CERN-TH-5961/90
December 1990

Complete Standard Model predictions for the muon forward-backward asymmetry at LEP

STANISŁAW JADACH^{*}

CERN, Theory Division, 1211 Geneva 23, Switzerland,

B. F. L. WARD

Department of Physics and Astronomy,

The University of Tennessee, Knoxville, Tennessee 37996-1200, USA, and

SLAC, Stanford University, Stanford, California 94309 USA,

ZBIGNIEW WĄS

Institute of Nuclear Physics, ul. Kawory 26a, PL-30055 Kraków, Poland, and

CERN, PPE Division, 1211 Geneva 23, Switzerland,

ABSTRACT

We discuss predictions of the Standard Model for the forward-backward asymmetry of muons at LEP. Our aim is to obtain as realistic predictions of the Standard Model as possible. We discuss the asymmetry as a function of centre-of-mass energy and of the Z, t-quark and Higgs masses. We include in our analysis all relevant effects, that is, electroweak corrections, QED corrections and QCD/hadronic corrections as well as effects of the typical *experimental* cuts. In our analysis we use the combined analytical and Monte Carlo calculation.

To appear in Phys. Lett. B

CERN-TH-5961/90
December 1990

^{*} On leave of absence from Institute of Physics, Jagellonian University, Kraków, ul. Reymonta 4, Poland

1. Introduction

The precision tests of the Standard Model of the LEP machine [1], were always considered as one of the main purposes [2]. The principle of such tests is very simple; one has to calculate a certain observable such as for example the forward-backward asymmetry of muons or polarization of τ within the framework of the Standard Model and then confront it with the measured quantity.

In practice, such comparison requires simultaneous treatment of all kinds of theoretical and experimental corrections and the complications increase rapidly with the precision. Let us concentrate, in the following, on the forward-backward asymmetry for muons, one of the simplest tests of the Standard Model. We can classify the corrections into several groups:

1. Experimental biases, cut-off effects, background etc.
2. Hadronic and pure weak corrections.
3. QED bremsstrahlung corrections.

Each type of the corrections has its specific properties and requires a different treatment. The genuine pure weak effects are usually the main target of the measurement, and they should be treated in a different way from the QED corrections. In principle, it does not make sense to talk about QED and remaining electroweak corrections separately. The separation between QED and the remaining electroweak corrections breaks down in the non-leading (non-logarithmic) terms of the $\mathcal{O}(\alpha^2)$. It should be stressed that close to the Z resonance the pure $(\frac{\alpha}{s})^2$ corrections can be *consistently* calculated only in the complete electroweak theory. Fortunately those corrections are expected to be very small $\sim 10^{-5}$. Thus, they are not of interest for the phenomenology of LEP experiments. In practice, one can divide electroweak corrections into two classes: QED corrections and the remaining electroweak corrections. Because QED bremsstrahlung corrections depend on the experimental cuts, the Monte Carlo approach to these corrections is definitely the most appropriate one. On the other hand, analytical calculations of the theoretical predictions are often easier to handle because they are not prone to the statistical error of the Monte Carlo method. This uncertainty can become especially awkward if one wants to study a rather mild dependence of the observables on top and Higgs masses. On the other hand, Monte Carlo technique is *inescapable* if one wants to include realistic cuts in the discussion.

In this paper we will present the strategy for handling theoretical predictions for the muon forward-backward asymmetry developed in [3,4]. The idea of this approach consists of treating all pure weak corrections together with the leading QED corrections with the help of the semi-analytical program. The remaining QED corrections will be calculated using the Monte Carlo method. They depend on the experimental cut-offs. This paper should be considered as a practical application of the strategy proposed in [3,4] to LEP experiments.

In chapter 2 we will describe the physical content of the Monte Carlo generator KORALZ [5] and semi-analytical program CALASY [6]. In chapter 3 we will discuss the numerical results obtained with the help of the CALASY program. In chapter 4 we will turn to the question of the effects of residual cut-off dependent QED corrections, calculated with the help of the Monte Carlo program KORALZ. Chapter 5 summarizes the paper.

