not depend on these factors owing to the gauge invariance. Therefore, the quantization

and formulation of the perturbation theory are accomplished in terms of ‘bare’ variables and do not differ from the standard quantization [Refs. [3],[5]]:

$$\mathcal{L}(A^{\mu h}, \psi^{\mu h}) + \bar{\eta}^{\mu h} \psi^{\mu h} + \bar{\psi}^{\mu h} \eta^{\mu h} = \mathcal{L}(A^T, \psi^T) + (\bar{\eta}^T W^{-1}) \psi^T + \bar{\psi}^T (W \eta^T)$$

The only difference consists in the definition of physical states which turn out to be ‘dressed’ in phase factors and degenerate:

$$\psi_{asympt, t \rightarrow \infty}^{phys}(\vec{x}, t) = W_{(n)}(\vec{x}) \psi_{asympt}^T(\vec{x}, t)$$

where ψ_{asympt}^T is a usual plane wave state.

Owing to the fact that all the computed physical quantities in quantum field theory, cross-sections, probabilities of decays, etc., are normalized in a finite space-time, the transition to infinite space should be accomplished after removing the degeneracy, i.e. after summing the generating functional of the Green functions over all degeneracy parameters (Refs. [1]–[3]), for example,

$$\lim_{R \rightarrow \infty} \lim_{L \rightarrow \infty} \frac{1}{2L} \sum_{n=-L}^L \exp\left(i \frac{x^0 \tau^a}{R} \pi n\right) = \begin{cases} 1, & \vec{x} = 0 \\ 0, & \vec{x} \neq 0 \end{cases}$$

In that case, due to this complete destructive interference of the Weyl factors all amplitudes of colour particle production turn into zero (for more details, see Refs. [1]–[3]).

At the same time colourless correlators of electromagnetic and weak currents are expressed in terms of propagators of ‘bare’ quarks (ψ^T) and gluons (A^T) and they coincide with the usual expressions of perturbative QCD. As has been shown in Refs. [16],[17], the situation does not contradict the unitarity of the S-matrix, and leads just to the principle of quark-hadron duality in Minkowski space widely used in parton phenomenology.

The topological mechanism of dressing physical quarks explains the main hypothesis of this phenomenology from first principles: the probability of hadronization being unity is explained by the independence of the hadronization of the confinement.

Such a confinement mechanism is a specific feature only of the minimal quantization (it cannot appear in the Dirac method with fixing an arbitrary gauge).

However, there is a physical difficulty in this explanation, namely the breaking of the gauge symmetry does not eliminate the topological confinement, in particular, in $SU(2) \times U(1)$ theory. This means that we cannot observe directly the $SU(2)$ sector of the Salam–Weinberg–Glashow theory.

In any case, the methods of measurement of quantum numbers of quarks and of the $SU(2)$ sector of electroweak theory coincide and both differ from the one of the $U(1)$ sector of charged massive particles and γ -quanta.

4 CONCLUSION

The History of the development of the quark theory of hadrons can be expressed in three words: in the sixties the dominating problem was of quark ‘existence’; in the seventies, the ‘confinement problem’; in the eighties, the ‘hadronization’ problem.

The physical statement of the problem of ‘confinement’ of quarks is dictated by experiments proving their ‘existence’.

The theoretical proof of confinement in QCD means giving answers from first principles to two questions:

- 1) Why are physical states of the theory limited by a hadron (colourless) sector?
- 2) Why does the sum over finite hadron states reproduce the analytical properties of quark-gluon diagram in the Minkowski space?

We have seen that neither the potential confinement, nor the asymptotic freedom formula answer these questions.

We try to answer these questions from the point of view of hadronization, i.e., of the definition of hadrons as instantaneous bound states. Consistent description of such hadrons is possible in the minimal scheme of quantization of gauge theory with the explicit solution of constraints. We have shown that this method contains additional physical information—the topological degeneracy of the colour physical states.

Phenomenological confinement can be explained by the complete destructive interference of the gauge (phase) factor of the topological degeneracy of the physical variables constructed by the explicit solutions of the gauge theory constraints.

