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The Search for Intermittency in the Finite-Size Ising Model

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Abstract

Using a new form of moments proposed recently by Novak, we present first clear evidence for intermittent behaviour in the two dimensional Ising model of sizes 64^2 to 1024^2 . Both factorial and standard moments are calculated numerically using a Monte Carlo cluster algorithm. We find that the resulting fractal dimension agrees with that for a site correlated percolation transition.

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1. Introduction

The study of intermittent behaviour has attracted a lot of attention recently, triggered by investigations in the context of high energy multiparticle production [1]. This led to intermittency studies in critical systems, specifically in systems with a second order phase transition [2-5].

In multiparticle production, intermittent behaviour is attributed to the branching structure of jets. Fluctuations in rapidity on all scales lead to a power-law behaviour of the factorial moments as functions of the linear bin size.

In critical systems, intermittent behaviour arises at the critical point because the correlation length of the system diverges there, giving rise to domains of ordered spins of all sizes. This implies some type of fractal structure; the corresponding fractal dimension d_f depends on the dimension of the system and on the definition of the spin-domains. As emphasized in [3], intermittent behaviour should only arise at the critical point of the system under consideration; for finite systems, this means at a point at which there is pseudo-critical behaviour, such as a peak in the susceptibility or in the specific heat.

The first numerical investigations of intermittency in the two dimensional finite-size Ising model were carried out by Wosiek [2]. Through Kadanoff scaling methods, intermittency was subsequently established analytically for the d-dimensional Ising system in the thermodynamic limit [3]. This led to a relation between the intermittency index λ_i and critical exponents.

In the context of investigations of global universality at the onset of chaos in a Raleigh-Bernard system, a general relation linking the intermittency index λ_i to the fractal dimension of the system was obtained [6].

Although considerable freedom exists in defining the block variables (see section 2), it has become apparent from previous numerical studies that care must be exercised not to use definitions that break the internal global symmetry of the system, here $Z(2)$ [4]. As we shall see in section 3, such a breaking can give rise to a behaviour very similar to intermittency, but not of dynamical origin. In addition, to measure the dynamical fluctuations of the system, an appropriate definition of the moments has to be used in order to eliminate statistical noise.

In the following we investigate intermittency in the two dimensional Ising model. In section 2 we introduce the relevant formalism and discuss possible choices of the moments. In section 3 we then give details of our numerical calculations and discuss the results. In section 4 we introduce and discuss a new definition for the block variables based on cluster ideas from percolation theory, and compare its effect with the results of section 3. In section 5, the intermittency indices and fractal dimensions obtained from the numerical results of sections 3 and 4 are considered. We conclude with a short summary in section 6.

2. Formulation of intermittency for the Ising model

We consider an Ising spin system defined on a two dimensional square lattice of linear size R . At each vertex we define a spin variable s_i , with $s_i = \pm 1$. In the numerical Monte

Carlo evaluation, we sample configurations weighted with the Boltzmann factor

$$\exp(K \sum_{i,j} s_i s_j), \quad (1)$$

where the sum is over all nearest neighbour sites on the lattice and K is the dimensionless nearest neighbour coupling

$$K = J/kT. \quad (2)$$

The first step in the numerical calculation is to divide the lattice into blocks of linear size L . For a given L , the number of blocks is therefore

$$M = (R/L)^2. \quad (3)$$

For each such block we then measure a "block spin" k_m ($m=1, \dots, M$); various definitions for the block variables k_m exist in the literature. Here, following ref. [4], we define k_m to be the number of up spins projected onto the total magnetization in block m ,

$$k_m = \frac{1}{2} \sum_{i=1}^{L^2} (\text{sgn}(s) s_i + 1). \quad (4)$$

where s is the total magnetization

$$s = \frac{1}{R^2} \sum_{i=1}^{R^2} s_i. \quad (5)$$

The definition of the block variables k_m given above respects the $Z(2)$ symmetry of the system: a measurement obtained on a given configuration equals one obtained after a global $Z(2)$ transformation of all spins. In refs. [2] and [5], the spin multiplicity was defined to be the number of up spins in a block, i.e.,

$$k_m = \frac{1}{2} \sum_{i=1}^{L^2} (s_i + 1). \quad (6)$$

It is immediately apparent that this definition is not $Z(2)$ symmetric in the sense mentioned above; the resulting consequences will be discussed in more detail in the next section.