2. KORALZ and CALASY programs

Before going into discussion of the numerical results, let us recall here the main properties of the KORALZ and CALASY programs. The KORALZ is a Monte Carlo program capable of generating any pair of elementary fermions of the Standard Model (except electrons and t quarks) at LEP energies. It is described elsewhere in detail [5]. The biggest QED correction is the initial state bremsstrahlung which is generated according to the multi-bremsstrahlung generator YFS2 [7], which implements $\mathcal{O}(\alpha^2)$ exponentiated QED. Strictly speaking, the present version included in KORALZ includes a QED matrix element with corrections of order $(\frac{\alpha}{s})$, $(\frac{\alpha}{s}) \log \frac{s}{m_Z^2}$, $(\frac{\alpha}{s})^2 \log^2 \frac{s}{m_Z^2}$. This ensures precision sufficient for LEP 1. Indeed, in [8] it has been shown that the difference between the exact second order double bremsstrahlung result and the leading log expression used to construct the two hard photon residual β_2 in [7] is less than 10% throughout the relevant phase space; this means that the attendant error in the cross section is well below 0.1%, for example. To improve further on the precision one should add simultaneously corrections of order $(\frac{\alpha}{s})^2 \log \frac{s}{m_Z^2}$, $(\frac{\alpha}{s})^3 \log^3 \frac{s}{m_Z^2}$ which are of the same size, see ref. [10] for discussion. In addition, since the program is based on the work of Yennie-Frautschi-Suura [11], the QED matrix element in the program is valid to infinite order for soft real and virtual photons (very important close to Z resonance). At our precision level, the prediction of light real fermion pairs is negligible [9] and is not included in the Monte Carlo.

The final state bremsstrahlung corrections are included at the $\mathcal{O}(\alpha)$ (no exponentiation). These are combined with the initial state bremsstrahlung in a rather simple way [12]. Initial and final state bremsstrahlung are treated separately. The interference, correction thus cannot be included. It is well known, however, that the short non-zero Z lifetime allows us to neglect (for mild cut-offs) initial/final state bremsstrahlung interference [4,13]. The overall uncertainty of the QED corrections for A_{FB} embodied in the main mode of operation of KORALZ is smaller

* Most of the corrections of order $(\frac{\alpha}{s})^2 \log \frac{s}{m_Z^2}$ are already in the program (two virtual photons, two hard photons). The correction with one real and one virtual photon is partially missing. The exact formula is available in the literature only in the averaged form [9] and not in the full differential form, as required for the exclusive Monte-Carlo.

than 0.1% in the region of the Z resonance. The initial/final state interference correction is therein neglected. It has to be calculated separately, e.g. with the help of KORALZ running in the single bremsstrahlung mode.

CALASY is the semi-analytical program which calculates basic observables at LEP, such as the total cross section or forward-backward asymmetries for τ or μ leptons. It is also capable of calculating basic asymmetries and distributions for the τ decay products. The main aim of this program is to provide a fast prediction for the basic observables with high precision. In ref. [3] it was carefully discussed what are the differences between results for the forward-backward asymmetry obtained from KORALZ and CALASY.

Both CALASY and KORALZ can be run with *exactly* the same library of electroweak corrections. In the results presented below, we will use the electroweak library of W. Hollik [14]. This program was recently carefully tested and compared with other calculations, see refs. [15,16]. We have reproduced those results. We have also obtained similar results when other electroweak libraries [17,18] were used instead of W. Hollik's program. All results agree sufficiently well for the analysis of 1990 data. The discussion of the differences is out of the scope of this paper. In particular the discussion of the interplay between QCD effects [19,20,21] and electroweak corrections is not covered in this paper; it can be found, e.g. in [22,23].

