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Radiative corrections to the masses of supersymmetric Higgs bosons

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Abstract

The lightest neutral Higgs boson in the minimal supersymmetric extension of the Standard Model has a tree-level mass less than that of the Z^0 . We calculate radiative corrections to its mass and to that of the heavier CP-even neutral Higgs boson. We find large corrections that increase with the top quark and squark masses, and vary with the ratio of vacuum expectation values v_2/v_1 . These radiative corrections can be as large as $\mathcal{O}(100)$ GeV, and have the effect of (i) invalidating lower bounds on v_2/v_1 inferred from unsuccessful Higgs searches at LEP I, (ii) in many cases, increasing the mass of the lighter CP-even Higgs boson beyond m_Z , (iii) often, increasing the mass of the heavier CP-even Higgs boson beyond the LEP reach, into a range more accessible to the LHC or SSC.

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One of the key problems in particle physics today is that of the origin of mass. In the Standard Model, the generation of vector boson and fermion masses is related to the existence of an elementary Higgs boson that is the object of many searches, so far in vain [1]. Unfortunately, there is no model-independent upper bound on the mass of such an elementary Higgs, and it seems prudent to plan searches reaching up to $O(1)$ TeV. In fact, the mass of any elementary Higgs is very sensitive to radiative corrections, which are even quadratically divergent in the Standard Model. This has been one of the primary motivations for introducing phenomenological supersymmetry [2], effectively broken at some mass scale $\lesssim 1$ TeV, which cancels the quadratic divergences and leaves behind logarithmic divergences that can be tamed using the familiar renormalization group.

The minimal supersymmetric extension of the Standard Model (MSSM)¹ contains in its physical spectrum three neutral Higgs bosons, two of which are CP-even and the third CP-odd. The lighter CP-even state is the lightest of all the Higgs bosons in the MSSM, and its tree-level mass is less than that of the Z^0 [4], but very close to it in a large region of parameter space. The heavier CP-even state has a tree-level mass which is always greater than that of the Z^0 , but is very close to it in another significant region of parameter space. These facts are of great experimental significance, since there is always a neutral Higgs boson with a tree-level mass less or at most a few GeV more than the Z^0 mass, with standard-model-like couplings to vector bosons², which can be produced and hopefully detected at LEP. If such a standard-model-like Higgs boson had a mass significantly above the Z^0 mass, it would be outside the LEP discovery range. However, it could be detected at a high energy hadron-hadron collider, such as the LHC or SSC, although the mass region between 90 and 120 GeV appears to pose a non-trivial experimental challenge. It is therefore of the utmost theoretical importance to calculate radiative corrections to the masses of the CP-even neutral Higgs bosons in the MSSM, and to determine whether they push their masses down, into a range where detection at LEP would be ‘easy’, or up, into a range where detection at LEP would be ‘hard’ or even impossible, but the LHC/SSC could play a major rôle.

¹For reviews and references, see for example ref. [3].

²The basic formulae for the susy Higgs couplings can be found, for example, in ref.[5]

We are aware of several partial results of such calculations. Several years ago, Li and Sher [6] calculated radiative corrections using an effective potential approach, but evaluated them only for the low top quark and squark masses which were then fashionable. More recently, Gunion and Turski [7] and Berger [8] have given a large number of general formulae focused primarily on violations of the tree-level sum rules linking CP-even, CP-odd and charged Higgs boson masses with those of the electroweak bosons. In parallel to this, experiment has pushed lower limits on the top [9] and squark [10] masses up above the Z^0 mass. Now therefore seems to be an opportune moment to revive the effective potential approach to calculate radiative corrections to Higgs masses in the case of large m_t and $m_{\tilde{t}}$. One could expect *a priori* to find large effects in this limit as relics of the quadratic divergences that cancel in the limit of exact supersymmetry.

Indeed, we find large corrections that increase with the top quark and squark masses, and vary with the ratio of vacuum expectation values v_2/v_1 . These radiative corrections can be as large as $O(100)$ GeV, and have the effect of (i) invalidating lower bounds on v_2/v_1 inferred from unsuccessful Higgs searches at LEP I, (ii) increasing the mass of the lighter CP-even Higgs boson beyond m_Z in many cases, (iii) often, increasing the mass of the heavier CP-even Higgs boson beyond the LEP reach, into a range more accessible to the LHC or SSC [11].