For the moments, the situation is the following. To eliminate statistical fluctuations in multiparticle production, Bialas and Peschanski [1] consider the normalized factorial moments

$$F_i(L) = M^{i-1} \left(\frac{\sum_{m=1}^M k_m(k_m-1) \dots (k_m-i+1)}{N(N-1) \dots (N-i+1)} \right), \quad (7)$$

with

$$N = \sum_{m=1}^M k_m. \quad (8)$$

Here $\langle \dots \rangle$ denotes the average over different configurations ("events"). If the total multiplicity N is constant and statistical fluctuations due to finite multiplicity can be described by a multinomial distribution, then the factorial moments (7) eliminate statistical noise. This definition was used in both the numerical studies [2] and [4].

For a variable multiplicity, the N dependence can be eliminated by defining

$$\bar{F}_i(L) = \frac{1}{M} \sum_{m=1}^M \bar{F}_i^{(m)} \quad (9)$$

with

$$\bar{F}_i^{(m)}(L) = \frac{\langle k_m(k_m-1)\dots(k_m-i+1) \rangle}{\langle k_m \rangle^i} \quad (10)$$

Here one thus performs first a "vertical" averaging over different configurations (events) in a given block m and then a "horizontal" average over blocks (bins).

An expression similar to the one above was derived by Novak specifically for the Ising model [5]. Here the total multiplicity is just the magnetization, and hence is generally not fixed. The total sum of spins in each block is, however, an independent random variable; the fluctuations in each block can therefore be described by a binomial distribution. This leads for the normalized moments in a fixed block m to

$$\bar{F}_i^{(m)}(L) = \frac{\langle \rho_m^i \rangle}{\langle \rho_m \rangle^i}, \quad (11)$$

with

$$\langle \rho_m^i \rangle = \frac{\langle k_m(k_m-1)\dots(k_m-i+1) \rangle}{L^i(L^2-1)\dots(L^2-i+1)}. \quad (12)$$

As before, L denotes the block size, while k_m is the spin multiplicity in block m . The lattice moments are obtained by averaging over all blocks,

$$\bar{F}_i(L) = \frac{1}{M} \sum_{m=1}^M \bar{F}_i^{(m)} = \left\{ \frac{L^{2i}}{L^2(L^2-1)\dots(L^2-i+1)} \right\} \frac{1}{M} \sum_{m=1}^M \bar{F}_i^{(m)}. \quad (13)$$

The expression (13) for the normalized moments thus differs from the form (9) only by a different normalization factor, which depends on the block size. It is claimed in [5] that this definition of the factorial moments should remove statistical fluctuations due to finite block size, even for lattices as small as $R = 16$.

We have investigated the factorial moments (7) and (13), as well as the corresponding standard moments

$$C_i(L) = M^{i-1} \left(\frac{\sum_{m=1}^M k_m^i}{N^i} \right) \quad (14)$$

and

$$\bar{C}_i(L) = \bar{C}_i(L) = \frac{1}{M} \sum_{m=1}^M \frac{\langle k_m^i \rangle}{\langle k_m \rangle^i}. \quad (15)$$

To simplify the nomenclature, we shall refer to eqs.(7) and (14) as *global* moments and to eqs. (13) and (15) as *block* moments.

Before turning to numerical studies, we recall that intermittency means a power law dependence of the factorial moments on the block size,

$$F_i(L) = c_i L^{-\lambda_i}, \quad (16)$$

where c_i and λ_i are order dependent constants; λ_i is the intermittency index for the i -th moment. Analogous relations hold, of course, for the other forms considered.

