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On Electroweak Theories with an Extra $SU(2)_V$ Vector-Boson Triplet *

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Abstract

We examine an extended electroweak theory based on $SU(2)_L \times SU(2)_V \times U(1)_Y$ and constrained by $\rho = 1$ at tree level. Various models previously considered in the literature appear as special cases of our framework. The non-decoupling of the additional neutral vector boson, V_0 , for $m_V \rightarrow \infty$ is traced back to the extra pointlike photon interaction of leptons and quarks contained in the Hung-Sakurai electroweak model based on γW^3 mixing. A comparison of the predictions of the present model with precision electroweak data (M_{Z_0} , $M_{W^\pm}$, $\bar{s}_W^2$) will allow one to quantify how firmly $SU(2)_L \times U(1)_Y$ symmetry of vector-boson fermion interactions is established by these data.

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1. Introduction

The question of the possible existence of extra weak-vector-bosons, in addition to the $W^\pm$, Z^0 triplet, has recently found renewed interest in connection with precision electroweak experiments at the Z^0 peak ¹⁾, as well as in connection with plans for future proton-proton scattering experiments in the multi-TeV energy range ²⁾. Within the present paper we will examine in some detail a class of electroweak theories containing one extra triplet of vector bosons, $V^\pm$, V^0 . This class of theories is characterized by the requirement that the $q^2 \rightarrow 0$ neutral-to-charged current ratio, the ρ parameter measured in neutrino scattering, be equal to unity, $\rho = 1$, at tree level. The model to be investigated, will in general contain three additional parameters, when compared with the standard $SU(2)_L \times U(1)_Y$ electroweak theory ³⁾. These parameters are associated with, e.g., a coupling constant, the mass of the charged bosons, $V^\pm$, and a mixing parameter of the neutral boson, V^0 , with the photon.

The question of the possible existence of more than the usual single triplet of weak vector-bosons, and the compatibility of an extra triplet with the $q^2 \rightarrow 0$ neutral current structure (in particular with the empirical value of the ρ parameter, $\rho \simeq 1$) was much investigated about ten years ago within the framework of extended gauge theories ^{4,5)} as well as within the framework of γ -weak-boson mixing ⁶⁾. It was shown that, under suitable assumptions, the $q^2 \rightarrow 0$ neutral-current structure of theories with extra weak bosons coincides with the one of the $SU(2)_L \times U(1)_Y$ theory apart from an additional contribution of the form ⁴⁾ $C j_{em}^2$, where the parameter C measures the presence of more than a single neutral boson, and j_{em} denotes the electromagnetic current of leptons and quarks. The parameter C thus fulfills $C = 0$ in the single-neutral-boson limit, and it has been experimentally constrained ^{7,8)} to $C \leq 0.01$ in e^+e^- total cross-section measurements. Before the discovery ⁹⁾ of the Z^0 and $W^\pm$ bosons, the interest in multi-boson scenarios was essentially focused on deriving upper bounds for the least massive intermediate vector boson and on possible effects of such bosons observable in lepton scattering for $|q^2| \ll M_{Z^0}^2$. In contrast, the present interest centers on the question of how strongly the characteristic parameters of (necessarily) heavier vector bosons are being constrained by precision data on Z^0 and $W^\pm$ properties.

In this paper we will examine a theoretical model allowing for a single extra triplet of vector bosons in addition to the $W^\pm$ and Z^0 and fulfilling the constraint of $\rho = 1$ at tree-level. Formulating the theory in terms of mass mixing with respect to the conventional $\{\bar{W}, \bar{V}, B\}$ basis [†] and, equivalently, in terms of current mixing (supplemented by symmetry or vector-boson-dominance constraints) with respect to the $\{\bar{w}, \bar{v}, \gamma\}$ basis ^{††}, will allow for a particularly transparent deduction of the phenomena contained within the theory. Our framework with three additional parameters (compared with the $SU(2)_L \times U(1)_Y$ standard case) will be seen to be general enough to contain various models ^{10,11,12,13)} which have been studied in the past as special cases. Among other things, we will clarify the relation

[†] Here, $\bar{W}$, $\bar{V}$ and B refer to the vector bosons associated with $SU(2)_L \times SU(2)_Y \times U(1)_Y$, respectively.

^{††} Here, $\bar{w}$ and $\bar{v}$ refer to the vector bosons associated with the gauge group $SU(2)_L \times SU(2)_Y$ and γ refers to the (primordial) photon, associated with $U(1)_{em}$.

of the BESS model ¹²⁾, originally motivated by considerations on non-linear realizations of electroweak symmetry breakdown in strongly interacting Higgs scenarios, to conventional (linear) gauge extensions of the electroweak theory. The physical implications of our theory for the properties of the Z^0 and its interactions become particularly transparent by studying the limit in which the V -boson mass, m_V , is sufficiently much above the $W^\pm$ mass, $m_W \gg m_V$, to be well approximated by $m_V \rightarrow \infty$. In this limit, the theory will be seen to coincide with the model of Hung and Sakurai based on γw^3 -mixing ¹³⁾.

In Section 2, the theory will be developed as an effective theory in the mass-mixing and current-mixing formalisms. In Section 3, we will briefly comment on how the model can be formulated as a conventional spontaneously broken gauge theory, and we will also discuss special cases of our framework, which coincide with models given in the literature. Section 4 will contain the discussion of the limit $m_V \rightarrow \infty$, and in Section 5 we will describe the trilinear and quadrilinear vector-boson self interactions contained within the theory. A few concluding remarks will be given in Section 6.

2. A Theory Containing an Extra $SU(2)_Y$ Triplet of Vector Bosons

In Section 2.1, the theory will be presented on the basis of the mass-mixing formalism usually employed in electroweak theories in which masses are introduced via the Higgs mechanism. The basic vector boson field, $\bar{W}_\mu$, $\bar{V}_\mu$, and B_μ will be associated with the gauge group $SU(2)_L \times SU(2)_Y \times U(1)_Y$. A second, equivalent, formulation of the theory based on current mixing will be given in Section 2.2. In addition to the two gauge boson triplets, $\bar{w}_\mu$, $\bar{v}_\mu$ transforming under $SU(2)_L \times SU(2)_Y$, the photon field will be introduced as transforming under $U(1)_{em}$ gauge transformations. The presentation of the theory in two equivalent forms will very much facilitate its physical interpretation. In particular, it will lead to a representation of the theory in terms of a convenient set of parameters, chosen as the masses of the charged vector bosons, m_W and m_V , their couplings to the fermions, g_W and g_V , and two mixing parameters, λ_W and λ_V , which arise precisely via the development of the theory in the current mixing formalism. The transparent formulation of the Lagrangian of the theory in terms of the "physical" parameters mentioned above will be *essential* for a clear discussion of the important limiting case when the mass of the additional vector boson, m_V , is large compared with the $W^\pm$ mass, $m_V \gg m_W$. In Section 2.3, the one-to-one correspondence between the primordial parameters used in the mass-mixing formalism ($g_0, g_1, g_2, v_1, v_2, v_3$) and the set ($g_W, g_V, \lambda_W, \lambda_V, m_W, m_V$) will be explicitly and completely displayed.

2.1 Extended Gauge Theory, Mass Mixing Approach.

Extending the gauge group $SU(2)_L \times U(1)_Y$ of the standard electroweak theory to the group $SU(2)_L \times SU(2)_V \times U(1)_Y$, we immediately obtain the Lagrangian †

$$L = -\frac{1}{4} \bar{W}_{\mu\nu}^i \bar{W}^{\mu\nu}_i - \frac{1}{4} \bar{V}_{\mu\nu}^i \bar{V}^{\mu\nu}_i - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + g_0 \bar{W}_\mu^i j_\mu^i + g_1 B_\mu j_\mu^B, \quad (2.1)$$

where the kinetic terms of the $\bar{W}_{\mu\nu}$ and $\bar{V}_{\mu\nu}$ fields are of non-Abelian form and contain the gauge couplings g_0 and g_2 , respectively. The $U(1)_Y$ field has been denoted by B_μ in (2.1), and j_μ^i and j_μ^B denote the usual weak-isospin,

$$j_\mu^i = \sum_L \bar{\psi}_L \gamma_\mu \frac{\tau^i}{2} \psi_L, \quad (2.2)$$

and weak-hypercharge currents

$$j_\mu^Y = Y_L \sum_L \bar{\psi}_L \gamma_\mu \psi_L + Y_R \sum_R \bar{\psi}_R \gamma_\mu \psi_R, \quad (2.3)$$

of the leptons and quarks, respectively. The standard weak-hypercharge quantum numbers have been denoted by Y_L and Y_R . The ansatz (2.1) to (2.3) is somewhat special insofar as a direct coupling of the $\bar{V}$ -boson triplet to the known leptons and quarks has been excluded. Mixing between the $SU(2)_L$ and $SU(2)_V$ triplets, to be introduced below, will lead, however, to a non-vanishing coupling of the “physical” V -boson triplet to the known fermions. The vanishing primordial coupling of the extra vector bosons to leptons and quarks corresponds to classifying the known fermions as singlets under $SU(2)_V$, implying that the general formula for the charge quantum number,

$$Q = T_L^3 + Y + T_V^3, \quad (2.4)$$

reduces to the standard one, involving the first two terms only, for the known leptons and quarks.

The assignment of the known fermions as $SU(2)_V$ singlets is well-known⁴⁾ to be an essential ingredient for obtaining $\rho = 1$ (where ρ is the neutral-to-charged-current ratio measured in neutrino scattering) at tree level. The presence of the extra $SU(2)_V$ group with the associated vector boson has to be viewed as either originating from the breakdown of a larger group containing, e.g., hitherto unknown fermions, or else, as an effective description of an additional vector boson of dynamical origin.

Turning to the mass terms for the $\bar{W}$ and $\bar{V}$ vector-boson triplets in (2.1), we will adopt the most general ansatz compatible with a remaining global $SU(2)_{L+V}$ “weak-isospin” symmetry. This allows for $\bar{W} - \bar{V}$ mixing and contains two masses and a $\bar{W} - \bar{V}$

† The kinetic terms for the fermions have been suppressed in (2.1). The notation $\bar{W}$, etc has been introduced to discriminate these primordial fields from the ones to be introduced via diagonalization of the mass matrix to be presented below.

mass-mixing parameter, thus three arbitrary parameters, v_1, v_2 , and v_3 , altogether. It is written as

$$\frac{v_1^2}{8} g_0^2 \left(\bar{W}_\mu^i \right)^2 + \frac{v_2^2}{8} g_2^2 \left(\bar{V}_\mu^i \right)^2 + \frac{v_3^2}{8} \left(g_0 \bar{W}_\mu^i - g_2 \bar{V}_\mu^i \right)^2. \quad (2.5)$$

The specific form of (2.5), with gauge couplings entering the definition of the three arbitrary mass scales, without loss of generality as regards the $SU(2)_{L+V}$ global invariance property, has been chosen for convenience in view of the subsequent discussion of the introduction of electromagnetism into the theory. Moreover, the form (2.5) renders manifest the different rôle of symmetry-breaking terms, as, in the absence of the first two terms, (2.5) would also be invariant under *local* $SU(2)_{L+V}$ transformations. The requirement of *global* $SU(2)_{L+V}$ symmetry is analogous to the requirement of global “residual” $SU(2)$ in the $SU(2)_L \times U(1)_Y$ standard electroweak theory and will be another ingredient necessary to obtain $\rho = 1$ at tree level, even in the presence of electromagnetism to be considered next.

