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Anyon Superconductivity Beyond the Random Phase Approximation

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Abstract

We carry out a revision of the mechanism of anyon superconductivity found in the Random Phase Approximation. It is shown that the Debye screening of a statistical Coulomb-like interaction produced by contributions beyond the RPA destroys the abovementioned mechanism. The linear mode appeared in the particle-hole channel within the RPA is identified with a finite damping zero-sound. As an alternative mechanism an attraction of anyons which could lead to a formation of boson composites is observed. A finite temperature phase transition, hydrodynamics and electrodynamics of anyon system are described from this alternative point of view.

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1 Introduction

A theory of fractional statistics which can be realized in 2+1-dimensional space-time is now being intensively investigated. The first discussion of the principal possibility of fractional statistics was undertaken more than a decade ago [1], [2], [3]. A recent renewal of activity in this field was caused by a discovery of the High Temperature Superconductivity (HTSC) and numerous attempts to explain the phenomenon by means of a transmutation of statistics of electrons as a result of their strong interaction [4], [5], [6], [7]. For instance, it was suggested that charge degrees of freedom are described by semions - half-fermions [8], [9].

It was also conjectured that a relevance of fractional statistics to HTSC could be more implicit. It was first stated by Fetter, Hanna and Laughlin [10], [11], that a gas of particles obeying fractional statistics (anyons) has a superconducting ground state for a special values of a statistical parameter $\delta = \pi(1 - 1/N)$. The authors of papers [10], [11] showed that a correlation function of electromagnetic currents $K_{\mu\nu}(\omega, \vec{k}) = \langle J_\mu(\omega, \vec{k}) J_\nu(-\omega, -\vec{k}) \rangle$ found within the Random Phase Approximation (RPA) behaves like that of a superconductor at zero temperature. Namely, it was found that the spatially transverse component $K_1(\omega, \vec{k})$ approaches a non-zero constant at small ω and $\vec{k}$: $K_1(\omega = \vec{k} = 0) = 1/\lambda^2$. It is nothing but a manifestation of the Meissner effect. This result was rededuced in the RPA in a number of subsequent papers [12], [13], [14]. Additionally, in the same approximation it was shown that a superconducting behaviour of the current-current correlator $K_{\mu\nu}$ ceases to hold at any finite temperature and no phase transition could take place [15], [16]. Should this be so, anyon superconductivity should be considered as some anomaly at zero temperature which has nothing in common with HTSC. Later the statement of papers [15], [16] was specified in the form that although there is no phase transition in the mathematically rigorous sense, nevertheless there exists a temperature T_c^* above which an arbitrary small external magnetic field B_{ext} penetrates the material [17]. At smaller temperatures an external magnetic field should exceed some critical value H_c^* to penetrate homogeneously inside a bulk sample.

As was stated in a set of papers the validity of RPA remains an open question. In paper [18] it was shown that a homogeneous background may be far from the correct ground state (this was also suggested in [19]) in the

case of a large number of filled Landau levels. In our recent paper [20] we demonstrated that in a self-consistent approach, when you renormalize the mean field, the naive RPA results may be wrong. In the present paper we show that a consideration based on the RPA is not complete and advanced calculations beyond this approximation provide new alternative results.

The organization of the paper is as follows. In Section 2 we consider anyons as usual particles interacting via statistical gauge field. Then we discuss the RPA and demonstrate that higher order loop corrections violate this approximation, smearing the mean field Landau levels. In Section 3 we discuss the electromagnetic properties of the system beyond the RPA. The new mechanism for anyon superconductivity is suggested in Section 4. In Section 5 we discuss the hydro- and electrodynamics in terms of the new order parameter as well as the phase transitions in the anyon system and the possible connection between anyons and 2d Conformal Field Theories (CFT). Section 6 summarizes basic conclusions and outlines a program for future investigations.

2 Within and beyond the RPA.

An appropriate framework for studying anyons is given by a 2+1-dimensional topologically massive electrodynamics [21], [22], [23]. We shall consider fermions as non-relativistic particles with a given density ρ . Here it is worth noticing that namely a non-relativistic case arises in the majority of condensed matter problems.

The Lagrangian of the system has the form:

$$L = \psi^\dagger (i\partial_0 - a_0 - eA_0 - \mu) \psi + \frac{1}{2m} \psi^\dagger (\vec{\partial} + i\vec{a} + ie\vec{A})^2 \psi + \frac{\theta}{4\pi^2} \epsilon_{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + e\rho A_0, \quad (1)$$

where ψ describes spinless non-relativistic fermions with a chemical potential μ and an electric charge e . The dynamics of an Abelian gauge field a_μ responsible for the "statistical" interaction is governed by the Chern-Simons term. Another Abelian field A_μ corresponds to the external electromagnetic potential, and the last term describes the electrical neutrality of the system. The Lagrangian (1) admits a natural regularization by means of an addition

of a higher order term $-1/4\gamma(\partial_\mu a_\nu - \partial_\nu a_\mu)^2 = -1/4\gamma f_{\mu\nu}^2$, which makes a statistical field to be the topologically massive gauge field, but the low-energy properties are the same as in the pure Chern-Simons theory. Such a regularization allows us to produce physically sensible results if any problems arise in the original theory.

The Lagrangian (1) can be considered as the field-theoretical realization of anyons in the sense that in the limit of zero fermion density isolated fermions behave like anyons with a statistical parameter $\delta = \pi(1 - \pi/\theta)$. This fact was first suggested in papers [24], [25] and based on the Aharonov-Bohm interaction of the charged particles in the low-energy limit of the topologically massive gauge theory found in classical papers [22], [23]. This fact can be also readily obtained from a calculation of an expectation value of a product of Wilson lines $W(C_i) = \exp(i \oint_{C_i} a_\mu dx^\mu)$ taken along worldlines $[C_i]_{i=1, \dots, n}$ of a set of distinct sources [26]:

$$\langle W(C_1) \cdots W(C_n) \rangle = \exp\left(\frac{i\pi^2}{\theta} \sum_{ij} \Gamma_{ij}\right), \quad (2)$$

where for, $i \neq j$, Γ_{ij} is a linking number (Gauss invariant) of a pair of contours C_i, C_j . The very notion of a statistics can be introduced on the basis of braiding properties of the world line, which may be non-trivial only for planar particles [1], [2], [3]. From this point of view we are dealing with an Abelian representation of a braid group with a character $\exp(i\delta)$. The notion of anyon spin could be connected with diagonal terms $i = j$ on the right-hand side exponent in (2). The numbers Γ_{ii} , which are not topological invariants, can be determined as wristling numbers of contours C_i and this interpretation leads to the natural appearance of induced spin [27] (see also [28]). However, this can be done unambiguously only for a particular value $\theta = \pi$; this value corresponds to Bose-Fermi transmutation. Only in this case is there some progress towards a strictly local theory with short range interactions [29]. Let us note that the non-Abelian representations of the braid group and, connected with them, new link invariants (Jones and HOMFLY polynomials) can be obtained from the non-Abelian Chern-Simons theory [30] which leads to a rather nontrivial connection with 1+1 Conformal Field Theory (CFT).

However, these definitions of spin and statistics are essentially "classical" and, in principal, could be subject to renormalization. The only relevant distance scale where some kind of crossover could take place is expected to be

of the order of a magnetic length $\sqrt{\theta/\rho}$. As the simplest example of the non-trivial scale dependence of a statistical parameter $\delta(r)$, one can consider the theory (1) with Maxwell term $-1/4\gamma f_{\mu\nu}^2$, i.e. the topologically massive gauge theory (TMGT). In that case braiding properties of Wilson lines preserve only for contours separated everywhere by distances larger than $(\gamma\theta/2\pi^2)^{-1}$. Otherwise, their linking doesn't result in the appearance of any non-trivial phase factor [22].

We would like to underline the following important point. It seems, at first glance, that if a condition $\gamma\theta \gg \sqrt{\rho}$ is satisfied, then one could readily treat the Lagrangian (1) without any additional Maxwell term. Naively, one could hope to use a sort of Born-Oppenheimer approximation and to consider any configuration of the gauge field as completely determined by the positions and momenta of fermions. The "dynamical" gauge degrees of freedom (topologically massive statistical "photons") have gap $\theta\gamma/2\pi^2$. It is the statement which is violated by the radiative corrections. It happens that, in spite of its bare large value, the charge γ acquires the natural order of $\rho/m\theta^2$ and no Born-Oppenheimer approximation is applicable. Consequently, the statistical gauge field fluctuations become important and modify significantly the mean field results obtain within the RPA.