3. Numerical results

3.1 Simulation details

We have investigated the two possible types of moments outlined in the previous section for various lattice sizes, ranging from $R = 64$ to $R = 1024$. In all cases, periodic boundary conditions were imposed on the lattice as a whole. A range of coupling values K , including the pseudo-critical point, was studied on all lattice sizes up to $R = 256$; for $R = 512$ and 1024 , we have performed calculations only at the pseudo-critical point. The pseudo-critical coupling is here defined at the point at which the susceptibility attains its maximum value. For lattice sizes $R = 64, 128$ and 256 , the pseudo-critical couplings are known from ref.[4]; the values for $R = 512$ and 1024 were obtained from the finite-size scaling relation

$$\left| \frac{K_c^R - K_c^\infty}{K_c^\infty} \right| = c R^{-1/\nu}. \quad (17)$$

All values are listed in table 1.

For the numerical calculation we used a cluster-Monte Carlo algorithm [7]. The advantages of this algorithm, especially in reducing critical slowing down (correlations between successive measurements at the critical point), are well known [8]. In all simulations at least the first 1000 updates were discarded for thermalization, and measurements were then made at every fifth update. Since the autocorrelation time is of the order 7 for lattice size $R = 256$, this suffices to minimize measurement-to-measurement correlations. All our results, except those for the $R = 1024$ lattice, are based on at least 30,000 measurements. The statistical errors for the block moments were obtained by a Jackknife procedure [9] implemented on the raw data.

3.2 Results for the block moments

In our numerical calculations, we use the approximation

$$\langle k_m \rangle \simeq \frac{1}{M} \left\langle \sum_{m=1}^M k_m \right\rangle \quad \forall \quad m \quad (18)$$

in the denominator of both the factorial and standard moments. For block moments, the normalization is carried out only after the configurational averaging. In this way the

problem of dealing with a fluctuating spin multiplicity is avoided. Note that the block moments (eqs. (13) and (15)) are normalized to one for $L = R$ only in the limit $R \rightarrow \infty$.

We have investigated the block moments (both standard and factorial), using the $Z(2)$ symmetric definition for the block variables (eq.(4)). In figs. 1 - 3 we show the results for lattice sizes $R = 64, 256$ and 1024 at the pseudo-critical points, together with a best linear fit to the data for the factorial moments. Both the standard and the factorial moments show a power law behaviour in agreement with that predicted by eq. (16), with slight deviations from linearity for block sizes $L \leq 8$. The numerical values of the slopes, which give the intermittency indices, will be discussed in section 5. The linear behaviour of the block moments in figs. 1 - 3 should be compared to the results obtained in [4] for the global moments. In fig. 4, we show the fourth order factorial moments from both global (7) and block (13) definitions for $R = 128$ at K_c^R ; only the block moments of Novak show clear evidence of intermittent behaviour.

The behaviour of the moments at non-critical temperatures is shown in figs. 5 and 6 for lattice size $R = 256$. Above and below the critical point it is similar to that found in [4] for the global moments. However, in the block formulation, factorial and standard moments now resemble each other more, as was also found at the pseudo-critical point in figs. 1 - 3. This is especially apparent for $K > K_c^R$ (fig. 5). For $K < K_c^R$ (fig. 6) we see the effect which various levels of magnetization have on the behaviour of the moments, especially for the smaller block sizes. Similar to the behaviour of the global moments [4], finite size effects here remain also in the block formulation. However, the range in R/L , for which $\ln \tilde{F}_i(L)$ is essentially zero, becomes larger with increasing lattice size, as seen in fig. 7.

From the results shown, it is clear that intermittency is linked to the presence of critical behaviour in the system. For coupling values away from the pseudo-critical point, no evidence for intermittent behaviour is found.