It should be noted that in our treatment we separate as strongly as possible the QED corrections from the rest. The first ones (QED) we assume to be known and calculable. The latter exhibit properties of the Standard Model we want to measure. As the aim of the LEP community is to *verify* the Standard Model, one will not necessarily assume the validity of its predictions. The method, where data analysis is performed as independently of properties of the SM as possible, has to be clearly defined. The power of the formalism developed in [3] consists of the possibility of using a *simple* formula involving the convolution of the Born (with respect to bremsstrahlung corrections) cross section with the (observable independent) bremsstrahlung distribution. The remaining part of the QED corrections is, however, very carefully analyzed and can be easily included in consideration whenever it is big enough. The corresponding systematic error (QED) is known.

3. A_{FB} as a function of Standard Model parameters

In the following we use the CALASY program to discuss dependence of the forward-backward asymmetry on the top mass, Higgs mass and Z mass as well as the $\sqrt{s}$. Because of the rather mild dependence of the forward-backward asymmetry we have chosen for the Higgs mass only two values, 20 and 500 GeV. On the other hand, for the top mass we have chosen points every 50 GeV, from 50 to 200 GeV. The Z mass of 91.1 GeV, (90.9 and 91.3 GeV) was chosen on the basis of its present experimental value. The centre-of-mass energies follow the choice of the 1990 run of the LEP machine.

In the numerical integration of the CALASY program it is possible to introduce only one simple cut on the invariant mass of the pair of final state muons. We will always require it to be bigger than 3 GeV, independently of the particular choice of cuts for KORALZ, see the later discussion in section 4. This cut is less restrictive than any of the more realistic cuts discussed in the next Section. This is the reason why we apply the same cut to set the limit on the invariant mass of the muon pair in the KORALZ generation. In the following tables we collect numerical values of A_{FB} (in %) for different values of M_Z , M_H , M_{top} and $\sqrt{s}$.

Table 1a, A_{FB} in per cent units, for $m_{Higgs} = 20GeV$ calculated from the GALASY program.

M_Z	$\sqrt{s} - M_Z$	$m_{top} = 50$	$m_{top} = 100$	$m_{top} = 150$	$m_{top} = 200$
	-0.80	-8.12	-7.97	-7.73	-7.39
	-0.05	-1.14	-1.02	-0.81	-0.52
90.9	0.20	0.96	1.08	1.27	1.55
	0.45	2.90	3.02	3.20	3.46
	1.20	7.76	7.87	8.03	8.27
	-0.80	-7.85	-7.69	-7.44	-7.07
	-0.05	-0.93	-0.80	-0.58	-0.26
91.1	0.20	1.15	1.28	1.49	1.79
	0.45	3.08	3.21	3.40	3.69
	1.20	7.91	8.03	8.21	8.46
	-0.80	-7.57	-7.40	-7.13	-6.74
	-0.05	-0.71	-0.56	-0.33	0.01
91.3	0.20	1.36	1.50	1.72	2.04
	0.45	3.27	3.41	3.62	3.93
	1.20	8.07	8.21	8.40	8.68

Table 1b, A_{FB} in per cent units, for $m_{Higgs} = 500GeV$ calculated from the GALASY program.

M_Z	$\sqrt{s} - M_Z$	$m_{top} = 50$	$m_{top} = 100$	$m_{top} = 150$	$m_{top} = 200$
	-0.80	-8.32	-8.18	-7.97	-7.64
	-0.05	-1.33	-1.22	-1.03	-0.76
90.9	0.20	0.77	0.88	1.05	1.31
	0.45	2.72	2.82	2.98	3.23
	1.20	7.58	7.68	7.82	8.04
	-0.80	-8.07	-7.93	-7.69	-7.34
	-0.05	-1.14	-1.02	-0.81	-0.52
91.1	0.20	0.94	1.06	1.25	1.53
	0.45	2.87	2.99	3.17	3.44
	1.20	7.71	7.82	7.98	8.22
	-0.80	-7.81	-7.65	-7.40	-7.03
	-0.05	-0.94	-0.80	-0.58	-0.26
91.3	0.20	1.13	1.26	1.47	1.77
	0.45	3.05	3.18	3.38	3.66
	1.20	7.86	7.98	8.16	8.42