To give masses to all charged fermions, and to avoid gauge anomalies, the MSSM contains two complex Higgs doublets (embedded in chiral supermultiplets as required by supersymmetry), whose $SU(3) \times SU(2) \times U(1)$ quantum numbers are summarized by

$$H_1 \equiv \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \sim (1, 2, -1/2), \quad H_2 \equiv \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \sim (1, 2, +1/2). \quad (1)$$

The starting point for a discussion of gauge symmetry breaking and Higgs boson masses is the tree-level Higgs potential

$$V_{Higgs} = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 (H_1 H_2 + \text{h.c.}) + \frac{1}{8} g^2 (H_1^\dagger \tilde{\sigma} H_2 + H_2^\dagger \tilde{\sigma} H_1)^2 + \frac{1}{8} g'^2 (|H_2|^2 - |H_1|^2)^2, \quad (2)$$

where we have assumed for simplicity that m_3^2 is real and we have chosen a field basis such that $m_3^2 \leq 0$ (then it will not be restrictive to assume

that the vacuum expectation values of H_1^0 and H_2^0 , denoted by v_1 and v_2 , are both real and non-negative). For the discussion of gauge symmetry breaking and of the neutral CP-even Higgs boson masses, it is convenient to consider the restriction of V_{Higgs} to the real components of the neutral Higgs fields, $\phi_1 \equiv \text{Re } H_1^0$ and $\phi_2 \equiv \text{Re } H_2^0$:

$$V_0 = m_1^2 \phi_1^2 + m_2^2 \phi_2^2 + 2m_3^2 \phi_1 \phi_2 + \frac{g^2 + g'^2}{8} (\phi_1^2 - \phi_2^2)^2. \quad (3)$$

After introducing the auxiliary variables

$$v^2 \equiv v_1^2 + v_2^2, \quad \tan \beta \equiv \frac{v_2}{v_1}, \quad (4)$$

the tree-level minimization conditions

$$\sin 2\beta = -\frac{2m_3^2}{m_1^2 + m_2^2}, \quad v^2 = \frac{8}{g^2 + g'^2} \frac{m_1^2 - m_2^2 \tan \beta}{\tan^2 \beta - 1}, \quad (5)$$

combined with the tree-level expression for the Z mass,

$$m_Z^2 = \frac{1}{2}(g^2 + g'^2)(v_1^2 + v_2^2), \quad (6)$$

give for the masses of the CP-even neutral Higgses

$$m_{h,H}^2 = \frac{1}{2} \left[m_A^2 + m_Z^2 \mp \sqrt{(m_A^2 + m_Z^2)^2 - 4m_A^2 m_Z^2 \cos^2 2\beta} \right], \quad (7)$$

where

$$m_A^2 = m_1^2 + m_2^2 \quad (8)$$

is the tree-level expression for the CP-odd neutral Higgs mass. It is immediate to observe that eq.(7) implies the inequalities

$$m_A < m_Z |\cos 2\beta| < m_Z < m_H, \quad (9)$$

implying in particular that, at the tree level, unsuccessful Higgs searches at LEP [1] can be used to infer a lower bound on $\tan \beta$ (which in the MSSM is always larger than 1). It is also important to realize that, at tree level, all Higgs masses and couplings can be expressed in terms of two parameters only: for example, one can choose as independent parameters m_A and $\tan \beta$.