Electromagnetic gauge invariance and the photon emerge by supplementing (2.5) with an appropriately chosen mass term for the B_μ field, upon postulating not only the B_μ (hypercharge) field,

$$B_\mu \rightarrow B_\mu + \frac{1}{g_1} \partial_\mu \chi(x), \quad (2.6)$$

but also the third component of the weak-isovector triplets,

$$\begin{aligned} \bar{W}_\mu^3 &\rightarrow \bar{W}_\mu^3 + \frac{1}{g_0} \partial_\mu \chi(x), \\ \bar{V}_\mu^3 &\rightarrow \bar{V}_\mu^3 + \frac{1}{g_2} \partial_\mu \chi(x), \end{aligned} \quad (2.7)$$

to transform under *electromagnetic* gauge transformations, $\chi(x) \in U(1)_{em}$. The transformation properties (2.6) and (2.7) allow one to modify (2.5) to take the manifestly $U(1)_{em}$ -invariant form

$$\begin{aligned} L_M = & \frac{v_1^2}{4} \text{tr} \left(g_0 \bar{W}_\mu^i \frac{\tau_i}{2} - g_1 B_\mu \frac{\tau_3}{2} \right)^2 + \frac{v_2^2}{4} \text{tr} \left(g_2 \bar{V}_\mu^i \frac{\tau_i}{2} - g_1 B_\mu \frac{\tau_3}{2} \right)^2 \\ & + \frac{v_3^2}{4} \text{tr} \left(g_0 \bar{W}_\mu^i \frac{\tau_i}{2} - g_2 \bar{V}_\mu^i \frac{\tau_i}{2} \right)^2, \end{aligned} \quad (2.8)$$

which is to be added to the basic Lagrangian (2.1). In (2.8), we introduced the trace and Pauli matrices in order to have a compact notation including both charged and neutral states. The orthogonal transformation relating the photon field to the basic fields $\bar{W}_\mu^3, \bar{V}_\mu^3$ and B_μ implies that it will transform under $U(1)_{em}$ via addition of

$$\frac{1}{e} \partial_\mu \chi(x) \quad (2.9)$$

with the electromagnetic coupling, e , being related to g_0, g_1, g_2 via

$$\frac{1}{e^2} = \frac{1}{g_0^2} + \frac{1}{g_1^2} + \frac{1}{g_2^2}. \quad (2.10)$$

The above introduction of mass terms is based on the observation that much of the empirical content of the (standard) electroweak theory, including ¹⁴ the dominant one-loop radiative corrections to four-fermion processes, is actually independent of the usually employed Higgs mechanism and only relies on a chain of specifically broken symmetries, i.e.,

$$SU(2) \times U(1)|_{\text{local}} \rightarrow SU(2)_{\text{global}} \times U(1)|_{\text{local}} \rightarrow U(1)_{\text{em}}. \quad (2.11)$$

The mass terms in (2.8) are nevertheless compatible with originating from a Higgs mechanism, which, in the case of (2.8) necessitates the introduction of a large number of observable scalar particles, essential for decent high-energy-behaviour in vector-boson scattering and for renormalizability at all orders of perturbation theory. We will comment on this point in Section 3.

The Lagrangian (2.1) with the mass term (2.8) is based on the six parameters [†]

$$(g_0, g_1, g_2, v_1, v_2, v_3) \quad (2.12)$$

and contains the three-parameter $SU(2)_L \times U(1)_Y$ theory as a special case, provided the $\bar{V}$ -boson term in (2.1) and the mass terms containing v_2 and v_3 in (2.8) are absent.

While referring to Sections 2.2 and 2.3 for a complete diagonalization of the theory, it will be useful to immediately remove the $\bar{V}$ $\bar{W}$ -mixing term in (2.8) via a two-by-two orthogonal transformation, $R_2(\varphi)$, the rotation angle φ being defined via

$$\frac{1}{2} \tan(2\varphi) \equiv \frac{\sin\varphi \cos\varphi}{\cos^2\varphi - \sin^2\varphi} = \frac{g_0 g_2 v_3^2}{g_0^2(v_1^2 + v_3^2) - g_2^2(v_2^2 + v_3^2)}. \quad (2.13)$$

Denoting the mass matrix originating from (2.8) by $\bar{m}^2$, the (diagonal) rotated mass matrix is given by

$$m^2 = R_2(\varphi) \bar{m}^2 R_2^T(\varphi), \quad (2.14)$$

where $R_2(\varphi)$ acts on the two-dimensional mass matrix determining $\bar{W}^\pm \bar{V}^\pm$ mixing in the charged sector and on the identical two-by-two-submatrix determining $\bar{W}^3 \bar{V}^3$ mixing in the neutral sector. The transformed fields, for later reference, will be denoted by W_μ^i and V_μ^i ; and, in the charged sector, they correspond to the physically observable particles of masses m_W and m_V given by

$$\begin{aligned} m_W^2 &= \frac{1}{4} \frac{g_0^2(v_1^2 + v_3^2) \cos^2\varphi - g_2^2(v_2^2 + v_3^2) \sin^2\varphi}{\cos^2\varphi - \sin^2\varphi}, \\ m_V^2 &= \frac{1}{4} \frac{g_2^2(v_2^2 + v_3^2) \cos^2\varphi - g_0^2(v_1^2 + v_3^2) \sin^2\varphi}{\cos^2\varphi - \sin^2\varphi}. \end{aligned} \quad (2.15)$$

Writing m_W^2 and m_V^2 in terms of $\sin\varphi$ and $\cos\varphi$ will be particularly convenient for the discussion to be given in Section 2.3.

[†] We are only counting the number of gauge couplings and gauge-boson mass parameters.

The vector boson triplets, $W^\pm$, W^3 and $V^\pm$, V^3 connected with m_W and m_V are related to $\bar{W}^\pm$, $\bar{W}^3$ and $\bar{V}^\pm$, $\bar{V}^3$ via

$$\begin{pmatrix} W_\mu^i \\ V_\mu^i \end{pmatrix} = R_2(\varphi) \begin{pmatrix} \bar{W}_\mu^i \\ \bar{V}_\mu^i \end{pmatrix} = \begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix} \begin{pmatrix} \bar{W}_\mu^i \\ \bar{V}_\mu^i \end{pmatrix} \quad (2.16)$$

and couple to leptons and quarks with the strengths

$$\begin{aligned} g_W &= g_0 \cos\varphi, \\ g_V &= g_0 \sin\varphi, \end{aligned} \quad (2.17)$$

respectively. Evidently, for $v_3 \rightarrow 0$, no $\bar{W}\bar{V}$ mixing will be present, and, consequently, the $V^\pm$ -boson shows the expected decoupling from leptons and quarks. Besides m_W , m_V and g_W , g_V , two more convenient parameters will be essential to characterise the physical content of the theory in the neutral sector. We will find it useful to employ the two mixing parameters λ_W and λ_V , which emerge from the current-mixing approach to be described next.

2.2 $\gamma\bar{W}^3$ and $\gamma\bar{V}^3$ Current Mixing

The starting point for the presentation of the theory is very similar to the one in Section 2.1: we introduce a Lagrangian invariant under local $SU(2)_L \times SU(2)_V$ symmetry which will be broken by mass terms which preserve global $SU(2)_{L+V}$ symmetry. The present approach differs from the one in Section 2.2 as regards the introduction of electromagnetism via the so-called current-mixing approach [†], which introduces mixing of the basic neutral fields, now denoted by $\bar{w}^3$ and $\bar{v}^3$, with the photon field in the kinetic terms of the theory. As this approach has been less popularized than the mass-mixing approach employed when using the Higgs mechanism, we start by a brief exposition of this approach for the case of the $SU(2)_L \times U(1)_Y$ theory.

Denoting the $SU(2)_L$ triplet by $\bar{w}_\mu^i$ and introducing a mass term which is invariant under global $SU(2)$, we have

$$L = -\frac{1}{4} \bar{w}_{\mu\nu}^i \bar{w}_i^{\mu\nu} + \frac{\bar{m}_W^2}{2} \bar{w}_\mu^i \bar{w}_i^\mu + g_0 \bar{w}_\mu^i j_\mu^i, \quad (2.18)$$

where the kinetic term is of non-Abelian form.

Note that the isovector vector-fields, $\bar{w}_\mu^i$, within the present approach are related to weak interactions only, and, in particular, the third component of the $\bar{w}_\mu^i$ triplet, $\bar{w}_\mu^3$, is unaffected by electromagnetic gauge transformations, in distinction from the transformation property under $U(1)_{\text{em}}$ of the field $\bar{W}_\mu^3$ which has been introduced in (2.1). As a

[†] As regards the interplay of electromagnetism and weak interactions this approach has first been used by Bjorken ⁶ and Hung-Sakurai ¹³.

second step, the photon field, a_μ , is introduced (without mass terms) and coupled to the electromagnetic current of the fermions via $e a^\mu j_\mu^{em}$, where

$$j_\mu^{em} = \sum_i Q_i \bar{\psi}_i \gamma_\mu \psi_i, \quad (2.19)$$

and to the currents of the charged vector bosons in a $U(1)_{em}$ -invariant manner via the replacement of ∂_μ in (2.18) by the covariant derivative

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - ieQ a_\mu, \quad (2.20)$$

where Q designates the charge of the different quarks, leptons, and vector bosons. The covariant derivative (2.20) obviously generates $U(1)_{em}$ -invariant trilinear and quadrilinear interactions of the photon with the charged bosons, $\bar{w}^\pm$. We also add a magnetic moment term, which is gauge-invariant by itself, but not generated by (2.20),

$$+ie \kappa a^{\mu\nu} \bar{w}_\mu^+ \bar{w}_\nu, \quad (2.21)$$

Here, κ denotes the so-called anomalous magnetic moment of the charged $\bar{w}^\pm$. Moreover, at the same time, we allow for a $\gamma\bar{w}^3$ mixing term of the form

$$-\frac{\lambda}{2} \bar{w}_{\mu\nu}^3 a^{\mu\nu} \quad (2.22)$$

with the strength λ .

The number of free parameters introduced so far is reduced, and the connection with the usual form of the standard $SU(2)_L \times U(1)_Y$ theory (apart from the Higgs scalar) is established by imposing local $SU(2)_L \times U(1)_Y$ symmetry of the resulting Lagrangian in the massless limit. Indeed, rewriting the Lagrangian in terms of the fields $\bar{W}_\mu$ defined via the linear field transformation

$$\begin{aligned} \bar{W}_\mu^3 &= \bar{w}_\mu^3 + \lambda a_\mu, \\ \bar{W}_\mu^\pm &\equiv \bar{w}_\mu^\pm, \end{aligned} \quad (2.23)$$

and imposing local $SU(2)_L$ symmetry of the Lagrangian (2.18) to (2.22) in the massless limit, thereby treating the fields $\bar{W}^\pm$ and $\bar{W}^3$ in (2.23) as the three components of an $SU(2)$ triplet, immediately implies[†]

$$\lambda = e/g_0 \quad (2.24)$$

as well as

$$\kappa = 1 \quad (2.25)$$

thus reducing the number, four, of coupling parameters (g_0 , e , λ and κ) to the canonical number, two. The manifestly $SU(2)_L \times U(1)_Y$ -invariant form of the Lagrangian (2.18) to

[†] Conditions (2.24) and (2.26) evidently eliminate terms proportional to $(\lambda - e/g_0)$ and $(\kappa - 1)$ which violate $SU(2)_L$ symmetry.

(2.22) (invariant in the massless limit), becomes indeed evident, when written in terms of the transformed fields (2.23)[†],

$$\begin{aligned} L &= -\frac{1}{4} \bar{W}_{\mu\nu}^i \bar{W}^{\mu\nu}_i + g_0 j_i^\mu \bar{W}_\mu^i + \frac{\tilde{m}_W^2}{2} \bar{w}_\mu^i \bar{w}_i^\mu \\ &\quad - \frac{1}{4} (1 - \lambda^2) a^{\mu\nu} a_{\mu\nu} + e j_\mu^Y a^\mu \end{aligned} \quad (2.26)$$

(with λ fulfilling (2.24)), where the hypercharge current (consistent with (2.3)) has been defined via

$$j_\mu^Y = j_\mu^{em} - j_\mu^3. \quad (2.27)$$

The agreement of (2.26) with the standard $SU(2)_L \times U(1)_Y$ Lagrangian is completed by introducing

$$B_\mu = \sqrt{1 - \lambda^2} a_\mu \quad (2.28)$$

and noting that under $U(1)_{em}$ transformations,

$$a_\mu \rightarrow a_\mu + \frac{1}{e} \partial_\mu \chi(x), \quad (2.29)$$

according to (2.23) and (2.24), $\bar{W}_\mu^3$ transforms as

$$\bar{W}_\mu^3 \rightarrow \bar{W}_\mu^3 + \frac{1}{g_0} \partial_\mu \chi(x), \quad (2.30)$$

and

$$B_\mu \rightarrow B_\mu + \frac{1}{g_1} \partial_\mu \chi(x), \quad (2.31)$$

where g_1 is related to e via

$$g_1 = \frac{e}{\sqrt{1 - \lambda^2}}, \quad (2.32)$$

and, by finally expressing the fields $\bar{w}_\mu^i$ in the mass term in (2.26) in terms of the fields $\bar{W}_\mu^i$ using (2.23) and (2.28).