4. Corrections with respect to the CALASY approximation

The results presented in the previous section contain all electroweak and the bulk of (QED) bremsstrahlung corrections in the convolution approximation of CALASY. These results are not complete and the remaining QED corrections (cut-off dependent) have to be separately included, as in ref [3]. They can be separated into two parts: (i) initial/final state bremsstrahlung corrections beyond convolution, (ii) initial/final state bremsstrahlung interference. In the following, we will calculate these corrections with the help of the KORALZ program for several types of experimental cuts*. Having in mind that the $\cos\theta$ distribution of muons is very close to the $1 + \cos\theta^2 + \eta \cos\theta$ shape [3], we will multiply effectively the prediction for A_{FB} of the CALASY program, by the following correction factor

$$F = \frac{4 \cos^2 \theta_{\max}}{3 \cos \theta_{\max} + \cos^3 \theta_{\max}}$$

before subtracting it from the results obtained from the Monte Carlo. The angle $\theta_{\max}$ defines the limit of the acceptance region. Like in Table 1 all values are given in %.

Table 2 The difference for A_{FB} (per cent units) between KORALZ and CALASY results, due to initial and final state bremsstrahlung.

$\sqrt{s} - M_Z$	C_1	C_2	C_3	C_4
-0.80	0.30 ± 0.03	0.37 ± 0.03	0.32 ± 0.03	0.57 ± 0.03
-0.05	0.08 ± 0.03	0.12 ± 0.03	0.11 ± 0.03	0.22 ± 0.03
0.2	0.04 ± 0.03	0.08 ± 0.03	0.09 ± 0.03	0.18 ± 0.03
0.45	0.01 ± 0.03	0.06 ± 0.03	0.06 ± 0.03	0.19 ± 0.03
1.20	-0.01 ± 0.03	0.02 ± 0.03	0.05 ± 0.03	0.10 ± 0.03

* To minimize statistical fluctuations, we use the same method as described in ref. [3]. We generate simultaneously with the KORALZ events, the distribution exactly the same as given by the CALASY program. The applied method allows us to reduce statistical fluctuations to $0.02 - 0.1\%$ even though typically only $1 \cdot 10^6$ events are generated per entry in the table.

The following sets of cuts were used

- C_1 : Harder of muons at least 35GeV , softer of muons at least 22GeV , acollinearity $< 15^\circ$, acceptance region for muons $26^\circ < \theta < 154^\circ$.
- C_2 : Each of muons at least 20GeV , acollinearity 10° , acceptance region for muons $45^\circ < \theta < 135^\circ$.
- C_3 : Each of muons at least 2GeV , acollinearity 10° . Acceptance for muons $|\cos\theta| < 0.8$. Visible energy $E_{\text{vis}} > 0.1\sqrt{s}$, photon can be detected if $E_\gamma > 0.2\text{GeV}$, $|\cos\theta_\gamma| < 0.95$
- C_4 : Each of muons at least 5GeV , acollinearity 5° . Acceptance for muons $|\cos\theta| < 0.96$.

It is interesting to notice that the corrections collected in Table 2, even though small, are actually bigger than the corrections of initial and final state bremsstrahlung calculated separately. The bulk of the effect originates from the $\mathcal{O}(\alpha^2)$ leading-log corrections, more precisely terms proportional to $(\frac{q}{m_l})^2 \log \frac{s}{m_l^2} \cdot \log \frac{s}{m_l^2}$. That is why, contrary to the case when initial state bremsstrahlung corrections are treated alone, it does not help too much to tune the cut-off on the invariant mass of the muon pair in CALASY to the cuts.

