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**SPONTANEOUS VERSUS EXPLICIT BREAKING
OF A CONTINUOUS GLOBAL SYMMETRY**

M. Lusignoli

Dipartimento di Fisica, Università di Roma
"La Sapienza" and INFN, Sez. di Roma, Italy

A. Masiero

INFN, Sez. di Padova- Padova, Italy

*M. Roncadelli**

CERN-Geneva

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UNIVERSITÀ DEGLI STUDI DI PADOVA
DIPARTIMENTO DI FISICA
"GALILEO GALILEI"

ISTITUTO NAZIONALE DI FISICA NUCLEARE
SEZIONE DI PADOVA

Via Marzolo, 8 - 35131 PADOVA (ITALIA)

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M. Lusignoli

Dipartimento di Fisica, Università di Roma
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A. Masiero

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ABSTRACT

We consider theories with continuous global symmetries which are very slightly explicitly broken. We show that in the presence of a spontaneous violation of these symmetries, the corresponding pseudo-Goldstone bosons may acquire masses much larger than the size of the explicit breaking. This result contradicts naive expectations. We discuss the relevance of this observation for physically interesting global symmetries like lepton number.

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* Permanent address: Dipartimento di Fisica Nucleare e Teorica, Università di Pavia, Pavia, Italy

There is a general consensus in particle physics that only gauge symmetries can be exact symmetries in Nature. Still, realistic extensions of the standard model-proposed with different motivations- very often contain continuous global symmetries. Thus, one expects invariances of the latter kind to be broken in some way.

Generally speaking, spontaneous breaking has been favoured (at least on aesthetical grounds) owing to its less arbitrary look. Obviously, the Goldstone theorem then implies the existence of massless scalar particles in the physical spectrum. Actually, a few years ago this possibility attracted considerable interest and axions ^[1], majorons ^[2], ^[3] ^[4] and familons ^[5] came into the bestiary of hypothetical elementary particles, making both high energy phenomenology and cosmology even more exciting. Unfortunately, the only "visible" particles among these Goldstone bosons -the doublet and triplet majoron ^[3], ^[4]- have been ruled out by LEP ^[6]. Perhaps even the "invisible" ones will be ruled out in the future.

One can certainly conclude that the whole idea of physical Goldstone bosons is simply wrong. Yet it might be that things are less trivial and more interesting -the above idea might not be the whole story. As a matter of fact, anybody would nowadays agree that even these "small" extensions of the standard model should be viewed as the low-energy manifestation of new underlying physics at some scale Λ much larger than the Fermi scale $G_F^{-1/2}$. An important point is that there is no reason to believe that continuous global symmetries giving rise to physical Goldstone bosons at low energy should play any role in the new physics at the scale Λ . Stated in a different way, these global symmetries are expected to be explicitly broken at the scale Λ . Looking at the above extensions of the standard model from an effective-theory point of view, non-renormalizable terms which explicitly break the would-be spontaneously-broken global symmetries should then be present. Moreover, these symmetries are not automatic *, so renormalizable soft-breaking terms are also expected. In either case, the additional violating terms should -just by definition- disappear as $\Lambda \rightarrow \infty$, which means that their

* As usual, by automatic symmetry we mean any global symmetry which is present in the most general renormalizable lagrangian invariant under the gauge group.

coefficients have to go like some inverse power of Λ .

An obvious question concerns the fate of the corresponding pseudo Goldstone bosons. General arguments support the idea that their masses should be quite tiny, because the amount of explicit breaking due to the above terms is certainly very small. Furthermore, very light pseudo-Goldstone bosons look natural in this context, since a non-linearly realized symmetry is recovered as $\Lambda \rightarrow \infty$ [7].

Our aim is to show that this expectation may be false. Specifically, taking the triplet majoron model [8] as an example, we show that the majoron acquires a mass much larger than the size of the explicit lepton number violation. A few comments about the physical relevance of our result will be offered toward the end of this Letter.

Consider the $SU(2)_L \otimes U(1)_Y$ standard model with an enlarged Higgs sector, containing, besides the usual doublet $\varphi \sim (2, 1/2, 0)$, a triplet of complex scalar fields $\Delta \sim (3, 1, -2)$ carrying two units of lepton number. The physical charged fields in Δ are

$$\begin{aligned}\Delta^{++} &= \frac{1}{\sqrt{2}}(\Delta_1 - i\Delta_2), \\ \Delta^+ &= \Delta_3, \\ \Delta^0 &= \frac{1}{\sqrt{2}}(\Delta_1 + i\Delta_2)\end{aligned}\quad (1)$$

The most general renormalizable Higgs potential invariant under $SU(2)_L \otimes U(1)_Y$ and lepton number can be written as

$$\begin{aligned}V_0(\varphi, \Delta) &= a^2 \varphi^\dagger \varphi + b^2 \Delta^\dagger \Delta + \lambda (\varphi^\dagger \varphi)^2 + \nu (\Delta^\dagger \Delta)^2 + \zeta (\Delta^\dagger \Delta^*) (\Delta^\tau \Delta) + \\ &+ \rho (\varphi^\dagger \varphi) (\Delta^\dagger \Delta) + i \sigma \epsilon_{ijk} \varphi^\dagger \tau_i \varphi \Delta_j^\dagger \Delta_k\end{aligned}\quad (2)$$