Comparing (2.18) with (2.26), one notes that the introduction of electromagnetism into the Lagrangian (2.18), under the requirement of local $SU(2)_L \times U(1)_Y$ symmetry in the massless limit, formally amounts to the replacement

$$\begin{aligned} \bar{w}_\mu^3 &\rightarrow \bar{W}_\mu^3 = \bar{w}_\mu^3 + \lambda a_\mu, \\ \bar{w}_\mu^\pm &\rightarrow \bar{W}_\mu^\pm \end{aligned} \quad (2.33)$$

in the kinetic and interaction terms. In addition, the kinetic term for the electromagnetic field is to be added as well as the interaction with the hypercharge current.

With the powerful substitution law (2.33) at hand, the generalization to the $SU(2)_L \times SU(2)_Y \times U(1)_Y$ case becomes a straightforward matter. Allowing for the most general

[†] The kinetic mixing term (2.22) is evidently hidden in (2.26) and only appears, if (2.26) is exclusively expressed in terms of $\bar{w}_\mu^i$.

mass term, invariant under global $SU(2)_{L+V}$, within a Lagrangian, which is invariant under local $SU(2)_L \times SU(2)_V$ in the massless limit, we have

$$L = -\frac{1}{4} \bar{w}_\mu^\mu \bar{w}_\mu^\mu - \frac{1}{4} \bar{v}_\mu^\mu \bar{v}_\mu^\mu + g_0 j_\mu^\mu \bar{w}_\mu^\mu + \frac{\bar{m}_W^2}{2} \bar{w}_\mu^\mu \bar{w}_\mu^\mu + \frac{\bar{m}_V^2}{2} \bar{v}_\mu^\mu \bar{v}_\mu^\mu + \bar{m}_{VW}^2 \bar{v}_\mu^\mu \bar{w}_\mu^\mu, \quad (2.34)$$

where, as in Section 2.1, the extra vector boson has vanishing direct coupling to the usual weak-isospin current. Extending the previous reasoning amounts to generalizing the substitution rule (2.33) to become

$$\begin{aligned} \bar{w}_\mu^3 &\rightarrow \bar{W}_\mu^3 = \bar{w}_\mu^3 + \lambda_W^0 a_\mu, \\ \bar{v}_\mu^3 &\rightarrow \bar{V}_\mu^3 = \bar{v}_\mu^3 + \lambda_V^0 a_\mu, \end{aligned} \quad (2.35)$$

and

$$\begin{aligned} \bar{w}_\mu^\pm &\rightarrow \bar{W}_\mu^\pm \equiv \bar{w}_\mu^\pm, \\ \bar{v}_\mu^\pm &\rightarrow \bar{V}_\mu^\pm \equiv \bar{v}_\mu^\pm, \end{aligned} \quad (2.36)$$

where

$$\lambda_W^0 g_0 = \lambda_V^0 g_2 = e \quad (2.37)$$

guarantees the correct coupling of the photon to the weak-isospin part of the electromagnetic fermion current and to the charged $\bar{w}_\mu^\pm$ and $\bar{v}_\mu^\pm$ fields.

In analogy to (2.26), the Lagrangian is complemented by adding the kinetic term of the photon field and the coupling of the photon to the weak-hypercharge current, thus leading to

$$\begin{aligned} L = & -\frac{1}{4} \bar{W}_{\mu\nu}^\mu \bar{W}_\mu^\mu - \frac{1}{4} \bar{V}_{\mu\nu}^\mu \bar{V}_\mu^\mu + g_0 j_\mu^\mu \bar{W}_\mu^\mu \\ & - \frac{1}{4} \left(1 - \lambda_W^0 - \lambda_V^0 \right) a_{\mu\nu} a^{\mu\nu} + e j_\mu^Y a^\mu \\ & + \frac{\bar{m}_W^2}{2} \bar{w}_\mu^\mu \bar{w}_\mu^\mu + \frac{\bar{m}_V^2}{2} \bar{v}_\mu^\mu \bar{v}_\mu^\mu + \bar{m}_{VW}^2 \bar{v}_\mu^\mu \bar{w}_\mu^\mu. \end{aligned} \quad (2.38)$$

This Lagrangian is obviously invariant under local $SU(2)_L \times SU(2)_V \times U(1)_V$ transformations in the massless limit. To establish the equivalence of (2.38) in the massless limit with (2.1) we introduce

$$B_\mu = \sqrt{1 - \lambda_W^0 - \lambda_V^0} a_\mu \quad (2.39)$$

in (2.38) and the hypercharge coupling, g_1 , related to e via

$$g_1 = e \left(1 - \lambda_W^0 - \lambda_V^0 \right)^{-1/2}. \quad (2.40)$$

We then note that under electromagnetic gauge transformations,

$$a_\mu \rightarrow a_\mu + \frac{1}{e} \partial_\mu \chi(x), \quad (2.41)$$

the fields $\bar{W}_\mu^3$ and $\bar{V}_\mu^3$ transform as

$$\begin{aligned} \bar{W}_\mu^3 &\rightarrow \bar{W}_\mu^3 + \frac{1}{g_0} \partial_\mu \chi(x), \\ \bar{V}_\mu^3 &\rightarrow \bar{V}_\mu^3 + \frac{1}{g_2} \partial_\mu \chi(x), \end{aligned} \quad (2.42)$$

where (2.37) has been inserted. The equivalence of the mass matrix in (2.38) with the mass matrix in (2.8) may also be established, essentially, by expressing $\bar{w}_\mu^i$, $\bar{v}_\mu^i$ in terms of $\bar{W}_\mu^i$, $\bar{V}_\mu^i$ according to the equalities in (2.35) and (2.36). The basic six parameters in the Lagrangian in the present formulation may be chosen as

$$(g_0, g_2, e, \bar{m}_W, \bar{m}_V, \bar{m}_{VW}), \quad (2.43)$$

or, equivalently,

$$(\lambda_W^0, \lambda_V^0, e, \bar{m}_W, \bar{m}_V, \bar{m}_{VW}). \quad (2.44)$$

Clearly, the mixing with the photon field in the kinetic terms in (2.38) becomes manifest only upon explicitly expressing $\bar{W}$, $\bar{V}$ in terms of $\bar{w}$, $\bar{v}$ according to (2.35) and (2.36). We refrain from explicitly carrying out this substitution. It is much more profitable to proceed with the Lagrangian (2.38) and to carry out the transition to the physical fields.

As a first step, as at the end of Section 2.1, we diagonalize the two-by-two mass matrix in (2.38) to obtain the fields which describe the physical particles of masses m_W and m_V in the charged sector. Explicitly we have[†]

$$\begin{pmatrix} w_\mu^i \\ v_\mu^i \end{pmatrix} = R_2(\varphi) \begin{pmatrix} \bar{w}_\mu^i \\ \bar{v}_\mu^i \end{pmatrix}, \quad (2.45)$$

or, in terms of the fields (2.35) and (2.36),

$$\begin{pmatrix} W_\mu^i \\ V_\mu^i \end{pmatrix} = R_2(\varphi) \begin{pmatrix} \bar{W}_\mu^i \\ \bar{V}_\mu^i \end{pmatrix}, \quad (2.46)$$

which, for the neutral fields is explicitly written as

$$\begin{pmatrix} W_\mu^3 \\ V_\mu^3 \end{pmatrix} = \begin{pmatrix} w_\mu^3 + \lambda_W a_\mu \\ v_\mu^3 + \lambda_V a_\mu \end{pmatrix}, \quad (2.47)$$

where by definition

$$\begin{pmatrix} \lambda_W \\ \lambda_V \end{pmatrix} = R_2(\varphi) \begin{pmatrix} \lambda_W^0 \\ \lambda_V^0 \end{pmatrix}. \quad (2.48)$$

[†] For the sake of clarity, we note that $R_2(\varphi)$ in (2.45), even though being identical conceptually to $R_2(\varphi)$ in Section 2.1, in the present case is to be expressed in terms of $\bar{m}_W$, $\bar{m}_{VW}$, and $\bar{m}_V$, rather than in terms of the parameters appearing in (2.13). The explicit expression for $R_2(\varphi)$ in (2.45) is unimportant for what follows, while (2.13) will be used later on to connect the different sets of parameters used in Section 2.1 and the present Section.

We note that (2.47), written in terms of $SU(2)_{L+V}$ mass eigenstates, w_μ^3, v_μ^3 , is identical in its form to the original relation (2.35).

In terms of the rotated fields Lagrangian (2.38) takes the form

$$\begin{aligned}
 L = & -\frac{1}{4} W_{\mu\nu}^i W_{\mu\nu}^{i\mu} + \frac{m_W^2}{2} w_\mu^i w_\mu^{i\mu} \\
 & -\frac{1}{4} V_{\mu\nu}^i V_{\mu\nu}^{i\mu} + \frac{m_V^2}{2} v_\mu^i v_\mu^{i\mu} \\
 & + j_\mu^{\mu i} (g_W W_\mu^i + g_V V_\mu^i) \\
 & -\frac{1}{4} (1 - \lambda_W^2 - \lambda_V^2) a_{\mu\nu} a^{\mu\nu} + e j_\mu^Y a^\mu \\
 & + \text{"non-Abelian part"} ,
 \end{aligned} \tag{2.49}$$

where the $W_{\mu\nu}^i, V_{\mu\nu}^i$ kinetic terms have *Abelian* form. Here, we have introduced the masses of the charged bosons, m_W and m_V , obtained via diagonalization of the mass matrix, as well as the transformed coupling constants,

$$\begin{aligned}
 g_W &= g_0 \cos \varphi, \\
 g_V &= g_0 \sin \varphi,
 \end{aligned} \tag{2.50}$$

both sets of parameters coinciding with (2.15) to (2.17) with respect to their physical significance. Note that the linear transformation (2.46) introduces a fairly involved structure in the kinetic terms in (2.49) due to the non-Abelian nature of the kinetic term in (2.38). The trilinear and the quadrilinear interactions implicitly contained in the “non-Abelian part” in (2.49) will be analysed in detail in Section 5.

We note that the field equation for the (primordial) photon field derived from (2.49) takes the form

$$\partial^\nu a_\nu^\mu = \frac{\lambda_W m_W^2 w_\mu^3 + \lambda_V m_V^2 v_\mu^3 - e j_\mu^Y}{1 - \lambda_W^2 - \lambda_V^2}, \tag{2.51}$$

while from (2.37), (2.48) and (2.50) we conclude that

$$g_W \lambda_W + g_V \lambda_V = e. \tag{2.52}$$

Equations (2.51) and (2.52) simply say that the source of the isovector photon is *exclusively* due to the neutral components of the w_μ^i and v_μ^i vector fields of masses m_W and m_V , respectively. The theory incorporates w_μ^3, v_μ^3 dominance of photon interactions with fermions and bosons[†]. The $SU(2)_L \times U(1)_Y$ case, as is well-known, corresponds to $\lambda_W g_W = e$, i.e. W dominance^{15,16}.