Later on, with the same sets of cuts, using this time the single bremsstrahlung mode of the program, we have calculated effects due to QED initial/final state interference. To this end we have run KORALZ Monte Carlo in the single bremsstrahlung option and interference correction switched off (but corresponding interference weight calculated). In the following table we present QED initial/final state interference corrections calculated as a difference between cross sections calculated with and without interference weight. For the minimal energy of the generated real photons we have used $k_0 = E_{\text{min}}^T / E_{\text{beam}} = 0.002$. Due to properties of our method of generation (we are treating properly events with negative weights) the k_0 bias is practically excluded. We have checked that the results for $k_0 = 0.004$ and $k_0 = 0.002$ agree within our statistical error.

Table 3 Initial-final state interference correction to A_{FB} from KORALZ in single photon emission mode (in per cent units).

$\sqrt{s} - M_Z$	C_1	C_2	C_3	C_4
-0.80	0.13 ± 0.03	0.13 ± 0.03	0.15 ± 0.03	0.35 ± 0.03
-0.05	0.07 ± 0.03	0.06 ± 0.03	0.06 ± 0.03	0.14 ± 0.03
0.2	0.04 ± 0.03	0.04 ± 0.03	0.05 ± 0.03	0.05 ± 0.03
0.45	0.01 ± 0.03	0.01 ± 0.03	0.01 ± 0.03	-0.02 ± 0.03
1.20	-0.01 ± 0.03	-0.01 ± 0.03	-0.02 ± 0.03	-0.02 ± 0.03

As we expect (keeping in mind results of refs. [3,4]), these corrections are in all cases below the statistical resolution of the presently available experimental samples ($3 \cdot 10^4$). In future, for the bigger samples one will have to add to the results presented in Tables 1a, 1b the corrections summarized in Tables 2 and 3.

5. Summary

In this paper we have thoroughly discussed predictions of the Standard Model for the forward-backward asymmetry of muons. We have reviewed the dependence of theoretical predictions on the parameters of the Standard Model and centre-of-mass energies. Discussion of uncertainties due to weak corrections was not included here but can be found elsewhere. Strong emphasis was put on the discussion of QED corrections. It should be noted that the approach developed in [3,4] is the first and, so far, the only approach to the forward-backward asymmetry performed in a complex way including discussion of the pure weak corrections and the QED theoretical systematic errors down to the per mille level. The latter papers [24,25] do not provide the strategy to handle the QED corrections (i.e. do not discuss how to calculate them or to calculate the systematic error due to their omission) specific to certain detector efficiencies which are detector dependent and cannot be absorbed into semi-realistic cuts. The present paper, to some extent validates, however, approaches of refs. [24,25] from that perspective.

The present paper does not include discussion of the qualities of the CALASY program for predicting the τ decay product spectra; this topic will be covered elsewhere.

We should note that the multiple photon effects in the μ and b asymmetries, as they interplay with detector cuts and fermion flavour, have been discussed in ref. [26]. In that work, the pure weak effects were not considered and the detailed analysis of the initial/final and final state radiative effects was not made. Thus, the present work represents the completion for the μ forward-backward asymmetry of the multiple photon analysis in [26]. Analogous results for the b case are under investigation.

Acknowledgements Useful discussions on electroweak corrections with W. Hollik, T. Riemann, M. Sachwitz and members of the ALEPH collaboration are acknowledged. One of the authors (ZW) acknowledges the warm hospitality of CERN-PPE Division, Geneva (ALEPH collaboration), and CPPM Marseille, where part of this work was done. Work supported in part by the Polish Government Grants CPBP.01.09, CPBP.01.03 and RRI.14.1. Work supported in part by the US DOE contracts DE-AS05-76ER03956 and DE-AC03-76SF00515.