In ref.[5], where the use of the block moments was first proposed, a numerical study was carried out using the spin multiplicity defined by eq.(6), i.e., it was taken to be the number of up spins in a given block m . As mentioned, this definition of the block variables is not $Z(2)$ symmetric. Moreover, the results in [5] were obtained on a $R=64$ lattice at $K = K_c^\infty \approx 0.441$. For a 64^2 lattice, this is quite far from the pseudo-critical point (see table 1), so that we do not expect intermittent behaviour at this value of the coupling. In fig. 8, we show our results at $K = 0.441$, using the $Z(2)$ -breaking form (6) of the block variables. The contribution from configurations with negative magnetization, i.e., those for which block spins (k_m) are anti-parallel to the overall magnetization, give strong but "non-linear" fluctuations. The configurations for which the block spins and the overall magnetization are parallel, give $\ln \tilde{F}_i(L) \approx 0$, as expected. The average of these two patterns leads to something resembling intermittent behaviour, but apparently of non-dynamical origin.

4. Intermittency in the framework of percolation

As the spin multiplicity is not uniquely defined, it is clear that as long as the global $Z(2)$ symmetry is not violated, alternative definitions to those given in section 2 may also be used.

In ref. [10], the concept of connectivity was introduced by considering the sum of spins in the largest "cluster" in each block, i.e., in the largest connected region of like-sign spins. In this vein, we now propose the following definition for the block variables, based on ideas taken from percolation theory. For each configuration of spins the total magnetization is calculated first. The blocking procedure is implemented as before by dividing the lattice successively into blocks. At each level we construct clusters between n.n. equal spins, but only between those whose spin direction is the same as the overall magnetization. The block variable k_m is then defined as the number of spins in the largest cluster c_m in block m ,

$$k_m = \text{sgn}(s) \sum_{i \in c_m} s_i, \quad (19)$$

where s is again the total magnetization. This definition of the block variables also respects the $Z(2)$ symmetry of the system.

So far, the clusters simply consist of all n.n.-equal spins. More generally, we can introduce a bond probability $P_B \leq 1$, which determines the weight for assigning two n.n.-equal spins to the same cluster. The Ising clusters considered above are then defined as clusters with $P_B = 1$. In this way one is able to characterize the onset of the ordered phase by explicitly measuring the domains of ordered spins. In particular, one can interpret the critical point as the point at which, in the thermodynamic limit, the cluster-size diverges. In percolation theory, this problem is studied purely on a geometrical level: for what relative fraction of up spins do we get infinite clusters?

The idea of describing the thermodynamic transition in spin systems geometrically in terms of clusters is already fairly old [11]. However [12], in order to obtain the thermodynamic phase transition in terms of cluster percolation, one has to consider "Ising-droplets", i.e., clusters of spins defined with the conditional bond probability

$$P_B = 1 - e^{-2K}, \quad (20)$$

with K is given by eq.(2). To arrive at the correct critical behaviour for the Ising model, the original site-correlated percolation model (SCPM) with Ising clusters thus has to be replaced by a site-bond-correlated percolation model (SBCPM) with Ising droplets. The SBCPM has the Ising model critical point in any dimension, with the correct critical exponents, provided the critical bond probability is given by eq. (20), with $K = K_c$. For the SCPM in two dimensions, the critical point coincides with that of the two dimensional Ising model, but the critical exponent γ associated with the susceptibility is different in the two cases. In higher dimensions, not even the critical couplings coincide. Away from the critical point, both the SCPM and SBCPM have the critical exponents of random percolation.

We have investigated both possible choices for P_B corresponding to the percolation models outlined above, using the block variables as defined in eq.(19). For the blocks we impose periodic boundary conditions. We first consider the case with $P_B = 1$. In fig. 9 the results for the global moments on the $R = 128$ lattice at K_c^R are compared to those using block moments. A linear fit to the block factorial moments is also shown. The differences between these results are quite pronounced; the block moments show a power-like behaviour in terms of the block size, similar to that found in section 3, while the results obtained from the global moments show no sign of intermittency. This again supports the claim that block moments effectively eliminate, or at least greatly reduce, the statistical noise present in the Ising model. - The behaviour of the block moments for non-critical couplings is in the SCPM formalism similar to that found in section 3.