We proceed to the physical base in (2.49) by first of all introducing W_μ^3, V_μ^3 also in the mass terms in (2.49), replacing, at the same time, a_μ by B_μ . According to (2.47) and (2.39), the corresponding transformation may be written as

$$\begin{pmatrix} a_\mu^3 \\ w_\mu^3 \\ v_\mu^3 \end{pmatrix} = D^T \begin{pmatrix} B_\mu^3 \\ W_\mu^3 \\ V_\mu^3 \end{pmatrix}, \tag{2.53}$$

[†] The asymmetry, as regards j_μ^Y , in (2.51) lead to the conjecture of the existence of additional isoscalar partners¹⁶, $Y_\mu(Y_{L,\mu})$ of the isovector w_μ^i triplet.

where

$$D = \begin{pmatrix} \frac{1}{\sqrt{1-\lambda_W^2-\lambda_V^2}} & \frac{-\lambda_W}{\sqrt{1-\lambda_W^2-\lambda_V^2}} & \frac{-\lambda_V}{\sqrt{1-\lambda_W^2-\lambda_V^2}} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{2.54}$$

and

$$(D^T)^{-1} = \begin{pmatrix} \sqrt{1-\lambda_W^2-\lambda_V^2} & 0 & 0 \\ \lambda_W & 1 & 0 \\ \lambda_V & 0 & 1 \end{pmatrix}. \tag{2.55}$$

The resulting non-diagonal mass-matrix in the neutral current sector

$$M_V^2 = D M^2 D^T, \tag{2.56}$$

where

$$M^2 = \begin{pmatrix} 0 & 0 & 0 \\ m_W^2 & 0 & 0 \\ 0 & 0 & m_V^2 \end{pmatrix}, \tag{2.57}$$

is finally diagonalized by an orthogonal transformation R yielding the neutral vector boson masses m_{Z_0} and m_{V_0} via

$$R M_V^2 R^T = \begin{pmatrix} 0 & 0 & 0 \\ m_{Z_0}^2 & 0 & 0 \\ 0 & 0 & m_{V_0}^2 \end{pmatrix} \tag{2.58}$$

and the (physical) photon, Z_0 , and V_0 fields according to

$$\begin{pmatrix} A_\mu^0 \\ Z_\mu^0 \\ V_\mu^0 \end{pmatrix} = R \begin{pmatrix} B_\mu^3 \\ W_\mu^3 \\ V_\mu^3 \end{pmatrix}. \tag{2.59}$$

The resulting neutral-current interaction Lagrangian, together with the charged-current Lagrangian for the interaction of the vector bosons with leptons and quarks is given in Table 1. For completeness and later reference we also include the explicit form of the transformation R .

We note that the interaction Lagrangians in Table 1 are explicitly expressed in terms of the set of parameters

$$(g_W, g_V, \lambda_W, \lambda_V, m_W, m_V) \tag{2.60}$$

supplementing the empirically observable masses of the charged bosons and their fermionic couplings with the two mixing parameters λ_W and λ_V which describe the mixing of the (primordial) photon with the neutral partners of the charged $W^\pm$ and $V^\pm$ bosons. The trilinear and quadrilinear vector-boson self-interactions will be treated in Section 5.

$$\begin{aligned}
L_{CC} &= \frac{g_W}{\sqrt{2}} (j_+^\mu W_\mu^+ + j_-^\mu W_\mu^-) + \frac{g_V}{\sqrt{2}} (j_+^\mu V_\mu^+ + j_-^\mu V_\mu^-), \\
L_{NC} &= e j_\mu^{em} A^\mu + g_{Z_0} (j_\mu^3 - X_Z j_\mu^{em}) Z_0^\mu + g_{V_0} (j_\mu^3 - X_V j_\mu^{em}) V_0^\mu, \\
&\quad \text{where } g_{Z_0} = e \frac{g_Z}{X_Z}, \quad g_{V_0} = e \frac{g_V}{X_V}, \\
X_Z &= e \left(\frac{g_W \lambda_W}{1 - \frac{g_W^2}{m_{Z_0}^2}} + \frac{g_V \lambda_V}{1 - \frac{g_V^2}{m_{Z_0}^2}} \right)^{-1}, \quad X_V = e \left(\frac{g_W \lambda_W}{1 - \frac{g_W^2}{m_{V_0}^2}} + \frac{g_V \lambda_V}{1 - \frac{g_V^2}{m_{V_0}^2}} \right)^{-1}, \\
&\quad \text{with } e = g_W \lambda_W + g_V \lambda_V.
\end{aligned}$$

$$\begin{aligned}
n_Z &= \left(1 - \lambda_W^2 - \lambda_V^2 + \frac{\lambda_W^2}{\left(1 - \frac{m_{Z_0}^2}{m_W^2}\right)^2} + \frac{\lambda_V^2}{\left(1 - \frac{m_{Z_0}^2}{m_V^2}\right)^2} \right)^{-1/2}, \\
n_V &= \left(1 - \lambda_W^2 - \lambda_V^2 + \frac{\lambda_W^2}{\left(1 - \frac{m_{V_0}^2}{m_W^2}\right)^2} + \frac{\lambda_V^2}{\left(1 - \frac{m_{V_0}^2}{m_V^2}\right)^2} \right)^{-1/2}, \\
&\quad \text{with } m_{Z_0}^2 + m_{V_0}^2 = m_W^2 + m_V^2 + \frac{\lambda_W^2 m_W^2 + \lambda_V^2 m_V^2}{1 - \lambda_W^2 - \lambda_V^2}, \\
&\quad m_{Z_0}^2 m_{V_0}^2 = \frac{m_W^2 m_V^2}{1 - \lambda_W^2 - \lambda_V^2}.
\end{aligned}$$

$$R = \begin{pmatrix} \sqrt{1 - \lambda_W^2 - \lambda_V^2} & \lambda_W & \lambda_V \\ -n_Z \sqrt{1 - \lambda_W^2 - \lambda_V^2} & -n_Z \lambda_W \frac{m_W^2}{m_W^2 - m_{Z_0}^2} & -n_Z \lambda_V \frac{m_V^2}{m_V^2 - m_{Z_0}^2} \\ -n_V \sqrt{1 - \lambda_W^2 - \lambda_V^2} & -n_V \lambda_W \frac{m_W^2}{m_W^2 - m_{V_0}^2} & -n_V \lambda_V \frac{m_V^2}{m_V^2 - m_{V_0}^2} \end{pmatrix}$$

Table 1: The charged-current and neutral-current Lagrangians of the $SU(2)_L \times SU(2)_V \times U(1)_Y$ theory in terms of the couplings, mixing parameters and (charged-boson) masses $(g_W, g_V, \lambda_W, \lambda_V, m_W, m_V)$.

A straightforward calculation on the basis of Table 1 yields the useful charged-current and neutral-current $q^2 \rightarrow 0$ four-fermion interactions

$$\begin{aligned}
L_{CC}(q^2 \rightarrow 0) &= \left(\frac{g_W^2}{m_W^2} + \frac{g_V^2}{m_V^2} \right) \frac{j_+^\mu j_-^\mu}{\sqrt{2} \sqrt{2}}, \\
L_{NC}(q^2 \rightarrow 0) &= \frac{e^2}{q^2} (j_\mu^{em})^2 + \left(\frac{g_{Z_0}^2}{m_{Z_0}^2} + \frac{g_{V_0}^2}{m_{V_0}^2} \right) (j_\mu^3 - \Sigma \delta j_\mu^{em})^2 + C (j_\mu^{em})^2,
\end{aligned} \tag{2.61}$$

where

$$\Sigma_\theta^2 = \frac{e \left(\frac{g_W \lambda_W}{m_W^2} + \frac{g_V \lambda_V}{m_V^2} \right)}{\frac{g_W^2}{m_W^2} + \frac{g_V^2}{m_V^2}} \tag{2.62}$$

and

$$C = \frac{e^2 \left(\frac{\lambda_W^2}{m_W^2} + \frac{\lambda_V^2}{m_V^2} \right)}{\frac{g_W^2}{m_W^2} + \frac{g_V^2}{m_V^2}} - \Sigma_\theta^4. \tag{2.63}$$

Furthermore, we have for the ratio of NC to CC strengths

$$\rho \equiv \frac{\frac{g_{Z_0}^2}{m_{Z_0}^2} + \frac{g_{V_0}^2}{m_{V_0}^2}}{\frac{g_W^2}{m_W^2} + \frac{g_V^2}{m_V^2}} = 1. \tag{2.64}$$

This result is a consequence of the underlying $SU(2)_{L+V}$ symmetry which is broken by mixing with the photon only, combined, with the assumed $SU(2)_V$ -singlet nature of the known leptons and quarks, excluding a direct coupling to the vector boson triplet, $\vec{V}$, of these particles. All these ingredients enter the form of the transformation $R(D^T)^{-1}$ which is responsible for the diagonalization of the $q^2 \rightarrow 0$ neutral current propagator which implies $\rho = 1$.

2.3 The Relation between the Mass-Mixing and the Current-Mixing Parameters.

In this Section, we will briefly collect the relations between the primordial parameters

$$(g_0, g_1, g_2, v_1, v_2, v_3) \tag{2.65}$$

of the mass-mixing approach of Section 2.1 and the set of parameters

$$(g_W, g_V, \lambda_W, \lambda_V, m_W, m_V) \tag{2.66}$$

which contains the mixing strengths, λ_W and λ_V , characteristic for the current-mixing approach of Section 2.2.

The expressions for the set (2.66) in terms of the set (2.65) have essentially been given and have to be collected from (2.13) to (2.17), and, concerning λ_W and λ_V , from (2.37) and (2.48). For easy reference these relations have been collected in Table 2a.

$$\begin{aligned}
\lambda_W &= \frac{e}{g_0} \cos \varphi - \frac{e}{g_2} \sin \varphi, & g_V &= g_0 \sin \varphi, \\
m_W^2 &= \frac{1}{4} \frac{g_0^2(v_1^2+v_3^2) \cos^2 \varphi - g_2^2(v_2^2+v_3^2) \sin^2 \varphi}{\cos^2 \varphi - \sin^2 \varphi}, \\
m_V^2 &= \frac{1}{4} \frac{g_2^2(v_2^2+v_3^2) \cos^2 \varphi - g_0^2(v_1^2+v_3^2) \sin^2 \varphi}{\cos^2 \varphi - \sin^2 \varphi}, \\
\text{where } \frac{\sin \varphi \cos \varphi}{\cos^2 \varphi - \sin^2 \varphi} &= \frac{g_0 g_2 v_3^2}{g_0^2(v_1^2+v_3^2) - g_2^2(v_2^2+v_3^2)} \equiv \frac{1}{2} \tan(2\varphi) \\
&\text{and } e^{-2} = g_0^{-2} + g_1^{-2} + g_2^{-2}.
\end{aligned}$$

Table 2a: The current mixing parameters in terms of the mass mixing parameters.

The reciprocal relations are most easily obtained upon identifying the mass matrix (2.14), obtained from the original mass matrix (2.8) of the mass-mixing approach by applying $R_2(\varphi)$, with the matrix (2.56) derived within the current-mixing approach, i.e.,

$$R_2(\varphi) \bar{m}^2 R_2^T(\varphi) = D \bar{M}^2 D^T. \quad (2.67)$$

From this relation one may extract the results shown in Table 2b.

$$\begin{aligned}
g_0 &= (g_W^2 + g_V^2)^{1/2}, & g_1 &= e(1 - \lambda_W^2 - \lambda_V^2)^{-1/2}, & g_2 &= \frac{e(g_W^2 + g_V^2)^{1/2}}{g_W \lambda_V - g_V \lambda_W}, \\
v_1^2 &= 4 \frac{g_W \lambda_W m_W^2 + g_V \lambda_V m_V^2}{e(g_W^2 + g_V^2)}, \\
v_2^2 &= 4 \frac{(g_W \lambda_V - g_V \lambda_W)(g_W \lambda_V m_V^2 - g_V \lambda_W m_W^2)}{e^2(g_W^2 + g_V^2)}, \\
v_3^2 &= 4 \frac{g_W g_V (g_V \lambda_W - g_W \lambda_V)(m_V^2 - m_W^2)}{e(g_W^2 + g_V^2)^2} \\
&\text{where } e = g_W \lambda_W + g_V \lambda_V.
\end{aligned}$$

Table 2b: The mass-mixing parameters in terms of the current-mixing parameters.