To begin with the renormalization of the Lagrangian (1) we shall recall the results of the RPA calculations. A variation of the Lagrangian over a_0 gives a constraint (let us note that the two-dimensional $\text{rot} \vec{a}$ is a scalar)

$$\frac{\theta}{2\pi^2} \text{rot} \vec{a} - \psi^+ \psi = 0 \quad (3)$$

It means that the total flux of the magnetic statistical field is proportional to the number of fermions with the coefficient $2\pi^2/\theta$. To study the Lagrangian (1) one should divide the field a_μ into a background part corresponding to a homogeneous magnetic field $B = 2\pi^2\rho/\theta$ and fluctuations around it, which we shall denote by the same symbol a_μ . Then, rewriting the Lagrangian in terms of these fluctuations and using the spatially transverse (Coulomb) gauge $\partial_i a_i = 0$, we get

$$L = \psi^+ (i\partial_0 - a_0 - eA_0 - \mu)\psi + \rho(eA_0 + a_0) + \frac{1}{2m}\psi^+ \vec{\nabla}^2 \psi - \bar{J}(\vec{a}_\perp + e\vec{A}_\perp) - \frac{1}{2m}(\vec{a}_\perp + e\vec{A}_\perp)^2 (\psi^+ \psi - \rho) + \quad (4)$$

$$+ \frac{\theta}{2\pi^2} a_0 r o i \vec{a} - \frac{\rho}{2m} (\vec{a}_\perp + e \vec{A}_\perp)^2$$

where the time derivative term $\epsilon_{ij} a_i \dot{a}_j$ identically equals zero for transverse fluctuations $\vec{a}_\perp$, current $J_i = 1/2im(\psi^\dagger \nabla_i \psi - \nabla_i \psi^\dagger \psi)$, $i = 1, 2$ and ∇_i are covariant derivatives obeying the commutation relation $[\nabla_x, \nabla_y] = -iB$. It can be readily seen from the last two terms in (5) that a bare statistical field propagator is given by the formula

$$D_{\mu\nu}^0 = -i \langle a_\mu(\vec{q}, \omega) a_\nu(-\vec{q}, -\omega) \rangle = \frac{(2\pi^2)^2}{\theta^2 q^2} \begin{pmatrix} \rho/m & \frac{\theta}{2\pi^2} \epsilon_{ij} q_j \\ -\frac{\theta}{2\pi^2} \epsilon_{ij} q_j & 0 \end{pmatrix} \quad (5)$$

It describes a static Coulomb repulsion between fermions as well as their instantaneous Aharonov-Bohm -type interaction.

To obtain an effective theory of two gauge fields A_μ and a_μ one should first integrate over fermions. An effective Lagrangian can be expanded over the sum $(A_\mu + a_\mu)$

$$L_{eff}(a_\mu, A_\mu) = \frac{\theta}{2\pi^2} a_0 r o i \vec{a} - \frac{\rho}{2m} (\vec{a}_\perp + e \vec{A}_\perp)^2 + \frac{1}{2} (A_\mu + a_\mu) \Pi_{\mu\nu} (A_\nu + a_\nu) + \dots, \quad (6)$$

where dots stand for the higher order terms. Of course, this expansion is sensible if fluctuations a_μ as well as an external field A_μ are small enough. In the non-relativistic case the polarization operator $\Pi_{\mu\nu}$ can be expanded over three independent tensor structures

$$\Pi_{\mu\nu}(\vec{q}, \omega) = (\eta_{\mu\nu} - \frac{q_\mu q_\nu}{\omega^2 - q^2}) \Pi_1(q, \omega) + \delta_{\mu i} \delta_{\nu j} (\delta_{ij} - \frac{q_i q_j}{q^2}) \Pi_2(q, \omega) + i \Pi_3(q, \omega) \epsilon_{\mu\nu\lambda} q^\lambda \quad (7)$$

In the transverse gauge it is convenient to deal with combinations

$$\Pi_{00} \equiv -\frac{q^2}{\omega^2 - q^2} \Pi_1, \quad \Pi_\perp \equiv \Pi_1 - \Pi_2 \quad (8)$$

instead of Π_1 and Π_2 .

The lowest order contribution to $\Pi_{\mu\nu}$ comes from a simple fermionic loop in an external field $B = 2\pi^2 \rho / \theta$ (see Fig.1). To simplify the consideration we shall take the coefficient θ to be an integer multiple of π : $\theta = \pi N$. Then in the mean field picture we have N completely filled Landau levels in the ground state and a finite gap $\omega_c = 2\pi \rho / Nm$ for one-particle excitations.

The diagram of Fig.1 was computed in a number of papers [12], [15], [16]. The result can be represented in the form

$$\begin{aligned} \Pi_\perp &= \epsilon(\omega, q) \omega^2 - \chi(\omega, q) q^2 - \frac{\rho}{m}, \\ \Pi_{00} &= \epsilon(\omega, q) q^2, \\ \Pi_3 &= \frac{1}{2} \sigma_{xy}(\omega, q). \end{aligned} \quad (9)$$

All the functions $\epsilon(\omega, q)$, $\chi(\omega, q)$ and σ_{xy} are analytic at small $q < q_H = \sqrt{2\pi \rho / N}$ and $\omega < \omega_c$. These functions have poles at $\omega = n\omega_c$, $n = \pm 1, 2, \dots$ and have the general form

$$\exp\left(-\frac{q^2}{2q_H^2}\right) \sum_{n=1}^{\infty} \frac{f_n(q/q_H)}{n^2 \omega_c^2 - \omega^2},$$

where $f_n(x)$ are polynomials of the order $(n-1)$. Limiting values of a dielectric constant $\epsilon(0, 0)$, a magnetic susceptibility $\chi(0, 0)$ and a Hall conductivity $\sigma_{xy}(0, 0)$ are given by the formulae [12], [15], [16]:

$$\epsilon = \frac{N^2 m}{2\pi^2 \rho}, \quad \chi = \frac{N^2}{\pi m}, \quad \sigma_{xy} = -\frac{N}{2\pi}. \quad (10)$$

The first corrections to these results are given by the diagrams depicted in Fig.2. These contributions are already beyond the standard RPA. Summing further all the corrections to the internal gauge field propagator one should use for wavy lines a renormalized expression

$$D_{\mu\nu}(\omega, \vec{q}) = \left(\frac{\Pi_\perp}{i(\Pi_3 - \frac{\theta}{4\pi^2}) \epsilon_{ij} q_j} - i(\Pi_3 - \frac{\theta}{4\pi^2}) \epsilon_{ij} q_j \right) \times \frac{1}{\Pi_{00}(\omega, q) \Pi_\perp(\omega, q) - (\Pi_3(\omega, q) - \frac{\theta}{4\pi^2})^2 q^2} \quad (11)$$

It would be the first step to a complete self-consistent formula for $D_{\mu\nu}$ to use the results of (9) and (10). Then (11) amounts to

$$D_{\mu\nu}(\omega, \vec{q}) = \begin{pmatrix} \frac{1}{\omega^2} & 0 \\ 0 & \frac{\delta_{\mu\nu}^T}{\omega^2 - \vec{q}^2} \end{pmatrix} \quad (12)$$

Let us note that an essential property of the cited results is an exact cancellation of the bare Chern-Simons term by another one produced after an integration over fermions. It was claimed by Banks and Lykken [31] that this cancellation is sufficient to provide a superconducting behaviour of the electromagnetic current-current correlation function. This statement was confirmed by the RPA results and is the basis for speculations about a principally new mechanism of superconductivity.

However, the RPA results cease to be self-consistent. Indeed, it follows from (12) that the cancellation of the Chern-Simons term leads to a long-range interaction between fermions. Moreover, being P and T even the effective interaction coincides with that of a pure 2 + 1 electrodynamics without the Chern-Simons term. It is well known, that at finite fermion density the latter theory suffers from severe infrared divergences [32]. Their origin (in two as well as in three dimensions) lies in an interaction mediated by transverse photons (current-current interaction) [33] (see also [34], [35] where infrared problems of 2+1 electrodynamics are discussed in the field-theoretical aspects). Since a long wavelength behaviour of the effective gauge field propagator (12) resembles that of the pure electrodynamics, one could expect that something similar could take place. On the other hand, it was recently conjectured [36] that a spontaneous violation of parity and time reversal symmetry as a result of a dynamical generation of a magnetic flux could be a cure saving a pure electrodynamics from singularities (which was one of the reasons for the introduction of a topological mass term a decade ago in field theory [22], [35], [23]). In our paper [20] we demonstrated that the abovementioned cancellation of the bare CS term is unstable, i.e. if renormalization of the background field is taken into account there is no cancellation and no infrared singularities.