It is well known that the tree-level potential of eq.(3) is subject to radiative corrections. Indeed, one of the nicest features of the MSSM is the possibility of describing $SU(2)_L \times U(1)_Y$ breaking as an effect of radiative corrections [12], via a generalization of the Coleman-Weinberg mechanism [13]. One starts with boundary conditions at the unification scale M_U

$$m_1^2 = m_2^2 = \mu_0^2 + m_0^2, \quad m_3^2 = B_0 \mu_0, \quad (10)$$

such that the potential is bounded from below:

$$S \equiv m_1^2 + m_2^2 + 2m_3^2 \geq 0, \quad (11)$$

and the symmetric point $v_1 = v_2 = 0$ is a minimum:

$$B \equiv m_1^2 m_2^2 - m_3^4 > 0. \quad (12)$$

Then one defines, at scales Q around the electroweak scale, a renormalization-group-improved tree-level potential, $V_0(Q)$, which incorporates the large logarithmic corrections, proportional to $\alpha \log(M_U/Q)$, into running masses $[m_i^2(Q), i = 1, 2, 3]$ and gauge couplings $[g^2(Q), g'^2(Q)]$. If the running parameters are such that $S(Q) \geq 0$ but $B(Q) < 0$, one would say that the gauge symmetry is broken and one would compute v_1, v_2 and the associated gauge and Higgs boson masses according to eqs.(4-8).

It is not obvious that, in the presence of a rich particle spectrum like the one of the MSSM, the use of $V_0(Q)$ is adequate for the determination of the VEVs v_1 and v_2 and of the gauge and Higgs boson masses. Indeed, it was recently shown [14] that the naive use of the minimization conditions of eq.(5), applied to the running parameters of $V_0(Q)$ at the generic scale $Q \sim m_Z$, is in general a bad approximation for the determination of v_1 and v_2 . A much more reliable procedure is the use of the full one-loop effective potential

$$V_1(Q) = V_0(Q) + \Delta V_1(Q), \quad (13)$$

where, exploiting the fact that the MSSM has softly broken supersymmetry, one can neglect field-independent contributions proportional to $\text{Str } M^2$, and write

$$\Delta V_1(Q) = \frac{1}{64\pi^2} \text{Str } M^4 \left(\log \frac{M^2}{Q^2} - \frac{3}{2} \right). \quad (14)$$

In the above formula, $\mathcal{M}^2$ is the field-dependent generalized squared mass matrix of the model: it contains mass terms for all spin-0, spin- $\frac{1}{2}$ and spin-1 particles of the MSSM, and one is interested in its dependence on the fields ϕ_1 and ϕ_2 . The supertrace is defined as usual according to

$$\text{Str } f(\mathcal{M}^2) = \sum_i (-1)^{2J_i} (2J_i + 1) f(m_i^2), \quad (15)$$

where m_i^2 is the field-dependent mass eigenvalue of the i -th particle of spin J_i . The logarithmic terms in eq.(14) play a crucial rôle in making $V_1(Q)$ independent of the renormalization scale Q , up to higher loop corrections. The same applies to the corresponding vacuum expectation values v_1 and v_2 if one neglects small wave function renormalization effects.

Since the use of the full one-loop effective potential is essential for the determination of the VEVs v_1 and v_2 , it is natural to use it to address the related issue of the neutral CP-even Higgs boson masses.

Indeed, the effective potential V can be used to obtain an estimate (quite accurate, as we shall see) of the Higgs boson masses. To see this, let us consider, as a simple example, a theory with a single real scalar field ϕ . The effective potential $V(\phi)$ is the first term in the derivative expansion of the effective action $\Gamma(\phi)$:

$$\Gamma(\phi) = -V(\phi) + \frac{1}{2} Z(\phi) \partial_\mu \phi \partial^\mu \phi + \text{higher order derivatives}. \quad (16)$$

On the other hand, $\Gamma(\phi)$ can be expanded in powers of ϕ around $\phi = 0$. The coefficients of such an expansion,

$$\Gamma^{(n)} = \left[\frac{\delta^n \Gamma(\phi)}{\delta \phi^n} \right]_{\phi=0} \quad (17)$$

are called the proper vertices of the theory³. The Fourier transform of $\Gamma^{(2)}$, the inverse propagator, can be written as

$$\tilde{\Gamma}^{(2)}(p) = p^2 - m^2 + \Sigma(p^2), \quad (18)$$

where m^2 is twice the coefficient of ϕ^2 in the lagrangian. The physical mass is just the value of p^2 at which $\tilde{\Gamma}^{(2)}$ is zero:

$$m_{phys}^2 = m^2 - \Sigma(m_{phys}^2). \quad (19)$$

³Here $\delta^n / \delta \phi^n$ stands for $\delta / \delta \phi(x_1) \dots \delta / \delta \phi(x_n)$.