As mentioned, the intermittency indices even at the pseudo-critical point are in the SCPM related to those of the percolation transition; only the SBCPM should give the correct Ising model intermittency indices [12]. A discussion of our numerical results for the intermittency indices follows in the next section. To conclude this section, we note here only that a preliminary study of intermittency in the SBCPM formalism for lattice sizes $R \leq 128$ has not provided reasonable evidence for intermittent behaviour, even for the block moments. It is not clear whether in this case, when the cluster size is reduced by the conditional bond probability, one has to go to much larger lattices in order to see signs of intermittent behaviour. Moreover, the critical bond probability, which is defined for the system as a whole, may have to be modified before a blocking of the system can be implemented.

5. Intermittency indices and fractal dimensions

We now consider the intermittency indices and fractal dimensions obtained for the block moments studied in sections 3 and 4. Intermittent behaviour, characterized by the power law behaviour given in eq.(16), is directly linked to the presence of an underlying fractal structure in the system under consideration. These two concepts can be related through the expression [6]

$$\lambda_i = (i-1)(d - d_f(i)) \quad (21)$$

where $d_f(i)$ is the fractal dimension associated with the i -th moment, while d is the dimension of the supporting space (here $d = 2$).

It follows from eq.(21) that, if the fractal dimension characterizing a particular system is order independent, the intermittency indices show a linear behaviour as function of the order i of the moments. In ref. [3], the intermittency indices for the d -dimensional Ising spin system were obtained in terms of the critical index η , the anomalous part of the scale dimension: $\lambda_i = (i-1)\eta/2$. For the two-dimensional model, $\eta = 1/4$; this gives

$$\lambda_i = (i-1)/8, \quad (22)$$

so that the corresponding fractal dimension is $d_f = 15/8$. For the SCPM formalism, the fractal dimension has recently been calculated, with the exact value $d_f = 187/96$ [13,14].

The corresponding intermittency indices thus become

$$\lambda_i = (i-1)5/96. \quad (23)$$

If we assume the power law behaviour of eq. (16), the intermittency indices can be obtained from the numerical data by calculating the ratios of the factorial moments

$$\ln[F_i(2L)/F_i(L)]/\ln 2 = \lambda_i, \quad (24)$$

for the different block sizes and order of the moments, or alternatively by performing a direct linear fit to the data ($\ln F_i(L)$ vs. $\ln(R/L)$). In fig. 10 we show the intermittency index λ_i obtained on a 128^2 lattice for a range of coupling values using the block moments. In the critical region, i.e., the region of coupling values where the absolute value of the magnetization changes most abruptly, we clearly see that for various block sizes the intermittency indices lie virtually on top of one another. This is in accordance with the linear behaviour found in the immediate vicinity of the pseudo-critical points for the various lattices. In order to obtain block-size independent estimates for the intermittency indices, we performed a χ^2 fit of the form $Y = a + bX$ to the data. In all fits we neglected the data for block size $L = 4$. The indices thus obtained, and divided by $(i-1)$, are listed in table 2 for lattice sizes $R = 64 - 1024$. The errors quoted here (and in all subsequent tables) correspond to a 95 percent confidence level. The intermittency indices for the SCPM formulation, obtained from the same fitting procedure, are listed in table 3; the values are very similar to those shown for the pure spin formulation in table 2.

It is clear that for increasing lattice size, the values for the indices are approaching those predicted by eq. (23) for the SCPM formalism. Indeed, by making a linear fit to the indices λ_i in tables 2 and 3, we obtain directly an estimate for the factor $(d - d_f)$ for each lattice size R . The resulting values for the fractal dimension are listed in table 4. The agreement with the analytically obtained value [13], namely $d_f = 187/96 \simeq 1.948$, is remarkable.

6. Conclusions

Our results provide for the first time unambiguous numerical evidence for intermittency in the two-dimensional Ising model. Using the block definition proposed by Novak [5], we find that the moments in terms of the block size show the predicted power-law behaviour for lattices from size 1024^2 down to 64^2 . For this conclusion, it is essential to use a $Z(2)$ -symmetric definition of the block spin; the use of a block variable violating the intrinsic symmetry of the system leads to strong fluctuations, but not to intermittency.