Anyway, the effective interaction (11) should be taken into account quite seriously and treated in a self-consistent way. Without solving this problem exactly, we shall make an attempt in this direction to demonstrate the actual importance of the question.

We shall start with an estimation of corrections to the fermion spectrum given by the diagram Fig.3. The result will be used later to compute the diagram Fig.2 a. To preserve $O(2)$ space rotational invariance of our results we apply the approach of [15]. Choosing the Landau gauge of the homogeneous magnetic field, we introduce two pairs of canonically conjugated variables

$$Q = \frac{1}{i} \nabla_1 = \frac{1}{i} \partial_1 + Bx_2, \quad P = \frac{1}{i} \nabla_2 = \frac{1}{i} \partial_2 \\ q = \frac{1}{i} \partial_2 + Bx_1, \quad p = \frac{1}{i} \partial_1 \quad (13)$$

The mean field one-particle Hamiltonian has the form of an oscillator in variables (Q, P)

$$H = \frac{1}{2m} (Q^2 + P^2) \quad (14)$$

the other pair (q, p) describing some "shadow" oscillator Hamiltonian, which yields no contribution to the energy.

The rotational invariance of the results is guaranteed if one uses a basis of 2-oscillator eigenstates

$$|n, n'\rangle = \frac{1}{\sqrt{n!}} \left(\frac{Q - iP}{\sqrt{2}} \right)^n \frac{1}{\sqrt{n'!}} \left(\frac{q - ip}{\sqrt{2}} \right)^{n'} |0\rangle \quad (15)$$

In the coordinate representation these wave functions factorize

$$\langle Q, q | n, n' \rangle = \langle Q | n \rangle \langle q | n' \rangle \quad (16)$$

where

$$\langle Q | n \rangle = \frac{\sqrt{qH}}{\sqrt{\pi^{1/2} 2^n n!}} H_n \left(\frac{Q}{\sqrt{qH}} \right) \exp \left(-\frac{Q^2}{2qH} \right) \quad (17)$$

where H_n are the standard Hermite polynomials. Matrix elements of a current $J_\mu = (1, 1/2im \nabla_i) = (1, 1/2im(Q, P))$ taken over the basis (15) also admit factorization, for example,

$$\langle n, n' | \exp(-i\vec{k}\vec{r}) | m, m' \rangle = \langle n | \exp \left(\frac{ik_1 P - ik_2 Q}{B} \right) | m \rangle \langle n' | \exp \left(\frac{-ik_1 q + ik_2 p}{B} \right) | m' \rangle$$

One-dimensional matrix elements are given by the formulae (we take for definiteness $m \geq n+1$):

$$\begin{aligned} \langle n | \exp(\frac{ik_1 P - ik_2 Q}{B}) | m \rangle &= (\frac{k_1 + ik_2}{q_H})^{m-n} (\frac{\vec{k}^2}{4q_H^2})^{L_n} \exp(-\frac{\vec{k}^2}{2q_H^2}), \\ \langle n | \frac{1}{2im} [\nabla_1 + i\nabla_2, \exp(\frac{ik_1 P - ik_2 Q}{B})]_+ | m \rangle &= \frac{q_H}{2m} (-\frac{k_1 + ik_2}{2q_H})^{m-n-1} \\ &\quad (\frac{2^{m-n-2} n!}{m!})^{1/2} \exp(-\frac{\vec{k}^2}{4q_H^2}) ((n+1) L_{n+1}^{m-n-1} (\frac{\vec{k}^2}{2q_H^2}) + m L_n^{m-n-1} (\frac{\vec{k}^2}{2q_H^2})), \\ \langle n | \frac{1}{2im} [\nabla_1 - i\nabla_2, \exp(\frac{ik_1 P - ik_2 Q}{B})]_+ | m \rangle &= \frac{q_H}{2m} (-\frac{k_1 + ik_2}{2q_H})^{m-n+1} \\ &\quad (\frac{2^{m-n} n!}{m!})^{1/2} \exp(-\frac{\vec{k}^2}{4q_H^2}) (L_{n-1}^{m-n+1} (\frac{\vec{k}^2}{2q_H^2}) + L_n^{m-n+1} (\frac{\vec{k}^2}{2q_H^2})), \end{aligned}$$

where

$$L_n^m(x) = \frac{1}{n!} x^n \frac{d^n}{dx^n} (e^{-x} x^{n+m}) \quad (19)$$

is an adjoint Laguerre polynomial. The bare fermion Green function has the form

$$G(\epsilon) = \sum_{n, n'} G_n(\epsilon) |n, n'\rangle \langle n, n'|,$$

where

$$G_n(\epsilon) = \frac{1}{\epsilon - \omega_c(n - N + 1) + i0 \text{sgn} \epsilon} \quad (20)$$

Before the calculation of the self-energy diagram of the Fig.3, it could be anticipated that an interaction described by the gauge field propagator (12) is able to cause a complete smearing of Landau levels. The reason is that a scale of the current-current interaction is of the order of ω_c , while the relative strength of the Coulomb interaction is N times weaker.

The contribution of the diagram of Fig.3 to self-energy is given by an expression

$$\Sigma_{nm}(\epsilon) = i \int \frac{d^2 q}{(2\pi)^2} \frac{d\omega}{2\pi} D_{\mu\nu}(\vec{q}, \omega) \text{Tr}[J^\mu(\vec{q}) G(\epsilon + \omega) J^\nu(-\vec{q})], \quad (21)$$

where Tr stands for $q_H^2 \Sigma_{n=0}^\infty$. Notice that other second order contributions given by Figs.4a,b are actually equal to zero because in the Lagrangian (5) a mean value of $\langle \psi^\dagger \psi - \rho \rangle$ is zero and a closed fermionic loop vanishes. The diagram of Fig.4c, which is a counterpart of the Fig.4a with $D_{00}(0, \vec{q})$ replaced by $D_\perp(0, \vec{q})$, can only produce a shift of the chemical potential.

To observe that a broadening of Landau levels holds it suffices to calculate a density of states

$$\nu(\epsilon) = \text{Tr} \text{Im} G(\epsilon) = q_H^2 \sum_{n=0}^\infty \frac{\text{Im} \Sigma_{nn}(\epsilon)}{(\text{Im} \Sigma_{nn}(\epsilon))^2 + (\epsilon - \omega_c(n - N + 1))^2} \quad (18)$$

The imaginary part of a diagonal matrix element of the self-energy operator is given by the formula

$$\begin{aligned} \text{Im} \Sigma_{nn}(\epsilon) &= \sum_{m=0}^\infty \int \frac{d^2 q}{(2\pi)^2} \frac{d\omega}{\pi} \text{Im} D_\perp(\vec{q}, \omega) |J_{nm}^\perp(\vec{q})|^2 \times \\ &\quad \times (\theta(-\epsilon) \theta(\xi_m) \delta(\epsilon + \omega - \xi_m) - \theta(\epsilon) \theta(-\xi_m) \delta(\epsilon - \omega - \xi_m)), \end{aligned} \quad (23)$$

where $J_{nm}^\perp(\vec{q}) = \epsilon_{ij} q_j / 2q \langle n | [J_1, \exp(iq_1 P - iq_2 Q)]_+ | m \rangle$.