Now, from eq.(16) we infer

$$\tilde{\Gamma}^{(2)}(p=0) = - \left[\frac{\partial^2 V(\phi)}{\partial \phi^2} \right]_{\phi=0} \quad (20)$$

or

$$\left[\frac{\partial^2 V(\phi)}{\partial \phi^2} \right]_{\phi=0} = m^2 - \Sigma(0) = m_{phys}^2 + \Sigma(m_{phys}^2) - \Sigma(0). \quad (21)$$

We now turn our attention to the MSSM, and we concentrate on the fields ϕ_1 and ϕ_2 . The above reasoning can be repeated with slight modifications: the second derivative of V at $\phi = 0$ must be replaced by the eigenvalues of the matrix

$$\frac{1}{2} \left[\frac{\partial^2 V(\phi_1, \phi_2)}{\partial \phi_i \partial \phi_j} \right]_{\phi_1=v_1, \phi_2=v_2} \quad (22)$$

In this case, we know that the most important contributions to the Higgs self-energies $\Sigma_{ij}(p^2)$ at the one-loop level come from the diagrams with the top quark (or its scalar partners) circulating in the loop, due to the large top Yukawa coupling. The analytic structure of these contributions in the complex p^2 plane is such that $\Sigma_{ij}(p^2) \simeq \Sigma_{ij}(0)$ if $p^2 \ll 4M^2$, where M is the mass of the particle circulating in the loop. So, the physical masses of the Higgs scalars are given by the eigenvalues of the matrix (22), provided that

$$m_{phys}^2 \ll 4 \min\{m_t^2, m_{\tilde{t}}^2\}. \quad (23)$$

This condition is certainly fulfilled by the lightest CP-even Higgs scalar, whose tree-level mass is forced to be less than the Z boson mass, but can be violated by the other Higgs particles in the theory.

Before describing our results, we want to make a comment about the relationship between the vacuum expectation values v_1, v_2 and the masses of the vector bosons Z and W . Since v_1 and v_2 are computed minimizing the *one-loop* effective potential V_1 , one should relate them to m_Z and m_W in a way which is correct at the *one-loop* order. The tree-level relations

$$m_W^2 = \frac{1}{2} g^2 (v_1^2 + v_2^2), \quad m_Z^2 = \frac{1}{2} (g^2 + g'^2) (v_1^2 + v_2^2) \quad (24)$$

(with the running gauge couplings evaluated at m_W and m_Z respectively) turn out to be the correct answer, up to wave function renormalization of

the scalars, which is known to be rather small [14]. The reason of this is that the terms in the lagrangian that give mass to the vector bosons are contained in the squared covariant derivative of the scalar fields, and therefore must receive the same renormalization as the scalar kinetic terms to preserve gauge invariance.

In view of these considerations, we will compute in the following the physical masses of the CP-even Higgs bosons of the MSSM in the effective potential approach.

In order to understand our numerical results, it is instructive to look at exact analytic results in the following simplifying limit of the MSSM: $m_3^2 = 0$, $m_1^2 \rightarrow +\infty$, $m_2^2 < 0$ and finite. In this limit, all the Higgses are superheavy apart from $H_2 \equiv (H_2^+, H_2^0)^T$, so that the light state after removing the Goldstone bosons is $h \equiv \sqrt{2}\phi_2$. It is consistent to assume that $v_1 = 0$ and $v_2 \neq 0$, so after renaming $m_2 \rightarrow m$, $\phi_2 \rightarrow \phi$, $v_2 \rightarrow v$ we can write

$$V_0 = m^2\phi^2 + \frac{g^2 + g'^2}{8}\phi^4 \quad (25)$$

$$\frac{\partial V_0}{\partial \phi} = 2m^2\phi + \frac{g^2 + g'^2}{2}\phi^3 \quad (26)$$

$$\frac{\partial^2 V_0}{\partial \phi^2} = 2m^2 + 3\frac{g^2 + g'^2}{2}\phi^2 \quad (27)$$

and the tree-level minimization would give

$$m^2 = -\frac{g^2 + g'^2}{4}v^2 = -\frac{m_Z^2}{2}, \quad m_h^2 = \left[\frac{1}{2} \frac{\partial^2 V_0}{\partial \phi^2} \right]_{\phi=v} = m_Z^2. \quad (28)$$