REFERENCES

1. S. Glashow, Nucl. Phys. **22** (1961) 579; A. Salam, in Elementary Particle Theory, ed. H. Svartholm (Almqvist, Stockholm, 1968); S. Weinberg, Phys. Rev. Lett. **19** (1967) 1264.
2. F. Dydak et al., ECFR Workshop on LEP 200, CERN 87-08 (1987) Vol. 1, p. 157.
3. S. Jadach and Z. Wąs, Phys. Rev. **D41** (1990) 1425.
4. S. Jadach and Z. Wąs, Phys. Lett. **B219** (1989) 103.
5. S. Jadach, B. Ward and Z. Wąs, KORALZ program, CERN preprint, in preparation, to be submitted to Comp. Phys. Comm. (1991).
6. S. Jadach and Z. Wąs, CALASY program, unpublished, CERN preprint, in preparation, to be submitted to Comp. Phys. Comm. (1991) The approach to QED was developed in ref. [3]; a short description can be found in: "PHYSICS at LEP 1" eds G. Altarelli R. Kleiss and C. Verzegnassi, vol. 3, page 62, CERN, Geneva 1989.
7. S. Jadach and B. F. L. Ward, Comp. Phys. Comm. **56** (1990) 351.
8. S. Jadach et al., Phys. Rev. **D42** (1990) 2977.
9. F. A. Berends, W. L. Van Neerven and G. J. H. Burgers, Nucl. Phys. **B297** (1988) 429, Erratum. Ibid. **B304** 1988, 921.

10. S. Jadach, M. Skrzypek and B. F. L. Ward, "Is there a better way of exponentiating QED corrections?" CERN preprint (1990) CERN-TH 5858/90, Phys. Letters B. (1991) in press.
11. D. R. Yennie, S. C. Frautschi and H. Suura, Ann. Phys. (N.Y.) **13** (1961) 379.
12. F. Boillot and Z. Wąs, Z. Phys C**43** (1989) 109.
13. T. Riemann and Z. Wąs, Mod. Phys. Lett. **A4** (1989) 2487.
14. W. Hollik, Fortsch. der Phys. **38** (1990) 165.
15. See, for example, D. Y. Bardin, W. Hollik and T. Riemann, "Bhabha scattering with higher order weak loop corrections", MPI-PAE-PTh 32/90 (1990), Munich preprint.
16. G. Degrassi and A. Sirlin, "Comparative Analysis of Electroweak Corrections to $e^+e^- \rightarrow f\bar{f}$ in On-Shell and $\overline{MS}$ Frameworks", Munich preprint MPI-PAE/PTh 48/90 (1990).
17. R. Stuart, "Radiative corrections to $e^+e^- \rightarrow \mu^+\mu^-$ in the Glashow-Salam-Weinberg model", PhD Thesis, Oxford University, Preprint RAL-T-008 (1985) unpublished.
18. D. Y. Bardin et al., Comp. Phys. Comm. **59** (1990) 303.
19. F. Halzen and B. Kniehl, " Δr beyond one loop" Wisconsin University preprint (1990) MAD-PH-588.
20. B. Kniehl and J.H. Kühn, Nucl. Phys. **B329** (1990) 547; B. Kniehl, Comp. Phys. Comm. **58** (1990) 293.
21. A. Djouadi and C. Verzegnassi, Phys. Lett. **B195** (1985) 265, A. Djouadi, Nuovo Cim. **A100** (1988) 357.
22. T. Riemann, M. Sachwitz and Z. Wąs, "A comment on $O(\alpha_s)$ corrections and phenomenology of LEP experiments", CERN preprint (1990) CERN-TH 5964/90.
23. W. Hollik, private communication.
24. D.Y. Bardin et al. "Analytic approach to the complete set of QED corrections to fermion pair production in e^+e^- annihilation", Zeuthen preprint PHE-89-19 (1990).
25. W. Beenakker, F. Berends and S. van der Merck, "Higher order corrections to the forward-backward asymmetry", Leiden preprint 1990.
26. S. Jadach and B.F.L. Ward, Phys. Rev. **D42** (1990) 1404.