It can be seen that for $\epsilon < 0$ lying between $-\omega_c(N-n)$ and $-\omega_c(N-n-1)$ only $(N-n)$ higher levels contribute. Using (19) we get the result

$$\begin{aligned} \text{Im} \Sigma_{nn}(\epsilon) &= \left(\frac{\epsilon}{v} \right)^2 \frac{1}{2\epsilon^2 m N^2} (L_{n-1}^1(x_n) + L_n^1(x_n))^2 e^{-x_n} + \\ &\quad + \frac{q_H^2}{m N^2} \sum_{m=n+1}^{N-1} e^{-x_m} x_m^{m-n-1} \frac{n!}{8m!} \{ [(n+1) L_{n+1}^{m-n-1}(x_m) + m L_n^{m-n-1}(x_m)]^2 + \\ &\quad + \frac{x_m^2}{4} [L_{n-1}^{m-n+1}(x_m) + L_n^{m-n+1}(x_m)]^2 \} \end{aligned} \quad (24)$$

where $x_m = (\epsilon - \omega_c(m - N + 1))/v q_H$ and $v = \sqrt{\chi(0,0)/\epsilon(0,0)}$ is a speed of statistical gauge field quanta. In particular, for $n = N-1$ and ϵ close to zero one has $(|\epsilon| < \omega_c)$

$$\text{Im} \Sigma_{N-1, N-1}(\epsilon) \approx -\frac{\epsilon |\epsilon|}{m v^2} \frac{(2N-1)^2}{2\epsilon N^2} \sim -\frac{\epsilon |\epsilon|}{N \omega_c} \quad (25)$$

The correction to the real part of the spectrum given by eq.(21) is

$$\text{Re} \Sigma_{N-1, N-1}(\epsilon) \sim \frac{\epsilon}{N}$$

Thus we have got that the contribution to the damping of quasiparticles with energies near the chemical potential is of the order of ϵ^2 like in the case of a conventional Fermi liquid. It also means that a density of states remain finite as ϵ tends to zero (but not equal to zero exactly)

$$\nu(\epsilon) \approx q_H^2 \frac{Im\Sigma_{N-1,N-1}(\epsilon)}{\epsilon^2} \approx \frac{m}{16N} \quad (26)$$

For energies far from the chemical potential the damping of quasiparticles increases. The density of states is a continuous function of ϵ , which reaches values of the order of m at $\epsilon \leq \omega_c\sqrt{N}$ and preserves this mean value at $\epsilon \geq \omega_c\sqrt{N}$.

Notice that the lowest order corrections which we deal with don't produce any features of the "marginal Fermi liquid"-type behaviour [37], [38]. For instance, a wave function renormalization factor $Z = (1 - \partial Re\Sigma(\epsilon)/\partial\epsilon)^{-1}$ is finite, because we get the normal damping $\sim \epsilon^2$. It was shown in [39] that at the two-loop order in a theory with local interaction the damping is proportional to $\epsilon^2 \ln \mu/\epsilon$; however, even this log deviation does not lead to a "marginal Fermi liquid", contrary to the 1D case, when damping is large, $Im\Sigma \sim |\epsilon|$ [40] and no normal Fermi-liquid ground state exists.

Formally, we have a small parameter $1/N$ in our calculations. However, it would not be surprising if the $1/N$ expansion appeared to be singular. We shall discuss some motivations for such a conclusion below. But, although we cannot control reliably the relevance of higher order corrections for the result (26), it nevertheless seems to be reasonable from the physical point of view. In fact, we have observed that a strong current-current interaction lifts up the degeneracy of Landau levels and the one-particle spectrum becomes continuous. An essential conclusion which one could draw from this fact is that the Debye screening of the "electric" statistical field occurs. Such a circumstance was completely forbidden in the RPA calculations, when the Landau level degeneracy and a gapful fermion spectrum were believed to be maintained. To get the Debye screening one could calculate the diagram of Fig.1 with fermion propagators corrected by means of eq.(25). Then, taking the limits of zero momentum $k \rightarrow 0$, $\omega/k \rightarrow 0$, one gets

$$\Pi_{00} = \frac{i}{2} \int \frac{d\epsilon}{2\pi} \text{Tr}(G(\epsilon))^2 = \frac{d\rho}{d\mu} = \frac{1}{2} (\nu(\epsilon)|_{\epsilon \rightarrow +0} + \nu(\epsilon)|_{\epsilon \rightarrow -0}) \sim m/N. \quad (27)$$

Formally the absence of Debye screening in RPA follows from the independence of ρ on μ in a whole range of values of the chemical potential.

By estimating the diagrams of Fig.2, one can also obtain a nonzero imaginary part of $\Pi_{\mu\nu}$, which corresponds to the damping of statistical photons. The leading contributions are given by

$$\delta\Pi_{\mu\nu}(\omega, \vec{k}) = \frac{1}{2} \int \frac{d\epsilon d\epsilon'}{2\pi} \frac{d^2q}{(2\pi)^2} \times \quad (28)$$

$$\{ J^\mu(\vec{k}) G(\epsilon) J^\nu(-\vec{k}) G(\epsilon') J^\mu(-\vec{q}) G(\epsilon + \omega) +$$

$$2 J^\mu(\vec{k}) G(\epsilon) J^\nu(-\vec{k}) G(\epsilon + \omega) J^\mu(\vec{q}) G(\epsilon' + \omega) J^\nu(-\vec{q}) G(\epsilon' + \omega) \}$$

Then taking the imaginary part of (29) in the limit $\omega/vk \ll 1$ one obtains for the transverse part

$$Im\Pi_\perp(\omega, \vec{k}) \approx \frac{k^2}{2^4 N m} \left(\frac{2N+1}{N} \right)^2 \quad (29)$$

Such an intermediate asymptotics is valid for wave vectors belonging to the interval $\omega \geq vk \gg 1/\tau(\omega)$, where $\tau(\omega)$ is a quasiparticle life-time.

It follows from (25) that $1/\tau(\omega) \sim \omega^2/N\omega_c$. To get the imaginary part of $\Pi_\perp$ for $k \leq 1/\tau\omega v$ one should sum all the ladder corrections to the vertex (see Fig.5). The result can be expressed by means of transport time $\tau_r(\omega)$

$$Im\Pi_\perp(\omega) = \Pi_\perp(0,0) \frac{\omega\tau_r(\omega)}{1 + (\omega\tau_r(\omega))^2} \approx \frac{\rho}{m} \frac{1}{\omega\tau_r(\omega)} \quad (30)$$

The ratio $\sigma_{xx}(\omega) = Im\Pi_\perp(\omega)/\omega$ is actually a diagonal component of a frequency-dependent conductivity. The transport time is given by the integral of the eq.(21) with an additional factor $\vec{q}^2/2\rho$. It can be estimated by the order of magnitude as

$$\frac{1}{\tau_r(\omega)} \sim \frac{\omega^4}{N^3\omega_c^3} \quad (31)$$

The regimes (29) and (30) sew together at $kv \sim \omega_c(\omega/\omega_c)^{3/2}$ and represent the "anomalous skin effect" and "normal skin effect" respectively.

One more crossover in the behaviour of the $Im\Pi_\perp(\omega, \vec{k})$ takes place at $k \geq \omega/v$. The resulting expression for $Im\Pi_\perp$ should be very sensitive to the

ratio ω/ω_c . It is the point where it is worth commenting on the applicability of the approach starting from Landau levels. Although we have found that the density of states is actually finite everywhere, it seems that our previous consideration based on the bare Landau level structure is, at least qualitatively, reasonable at small ω/ω_c . To get a bound value of this ratio we shall recall an estimate for the quasiparticle life-time in pure electrodynamics. As was shown in [32],

$$\tau^{-1}(\epsilon) \sim \frac{\rho}{m} \left(\frac{mc}{p}\right)^{2/3} \quad (32)$$

The effect of the magnetic field is visible and Landau levels are distinguishable if $\tau^{-1}(\epsilon) \leq \omega_c$. Hence, we believe that our consideration is correct, at least, at

$$\omega \leq \frac{\rho}{m} \frac{1}{N^{3/2}} \quad (33)$$

We also suggest that for frequencies violating (33) it could be adequate to treat the magnetic field perturbatively. In particular, at $\omega \gg \frac{\rho}{m} \frac{1}{N^{3/2}}$ one could expect the following asymptotics for the transverse damping

$$\text{Im}\Pi_{\perp}(\omega, \vec{k}) \sim \frac{\rho}{m} \left(\frac{\omega m}{p}\right)^{1/3} \quad (34)$$

if $k \leq \rho/vm(\frac{\omega m}{p})^{2/3}$, or otherwise

$$\text{Im}\Pi_{\perp}(\omega, \vec{k}) \sim \sqrt{\rho} \frac{\omega}{k} \quad (35)$$

which is nothing but the Landau damping. The result (35) could also be obtained from the simple fermion loop (Fig.1) giving

$$\text{Im}\Pi_{\perp}(\omega, \vec{k}) = \sum_{n=1}^{\infty} f_n \left(\frac{k}{q_H}\right) \exp\left(-\frac{k^2}{2q_H^2}\right) \delta(n\omega_c - |\omega|)$$

after replacement of a summation by an integral over n [41]. Such a procedure becomes legitimate if the level's width is taken into account.