Consider now the limit $m_t^2, m_i^2 \gg m_Z^2$, in which the dominant field-dependent masses are

$$m_t^2 = h_t^2\phi^2, \quad m_i^2 = h_i^2\phi^2 + m_i^2 \quad (29)$$

where D-terms have been neglected. We have also neglected possible stop mixing, and assumed degenerate stop masses. We find from eqs.(14,15) that the dominant part of the one-loop potential is:

$$\Delta V_1 = \frac{3}{16\pi^2} \left[m_t^2 \left(\log \frac{m_t^2}{Q^2} - \frac{3}{2} \right) - m_i^4 \left(\log \frac{m_i^2}{Q^2} - \frac{3}{2} \right) \right]. \quad (30)$$

Minimization of $V_1 = V_0 + \Delta V_1$ now yields

$$\frac{\partial V_1}{\partial \phi} = \frac{\partial V_0}{\partial \phi} + \frac{3}{8\pi^2} \frac{\partial m_i^2}{\partial \phi} \left[m_i^2 \left(\log \frac{m_i^2}{Q^2} - 1 \right) - m_i^2 \left(\log \frac{m_t^2}{Q^2} - 1 \right) \right] = 0, \quad (31)$$

where we have used the fact that $\partial m_t^2 / \partial \phi = \partial m_i^2 / \partial \phi$ after neglecting the D-terms. It is convenient to choose a renormalization scale $\hat{Q}$ such that the term in square brackets in eq.(31) vanishes, so that $\partial V_0 / \partial \phi = 0$ can be retained as the minimization condition:

$$\left[m_i^2 \left(\log \frac{m_i^2}{\hat{Q}^2} - 1 \right) - m_i^2 \left(\log \frac{m_t^2}{\hat{Q}^2} - 1 \right) \right]_{\phi=v} = 0. \quad (32)$$

Evaluating the second derivative of the potential at $\hat{Q}$ we find

$$\left[\frac{1}{2} \frac{\partial^2 V_1}{\partial \phi^2} \right]_{\phi=v} = \left[\frac{1}{2} \frac{\partial^2 V_0}{\partial \phi^2} + \frac{3g^2}{16\pi^2} \frac{m_i^2}{m_W^2} \log \frac{m_i^2}{m_t^2} \right]_{\phi=v} \quad (33)$$

where the tree-level piece of the potential should be evaluated with $g^2(\hat{Q})$ and $g'^2(\hat{Q})$.

We see in eq.(33) a contribution proportional to m_i^4 . It is easy to check that the $\Sigma(m_{p_{Higgs}}^2) - \Sigma(0)$ term in eq.(21) does not have such quartic behaviour, and hence that the one-loop effective potential contribution to m_h^2 is the most important for large m_t . Moreover, the fact that direct searches [9] and analyses of precision electroweak data including radiative corrections [15] indicate that $m_t \simeq 100 \div 180$ GeV means that the one-loop correction to m_h will be much larger than has been estimated previously [6] assuming that $m_t \sim 40$ GeV. These conclusions apply not only to the simplified analytic limit, but also to the realistic case where v_1 and v_2 are different from zero and all contributions to ΔV_1 are taken into account, including squark mixing.

We present numerical results for two different types of input: (A) *Low-energy*, where we simply use some test values of the physical supersymmetry-breaking parameters such as the soft squark mass $m_{\tilde{q}}$ (in this case we include only the $t, \tilde{t}, b, \tilde{b}$ contributions to ΔV_1) and (B) *High-energy*, where we assign the soft supersymmetry-breaking parameters at the conjectural scale of supersymmetric grand unification and run them down to low energies

using the renormalization group. In this case, we are able to compute all the contributions to ΔV_1 , taking into account the full structure of supersymmetric particle masses and mixing angles. The second type of input focuses attention on a subspace of the available parameter space which is well-motivated.