A useful consequence of the current-mixing formalism appears when considering limiting cases of the values of the parameters. For $\lambda_V \rightarrow 0$ and $g_V \rightarrow 0$, the Lagrangian of Table 1 can be shown to correspond to the standard model limit, *independently* of whether m_V is finite or not. Although from Tables 2a and 2b we conclude that $\lambda_V \rightarrow 0$, $g_V \rightarrow 0$ is equivalent to $g_2 \rightarrow \infty$, $\varphi \rightarrow 0$, it is, in contrast, a somewhat delicate matter to establish the full connection of the limit of $\lambda_V \rightarrow 0$, $g_V \rightarrow 0$, m_V *arbitrary* with the corresponding limit in the mass-mixing set of parameters ($g_0, g_1, g_2, v_1, v_2, v_3$). Alternatively, as m_V is an independent parameter in the current-mixing set ($g_W, g_V, \lambda_W, \lambda_V, m_W, m_V$), the limit $m_V \rightarrow \infty$ does *not* lead to the standard $SU(2)_L \times U(1)_Y$ model, unless the above conditions $\lambda_V \rightarrow 0$, $g_V \rightarrow 0$ are fulfilled. A more detailed discussion of this “non-decoupling” property will be given in Section 4.

We refrain from explicitly expressing the Lagrangian of Table 1 in terms of the mass-mixing parameters, but we will give the $q^2 \rightarrow 0$ Lagrangian (2.61) in the mass-mixing base. Using Table 2a, the Lagrangian (2.61) becomes

$$L_{CC}(q^2 \rightarrow 0) = 4 \frac{v_2^2 + v_3^2}{v_1^2 v_2^2 + v_1^2 v_3^2 + v_2^2 v_3^2} \frac{j_\mu^+ j_\mu^-}{\sqrt{2} \sqrt{2}},$$

$$L_{NC}(q^2 \rightarrow 0) = \frac{e^2}{q^2} (j_\mu^{em})^2 + 4 \frac{v_2^2 + v_3^2}{v_1^2 v_2^2 + v_1^2 v_3^2 + v_2^2 v_3^2} \left((j_\mu^+ - \Sigma_\theta^2 j_\mu^{em})^2 + C (j_\mu^{em})^2 \right), \quad (2.68)$$

where

$$\Sigma_\theta^2 = \frac{(e^2/g_0^2) v_2^2 + (1 - e^2/g_1^2) v_3^2}{v_2^2 + v_3^2}, \quad (2.69)$$

$$C = \frac{e^4 \left(\frac{v_2^2 + v_3^2}{g_0^2} + \frac{v_1^2 + v_2^2}{g_2^2} + 2 \frac{v_1^2}{g_0 g_2^2} \right)}{v_2^2 + v_3^2} - \Sigma_\theta^4. \quad (2.70)$$

An important phenomenological consequence on possible deviations of the electroweak $q^2 \rightarrow 0$ mixing parameter, Σ_θ^2 , from its $SU(2)_L \times U(1)_Y$ value now follows immediately by employing the inequality

$$\frac{e^2}{g_0^2} \leq 1 - \frac{e^2}{g_1^2} \quad (2.71)$$

which follows from the expression for e in Table 2a. From (2.69) we obtain

$$\frac{e^2}{g_0^2} \leq \Sigma_\theta^2 \leq e^2 \left(\frac{1}{g_0^2} + \frac{1}{g_2^2} \right), \quad (2.72)$$

or, when passing to λ_V and g_V according to Table 2b,

$$\frac{e^2}{g_W^2 + g_V^2} \leq \Sigma_\theta^2 \leq \lambda_W^2 + \lambda_V^2. \quad (2.73)$$

The lower and upper bound on possible deviations of Σ_θ^2 , from its $SU(2)_L \times U(1)_Y$ value of $\Sigma_\theta^2 = \lambda_W^2 = e^2/g_W^2$ (reached for $\lambda_V = g_V = 0$) is thus entirely determined by the magnitude of g_V and λ_V and independent of the mass m_V . In other words, provided $\lambda_V g_V \neq 0$, an arbitrarily heavy vector-boson triplet can still yield non-vanishing deviations from the $SU(2)_L \times U(1)_Y$ theory via modifying the value of the effective mixing angle. This non-decoupling property for $m_V \rightarrow \infty$ appears as a simple consequence of the saturation (or w^3, v^3 dominance) condition (2.52). In other words: in the limit $m_V \rightarrow \infty$ with $\lambda_V g_V \neq 0$, the primordial photon has a pointlike interaction with leptons and quarks in addition to the interaction via mixing with the w^3 boson.

3. On Renormalizability and Specific Models Obtained by Restrictions on Parameters

In this Section, we will briefly address the question on whether the model of Section 2 can be enlarged by adding an appropriate number of Higgs scalars to become a renormalizable theory. We will also show that various theories containing an extra isovector vector-boson triplet that were considered in the previous literature, are actually obtained as special cases within the framework of the present paper.

The question of the renormalizability of the model of Section 2 amounts to constructing a Higgs potential which upon spontaneous symmetry breaking yields precisely the mass matrix (2.8). By generalizing earlier considerations on extended gauge theories^{4,5} such a construction of a suitable Higgs potential has in fact been carried out in ref. 10. The model, dubbed “BMW model”[†] by the authors, indeed contains a sufficiently general Higgs sector to precisely reproduce the mass matrix (2.8).

The BMW construction is based on three Higgs fields, two complex doublets transforming non-trivially under $SU(2)_L \times U(1)_Y$ and $SU(2)_V \times U(1)_Y$, respectively, as well as a self-dual quartet, transforming non-trivially under $SU(2)_L \times SU(2)_V$, i.e.,

$$\begin{aligned}\phi_1 &\sim \left(\frac{1}{2}, 0, Y = \frac{1}{2}\right), \\ \phi_2 &\sim \left(0, \frac{1}{2}, Y = \frac{1}{2}\right), \\ \phi_3 &\sim \left(\frac{1}{2}, \frac{1}{2}, Y = 0\right),\end{aligned}\quad (3.1)$$

under $SU(2)_L \times SU(2)_V \times U(1)_Y$ with

$$<\phi_1> = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_1 \end{pmatrix}, \quad <\phi_2> = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad <\phi_3> = \frac{1}{\sqrt{2}} \begin{pmatrix} v_3 & 0 \\ 0 & v_3 \end{pmatrix}, \quad (3.2)$$

where all vacuum expectation values squared have been assumed¹⁰ to be positive, $v_i^2 \geq 0$. For later reference, we note that this positivity requirement actually corresponds to a restriction of the parameter space of Section 2, as not all of the constants v_i^2 in (2.8) have necessarily to be positive, in order to yield positive mass eigenvalues. If the Higgs mechanism is employed, six of the twelve scalar degrees of freedom in (3.1) remain as physical scalar particles. They are necessary for renormalizability and enlarge the particle content of the effective theory described in Section 2.

A model which appears as a special case of our framework of Section 2 is the model of ref. 12, more recently called the “BESS” model¹⁷ by the authors. Upon rewriting its mass term^{12,17},

$$L_M^{BESS} = \frac{v^2}{4} \left(\text{tr} \left(g_0 W_\mu^{i\tau_i} - g_1 B_\mu \frac{\tau_3}{2} \right)^2 + \alpha \text{tr} \left(g_0 W_\mu^{i\tau_i} + g_1 B_\mu \frac{\tau_3}{2} - 2g_2 V_\mu^{i\tau_i} \right)^2 \right), \quad (3.3)$$

[†] Not to be confused with the German four-wheel luxury product.

in the form

$$L_M^{BESS} = \frac{v^2}{4} (1 - \alpha) \text{tr} \left(g_0 W_\mu^{i\tau_i} - g_1 B_\mu \frac{\tau_3}{2} \right)^2 + \left(\frac{v^2}{4} \right) 2\alpha \text{tr} \left(g_2 V_\mu^{i\tau_i} - g_1 B_\mu \frac{\tau_3}{2} \right)^2 + \left(\frac{v^2}{4} \right) 2\alpha \text{tr} \left(g_0 W_\mu^{i\tau_i} - g_2 V_\mu^{i\tau_i} \right)^2, \quad (3.4)$$

one realizes that it is a special case of (2.8), that is obtained via the restriction

$$v_2^2 = v_3^2. \quad (3.5)$$

Indeed, (2.8) is identical to (3.4) for

$$\begin{aligned}v_2^2 &= v_3^2 = 2v^2 \alpha, \\ v_1^2 &= v^2 (1 - \alpha).\end{aligned}\quad (3.6)$$

The constraint (3.5) thus restricts the number of three mass parameters to two such parameters which may be taken as v^2 and α . We note that the mass matrix corresponding to (3.4) yields positive eigenvalues for any $\alpha \geq 0$. According to Table 2b, the restriction (3.6) corresponds to

$$\lambda_V = \lambda_W \left(\frac{g_V}{g_W} \right) \frac{(g_V^2 + 2g_W^2) m_V^2 - g_W^2 m_V^2}{(g_W^2 + 2g_V^2) m_V^2 - g_V^2 m_W^2}. \quad (3.7)$$

We note in passing that (3.5), when inserted into the expression (2.69) for the low-energy mixing parameter, Σ_θ^2 , implies

$$\Sigma_\theta^2(BESS) = \frac{1}{2} \left(\frac{e^2}{g_0^2} + e^2 \left(\frac{1}{g_0^2} + \frac{1}{g_2^2} \right) \right), \quad (3.8)$$

or, upon using Table 2b,

$$\Sigma_\theta^2(BESS) = \frac{1}{2} \left(\frac{e^2}{g_W^2 + g_V^2} + \lambda_W^2 + \lambda_V^2 \right). \quad (3.9)$$

Comparing (3.8) and (3.9) with the upper and lower bounds on Σ_θ^2 given in (2.72) and (2.73), one sees that Σ_θ^2 in the BESS model is equal to the arithmetic mean between the upper and lower bounds on Σ_θ^2 derived within our six-parameter model. Relations (3.8) and (3.9) show that the effect of the additional vector boson, V^0 , is present for arbitrary m_V (including the case $m_V \rightarrow \infty$), as long as $\lambda_V, g_V \neq 0$. BESS thus constitutes a particular example of a model with the “non-decoupling” property suggested by (2.72) and (2.73).

With the constraint (3.5), the mass matrix (2.8), (5.4) develops a symmetry under the exchange

$$g_0 W_\mu^{i\tau_i} \leftrightarrow g_1 B_\mu \tau_3, \quad (3.10)$$

which is to be interpreted as a left-right symmetry of the mass matrix, as invariance under (3.10) implies that mass terms do not discriminate between the fields $W_\mu^{i\tau_i}$ coupled to left-handed fermions only, and the field B_μ , coupled to left-handed and right-handed ones.

The connection of (3.3) with left-right symmetry also becomes explicit when going back to the original construction ¹²⁾ of the BESS theory, which has been based on the “hidden” $SU(2)_V$ symmetry of the non-linear σ -model with its $SU(2)_L \times SU(2)_R$ symmetry.

The observation that BESS is obtained by specialization of the six-parameter model of Section 2 which can be formulated as a spontaneously broken gauge theory by employing the BMW construction, implies, that also BESS can be formulated as a standard $SU(2)_L \times SU(2)_V \times U(1)_Y$ renormalizable gauge theory by adding the appropriate Higgs sector and physical Higgs particles, at least as long as $v_1^2 \geq 0$, ($0 \leq \alpha \leq 1$), as assumed in the BMW construction. For $\alpha \geq 1$ the question of renormalizability is less clear, although it is in fact known ¹⁸⁾ that with several Higgs multiplets the most general Higgs mechanism can admit non-trivial solutions with arbitrary complex vacuum expectation values, which contribute, e.g., to CP-violating phases ¹⁸⁾.