It should also be mentioned that with all these non-trivial corrections to $\Pi_{\mu\nu}$ and $D_{\mu\nu}$ taken into account, the conventional theorems about a power

of divergency and external momenta dependence of a perturbation expansion with completely renormalized $D_{\mu\nu}$ could fail. As a result, one could expect the appearance of a contribution to $\Pi_3(0,0)$ beyond the one-loop level [20],[42], although in the case of zero fermion density it is prohibited by the Coleman-Hill theorem [43]. Were this to happen, the mechanism of anyon superconductivity based on the exact cancellation of the bare Chern-Simons term by the induced one [31] would cease to work. It is interesting to note that in papers [20] and [42] the same result was obtained using different approaches, that in the self-consistent approximation, instead of the cancellation of θ , we get the renormalization $\theta \rightarrow \theta/2$. As a by-product this leads to a more complicated picture of statistic transmutation, the so-called "two-step transmutation" (see [20]).

However here we shall not rely upon such a possibility to generate higher order corrections, but show immediately that an established existence of the Debye screening and the transverse photon's damping are already sufficient to destroy the abovementioned mechanism.

3 Electromagnetic response: perturbative analysis.

To prove our claim we shall first derive an effective action for a weak electromagnetic field with non-zero components A_0, A_x, A_y only. We shall also consider models with momenta lying in the basic plane (x,y). To get the desired Gaussian effective action sufficient for the linear response theory one should integrate the Lagrangian (6) over the statistical gauge field fluctuations. Using the Coulomb gauge for the electromagnetic potential ($\vec{\partial} \vec{A} = 0$) and introducing the bare kinetic term $1/e^2(E^2 - c^2 B^2)$ one can represent the result in the form

$$L(A_\mu) = A_1^2 \left(\frac{\omega^2 - c^2 k^2}{e^2} - \frac{c^2 k^2 \Pi_3(\omega, \vec{k})^2}{\Pi_{00}(\omega, \vec{k})} \right) + A_0^2 \left(\frac{c^2 k^2}{e^2} + \frac{c^2 k^2 \Pi_3(\omega, \vec{k})^2}{\Pi_{\perp}(\omega, \vec{k})} \right) + 2k \Pi_3(\omega, \vec{k}) A_0 A_{\perp}, \quad (36)$$

where c is the speed of light and a 2+1-dimensional charge e has a dimension $(\text{energy})^{1/2}$.

Thus we obtain the correlators

$$K_{00}(\omega, \vec{k}) = \langle j_0^{em}(\omega, \vec{k}) j_0^{em}(-\omega, -\vec{k}) \rangle = \frac{k^2 \Pi_3(\omega, \vec{k})^2}{\epsilon \omega^2 - \chi k^2 + \Pi_1(\omega, \vec{k})} \quad (37)$$

$$K_{\perp}(\omega, \vec{k}) = (\delta_{ij} - \frac{k_i k_j}{k^2}) \langle j_i^{em}(\omega, \vec{k}) j_j^{em}(-\omega, -\vec{k}) \rangle = \frac{k^2 \Pi_3(\omega, \vec{k})^2}{\epsilon k^2 + \Pi_{00}(\omega, \vec{k})} \quad (38)$$

$$K_{0ad}(\omega, \vec{k}) = \frac{\epsilon_{ij} k_j}{k^2} \langle j_i^{em}(\omega, \vec{k}) j_0^{em}(-\omega, -\vec{k}) \rangle = \Pi_3(\omega, \vec{k}) \quad (39)$$

As we have already mentioned, it cannot be excluded that the function $\Pi_3(\omega, \vec{k})$ varies at $\omega \sim \sqrt{B/\rho\omega_c}$. Nevertheless, in the static limit which is the only relevant thing for the discussion of the Meissner effect, we can neglect the possible ω dependence. Using (27) we get the low frequency and small momenta behaviour of the current-current correlation function

$$K_{\perp}(0, \vec{k}) = \frac{k^2 \Pi_3(0, \vec{k})^2}{\epsilon k^2 + \nu}. \quad (40)$$

Thus it is evident that the Debye screening of the statistical field prohibits the appearance of the mass of the transverse electromagnetic potential $A_{\perp}$ independently of the limiting value $\Pi_3(0, \vec{k})$.

Now let us discuss the function $K_{00}(\omega, \vec{k})$. By neglecting $\Pi_{\perp}$, one has a sound-like pole in $K_{00}(\omega, \vec{k})$. This result is reasonable until the fluctuations of the real electromagnetic field are included. It corresponds to the intuitively evident property of a compressibility of a system of electrically neutral anyons. One has to treat A_{μ} as an infinitely weak external field, otherwise density fluctuations have a plasmon gap $\Omega = \sqrt{\epsilon^2 \rho/m}$. Note that at not very large N the influence of the electromagnetic field can be made relatively small because of the smallness of the parameter $\epsilon^2 m N / \rho$ proportional to the fine structure constant. It seems to be important for anyons arising (if any) in the context of strongly correlated electron systems, where the natural value of N is two. Anyway, the conclusion about a linear mode is correct at wave vectors larger than the electrostatic Debye screening length. Thus a collective excitation observed in the particle-hole channel can be recognized as a zero-sound mode and not as a second sound (which appears due

to the superfluidity). In particular, in the regime $Im\Pi_{\perp}(\omega) \sim \omega^3$ (see (30)) the damping of the linear mode coincides with a conventional damping of sound in the Fermi liquid at zero temperature. Now we are able to draw the conclusion that in the case of neutral anyons this mode can hardly play the role of a Goldstone mode resulting from a spontaneous violation of the wave function phase symmetry and manifesting the superfluid long-range order at zero temperature. Similarly, in the case of electrically-charged anyons such a damp mode cannot produce the Higgs-Anderson phenomenon.

At the same time, the existence of some Goldstone mode which could give mass to the electromagnetic field follows from the theorem about a "spontaneous violation of a fact" [12], [44]. Namely, it was shown that due to the non-commutativity of the x and y components of a quasiparticle momentum $\vec{P}_f$ in the case of background magnetic field

$$[P_f^x, P_f^y] = iB \int d^2 r \psi^\dagger \psi$$

an additional boson should exist in the system to restore the original commutativity of the components of a total momentum $\vec{P} = \vec{P}_f + \vec{P}_b$, where $\vec{P}_b$ corresponds to the bosonic contribution. A gapless character of this mode follows from the fact that it should produce a non-zero contribution of the integral of the momentum density over a contour at spatial infinity. Let us note that in the case of the torus there are some states for which the "spontaneous violation of fact" leads not to the appearance of a bosonic mode, but to taking into account some specific Chern-Simons degree of freedom [45].

In our opinion, the Goldstone mode implementing "the spontaneous violation of a fact" could only be found as a collective excitation in the N -particle channel. We should say that this conjecture was made in ref [12] simultaneously with the observation of the zero-sound pole, but it was appropriately developed.

4 Non-perturbative mechanism of anyon superconductivity

The hypothetical phenomenon of bounding of N (for N even) or $2N$ (for N odd) anyons together could lie in the origin of a non-perturbative mechanism

of anyon superconductivity. The motivations for such a hypothesis could be found already in the mean field consideration. Let us have N levels completely filled. Then if one more anyon is injected into the system it can occupy a state on the $N+1$ level. The same will be valid for $2, 3, \dots, N-1$ anyons, but with the N -th anyon introduced a total flux of the system will increase onto the one flux quantum. As a result, exactly one new state will appear on each of the Landau levels and, consequently, N extra anyons will drop into N lowest levels. In a sense, N fermions on the $N+1$ level are attracted by one hole located at some point where there is an additional flux quantum, resulting from a fusion of fluxes carried by N anyons. In fact such a naive picture cannot be correct, since anyons cannot be separated into fermions and fluxes, but it could be used for some preliminary estimates. For instance, it shows that a range of attraction between anyons (if any) should be of the order of a magnetic length l_H and a bounding energy should be of the order of ω_c .