We will be looking at three physical consequences of the radiative corrections: (i) invalidation of the apparent lower bound on $\tan\beta$ that is derived from LEP data, since radiative corrections to a light Higgs boson mass can be large and positive, (ii) the likelihood for large $\tan\beta$ that radiative corrections push the lightest Higgs boson above the Z^0 mass, dimming the prospects that LEP II will find it, and (iii) the probability that the heavier CP-even Higgs mass m_H , when its tree-level value is only slightly above m_Z , will be shifted up, outside the LEP reach but possibly also out of the range $90 < m_H < 120$ GeV, where detection at the LHC or SSC would be problematic [11].

The parameter space of low-energy inputs is multi-dimensional: after restricting our attention to the $t - \bar{t}$, $b - \bar{b}$ sector, in addition to m_t , it includes $\tan\beta = v_2/v_1$, the tree-level CP-odd Higgs mass⁴ m_A , the soft supersymmetry-breaking mass⁵ $m_{\tilde{q}}$, and the magnitude of the trilinear soft supersymmetry-breaking parameter⁶ A and of the Higgs mixing mass μ relative to $m_{\tilde{q}}$. As a general rule, we find that the shift in the lighter CP-even Higgs boson mass m_h decreases as $\tan\beta$ increases, increases as m_A increases, increases as $m_{\tilde{q}}$ increases, and is almost insensitive to $A/m_{\tilde{q}}$ and $\mu/m_{\tilde{q}}$ in the range 0 to 1. Fig. 1 shows some representative results that illustrate these remarks. Comparing the $m_A = 50$ GeV examples in fig. 1a and fig. 1b, we see that a large positive shift for $\tan\beta = 1.1$ becomes negligible for $\tan\beta = 10$ and it can never become substantially negative for larger values of $\tan\beta$. If m_A is now increased from 50 GeV to 500 GeV as in figs. 1c and 1d, we see a sharp increase in the shift in m_h for $\tan\beta = 1.1$, and the shift for $\tan\beta = 10$ is now also large and positive. As for the shift in the heavier CP-even Higgs boson mass m_H , although it exhibits similar trends to decrease as $\tan\beta$ increases and to increase as $m_{\tilde{q}}$ increases, it is

⁴Of course, the parameter m_A will be different from the physical mass of the CP-odd eigenstate, but this is not relevant for the purpose of the present paper.

⁵For simplicity, we take the same numerical value for $(\tilde{t}, \tilde{b})$, $\tilde{t}^c$ and $\tilde{b}^c$.

⁶For simplicity, we take $A_t = A_b$.

more sensitive to A , and has a marked tendency to decrease as m_A increases. The first three of these points are illustrated in fig. 2a, where the increase in m_H is maximized by choosing $\tan\beta$ small, $m_{\tilde{q}}$ large, and $A = m_{\tilde{q}}$, whilst the case of larger $\tan\beta$ and smaller $m_{\tilde{q}}$ in fig. 2b exhibits a smaller increase in m_H .

Figs. 1 and 2 exhibit the three features advertized earlier in the paper: (i) the lower bound on $\tan\beta$ deduced from tree-level mass formulae and the absence of a light supersymmetric Higgs at LEP is completely invalidated by radiative corrections, (ii) the lighter CP-even Higgs may well weigh more than m_Z , and (iii) the heavier CP-even Higgs may be pushed away from mass values close to the Z^0 mass.