This confinement mechanism seems more adequate to account for quark-parton phenomenology utilizing the Minkowski space perturbation theory and to relativistic covariant description of hadrons.

Acknowledgements

The authors would like to thank Profs. N.N. Bogolubov, A. DiGiacomo, A.V. Efremov, V.G. Kadyshchey, for fruitful discussions. Prof. R. Stora is acknowledged for some conversations about the Coulomb gauge. One of the authors, Nguyen Suan Han, expresses his gratitude to Prof. John Ellis for warm hospitality and support.

References

- [1] V.N. Pervushin, Riv. Nuovo Cimento 8, No. 10 (1985).
- [2] R.I. Azimov, V.N. Pervushin, Teor. Math. Fiz. 67, 349 (1986).
- [3] Nguyen Suan Han, V.N. Pervushin, Fortschr. Phys. 37, 611 (1989).
- [4] Nguyen Suan Han, V.N. Pervushin, Mod. Phys. Lett. A240, 367 (1987) and N.P. Ilieva, Nguyen Suan Han, V.N. Pervushin, Yademaya Fizika 45, 1169 (1987).
- [5] M. Gell-Mann, Report CTSL-20, California Institute of Technology, 1961.
- [6] R.P. Feynman, 'Photon-Hadron Interactions', 1972; W.A. Benjamin, Inc. Reading, Massachusetts.
- [7] Yu.L. Dokshitzer, D.I. Dyakonov, S.L. Troyan, Phys. Rep. 58, 269 (1980).
- [8] M.K. Volkov, Ann. of Phys. 157, 282 (1984); D. Ebert, H. Prinhardt, Nucl. Phys. B271, 188 (1986).
- [9] V.N. Pervushin et al. Fortsch. Phys. 38, No. 4 (1990).
- [10] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, Nucl. Phys. B147, 385 (1979).

- [11] D. Gross, F. Wilczek, Phys. Rev. D8, 3633 (1973).
- [12] K.G. Wilson, Phys. Rev. D10, 2445 (1974).
- [13] J. Finger, J.E. Mandulla, Nucl. Phys. B199, 168 (1982).
- [14] S.L. Adler, A.C. Davis, Nucl. Phys. B244, 469 (1984).
- [15] A. L. Yaonauc, L. Oliver, S. Ono, O. Pene, and J.C. Raynal, D31, 137 (1985).
- [16] G. 't Hooft, Nucl. Phys. B72, 461 (1974); C.G. Callan (Jr.), N. Coote, D.J. Gross, Phys. Rev. D13, 1649 (1976).
- [17] V.N. Pervushin, H. Reinhardt, D. Ebert, Sov. J. Part. Nucl. 10, 444 (1979).
- [18] N. Nakanishi, Suppl. Progr. Theor. Phys. 43, 1 (1969).
- [19] E.E. Salpeter, Phys. Rev. 87, 328 (1952).
- [20] S. Love, Ann. Phys. 113, 135 (1978).
- [21] L.L. Nemenov, Yad. Phys. 16, 125 (1972).
- [22] W. Heisenberg and W. Pauli, Z. Phys. 59, 168 (1930).
- [23] M.A. Markov, J. Phys. USSR 3, 452 (1940); H. Yukawa, Phys. Rev. 77, 219 (1949).
- [24] J. Lukierski, and M. Oziwicz, Phys. Lett. 69, 339 (1977).
- [25] Yu.K. Chung and T. Fulton, Phys. Rev. A14, 552 (1976); G. Feldman, T. Fulton, and D. Heckathorn, Nucl. Phys. B167, 364 (1980).
- [26] Yu.L. Kalinovsky, L. Kaschlun and V.N. Pervushin, Phys. Lett. B231, 288 (1989).
- [27] P.A.M. Dirac, Lectures on Quantum Mechanics Belfer Graduate School of Science Yeshive University, New York, 1964.
- [28] N.P. Ilieva, L. Litov, 'Symplectic structure and quantization of gauge theories' in 'Selected Topics in QFT and Mathematical Physics', World Scientific (1989).
- [29] B. Zumino, J. Math. Phys. 1, 1 (1960).