A composite of N anyons behaves like a boson for N even or as a fermion for N odd, since its statistical parameter is $N^2\pi^2/\theta = N\pi$. A competition between the binding into bosons consisting of $2N$ anyons or into fermions of N anyons, which arises at odd N , is resolved in favour of bosons having a smaller energy at a given density of a system.

One implicit evidence in favour of this binding was presented by Girvin and MacDonald [46] and Jain and Read [47]. Studying a variational anyon ground-state wave function proposed by Laughlin, they succeeded in demonstrating that it exhibits a special type of the Off-Diagonal Long-Range Order (ODLRO). The wave function involved was taken in the form of a product of the Slater determinant for N filled Landau levels and a "monodromy factor" $\prod_{i < j} (z_i - z_j)^{1/N} / |z_i - z_j|^{1/N}$, where $z_i = x_i + iy_i$ is a complex coordinate of the i -th anyon. Then a k -particle density matrix was generated on the basis of this wave function

$$g(\vec{r}_1, \dots, \vec{r}_k; \vec{r}'_1, \dots, \vec{r}'_k) = \int \dots \int \Psi^*(\vec{r}_1, \dots, \vec{r}_k; \dots) \Psi(\vec{r}'_1, \dots, \vec{r}'_k, \dots) d^2r_{k+1} \dots,$$

where integrations are performed over the coordinates of the rest particles in the system. As was shown in [46], [47], the density matrix has a power-law decay at $|\vec{r}_i - \vec{r}'_j|, |\vec{r}'_i - \vec{r}'_j| < |\vec{r}_i - \vec{r}'_j|$ if and only if k is a multiple of

N . The exponent in the power law decay with respect to the difference of averaged coordinates $|\vec{r} - \vec{r}'|$

$$g(\{\vec{r}\}; \{\vec{r}'\}) \sim \frac{1}{|\vec{r} - \vec{r}'|^\alpha} \quad (41)$$

is given by $\alpha = N/2$. Such a behaviour of an equal time correlation function could be understood as a manifestation of a collective excitation having a linear spectrum and corresponding to the N -particle bound state. Let us note, that this correlation law means that the bound state in the N particle channel exists only at zero temperature $T = 0$. To get some nonzero critical temperature T_c we need $\alpha = 0$ at zero temperature $T = 0$, which gives some $\alpha(T) \neq 0$ at $T < T_c$. This contradiction may be connected with the fact, that (41) was obtained in the mean field approximation for N filled Landau levels with zero widths. The Debye screening was not taken into account. For realistic ground state wave function we hope to get $\alpha = 0$ at zero temperature $T = 0$.

Were the exact ground state to exhibit correlations of the type (41), it could be recognized as a homogeneous liquid of bosons with a core size about l_H and an effective mass about Nm . Note that at all values of N the resulting boson liquid remains dense, because a density of N -anyon compounds is $\rho/N \sim \rho_H^2$, that is a core size squared. Although at large N an oversimplified picture of huge composites is not adequate, one should realize that the phenomenon concerned is "strong" in the sense that all particles in the system are involved in a formation of some condensate.

In other words, the bounding phenomenon should have an essentially non-perturbative nature and it is not weak, because a typical condensation energy per particle could be of the order of ω_c .

Although such a scenario looks very amusing, convincing arguments in its favour remain to be found. Below we shall address the problem of possible attraction between two anyons, which could be relevant in the case $N = 2$ (sermons). Here it will be more convenient to use a basis of one-particle states characterized by the Landau level number n and a momentum k in the x direction

$$(\vec{r}|n, k) = \langle y|n \rangle \exp(ikx)$$

where $\langle y/n \rangle$ is given by (17). Let us consider a scattering of two fermions on the n -th and m -th Landau levels (see Fig.6). The amplitude of an elastic scattering of Fig.6a is given by

$$\begin{aligned} A_{nm, mn}(k', p'; p, k) &= (2\pi)^2 \delta(k' + p' - k - p) T_{nm, mn}(k', p'; p, k), \\ T_{nm, mn}(k', p'; p, k) &= \int \frac{d^2 q}{(2\pi)^2} \delta(q_z - k' + k) J_{nn}^\mu(\vec{q}) J_{mm}^\nu(-\vec{q}) D_{\mu\nu}(0, \vec{q}) = \\ &= \int \frac{d^2 q}{(2\pi)^2} \delta(q_z - k' + k) \exp(-\frac{\vec{q}^2}{2q_H^2}) \{ L_n(\frac{\vec{q}^2}{2q_H^2}) L_m(\frac{\vec{q}^2}{2q_H^2}) \frac{1}{\epsilon \vec{q}^2 + \nu} - \\ &\quad - [L_n^1(\frac{\vec{q}^2}{2q_H^2}) + L_{n-1}^1(\frac{\vec{q}^2}{2q_H^2})] [L_m^1(\frac{\vec{q}^2}{2q_H^2}) + L_{m-1}^1(\frac{\vec{q}^2}{2q_H^2})] \frac{1}{4m^2 \chi} \} \end{aligned} \quad (42)$$

We neglect here the amplitude of Fig.6b, because in the region of large momenta we shall be interested (see below) this amplitude will be small. The detailed investigation of the total amplitude will be done in the next publication. Let us consider the amplitude (43). Note, first of all, that at $Q = |k - k'| > q_H$ the current-current interaction is actually stronger. However, the total sign is different for $n - m = 0 \pmod{2}$ and $n - m = 1 \pmod{2}$, because Laguerre polynomials $L_n^1(x) \approx (-x)^n/n!$, $x \gg 1$ (see eq.(19)). The expression in brackets in (43) is then proportional to

$$(-1)^{n+m} \left(\frac{\vec{q}^2}{2q_H^2} \right)^{n+m} \left[\frac{1}{\epsilon \vec{q}^2} - \frac{1}{4m^2 \chi} \right], \quad (43)$$

here m_f is the fermion mass.

Let us note that this approximation is valid when $q > q_H = \sqrt{2\pi\rho/N}$ and current-current interaction becomes dominant at larger momenta $q > 2\sqrt{2\pi\rho}$. At large N we have two regions

1. $\sqrt{2\pi\rho/N} < q < 2\sqrt{2\pi\rho}$
2. $q > 2\sqrt{2\pi\rho}$

In region 1. the Coulomb interaction is dominate and we get the attraction for $n - m = 1 \pmod{2}$. In the region 2. the amplitude (43) is defined by current-current interaction and we get the attraction for $n - m = 0 \pmod{2}$. The sign of the integrand in the opposite limit of small q is given by the expression

(Laguerre polynomials has asymptotic $L_n^1(x) \approx (n+m)!/n!m!$, $x \ll 1$)

$$\left[\frac{1}{\nu} - \frac{1}{4m^2 \chi} (2n+1)(2m+1) \right]$$

For excited states this potential again gives us attraction ($nm > N^3$), but for $n, m < N$ it gives us repulsion. Anyway, we see that at intermediate distances (region 1.) anyons from the nearest levels $n - m = 1 \pmod{2}$ attract each other and at short distances (region 2.) there is attraction between anyons at the levels $n - m = 0 \pmod{2}$. Estimating the potential in the region 1. from (43) we get

$$V_{mn}(q) \sim (-1)^{n+m} \frac{1}{\epsilon q_H^2} \left(\frac{\vec{q}^2}{2q_H^2} \right)^{n+m-1} \sim (-1)^{n+m} \frac{1}{mN} \left(\frac{\vec{q}^2}{2q_H^2} \right)^{n+m-1} \quad (44)$$

which gives the approximate value $V(l_H/\sqrt{N}) < x < l_H \sim \rho/mN^2$. Estimating potential in the region 2. we get

$$V_{mn}(q) \sim (-1)^{n+m+1} \frac{1}{4m_f^2 \chi} \left(\frac{\vec{q}^2}{2q_H^2} \right)^{n+m} \sim (-1)^{n+m+1} \frac{1}{mN^2} \left(\frac{\vec{q}^2}{2q_H^2} \right)^{n+m} \quad (45)$$

Estimating the potential $V(x)$ at large x we get (independently of possible sign) $V(x \gg l_H) \sim \rho/mN^3 \exp(-x^2/l_H^2)$. Thus we see that independently of attraction or repulsion at large x , we have strong attraction in region 1. x (N times stronger then we get from the continuation of $V(x \gg l_H)$ to the region $x < l_H$) and less strong attraction in region 2..