(τ_i are the Pauli matrices). We assume that lepton number is explicitly broken by a trilinear term **, and so the whole scalar potential reads

$$V(\varphi, \Delta) = V_0(\varphi, \Delta) - (c \bar{\varphi}^\dagger \tau_i \varphi \Delta_i^* + h.c.) \quad (3)$$

** On general grounds, we should also consider non renormalizable terms of dimension five or more in the Higgs potential. Nevertheless, it can be shown that our general conclusions are unaffected by their presence.

($\tilde{\varphi} \equiv i\tau_2 \varphi^*$). This model has already been discussed in the literature [8], without any assumption about the size of the trilinear term. As usual, we parametrize the neutral scalar fields in terms of their VEVs v_D and v_T :

$$\begin{aligned}\varphi^0(x) &\equiv \frac{1}{\sqrt{2}}[v_D + \rho_D(x) + i\eta_D(x)], \\ \Delta^0(x) &\equiv \frac{1}{\sqrt{2}}[v_T + \rho_T(x) + i\eta_T(x)].\end{aligned}\quad (4)$$

Observe that v_D can be made real and positive by a suitable $SU(2)_L$ transformation, whereas the (global) phase factor multiplying Δ^0 can be reabsorbed in the (arbitrary) phase of the parameter c , so that v_T is real and positive as well. The extremum conditions can be written as

$$v_D [a^2 + \lambda v_D^2 + \frac{1}{2}(\rho - \sigma)v_T^2 - 2v_T \text{Re}c] = 0, \quad (5)$$

$$v_T [b^2 + \nu v_T^2 + \frac{1}{2}(\rho - \sigma)v_D^2] - v_D^2 \text{Re}c = 0, \quad (6)$$

$$v_D v_T \text{Im}c = 0, \quad (7)$$

$$v_D^2 \text{Im}c = 0 \quad (8)$$

Manifestly, in order to have a solution with non-vanishing v_D , c must be real. As can be seen from eq.(6), the explicit L -violating term forces v_T to be non-vanishing, thereby triggering also spontaneous lepton number breaking.

We now proceed to discuss the scalar spectrum, and in particular the conditions for the extremum to be a (local) minimum. The mass squared of the doubly-charged scalar is

$$M_{\Delta^{++}}^2 = \sigma v_D^2 + 2\zeta v_T^2 + c \frac{v_D^2}{v_T} \quad (9)$$

while the mass terms in the singly-charged sector are given by

$$(c + \frac{1}{2}\sigma v_T)(\varphi^+, \Delta^+) \begin{pmatrix} 2v_T & -\sqrt{2}v_D \\ -\sqrt{2}v_D & v_D^2/v_T \end{pmatrix} \begin{pmatrix} \varphi^- \\ \Delta^- \end{pmatrix}. \quad (10)$$

One of the eigenvalues is zero (corresponding to the Goldstone boson eaten up by $W^\pm$). The physical singly-charged Higgs has mass squared

$$M_{H^\pm}^2 = \left(\frac{\sigma}{2} + \frac{c}{v_T}\right)(v_D^2 + 2v_T^2), \quad (11)$$

implying

$$\frac{\sigma}{2} + \frac{c}{v_T} > 0. \quad (12)$$

Similarly, the mass terms in the pseudoscalar sector are

$$\frac{1}{2}c(\eta_D, \eta_T) \begin{pmatrix} 4v_T & -2v_D \\ -2v_D & v_D^2/v_T \end{pmatrix} \begin{pmatrix} \eta_D \\ \eta_T \end{pmatrix}. \quad (13)$$

Again, one eigenvalue is zero (corresponding to the Goldstone boson eaten up by Z), the other being

$$M_\eta^2 = \frac{c}{v_T}(v_D^2 + 4v_T^2). \quad (14)$$

Therefore

$$c > 0. \quad (15)$$

The η particle is the pseudo-Goldstone boson of our model - it becomes the majoron in the limit $c = 0$ with $v_T \neq 0$. As eq. (14) already shows, its mass can be large even if c is very small, provided v_T is also small. Finally, we write down the mass terms in the neutral scalar sector

$$\frac{1}{2}(\rho_D, \rho_T) \begin{pmatrix} 2\lambda v_D^2 & v_D[(\rho - \sigma)v_T - 2c] \\ v_D[(\rho - \sigma)v_T - 2c] & 2\nu v_T^2 + cv_D^2/v_T \end{pmatrix} \begin{pmatrix} \rho_D \\ \rho_T \end{pmatrix}. \quad (16)$$

Since λ, ν and c must be positive, for a large region of values in the parameter space the assumed extremum of $V(\varphi, \Delta)$ is indeed a (local) minimum.