The original construction of BESS has been motivated by the desire to avoid the existence of scalar (Higgs) particles altogether by realizing symmetry breaking in a non-linear way. As stated by the authors ^{12,17)}, it must be explicitly *assumed* within the non-linear approach that the “hidden” $SU(2)_V$ symmetry of the model constitutes a dynamical degree of freedom, the V-boson triplet. The basic Lagrangian given by (2.1) with (3.3), according to ref. 17 and the present paper, can be derived in a straightforward manner without referring to non-linearity, and the phenomenological consequences of BESS can thus hardly be viewed as consequences of non-linear symmetry breaking. On the other hand, nevertheless, the non-linear approach may be considered as providing a certain motivation for introducing an extra V-boson without direct coupling (“inspired” by non-linear symmetry breaking) to the known leptons and quarks.

A somewhat different model, also related to the general framework of Section 2, constitutes the W' model ¹¹⁾ motivated, or inspired, by compositeness [†]. If the weak bosons are of composite nature, one is to expect a spectrum of excitations, in particular spin 1 excitations, similar to the series $\rho, \rho', \rho'', \dots$ or $J/\psi, \psi', \psi'', \dots$. Accordingly, restricting oneself to the simplest case, the consequences of one additional vector-boson triplet, W' , have been investigated in ref. 11. As regards vector-boson fermion interactions, the W' model is again obtainable as a special case of the present framework by imposing the “duality” constraint

$$\lambda_V = \lambda_W \frac{m_W}{m_V} \quad (3.11)$$

abstracted from the duality between the energy dependence of e^+e^- annihilation into hadrons via the prominent vector-meson peaks, and e^+e^- annihilation into pointlike quarks. The coincidence of the W' model with a special case of the presently explored framework is restricted to vector-boson fermion interactions, the vector-boson self-interactions being less constrained in the W' model, which does not assume the local $SU(2)_L \times SU(2)_V \times U(1)_Y$ structure that is realized within the presently explored framework in the massless limit.

We will come back to the BESS and W' models when discussing the $m_V \rightarrow \infty$ limit of our six-parameter framework in Section 4.

[†] Concerning “compositeness”, one should remind oneself that once upon a time ¹⁹⁾ also the vector mesons ρ, ω and ϕ have been introduced as elementary gauge bosons.

4. The limit of $m_V \rightarrow \infty$ for Vector-Boson Fermion Interactions

The motivations for examining the limit of $m_V \rightarrow \infty$ are twofold. First of all, by studying the limit of $m_V \gg m_W$ within the $SU(2)_L \times SU(2)_V \times U(1)_Y$ model, we will be able to describe the influence of the additional V boson at the Z^0 energy in a simplified and transparent manner, as this influence will indeed be seen to remain even in the limit of $m_V \rightarrow \infty$. Secondly, this limit is of phenomenological relevance, taking into account the available negative results of direct searches for additional bosons (within a restricted energy range) and the well established consistency with all available precision data with the $SU(2)_L \times U(1)_Y$ predictions.

Altogether, we have six free parameters, and within a careful examination of the limit of $m_V \rightarrow \infty$, the remaining five free parameters have to be specified precisely. We will keep g_W, λ_W and m_W as finite constants, as otherwise the $SU(2)_L \times U(1)_Y$ theory would be excluded as a particular case of the $m_V \rightarrow \infty$ limit. As for g_V and λ_V , it will be sufficient to start by imposing the constraint

$$\lambda_V g_V = \text{const} \quad (4.1)$$

for $m_V \rightarrow \infty$. The constraint (4.1), apart from the obvious case of $\lambda_V = \text{const}, g_V = \text{const}$, also includes the limiting case of $\lambda_V \rightarrow 0, g_V \rightarrow \infty$, to be commented upon in connection with the W' model at a later stage.

A straightforward calculation of the neutral-current parameters of Table 1 in the limit of $m_V \rightarrow \infty$ under the constraint (4.1) yields the results given in Table 3.

$$\begin{aligned} m_{Z_0}^2 &= \frac{m_W^2}{1-\lambda_W^2}, & \frac{m_{Z_0}^2}{m_V^2} &= \frac{1-\lambda_W^2}{1-\lambda_W^2-\lambda_V^2}, \\ g_{Z_0}^2 &= \frac{g_W^2}{1-\lambda_W^2}, & g_{Z_0}^2 &= g_V^2 \frac{1-\lambda_W^2}{1-\lambda_W^2-\lambda_V^2} \left(1 + \frac{\lambda_V}{g_V} \frac{\lambda_W g_W}{(1-\lambda_W^2)} \right)^2, \\ \chi_Z &= \lambda_W \frac{e}{g_W}, & \chi_V &= \lambda_V \frac{e}{g_V} \frac{1}{(1-\lambda_W^2)} \left(1 + \frac{\lambda_V}{g_V} \frac{\lambda_W g_W}{(1-\lambda_W^2)} \right) \end{aligned}$$

where $e = g_W \lambda_W + g_V \lambda_V$.

Table 3: The neutral-current parameters as a function of $m_W, \lambda_W, g_W, \lambda_V, g_V$ in the limit of $m_V \rightarrow \infty$.

As to the physical interpretation of the results in Table 3, let us first specialize (4.1) to

$$\lambda_V g_V = 0. \quad (4.2)$$

From the expressions for e and the Z^0 mixing parameter, χ_Z , in Table 3, one immediately concludes that the Z^0 properties for $m_V \rightarrow \infty$ with the additional constraint (4.2) coincide[†] with the Z^0 properties within the $SU(2)_L \times U(1)_Y$ theory. In addition, as (2.62) and (2.63) imply $C = 0$ and $\Sigma_\theta^2 = e^2/g_W^2$ in this limit, also the $q^2 \rightarrow 0$ Lagrangian (2.61) is found to coincide with the $SU(2)_L \times U(1)_Y$ four-fermion interaction in the $q^2 \rightarrow 0$ limit. In the limit of $m_V \rightarrow \infty$ with the constraint (4.2), we thus obtain a complete decoupling of the heavy V-boson triplet from the electroweak theory in the q^2 range of $q^2 \simeq 0$ to $q^2 \simeq M_{Z_0}^2$. This result is consistent with the conclusions drawn from Table 1 for $\lambda_V \rightarrow 0$, $g_V \rightarrow 0$ and arbitrary values of m_V .

We add a remark on the properties of the “infinitely” ($m_V \gg m_W$) heavy V-boson triplet listed on the right-hand side in Table 3. The properties of the V-boson triplet depend on whether (4.2) is realized via $\lambda_V = 0$, $g_V \neq 0$, or, alternatively, via $g_V = 0$, $\lambda_V \neq 0$. For the case of $\lambda_V = 0$, $g_V \neq 0$, one obtains a heavy mass-degenerate triplet coupled to the three components of the weak-isospin current. For the case of $\lambda_V \neq 0$, $g_V = 0$, the only state remaining as coupled to the fermions, the neutral one, V^0 , turns out to be coupled to the weak hypercharge current via mixing.

A different situation results from taking the limit of $m_V \rightarrow \infty$ under the subsidiary constraint of

$$\lambda_V g_V = \text{const} \neq 0. \quad (4.3)$$

The $W^\pm$, Z^0 , γ sector of the theory, according to Table 3, now depends on four free parameters, $(m_W, g_W, \lambda_W, \lambda_V g_V)$ which may, equivalently, be replaced by the set (m_W, g_W, e, χ_Z) . The situation is thus different from the three-parameter standard case discussed above and based upon (m_W, g_W, λ_W) or (m_W, g_W, e) as free parameters. In terms of the set (m_W, g_W, e, χ_Z) we have

$$m_{Z_0}^2 = \frac{m_W^2}{1 - \chi_Z \frac{g_W^2}{g_V^2}}, \quad (4.4)$$

as well as

$$g_{Z_0}^2 = \frac{g_W^2}{1 - \chi_Z \frac{g_W^2}{g_V^2}}, \quad (4.5)$$

and the interaction Lagrangians for the $W^\pm$ and the Z^0 boson for $m_V \rightarrow \infty$ become

$$L_{CC} = \frac{g_W}{\sqrt{2}} (W_\mu^+ j_\mu^+ + W_\mu^- j_\mu^-), \quad (4.6)$$

$$L_{NC} = e A_\mu j_\mu^e + g_{Z_0} (j_3^\mu - \chi_Z j_\mu^e) Z_\mu^0.$$

As a consequence of (4.3), the influence of the additional heavy vector-boson on the Z^0 interaction has remained, even when $m_V \rightarrow \infty$. This “non-decoupling” property of the heavy V^0 state, as regards the Z^0 interaction with fermions, in the language of γw^3 , γv^3 mixing simply corresponds to the fact that the electromagnetic charge, e , is not saturated

[†] Under the assumption (4.2), this is even true for arbitrary finite values of m_V . For finite m_V , observable effects remain, however, due to $Z^0 V^0$ interference at $q^2 = 0$ as well as at $q^2 \simeq m_{Z_0}^2$, unless both g_V and $\lambda_V \rightarrow 0$ as mentioned in Section 2.3.

by γw^3 mixing. In distinction from the $SU(2)_L \times U(1)_Y$ case, where $\lambda_W g_W = e$, in the present case, we have $\lambda_W g_W + \lambda_V g_V = e$. The γw^3 mixing contribution for $m_V \rightarrow \infty$ yields an additional pointlike interaction of the (primordial) photon.

The interaction Lagrangian (4.6) with relations (4.4) and (4.5) for the Z^0 mass and coupling is actually known already. It coincides[†] with the Z^0 interaction and mass relation of the 1978 Hung-Sakurai electroweak model¹³ based upon γw^3 mixing without the additional requirement of $SU(2)_L \times U(1)_Y$ symmetry. Within the present context, the Lagrangian (4.6) is obtained as the $m_V \rightarrow \infty$ limit under the constraint $\lambda_V g_V \neq 0$ of our $SU(2)_L \times SU(2)_V \times U(1)_Y$ model.

Next, we turn to a brief discussion of the $m_V \rightarrow \infty$ limit of the two specific cases of five-parameter models introduced in Section 3. The BESS model is characterized by relation (3.7), which reduces the number of six free parameters of the $SU(2)_L \times SU(2)_V \times U(1)_Y$ model to five free parameters by relating λ_V to the remaining five parameters, $(g_W, \lambda_W, m_W, g_V, m_V)$. For

$$m_V \rightarrow \infty, \quad g_V = \text{const}, \quad (4.7)$$

the constraint (3.7) becomes

$$\lambda_V = \frac{-g_V \lambda_W g_W}{g_W^2 + 2g_V^2} = \text{const}, \quad (4.8)$$

i.e., the limit (4.7) within the five parameter BESS model implies

$$\lambda_V g_V \rightarrow \text{const}, \quad (4.9)$$

which coincides with condition (4.1), including the special cases (4.2) and (4.3). The limit (4.7) of the BESS model thus corresponds to a concrete realization of the limit of $m_V \rightarrow \infty$, supplemented by the constraint (4.1), of our six-parameter $SU(2)_L \times SU(2)_V \times U(1)_Y$ model. In other words, for $m_V \rightarrow \infty$, the BESS model coincides with the Hung-Sakurai model. This is also seen by analysing the $q^2 \rightarrow 0$ four-fermion interaction under the BESS constraints. Using (4.7) and (4.8), from (2.62) and (2.63) we immediately find

$$C = 0, \quad \Sigma_\theta^2 = \chi_Z, \quad (4.10)$$

and thus

$$L_{CC}(q^2 \rightarrow 0) = \frac{g_W^2}{m_W^2} \frac{j_\mu^+ j_\mu^-}{\sqrt{2} \sqrt{2}},$$

$$L_{NC}(q^2 \rightarrow 0) = \frac{e^2}{q^2} (j_\mu^e)^2 + \frac{g_W^2}{m_W^2} \left(j_\mu^3 - \frac{e}{g_W} \lambda_W j_\mu^e \right)^2, \quad (4.11)$$

which is to be supplemented with (4.4) and (4.5) and coincides with the four-fermion interaction of ref.13. We add the obvious remark that the $SU(2)_L \times U(1)_Y$ case appears again as the special case of $g_V = \lambda_V = 0$ in (4.7) and, consequently, in (4.8).