For small values of N we have no possibility to control the validity of our calculations; in particular, the values of ϵ, χ and ν , given by eqs.(10),(27) can be incorrect. However, the qualitative picture of large q attraction will hold and we get the effective potential $V(xq_H)$ which is negative at small x and small (both signs cannot be excluded) at large x . Assuming that the above results can be extended to the case of semions with $N = 2$, we find the desired mechanism for anyon correlations. Let us consider again the injection of some anyon pair into the system with two completely filled levels. Due to the additional flux quantum in the system we have two holes on the first and second levels, which attract each other in region 1. and repulse each other in region 2.. At the same time two excited anyons being on the same third level attract each other and they attraction is stronger at large

q (due to the power q^{n+m}). After all they occupy two states at the lowest levels and start to interact as particle at nearest levels. So they attract each other in intermediate region 1.. These two anyons will be correlated and in some sense this is the correlation between two different Landau levels. We can neglect now the finite widths of these levels, because taking them into account we get the same picture of correlations. If we consider the ground state the total momentum of the new pair equals zero, so we have two anyons with momenta k and $-k$. Because attraction exists in intermediate region of momenta we assume that correlation exists only for pairs with momenta $(k, -k)$, $2\sqrt{N}q_H > k > q_H$. Remembering the connection between the y coordinate of the particle in the magnetic field and the x -component of the momenta $y \sim k/B$, we get the correlation length $l_c = y_+ - y_- \sim q_H/B \sim 1/q_H \sim \sqrt{N}/\rho$. Thus the correlation length is $\sqrt{N}$ larger than the average distances between anyons. This picture is very similar to the usual BCS correlations and the new mechanism of attraction (also using the Fermi-surfaces - Landau levels here) looks like some new type of Cooper phenomenon. Using this analogy we suggest the ground state wave function

$$|\Psi\rangle = \prod_{k \geq q_H} \{\alpha_k + \beta_k a_1^+(k) a_2^+(-k)\} |0\rangle \quad (46)$$

where indices 1 and 2 describe the first and second Landau levels. It is possible to show that the superconducting gap will be of order ω_c .

We can only suggest now that the same attraction mechanism exists for the general case $N > 2$ also, but to prove it we must consider the scattering amplitude of N particles. If in this case we get the saturation of attraction forces for N particles from N lowest Landau levels we shall get the same correlation picture as in the case $N = 2$. Then the ground state wave function will be the generalization of (46)

$$|\Psi\rangle = \prod_{k_1, \dots, k_{N-1} \geq q_H} \{\alpha_{k_1, \dots, k_{N-1}} + \beta_{k_1, \dots, k_{N-1}} a_1^+(k_1) \dots a_{N-1}^+(k_{N-1}) a_N^+(-k_1 - \dots - k_{N-1})\} |0\rangle \quad (47)$$

where indices $1, \dots, N$ describe the first N Landau levels.

We also note that if the magnetic moment of non-relativistic fermions is taken into account then a principally new mechanism of attraction is possible

as was shown by one of the authors in [48]. Whether this mechanism obeys the saturation property is an open and important question. We also can not exclude more complicated correlation due to the same level anyon attraction at extreme small scales. To decide definitely what type of correlation is the most energetically favourable we must solve the general gap equation. We hope to consider this problem in the future publications, here we only mention, that in the case of one-level system (fermion-boson transmutation) the only possible attraction exists only in region 2., but in this case $N = 1$ we automatically get bosons and don't need any correlation.

Let us now briefly discuss the interaction of such correlated clusters (pairs in the $N = 2$ case). Let us consider two pairs which we can treat as some neutral molecules. If we try to place them near each other we get the attraction between fermions from different levels, but the repulsion between fermions from the same ones. Moreover, this repulsion will be stronger (it is possible to see this from the amplitude (44)), so we get some hard-core repulsion for the clusters. This picture gives us a reasonable description of a two-dimensional Bose liquid which we shall discuss in the next section.

5 Hydrodynamics, electrodynamics and phase transitions of anyons

Based on the above consideration, we could expect that the only correct hydrodynamical description of the anyon system at momenta $k \ll q_H$ and frequencies $\omega \ll \omega_c$ is possible in terms of a composite field $\Phi(\vec{r})$ corresponding to anyon compounds. It was first proposed in [19] that such an operator can be composed of a fusion of N (for N even) anyon creation operators $c^+(\vec{r})$:

$$\Phi^+(\vec{r}) = \int : c^+(\vec{r}_1) \dots c^+(\vec{r}_N) : F(\vec{r}_1, \dots, \vec{r}_N; \vec{r}) d^2r_1 \dots d^2r_N, \quad (48)$$

where the kernel $F(\vec{r}_1, \dots, \vec{r}_N; \vec{r})$ decays as a Gaussian exponent with respect to any difference $|\vec{r}_i - \vec{r}|$ with a characteristic length l_H . Based on the previous discussion we assume that a convolution with this function performs a projection of anyon creation operators onto the $n = 1, 2, \dots, N$ Landau

levels

$$c^+(\vec{r}) \rightarrow c_{n,k}^+ = \int d^2r' c^+(\vec{r}') \langle \vec{r}' | n, k \rangle$$

The bare anyon operator $c^+(\vec{r})$ could be expressed in the form of a non-local functional of the original fermionic field [29], [49]

$$c^+(\vec{r}) = \psi^+(\vec{r}) \exp\left(\frac{i\pi}{\theta} \int \arg(\vec{r} - \vec{r}') \psi^+(\vec{r}') \psi(\vec{r}') d^2r'\right) \quad (49)$$

Such a two-dimensional counterpart of the Jordan-Wigner transform appears when one expresses the gauge field in terms of fermionic matter in the bare theory (1).

The definitions (48), (49) are actually ill defined. Trying to overcome the usual problems with local operator products one unavoidably encounters all the complexity of anyon dynamics. However one can draw some conclusions with the help of (48). For instance, it should be noticed that our composite field behaves as one consisting of N anyons only at distances much larger than magnetic length l_H . Indeed, only at these distances is the total exponent $\exp\left(\frac{i\pi}{\theta} \int \arg(\vec{r} - \vec{r}') \psi^+(\vec{r}') \psi(\vec{r}') d^2r'\right)$ indistinguishable from unity. At distances of the order of the magnetic length our composite field exhibits a complex internal structure, being a superposition of states with an arbitrary large number of anyons. It agrees with an intuitive impression that the picture of N -anyon bounding could be only a simplified image of real correlations present in the anyon system.

In the case of electrically neutral anyons the Goldstone mode implementing the "spontaneous violation of a fact" could be identified with fluctuations of a phase of the complex scalar field $\Phi(\vec{r}) = \Delta(\vec{r}) \exp(i\phi(\vec{r}))$. Fluctuations of $\Delta(\vec{r})$ are expected to have a gap of the order of ω_c .

The next problem is to build an effective action for hydrodynamical fluctuations in terms of $\Phi(\vec{r})$. Using the qualitative picture of N -anyon bounding we could suppose that bosonic composites have only a short-range (screened) statistical Coulomb repulsion. No long-range Aharonov-Bohm interaction is present, so one could say that statistical forces exhibit a sort of "saturation" property, in the sense that the tensor product of N braid group representations, each with a character $\exp(i\pi(1 - 1/N))$, is a unit representation.

A problem of the nature of the ground state of a dense system of two-dimensional bosons with short-range repulsion is now under intensive consideration [50], [51]. It was conjectured that depending on density one could

find the system at zero temperature in the superfluid phase, or in a normal fluid phase with or without some sort of an orientational long-range order (say, hexatic order), or even in a solid ("anyon crystal", [7]) phase. The latter insulating phase seems to be less possible, because one could suspect that a dense crystal will be easily melted by means of coherent ring exchange processes [52], [53]. The existence of a normal fluid state, which was proposed as a 2 + 1-dimensional counterpart of a flux vortex fluid in the dual picture of a three-dimensional superconductor [54] cannot be excluded. In that case an electromagnetic response of the charged anyon system can be described, at least qualitatively, by the formulae (37) - (39). Again, a superconducting behaviour is absent.

