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THE K_L - K_S MASS DIFFERENCE

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Abstract

The matching of the long- and short-distance contributions to the K_L - K_S mass difference is investigated in the large N_c (N_c being the number of colours) limit. In this limit a physically well-defined separation between the two regimes exists. This allows for an investigation of the matching between the two contributions. We obtain that 0.8 ± 0.1 of the observed mass difference is due to the light and charm quark contribution. An estimate of the top and top-charm contributions is also given.

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The Kaon system has provided us with a lot of experimental surprises. In particular, the concepts of parity-violation, CP-violation and flavour conservation in neutral currents, three pillars of the now well-established standard model for electroweak interactions, were indeed triggered by the physics of strangeness. Fortunately, the pseudo-Goldstone boson nature of Kaons (and pions) allows us to extract a fair amount of theoretical information. One notable exception to this rule has been the field of non-leptonic weak interactions. In this letter we extend the method used for the electromagnetic $\pi^+\pi^-$ mass difference [1] to the $K_L - K_S$ mass difference to study the interplay between the strong and weak interactions.

In the absence of weak interactions there would be two equal mass states K^0 and $\bar{K}^0$, with strangeness $S = -1, +1$, respectively. The weak interactions cause transitions between these strong interaction eigenstates so that the mass eigenstates become

$$\begin{aligned} K_L &= \frac{1}{\sqrt{2}}(K^0 + \bar{K}^0) \\ K_S &= \frac{1}{\sqrt{2}}(K^0 - \bar{K}^0). \end{aligned} \quad (1)$$

These are the physical states if (tiny) CP-violation effects are neglected. We will do so in the remainder of this letter. The mass difference in terms of the transition element is given by

$$\Delta m = m_L - m_S = -\frac{2i}{m_K} \text{Re} A(K^0 \rightarrow \bar{K}^0) \quad (2)$$

This transition is an effect of second order in the weak interactions and can be parametrized as

$$\Delta m = \frac{1}{4\pi^2} \left(\frac{G_F}{\sqrt{2}} \right)^2 (\sin \theta_c \cos \theta_c)^2 f_K^2 m_K M^2. \quad (3)$$

Here, G_F is the Fermi constant and θ_c is the Cabibbo angle. The factor f_K^2 follows from chiral symmetry arguments, the Cabibbo angle dependence from the $\Delta S = 2$ nature of the transition and m_K because the transition element gives the difference of the masses squared.

Experimentally, the regeneration of the short-lived K_S via matter interactions allows a remarkably precise determination of the mass splitting :

$$(\Delta m)_{exp} = (3.521 \pm 0.014) \cdot 10^{-15} \text{ GeV} \quad (4)$$

From eqs. (3) and (4) we conclude that

$$M \approx 1.7 \text{ GeV}. \quad (5)$$

The theoretical estimate of M has a long and famous history.

In the late sixties current algebra and loop calculations suggested that the scale M was the weak cut-off fixed by the mass of the (then hypothetical) W -gauge boson. This inserted in (3) led to a low W -boson mass which was soon ruled out by experiments. In 1974 Gaillard and Lee [2] used the GIM mechanism [3] and a vacuum insertion estimate of the quark box diagram to obtain

$$M \approx m_c^2 - m_u^2 \quad (6)$$

This theoretical prediction for the charm quark mass was confirmed a few months later with the discovery of the J/ψ . The magnitude of the mass difference was understood.

The presence of the light quark mass m_u in (6) indicates that long-distance effects induced by virtual pion exchanges might be non-negligible [4]. The short-distance strong interaction corrections [5] introduce another source of uncertainty :

$$M^2 \approx \eta_{QCD} m_c^2 \quad ; \quad 0.7 \leq \eta_{QCD} \leq 1.0 \quad (7)$$

due to ignorance of the QCD scale Λ_{QCD} ($0.1 \text{ GeV} \leq \Lambda_{QCD} \leq 0.3 \text{ GeV}$). Finally, the appearance [6] of a (current or constituent ?) strange quark mass dependence in the estimate of the box diagram with non-zero external momenta totally obscures a possible separation of short- and long-distance contributions to Δm . In the large N_c limit, we indeed obtain for the standard box diagram :