We now turn to high-energy inputs. In this case, we choose a value for v_2/v_1 , m_t and for the common gaugino mass $m_{1/2}$, the common scalar mass m_0 and the common trilinear scalar coupling A at the unification scale⁷. A choice for the sign of μ must also be made. These parameters are sufficient to determine, with the help of the renormalization group equations, the masses of all the particles in the theory. This method has the advantage that all the contributions to ΔV_1 can be included, with a small number of input parameters. Furthermore, it is easy to check whether a given set of parameters is compatible with the present experimental limits on supersymmetric particle masses. We find that the effect of varying m_0 and A is negligible for $m_0/m_{1/2}$ and $A/m_{1/2}$ between 0 and 1. When we choose $m_{1/2} = 100$ GeV, we find phenomenologically acceptable solutions only for $\tan\beta \lesssim 3$ and $\mu < 0$. Fig. 3a shows a representative solution with $\tan\beta = 1.5$, which only exists for $m_t \leq 160$ GeV, and exhibits a modest increase in m_h . We find more solutions when $m_{1/2} = 150$ GeV, and two extreme examples are shown in fig. 3b, where $\tan\beta = 1.1$, $m_t \leq 140$ GeV and m_h may approach m_Z ; and fig. 3c, where m_h is shifted out of the LEP range. The increases in m_h are smaller for larger $m_{1/2}$ when $\tan\beta = 1.1$, as seen in fig. 3d, but larger when $\tan\beta = 10$, as seen in fig. 3e. This trend is accentuated in the case of $\tan\beta = 30$, as seen in fig. 3f, for which we find acceptable solutions only if $m_t \leq 120$ GeV. We do not reproduce any more graphs of the increases in the mass m_H of the heavier CP-even

⁷We have taken the unified gauge coupling constant α_U to be 0.041 at the unification scale $M_U = 2.29 \times 10^{16}$ GeV.

Higgs, but simply remark that it may well be shifted out of the ‘difficult’ range $90 < m_H < 120$ GeV into the ‘easier’ range $m_H > 120$ GeV, as seen previously using low-energy inputs.

We conclude that radiative corrections may alter dramatically the conventional view of supersymmetric Higgs phenomenology. The CP-even Higgs boson masses are in general shifted upwards, though they can also be reduced in some special cases. These shifts increase with m_t , $\cot \beta$ and m_A , and can even attain $\mathcal{O}(100)$ GeV. Three particularly important consequences for experimental searches are (i) the present lower limit on m_h cannot be considered as a lower limit on $\tan \beta$, which has implications for the allowed range of LSP masses; (ii) m_h may easily be pushed beyond m_Z , in particular for large values of $\tan \beta$; and (iii) the heavier CP-even Higgs mass may be pushed away from the Z^0 mass region, with important implications for Higgs searches at LEP and at the LHC/SSC.

Clearly more work is needed to map out more completely these effects of radiative corrections, and it may be necessary also to include them in discussions of LSP phenomenology. The implications for present and future Higgs searches need to be more fully digested, but it seems clear that the results of searches should be presented in a more model-independent way, as should strategies for Higgs searches at future accelerators.

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Note added: after completing the work described in this paper, we received another paper on a similar subject [17]. Ref.[17] does not discuss t mixing, the definition of the physical Z^0 mass, radiative corrections to m_H , or high-energy inputs, and does not minimize the full one-loop effective potential.

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Figure Captions

Figure 1

Values of the lighter CP-even Higgs boson mass before (dashed lines) and after (solid lines) including one-loop radiative corrections for 100 GeV $< m_t < 180$ GeV. The specific choices of the low-energy inputs m_A , A , μ , $m_{\tilde{g}}$ and v_2/v_1 are shown in each panel. We see in (a) a significant increase in m_h , in (b) no significant change, in (c) a larger increase traceable to the larger value of m_A , and in (d) an increase in m_h beyond m_Z .

Figure 2

As in fig. 1, but for the heavier CP-even Higgs boson mass. We see in (a) a large increase into the mass range more accessible to the LHC and SSC, and in (b) a more modest increase traceable to the larger value of $\tan \beta = v_2/v_1$.

Figure 3

As in fig. 1, but for the noted values of the high-energy inputs. The dotted lines are the maximum tree level values of m_h for the indicated values of $\tan \beta = v_2/v_1$. The renormalization group gave solutions only for limited ranges of m_t in cases (a), (b), (d) and (f). The cases (b) and (d) exemplify our observation that present lower bounds on m_h cannot be used to exclude low values of $\tan \beta$. We see in panels (c), (e) and (f) cases where m_h is increased beyond m_Z .

Fig. 1

Fig. 1 (cont.)

Fig. 2

Fig. 3

Fig. 3 (cont.)

Fig. 3 (cont.)