If the bosonic system has a chance to find itself in the superfluid phase, its static description is provided by the Higgs-like effective Ginzburg - Landau Lagrangian. With a weak electromagnetic field included the effective time-dependent action is given by

$$L_{eff} = i\Phi^+(\partial_0\Phi - ieNA_0\Phi) - \frac{1}{2mN}|\vec{\nabla}\Phi - ieN\vec{A}\Phi|^2 + \mu_B|\Phi|^2 - \lambda|\Phi|^4 \quad (50)$$

where $\lambda \approx D_0(0,0)$ and $\mu_B > 0$ denotes a chemical potential of composite bosons. In the static limit this Lagrangian gives us the Meissner effect with the London penetration depth $\lambda_L \sim \sqrt{(mc^2/Ne^2\Delta^2)}$. The presence of the anomalous expectation value $\langle \Phi(r) \rangle$ changes the electromagnetic response of the anyon system drastically. To recover the same result in terms of fermionic perturbation theory one should introduce the anomalous N -point Green functions into the diagrammatics and to take into account anomalous loops on an equal footing with normal ones. For instance, the Meissner effect is produced by the loop diagram of Fig.7 The complete time-dependent response is organized in a more sophisticated manner and we shall leave this problem for the future.

We would like to mention that this behaviour is preserved at non-zero temperatures up to the temperatures of the two-dimensional Bose condensation. Note that the power-like decay of correlations (41) even at zero temperature would mean that there are no condensates at finite temperatures. However the simplest variational function used in [47] doesn't take into account an intrinsic connection of the local field and density. We believe that should this be done, g should tend to a constant at $|\vec{r} - \vec{r}'| \rightarrow \infty$. Thus the

present picture assumes that the phase transition is of the BKT (Berezinsky - Kosterlitz - Thouless) type. Let us note that in paper [55] the Chern-Simons + Higgs system was considered, which after integration over the CS field gives some effective Higgs potential obeying a non-trivial vacuum average. It will be interesting to understand if this radiative correction mechanism describes the same physics as the bounding. The second interesting point is the connection which was established in [55] between anyon systems and transitions in 1+1 systems (strings), where the nature of the BKT transition is quite natural.

The critical temperature for the phase transition can be estimated by the formula

$$T_c \sim \frac{\rho_B}{M_B f(\rho_B a_B^2)} \quad (51)$$

where $\rho_B = \rho/N$ is a boson density, $M_B = mN$ is a boson mass and $a_B \sim l_H$ is a boson core size. The function $f(x)$ behaves as $|\ln(x)|$ at $x \ll 1$ and is a constant of order of unity at $x \geq 1$. Thus we could expect the following expression for T_c :

$$T_c \sim \frac{\rho}{mN^2} \quad (52)$$

The largest value of the function $T_c(\delta)$ taken at a series of points $\delta = \pi(1 - 1/N)$ is achieved at $N = 2$. It seems reasonable to suppose that at all rational points $\delta = \pi p/q$ the critical temperature is given by

$$T_c^{p/q} \sim \frac{\rho}{mq^2} \quad (53)$$

This function is plotted in Fig.8. Notice that the symmetry $\delta \rightarrow \pi - \delta$ is present. A very irregular dependence on δ could be anticipated in view of the hidden ultrametricity of the anyon problem, coming from a possible relevance of quantum geometry and a theory of representations of the quantum group $SL_q(2)$ with $q = e^{i\delta}$ for the construction of a consistent quantum theory of anyons. The appearance of the quantum group structure is due to the appearance of some quantum factorials $[n!]_q$ in the loop representation of anyonic dynamics [56].

For anyons in the superfluid ground state, the Gaussian critical behaviour seems to be natural and the only possible one. However, one could speculate

that at appropriate conditions the very type of criticality developed at the finite temperature phase transition could depend on statistics. For instance, a 2d Conformal Field Theory (CFT) approach gives us some number of primary fields $\chi_n(z)$ which are mutually non-local

$$\chi_n(z)\chi_m(0) = \exp(inm\frac{\pi}{N})\chi_n(0)\chi_m(z)$$

In the case of anyons placed on an infinite plane these fields can be found in the operator content of the central charge $c = 1$ CFT called the "level N rational torus" [57]. We suspect that in some situations the relevant CFT may be not $c = 1$ theory, but a complete algebra of Z_N parafermions which governs the critical behaviour of the $c = 2(N-1)/N + 2$ CFT (see for example review [58]). The relevance of a such type of model can be seen for anyons placed on a genus $g \geq 1$ Riemann surface. Each state of the system including the ground state is then N^g times degenerate [59]. One could understand the realization of one of these degenerate ground states as a result of a spontaneous discrete Z_N symmetry breaking at the critical temperature T_c . Below we shall restrict ourselves to the case of a torus $g = 1$. N distinct ground states correspond to different values of the N -valued "order parameter" in the Ginzburg-Landau - type description. At temperatures close to T_c this order parameter becomes a real scalar field $\phi(z)$ with an effective potential $V(\phi) = \sum_{k=1}^N g_k \phi^{2k}$, where g_k are some coupling constants. On some two-dimensional surface in the space of the parameters g_k the potential $V(\phi)$ has N degenerate minima (see Fig.9a) [58]. The point where $N-1$ first parameters g_k vanish is an N -critical point, where the Z_N symmetry is restored and some Z_N symmetrical CFT arises (see Fig.9b). So we can expect that at some special points, critical behaviour of the anyon system may be described by such a CFT.

In addition we shall remark about another possible application to CFT, based on the connection between 2+1 Chern-Simons theories and 1+1 CFT [30]. Using this connection we can represent the expectation values of the product of Wilson lines

$$W_q(z_i, z_j) = \exp(iq \int_{z_i}^{z_j} A dx)$$

with endpoints z_i, z_j (see Fig.10) as a correlation function of primary fields of some CFT (a Gaussian rational CFT with $c=1$ in the case of $U(1)$ Abelian

CS theory)

$$\int DA_\mu \exp(i \frac{N}{4\pi} \int AdA) \prod_{i=1}^M W_{q_i - q_{i+1}}(z_i, z_{i+1}) = \langle g_1(z_1) \cdots g_M(z_M) \rangle \quad (54)$$

where $g_i(z_i) =: \exp(iq_i \frac{g(z_i)}{N})$: and $\phi(z)$ is a free scalar field. It is nothing but the wave function of the Chern-Simons theory with static sources q_i attached at points z_i $\Psi(\{A_\mu\}; z_i)$. The case of mobile sources would reproduce the anyon theory (1). An integration over A_μ is equivalent to a transition to the conventional quantum mechanical problem with the Hamiltonian

$$H = \frac{1}{2} \sum_{i=1}^M \{Q_i, Q_i^\dagger\}_+ \quad (55)$$

where

$$Q_i = \frac{\partial}{\partial z_i} - \frac{1}{N} \sum_{i \neq j} \frac{1}{z_i - z_j} \quad (56)$$

Thus is not surprising that the anyon ground state wave function should be a solution of an abelian version of the Knizhnik-Zamolodchikov equation

$$\begin{aligned} Q_i \Psi(z_1, \dots, z_M) &= 0, \quad i = 1, \dots, M; \quad N > 0 \\ Q_i^\dagger \Psi(z_1, \dots, z_M) &= 0, \quad i = 1, \dots, M; \quad N < 0 \end{aligned} \quad (57)$$

The well-elaborated methods of CFT can be used to analyse the correlation properties of anyons and to provide a new insight into the hypotheses discussed above (see, for instance [45], [57], [60]).

6 Conclusion

In conclusion, we have shown that the interaction mediated by the fluctuations of the statistical gauge field smears the mean field Landau levels and lifts up their degeneracy. As a result, one has a finite density of states on the chemical potential level and a finite Debye screening length. This fact is sufficient to destroy the mechanism of anyon superconductivity based on statistical photon - electromagnetic photon conversion independent of possible

cancellation of the bare statistical Chern-Simons term. The gapless mode which appears in such an approach and can be identified with the massless statistical photon in the case of the bare statistical Chern-Simons term cancellation is actually a finite damping zero-sound mode. Nevertheless, the arguments are present that the anyon system could be a superconductor, but due to a quite different mechanism. The origin of this phenomenon is a bounding of anyons into composite bosons. We have observed an attraction between anyons which could be understood as a possible manifestation of this phenomenon and it looks like that attraction obeys some saturation principle, i.e. the real physical excitations have no long-range anyonic properties (some type of non-trivial braiding or fractional statistics confinement).

In the framework of this scenario we have discussed electrodynamics and the finite temperature critical behaviour of anyon matter.

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Fig. 3

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