[†] Here, we identify the on-mass shell mixing parameter, χ_Z , with “ $\sin^2 \theta_W$ ” of Hung and Sakurai.

We turn now to the other five-parameter case, the W' model of ref.11, which according to 11) is obtained by imposing the constraint

$$\lambda_V = \lambda_W \frac{m_W}{m_V}, \quad (4.12)$$

which implies $\lambda_V \rightarrow 0$ for $m_V \rightarrow \infty$. Consistency with the $q^2 \rightarrow 0$ four-fermion interaction implies

$$\lambda_V g_V \rightarrow \text{const} \quad (4.13)$$

for $m_V \rightarrow \infty$, and consequently [†]

$$\frac{g_V}{m_V} \rightarrow \text{const}. \quad (4.14)$$

While (4.13) immediately implies the Hung-Sakurai interaction Lagrangian (4.4) to (4.6) for the Z^0 and the $W^\pm$, the $q^2 \rightarrow 0$ Lagrangian is now more complicated, as

$$\Sigma_\theta^2 = \chi_Z \frac{1}{\left(1 + \frac{g_V^2}{m_V^2} / \frac{g_W^2}{m_W^2}\right)}, \quad (4.15)$$

as well as

$$C = \Sigma_\theta^4 \left(\frac{g_V^2}{m_V^2} / \frac{g_W^2}{m_W^2} \right). \quad (4.16)$$

The form (4.11) is regained, approximately, however, for $g_V^2/m_V^2 \ll g_W^2/m_W^2$, which due to the good agreement of the data with the $SU(2)_L \times U(1)_Y$ predictions is to be considered as the most important case.

Summarizing this Section, our $SU(2)_L \times SU(2)_V \times U(1)_Y$ six-parameter model for $m_V \rightarrow \infty$ (and g_V finite) coincides with the Hung-Sakurai model, which contains the $SU(2)_L \times U(1)_Y$ theory as a special case. The same result is obtained for the $m_V \rightarrow \infty$ limit of the five-parameter BESS model. In the W' case, the Z^0 and $W^\pm$ interactions for $m_V \gg m_{Z^0}$ also coincide with the ones of the Hung-Sakurai model, while certain non-vanishing effects remain in the $q^2 \rightarrow 0$ Lagrangian in the limit of large values of m_V .

The results of this Section are of obvious importance with respect to a detailed discussion of the significance of the electroweak precision data (Z^0 mass, $W^\pm$ mass, the effective weak angle, s_W^2) for the validity of the $SU(2)_L \times U(1)_Y$ theory. We will come back to this point ²⁰⁾.

[†] The W' model ¹¹⁾ was constructed as an approximation of a theory with a sequence of more massive bosons, $W, W', W'', \dots$. The limit $m_V \rightarrow \infty$ (where $V = W'$), strictly speaking, is meaningless. However, this limit still makes sense as an approximation of the case of large level spacing, $m_V \gg m_W$.

5. Trilinear and Quadrilinear Vector Boson Self Interactions

In this Section, we will explicitly give the trilinear and quadrilinear vector-boson self interactions among the $\gamma, Z^0, W^\pm$ and $V^0, V^\pm$ vector bosons. In the near future, the trilinear $Z^0 W^+ W^-$ coupling will be experimentally accessible at LEP 200. The (trilinear) couplings involving the $V^0, V^\pm$ states will be of interest in connection with the question of the observability of such particles at future colliders, e.g., at the LHC and the SSC.

Implicitly, the vector-boson self interactions (VBSI) are contained in Lagrangian (2.49). For their explicit form, in terms of the physical degrees of freedom, we have to start from the Lagrangian (2.1), or, equivalently, from Lagrangian (2.38). The non-Abelian part of the W, V kinetic terms reads explicitly

$$L_{NA} = -\frac{1}{2}g_0 (\vec{W}_\mu \times \vec{W}_\nu) \cdot \vec{W}^{\mu\nu} - \frac{1}{2}g_2 (\vec{V}_\mu \times \vec{V}_\nu) \cdot \vec{V}^{\mu\nu} - \frac{1}{4}g_0^2 (\vec{W}_\mu \times \vec{W}_\nu) \cdot (\vec{W}^\mu \times \vec{W}^\nu) - \frac{1}{4}g_2^2 (\vec{V}_\mu \times \vec{V}_\nu) \cdot (\vec{V}^\mu \times \vec{V}^\nu), \quad (5.1)$$

where $\vec{W}_{\mu\nu}, \vec{V}_{\mu\nu}$ now have Abelian form.

According to Section 2.1, the interaction Lagrangian in terms of the physical degrees of freedom is obtained in two steps,

i) by applying the rotation $R_2(\varphi)$, defined in (2.14), or, equivalently, in (2.46) and explicitly given in (2.16), which leads to the $SU(2)_{L+V}$ mass eigenstates (W_μ^i, V_μ^i) , of masses m_W and m_V , respectively, and thus completely specifies the *charged* physical states.

ii) By applying the three-dimensional orthogonal transformation R , given explicitly in Table 1, which leads to the *neutral* physical states $(A_\mu, Z_\mu^0, V_\mu^0)$.

After the first step, the Lagrangian (5.1) yields for the trilinear and quadrilinear interaction terms, respectively,

$$\begin{aligned} -2 L_{3-linear} = & f_{WWW} (\vec{W}_\mu \times \vec{W}_\nu) \cdot \vec{W}^{\mu\nu} + f_{VVV} (\vec{V}_\mu \times \vec{V}_\nu) \cdot \vec{V}^{\mu\nu} \\ & + f_{WVV} [(\vec{W}_\mu \times \vec{V}_\nu) \cdot \vec{W}^{\mu\nu} + 2(\vec{W}_\mu \times \vec{V}_\nu) \cdot \vec{V}^{\mu\nu}] \\ & + f_{VWV} [(\vec{V}_\mu \times \vec{W}_\nu) \cdot \vec{V}^{\mu\nu} + 2(\vec{V}_\mu \times \vec{W}_\nu) \cdot \vec{W}^{\mu\nu}], \end{aligned} \quad (5.2)$$

$$\begin{aligned} -4 L_{4-linear} = & h_{WWW} (\vec{W}_\mu \times \vec{W}_\nu) \cdot (\vec{W}^\mu \times \vec{W}^\nu) + h_{VVVV} (\vec{V}_\mu \times \vec{V}_\nu) \cdot (\vec{V}^\mu \times \vec{V}^\nu) \\ & + h_{WVVV} [(\vec{W}_\mu \times \vec{W}_\nu) \cdot (\vec{V}^\mu \times \vec{V}^\nu) + 2(\vec{W}_\mu \times \vec{V}_\nu) \cdot (\vec{W}^\mu \times \vec{V}^\nu)] \\ & + h_{VWVV} (\vec{W}_\mu \times \vec{W}_\nu) \cdot (\vec{W}^\mu \times \vec{V}^\nu) + h_{VWVW} (\vec{V}_\mu \times \vec{V}_\nu) \cdot (\vec{W}^\mu \times \vec{V}^\nu). \end{aligned} \quad (5.3)$$

The different couplings, f_i and h_i in (5.2) and (5.3), simply correspond to all possible interactions generated by $R_2(\varphi)$, i.e., the WWW, WVV, WVV, WVV and VVV interactions, and the WWW, WVV, WVV, WVV and VVV interactions, for the trilinear and quadrilinear cases, respectively. The explicit expressions for the couplings are given in Table 4 in terms of g_0, g_2 , and the rotation angle φ , which itself depends on g_0, g_2 as well as v_1, v_2, v_3 (compare (2.13) or Table 2a).

Trilinear Couplings	Quadrilinear Couplings
$f_{WWW} = g_0 \cos^3 \varphi - g_2 \sin^3 \varphi$	$h_{WWW} = g_0^2 \cos^4 \varphi + g_2^2 \sin^4 \varphi$
$f_{WWV} = (g_0 \cos \varphi + g_2 \sin \varphi) \cos \varphi \sin \varphi$	$h_{WWV} = 4(g_0^2 \cos^2 \varphi - g_2^2 \sin^2 \varphi) \cos \varphi \sin \varphi$
$f_{WVV} = (g_0 \sin \varphi - g_2 \cos \varphi) \cos \varphi \sin \varphi$	$h_{WVV} = 2(g_0^2 + g_2^2) \cos^2 \varphi \sin^2 \varphi$
$f_{VVV} = g_0 \sin^3 \varphi + g_2 \cos^3 \varphi$	$h_{VVV} = 4(g_0^2 \sin^2 \varphi - g_2^2 \cos^2 \varphi) \cos \varphi \sin \varphi$
	$h_{VVV} = 4(g_0^2 \sin^2 \varphi - g_2^2 \cos^2 \varphi) \cos \varphi \sin \varphi$

Table 4: Trilinear and Quadrilinear couplings in mass-mixing form.

The physical interpretation is facilitated by passing to the current-mixing parameters ($g_W, \lambda_W, g_V, \lambda_V, m_W, m_V$) of Section 2.2 and 2.3. From Table 4, making use of Tables 2a and 2b, one finds for the trilinear couplings

$$f_{WWW} \lambda_W + f_{WWV} \lambda_V = e, \quad (5.4)$$

$$f_{WVV} \lambda_W + f_{VVV} \lambda_V = e, \quad (5.5)$$

$$f_{WWW} \lambda_W + f_{WVV} \lambda_V = 0, \quad (5.6)$$

where

$$f_{WWW} = g_W - \frac{g_V^2 \lambda_V}{g_W \lambda_V - g_V \lambda_W}, \quad f_{WWV} = \frac{g_W g_V \lambda_V}{g_W \lambda_V - g_V \lambda_W},$$

$$f_{VVV} = g_V + \frac{g_W^2 \lambda_W}{g_W \lambda_V - g_V \lambda_W}, \quad f_{WVV} = \frac{-g_V g_W \lambda_W}{g_W \lambda_V - g_V \lambda_W}. \quad (5.7)$$

Note the analogy of (5.4), (5.5) to the fermionic relation (2.52), which relates e to the sum $g_W \lambda_W + g_V \lambda_V$. The quadrilinear couplings are obtained as

$$h_{WWW} = \frac{g_W^4 + g_V^4 (g_W \lambda_V - g_V \lambda_W)^2}{g_W^2 + g_V^2} \quad (5.8)$$

$$h_{WWV} = 4g_W g_V \left(\frac{g_W^2 - g_V^2 (g_W \lambda_V - g_V \lambda_W)^2}{g_W^2 + g_V^2} \right) \quad (5.9)$$

$$h_{WVV} = \frac{2g_W^2 g_V^2 (\lambda_W^2 + \lambda_V^2)}{(g_W \lambda_V - g_V \lambda_W)^2} \quad (5.10)$$

$$h_{VVV} = 4g_W g_V \left(\frac{g_V^2 - g_W^2 (g_W \lambda_V - g_V \lambda_W)^2}{g_W^2 + g_V^2} \right) \quad (5.11)$$

$$h_{VVV} = \frac{g_V^4 + g_W^4 (g_W \lambda_V - g_V \lambda_W)^2}{g_W^2 + g_V^2}. \quad (5.12)$$

As far as those interactions are concerned, which involve charged vector bosons only, the relevant couplings are directly given in (5.8) to (5.12).