We have determined the intermittency indices λ_i of the system; they show only weak finite-size remnants of an order dependence and extrapolate well to a common value d_f for the fractal dimension of the system. This value in turn shows only very little lattice size dependence. The largest lattice, 1024^2 , gives us $d_f = 1.942 \pm 0.002$; this value is in excellent agreement with the fractal dimension $d_f = 1.948$ for the site-correlated percolation transition, as suggested in a note by Peschanski [10].

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Figure Captions

- Fig. 1 Factorial and standard moments of order $l = 2(\circ)$, $3(\Delta)$, $4(\Diamond)$ and $l = 5(\nabla)$, on a 64^2 lattice at $K = 0.434$, using $Z(2)$ -symmetric block moments. The solid symbols show the standard moments.
- Fig. 2 As in fig. 1, but for $R = 256$ at $K = 0.439$.
- Fig. 3 As before, but for $R = 1024$ at $K = 0.4403$.
- Fig. 4 Comparison of the factorial moments for block and global moments on a 128^2 lattice at $K = 0.437$. Symbols for the moments as in fig. 1, with solid ones denoting global moments.
- Fig. 5 Behaviour of the block moments at $K = 0.355$ on a 256^2 lattice, with symbols as in fig. 1.
- Fig. 6 As in fig. 5, but for $K = 0.446$.
- Fig. 7 Behaviour of $\ln \tilde{F}_3(L)$ for lattice sizes $R = 64(\circ)$, $128(\Delta)$ and $256(\nabla)$ at $K = 0.355$.
- Fig. 8 Behaviour of $\ln \tilde{F}_3(L)$ using the $Z(2)$ -breaking block moments for $R = 64$ at $K = 0.441$, with (∇) the moments calculated at overall positive magnetization, (Δ) those for overall negative magnetization and $(\circ)$ the average over magnetizations.
- Fig. 9 The behaviour of $\ln \tilde{F}_4(L)$ in the SCPM formalism, using block $(\circ)$ and global moments (Δ) , for $R = 128$ at $K = 0.437$.
- Fig. 10 Intermittency index λ_4 , calculated from block moments for $R = 128$, $L = 8(\circ)$, $16(\Delta)$ and $32(\nabla)$, as function of the coupling K .

Table 1: Lattice size dependence of the critical coupling K_c^R .

R	K_c^R
∞	0.4407
1024	0.4403
512	0.4398
256	0.4390
128	0.4370
64	0.4340

Table 2: Intermittency Indices $\lambda_i/(i-1)$ ($i = 2, 3, 4, 5$) obtained from linear fits to the data for $R = 64 - 1024$ at K_c^R , using $Z(2)$ -symmetric block moments.

i	$R = 64$	$R = 128$	$R = 256$	$R = 512$	$R = 1024$
2	0.061 ± 0.001	0.060 ± 0.001	0.053 ± 0.001	0.050 ± 0.001	0.045 ± 0.001
3	0.069 ± 0.002	0.067 ± 0.001	0.060 ± 0.001	0.057 ± 0.002	0.051 ± 0.002
4	0.072 ± 0.003	0.070 ± 0.002	0.063 ± 0.002	0.059 ± 0.003	0.054 ± 0.002
5	0.074 ± 0.005	0.072 ± 0.003	0.065 ± 0.003	0.061 ± 0.004	0.056 ± 0.003

Table 3: As in table 2, but for the SCPM formalism, with $R = 64$ and 128 at K_c^R .

i	$R = 64$	$R = 128$
2	0.066 ± 0.001	0.065 ± 0.001
3	0.071 ± 0.002	0.069 ± 0.002
4	0.073 ± 0.003	0.070 ± 0.003
5	0.073 ± 0.004	0.070 ± 0.004

Table 4: Fractal dimension obtained from a linear fit to the intermittency indices listed in table 2.

R	d_f
64	1.923 ± 0.001
128	1.924 ± 0.001
256	1.932 ± 0.002
512	1.936 ± 0.002
1024	1.942 ± 0.002

Figure 1

Figure 3

Figure 2

Figure 3

Figure 4

Figure 6

Figure 7

Figure 9

Figure 8

Figure 10