$$M^2 \approx m_c^2 + \frac{2}{3} m_K^2 \log \frac{m_c^2}{m_K^2}. \quad (8)$$

A further uncertainty on the $K_L - K_S$ mass difference arises from the estimate of the hadronic matrix element for the short distance $\Delta S = 2$ weak operator. The B -parameter indeed measures the deviation from the vacuum insertion approximation used in ref. [2]. Fifteen years of theoretical investigation led to predictions ranging from [7] $B = -0.4$ to $B = 2.8$. Recently, new non-perturbative techniques have been developed, giving rise to a refined estimate. The range

$$1/3 \leq B \leq 1 \quad (9)$$

fairly summarizes the present status of the B parameter [8].

Unfortunately, the B parameter itself cannot be extracted directly from experimental data. The CP violating parameter ϵ_K is proportional to $B \sin \delta_{KM}$, with δ_{KM} the Kobayashi-Maskawa phase, and strongly depends on the top quark mass. Therefore, any prediction of the B parameter requires an estimate of the measured $K_L - K_S$ mass difference within the same non-perturbative framework. We present¹ here such an estimate within the simple $1/N_c$ approach, N_c being the number of colours of the strong interaction gauge group.

In the large N_c limit, single pseudoscalar exchange contributions and short-distance QCD corrections are $1/N_c^2$ and $1/N_c$ suppressed, respectively. On the other hand we have [10]

$$B = 3/4 + \mathcal{O}(1/N_c) \quad (10)$$

such that the uncertainties listed above simply disappear. Moreover, in this limit there also exists a clean separation of the long- and short-distance contributions. Since no quark loops are present, the flavour structure of all contributions is like in Fig. 1 with no gluonic connections across the dashed line. This allows for a unique identification of the momentum squared q^2 of the W loop independent of the parts involving the strong interaction.

Below the matching scale $\Lambda \approx 1 \text{ GeV}$ we use the non-linear sigma model coupled to external W fields. The only hadrons appearing at this level are the pseudoscalars π, K and η . The corresponding truncated Lagrangian is given by

$$\mathcal{L} = \frac{f^2}{8} \text{tr} \left(D_\mu \Sigma D^\mu \Sigma^\dagger + \chi \Sigma^\dagger + \Sigma \chi^\dagger \right) \quad (11)$$

¹This result has already been presented at the XXIX Cracow School of Theoretical Physics (June 1989)[9].

where

$$\begin{aligned}\Sigma &= \exp \frac{2i}{f} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \frac{K^0}{\sqrt{2}} & -\sqrt{\frac{2}{3}}\eta \end{pmatrix} \\ D_\mu \Sigma &= \partial_\mu \Sigma + i \frac{g_W}{2} \begin{pmatrix} 0 & \cos \theta_c W_\mu^+ & \sin \theta_c W_\mu^+ \\ \cos \theta_c W_\mu^- & 0 & 0 \\ \sin \theta_c W_\mu^- & 0 & 0 \end{pmatrix} \Sigma \\ \chi &= \begin{pmatrix} m_\pi^2 & 0 & 0 \\ 0 & m_\pi^2 & 0 \\ 0 & 0 & 2m_K^2 - m_\pi^2 \end{pmatrix}\end{aligned}\quad (12)$$

In this letter we work in the limit $m_\pi = 0$. Above the matching scale Λ we use the QCD quark Lagrangian coupled to external W -fields. The matrix element is then evaluated by deriving the couplings to the currents from (11). This is consistent to leading order in $1/N_c$ and corresponds to using $B = 3/4$ for this part of the contribution. We emphasize the unique identification of the W q^2 momentum here across the matching scale Λ . This complementary approach worked satisfactorily for the π^+ , π^0 mass difference [1].

Using the vertices from (11), the long-distance part can be calculated from the hadronic diagrams in Fig. 2. In particular, the diagram in Fig. 2c corresponds to the long-distance contribution advocated by Wolfenstein [4].