As for interactions which involve neutral particles, step ii), as mentioned above, is still to be carried out. The three-dimensional rotation R^Z amounts to the substitutions

$$W_3^\mu = \lambda_W \left(A^\mu - \frac{n_Z}{1 - \frac{m_{Z_0}^2}{m_W^2}} Z_0^\mu - \frac{n_V}{1 - \frac{m_{V_0}^2}{m_W^2}} V_0^\mu \right), \quad (5.13)$$

$$V_3^\mu = \lambda_V \left(A^\mu - \frac{n_Z}{1 - \frac{m_{Z_0}^2}{m_V^2}} Z_0^\mu - \frac{n_V}{1 - \frac{m_{V_0}^2}{m_V^2}} V_0^\mu \right), \quad (5.14)$$

to be carried out in (5.2) and (5.3) upon explicitly writing out all cross products and picking up all relevant terms. The trilinear couplings involving a photon and a Z^0 , respectively, are given by

$$L_{\gamma W V} = ie (A^\mu (W_+^\nu W_{\mu\nu}^- - W_-^\nu W_{\mu\nu}^+) + A^{\mu\nu} W_\mu^+ W_\nu^-) + ie (A^\mu (V_+^\nu V_{\mu\nu}^- - V_-^\nu V_{\mu\nu}^+) + A^{\mu\nu} V_\mu^+ V_\nu^-) \quad (5.15)$$

and

$$L_{Z_0 W V} = ig_{Z_0 W W} (Z_0^\mu (W_+^\nu W_{\mu\nu}^- - W_-^\nu W_{\mu\nu}^+) + Z_0^{\mu\nu} W_\mu^+ W_\nu^-) + ig_{Z_0 V V} (Z_0^\mu (V_+^\nu V_{\mu\nu}^- - V_-^\nu V_{\mu\nu}^+) + Z_0^{\mu\nu} V_\mu^+ V_\nu^-) + ig_{Z_0 W V} (Z_0^\mu (W_+^\nu V_{\mu\nu}^- - W_-^\nu V_{\mu\nu}^+) + Z_0^{\mu\nu} W_\mu^+ V_\nu^-) + ig_{Z_0 W V} (Z_0^\mu (V_+^\nu W_{\mu\nu}^- - V_-^\nu W_{\mu\nu}^+) + Z_0^{\mu\nu} V_\mu^+ W_\nu^-), \quad (5.16)$$

where

$$g_{Z_0 W W} = -n_Z \left(\frac{f_{WWW} \lambda_W}{1 - \frac{m_{Z_0}^2}{m_W^2}} + \frac{f_{WWV} \lambda_V}{1 - \frac{m_{Z_0}^2}{m_V^2}} \right), \quad (5.17)$$

$$g_{Z_0 V V} = -n_Z \left(\frac{f_{WVV} \lambda_W}{1 - \frac{m_{Z_0}^2}{m_W^2}} + \frac{f_{VVV} \lambda_V}{1 - \frac{m_{Z_0}^2}{m_V^2}} \right), \quad (5.18)$$

$$g_{Z_0 W V} = -n_Z \left(\frac{f_{WWV} \lambda_W}{1 - \frac{m_{Z_0}^2}{m_W^2}} + \frac{f_{WVV} \lambda_V}{1 - \frac{m_{Z_0}^2}{m_V^2}} \right). \quad (5.19)$$

The corresponding couplings of the V_0 boson, $g_{V_0 W W}$, $g_{V_0 V V}$ and $g_{V_0 W V}$, are obtained by the replacement $Z_0 \rightarrow V_0$, i.e., $n_Z \rightarrow n_V$ and $m_{Z_0} \rightarrow m_{V_0}$ in (5.17) to (5.19). It is noteworthy that all anomalous electromagnetic and weak moment couplings have the “standard” values of $\kappa_{\gamma W W} = \kappa_{\gamma V V} = \kappa_{Z_0 W W} = \kappa_{Z_0 V V} = 1$. This is related to the underlying local $SU(2)_L \times SU(2)_V$ symmetry of the kinetic energy terms in full analogy to the $SU(2)_L$ symmetry in the $SU(2)_L \times U(1)_Y$ theory. Within the current mixing formalism the result of $\kappa = 1$ is directly related to relations (5.4) to (5.6). In connection

with (5.15) and (5.6) we also note that the underlying symmetry forbids the decay of $V^{+(-)}$ into $W^{+(-)}\gamma$ (at tree-level).

Evaluating the trilinear Z_0 and V_0 couplings (5.17) to (5.19) in the limit of $\lambda_V \rightarrow 0$ and $g_V \rightarrow 0$ for arbitrary values of m_V , the $Z_0 W^+ W^-$ and the $\gamma W^+ W^-$ couplings are found to coincide with the $SU(2)_L \times U(1)_Y$ couplings

$$g_{\gamma WW} = e = g_W \lambda_W, \quad g_{Z_0 WW} = g_W \sqrt{1 - \lambda_W^2}, \quad (5.20)$$

while the couplings belonging to interactions with one or more V bosons become

$$\begin{aligned} g_{Z_0 VV} &= \frac{g_W}{\sqrt{1 - \lambda_W^2}} \left(\frac{1}{1 - \frac{g_W \lambda_V}{\lambda_W g_V}} \right) \left((1 - \lambda_W^2) + \left(\frac{\lambda_V}{g_V} \right) \frac{g_W \lambda_W}{1 - \frac{m_V^2}{m_W^2}} \right), \\ g_{Z_0 WV} &\rightarrow 0, \quad g_{V_0 WW} \rightarrow 0, \quad g_{V_0 VV} \rightarrow \infty, \\ g_{V_0 WV} &= \frac{g_W}{1 - \frac{g_W \lambda_V}{\lambda_W g_V}}. \end{aligned} \quad (5.21)$$

Some of the couplings obtained for $\lambda_V \rightarrow 0$, $g_V \rightarrow 0$ thus depend on the ratio of λ_V/g_V . Particularly simple results are obtained for specific values of this ratio, such as $\lambda_V/g_V \rightarrow \lambda_W/g_W$, $\lambda_V/g_V \rightarrow 0$ and $\lambda_V/g_V \rightarrow -\lambda_W/g_W$.

Similarly, one may deduce the quadrilinear couplings in the limit of $\lambda_V \rightarrow 0$, $g_V \rightarrow 0$ by making use of (5.3) with (5.13), (5.14) and (5.8) to (5.12). We only note the final expressions for the couplings involving the $W^\pm$, Z^0 and the photon. They coincide with the $SU(2)_L \times U(1)_Y$ values and are given by

$$\begin{aligned} h_{WWWW} &= g_W^2, \\ h_{\gamma\gamma WW} &= g_W^2 \lambda_W^2 = e^2, \\ h_{Z_0\gamma WW} &= g_W^2 \lambda_W \sqrt{1 - \lambda_W^2}, \\ h_{Z_0 Z_0 WW} &= g_W^2 (1 - \lambda_W^2). \end{aligned} \quad (5.22)$$

In the BESS model the value of λ_V/g_V is uniquely determined by the constraint (3.7). For $\lambda_V \rightarrow 0$, $g_V \rightarrow 0$ we obtain from (3.7)

$$\frac{\lambda_V}{g_V} \rightarrow -\frac{\lambda_W}{g_W} \left(1 - 2 \frac{m_W^2}{m_V^2} \right), \quad (5.23)$$

which is to be substituted into (5.21). The specific values of λ_V/g_V mentioned after (5.21) correspond to specific values of m_V .

A considerable simplification of the general expressions (5.15) to (5.19) for the trilinear couplings is also obtained in the limit of $m_V \rightarrow \infty$ for arbitrary values of λ_V and g_V . The results are collected in Table 5. We note that the $g_{Z_0 WW}$ coupling differs from its $SU(2)_L \times U(1)_Y$ value, unless the conditions $\lambda_V \rightarrow 0$ and $g_V \rightarrow 0$ are fulfilled. Under these conditions all results in Table 5 correctly reduce to the ones given in (5.20), (5.21).

Coupling	form for $m_V \rightarrow \infty$
$W^+ W^- \gamma$	$g_W \lambda_W + g_V \lambda_V = e$
$W^+ V^- \gamma$	zero from (5.6)
$V^+ V^- \gamma$	$g_W \lambda_W + g_V \lambda_V = e$
$g_{Z_0 WW}$	$f_{WWW} \sqrt{1 - \lambda_W^2} - f_{WVW} \frac{\lambda_W \lambda_V}{\sqrt{1 - \lambda_W^2}}$
$g_{Z_0 WV}$	$f_{WVW} \sqrt{1 - \lambda_W^2} - f_{WVV} \frac{\lambda_W \lambda_V}{\sqrt{1 - \lambda_W^2}}$
$g_{Z_0 VV}$	$f_{WVV} \sqrt{1 - \lambda_W^2} - f_{VVV} \frac{\lambda_W \lambda_V}{\sqrt{1 - \lambda_W^2}}$
$g_{V_0 WW}$	$f_{WWW} \sqrt{\frac{1 - \lambda_W^2 - \lambda_V^2}{1 - \lambda_W^2}}$
$g_{V_0 WV}$	$f_{WVW} \sqrt{\frac{1 - \lambda_W^2 - \lambda_V^2}{1 - \lambda_W^2}}$
$g_{V_0 VV}$	$f_{VVV} \sqrt{\frac{1 - \lambda_W^2 - \lambda_V^2}{1 - \lambda_W^2}}$

Table 5: Trilinear Couplings in the limit of $m_V \rightarrow \infty$. (Compare (5.7) for the explicit expressions for f_{WWW} , etc.)

In summary, for $\lambda_V \rightarrow 0$ and $g_V \rightarrow 0$, the trilinear and quadrilinear couplings of the known vector bosons, the $W^\pm$, Z^0 and the photon, reduce to their $SU(2)_L \times U(1)_Y$ values, independently of what value is chosen for m_V . Even for $m_V \rightarrow \infty$, the $SU(2)_L \times U(1)_Y$ couplings are obtained for $\lambda_V \rightarrow 0$ and $g_V \rightarrow 0$ only. The limiting values of the other couplings (involving a member of the V -boson triplet) in general depend on the $\lambda_V \rightarrow 0$, $g_V \rightarrow 0$ limit of the ratio λ_V/g_V , which in the case of BESS is given as a simple function of m_V .

6. Conclusions

We have formulated an effective gauge theory containing an extra V^0 , $V^\pm$ triplet based on the gauge group $SU(2)_L \times SU(2)_V \times U(1)_Y$, broken down to $U(1)_{em}$. Our essential points may be summarized as follows:

- i) Employing the mass-mixing as well as the current-mixing formalism (restricted by w^3 , v^3 -dominance conditions) allows for a transparent representation of the Lagrangian, including the vector-boson self interactions, directly in terms of the physically relevant masses, coupling constants and (current-) mixing parameters. The $SU(2)_L \times U(1)_Y$ theory appears as a special case, obtained for vanishing couplings to fermions and vanishing mixing parameter of the extra vector bosons. By adding an appropriately chosen Higgs sector,

the theory may be formulated as a renormalizable one, but this has not been the main concern of the present paper.

ii) The use of the γw^3 and γv^3 (current) mixing parameters is essential for a careful discussion of the "non-decoupling" of the additional vector boson triplet in the limit in which its mass, m_V , becomes large compared with the $W^\pm$ mass. In this limit of $m_V \rightarrow \infty$ our six-parameter model becomes identical to the four-parameter Hung-Sakurai model based on γw^3 mixing. The non-decoupling for $m_V \rightarrow \infty$ of the additional neutral boson, V^0 , is found to be related to the extra pointlike photon interaction with leptons and quarks, which is contained in the Hung-Sakurai model. Confronting the $m_V \rightarrow \infty$ limit of our model with the electroweak precision data on the Z^0 and $W^\pm$ masses and the weak angle, s_W^2 , will allow one to quantify $^{20)}$ to what extent these data confirm the $SU(2)_L \times U(1)_Y$ symmetry of vector-boson fermion interactions underlying the standard electroweak theory.

iii) The five-parameter BESS model and the five-parameter W' model previously considered in the literature appear as special cases of our theory, and accordingly, both of these models also coincide with the Hung-Sakurai model (the W' model in an approximate sense) in the $m_V \rightarrow \infty$ limit.

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