It is well known that the successful standard model relation $M_W = M_Z \cos \theta_w$ implies $v_T < v_D$. This can be naturally obtained in the present model: as can be seen from eq. (6), we have

$$v_T \sim c \quad (17)$$

barring possible fine-tunings (to be discussed below). We recall that c is related to the explicit breaking of (global) lepton number, and so we take $c < G_F^{-1/2}$. Moreover, c should vanish as $\Lambda \rightarrow \infty^{***}$. Given eq. (17), we recognize eq. (5) as the usual extremum condition of the standard model, so that $v_D \simeq (-a^2/\lambda)^{1/2} \simeq 246 \text{ GeV}$.

Our main result now follows by combining eqs. (14) and (17): the pseudo-majoron has mass $0(v_D)$ for arbitrary small explicit breaking size c ! Moreover, as eqs. (9), (10) and (16) show, all other Higgs scalars have masses $0(v_D)$.

One might also like to consider the effect of a tiny explicit L -violation starting from the original triplet majoron model ^[9]. In that case $c = 0$, and in order to have e.g. $v_T \simeq 1 \text{ KeV}$ the following fine-tuning had to be imposed:

$$\frac{1}{\nu}|\rho^2 + \frac{1}{2}(\rho - \sigma)v_D^2| \equiv V^{(0)2} \simeq -1 \text{ KeV}^2 \quad (18)$$

Correspondingly, eq. (6) would now read

$$v_T(v_T^2 - V^{(0)2}) = c \frac{v_D^2}{\nu} \quad (19)$$

Even assuming $c/\nu = v_D^2/M_{PLANCK} \simeq 10^{-8} \text{ KeV}$, the r.h.s. of eq. (19) is very large, namely $0((10^3 \text{ KeV})^3)$, implying that eq. (19) has only one real solution, $v_T \simeq 10^3 \text{ KeV}$. Now, the pseudo-majoron mass is

$$M_\eta \simeq \sqrt{\nu}v_T = 1 \text{ MeV} \quad (20)$$

Also in this case, we see that by starting with an extremely small explicit symmetry breaking ($c \simeq 10^{-5} \text{ eV}$), we end up with a considerable mass for the pseudo-Goldstone boson. Furthermore, in spite of the drastic increase of v_T due to the fine-tuning (18) - as compared to the "natural solution" (17) - the η mass is still larger than the explicit symmetry breaking parameter c by eleven orders of magnitude!

We briefly discuss a few phenomenological implications. Considering first the no fine-tuning case (eq. (17)) we find a scalar mass spectrum similar to the one previously discussed in ref. ^[8]. Notice that all Higgs bosons are naturally heavy

*** As a simple choice, we will assume $c \simeq v_D^2/\Lambda$

enough not to contribute to the Z width. A possible interesting consequence arises from the assumed smallness of the parameter c , and hence v_T : neutrino masses are predicted to be naturally small and may be in the range for the MSW ^[9] solution to the solar neutrino problem †. On the contrary, the fine-tuning option (eqs.(18), (19) and (20)) is phenomenologically unacceptable for the same reason that excludes the doublet and triplet majoron models ^{[3], [4]}. A more detailed analysis will be presented elsewhere.

The main conclusion of our investigation, namely that in a theory with a very small explicit violation of a global symmetry the pseudo-Goldstone masses can be much larger than the size of the breaking, may look rather surprising. Thus, we find it illuminating to briefly reconsider the whole issue from an alternative point of view. Suppose we start with exact L -conservation in the presence of the Higgs triplet Δ : this situation is realized for $c = 0$ and, e.g., b^2 large and positive (see eq. (6)). Now $v_T = c = 0$, and so the only mass scale is provided by the doublet VEV v_D . One can easily see that all scalars have mass $O(v_D)$ with the obvious exception of the Goldstone bosons eaten up by $W^\pm$ and Z . Introducing next an explicit L -violation in the Higgs potential by a tiny c (see eq.(3)), its presence drives v_T to small non-zero values. Nevertheless, the smallness of c can only slightly modify the original scalar mass spectrum. Hence, there is no reason why the pseudo-majoron should have a small mass! This somehow trivial observation merely shows that a theory with an exact global symmetry actually behaves in the way one expects if the symmetry is explicitly broken by a small amount. This is not the case if one starts from a spontaneously broken symmetry. Indeed, by adding to the lagrangian a tiny explicit breaking it may well happen that the corresponding pseudo-Goldstone bosons get a mass that is several orders of magnitude larger than the mass scale of the explicit violating term.

Almost thirty years ago Jona-Lasinio and Nambu ^[12] noted that, in their dynamical model of elementary particles based on an analogy with superconductivity, a small breaking of chiral symmetry - in the form of a bare nucleon mass of few MeV - could give the pion its physical mass, while all the other parameters

† A similar result ^[10] was obtained by introducing non-renormalizable Yukawa terms ^[11] suppressed by a factor $1/M_{PLANCK}$.

were changed by tiny amounts. In the model that we have discussed in this letter the effect is considerably amplified by the presence of two very different mass scales, v_D and v_T , as can be seen from eq.(14).

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