A standard one-loop calculation using an Euclidean ultraviolet cut-off Λ in the Feynman integrals leads to

$$M_{LD}^2 = \frac{7}{4}\Lambda^2 - \frac{3}{4}m_K^2 \log \frac{\Lambda^2}{m_K^2} + \frac{11}{24}m_K^2 \quad (13)$$

A similar calculation of the standard quark box diagrams with Λ as the infrared cut-off leads to

$$M_{SD}^2 = m_c^2 - \Lambda^2 + m_K^2 \log \frac{m_c^2}{\Lambda^2} - \frac{5}{6}m_K^2 \quad (14)$$

At this point it should be noted that the uncertainty on the strange quark mass present in (8) has disappeared. In the short distance the logarithm is bounded by the matching scale and in the long distance it is unambiguously m_K^2 which appears. The quadratic dependence on the matching scale Λ does not cancel when adding the two contributions. A similar observation was made for the π^+ , π^0 mass difference in [1]. There the dependence on the square of Λ disappeared after the virtual effects of vector and axial vector mesons were added.

In these expressions it is very important to include the effect of the external momenta going through the W loop. Integrating out the W propagators and treating them as point vertices from the start would lead to

$$M_{LD}^2 = \frac{3}{2}\Lambda^2 - \frac{3}{4}m_K^2 \log \frac{\Lambda^2}{m_K^2} + \frac{3}{8}m_K^2 \quad (15)$$

$$M_{SD}^2 = m_c^2 - \Lambda^2 + \frac{2}{3}m_K^2 \log \frac{m_c^2}{\Lambda^2} - \frac{5}{9}m_K^2 \quad (16)$$

Turning now to an evaluation of our result, we insert into (3)

$$M^2 = M_{LD}^2 + M_{SD}^2 \quad (17)$$

using (13,14) and the values $G_F = 1.6637 \cdot 10^{-5} \text{ GeV}^{-2}$, $f_K = 161 \text{ MeV}$ and $\sin \theta_c = 0.22$. The results are plotted in Fig. 3. The three lines correspond to $m_c = 1.4, 1.5$ and 1.6 GeV , respectively. The case where the W propagators have been integrated out and treated as operators from the start is numerically very similar. For most Λ 's the results of that approach are about 5 % below those of Fig. 3. This result therefore provides further confidence in the similar approximation used for the $\Delta I = 1/2$ rule and the B parameter (See e.g. Ref. [9]). The sensitivity to Λ is smallest for $\Lambda \approx 750 \text{ MeV}$. This best matching scale is very similar to the one found in [1]. Varying Λ between 0.5 and 1 GeV and taking the uncertainty on the charm mass into account leads us to :

$$\Delta m(uu, uc, cc) = (0.8 \pm 0.1) \Delta m_{exp}. \quad (18)$$

We have up to now neglected the top quark contributions to the mass difference. The associated Kobayashi-Maskawa angles are small but they are multiplied by m_t^2 . So, in principle, the corrections could become large. Assuming $90 \text{ GeV} \leq m_t \leq 170 \text{ GeV}$ as indicated by the LEP data and imposing the constraints on the KM matrix coming from B physics (semileptonic decays and oscillations) we obtain a positive contribution to the K_L - K_S mass difference from the top-top and the top-charm contributions.

$$0.03 \leq \Delta m(tt, tc) / \Delta m_{exp} \leq 0.10 \quad (19)$$

In the large N_c limit, the constraints on hypothetical particles can be improved because

$$\Delta m(\text{new physics}) \leq \frac{1}{4} \Delta m_{exp} \quad (20)$$

follows from our results. Since most new particles contribute to the mass difference as M_{hyp}^{-2} this leads in general to a bound on their masses improved by a factor of 2.

In conclusion we have calculated the K_L - K_S mass difference to leading order in the $1/N_c$ expansion taking into account all contributions, both long and short distance. We obtain a value in reasonable agreement with the observed one. The inclusion of vector mesons to improve the matching behaviour is in progress. Calculations of the K_L - K_S mass difference using other non-perturbative methods, e.g. lattice or QCD sum rules, are needed to check the validity of our simple approach. This approach has already led to some encouraging results in the non-leptonic weak and electromagnetic sector of the standard model at low energies (see e.g. Ref. [9]).

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Fig. 1. The generic contribution to the K_L - K_S mass difference in the large N_c limit.

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Fig. 3. The $K_L - K_S$ mass difference as a function of the matching scale Λ . The upper curve is for a charin quark mass of 1.6 GeV, the middle one for $m_c = 1.5$ GeV and the lower one for $m_c = 1.1$ GeV.