SIMPLE APPROACHES TO ANTIPROTONIC DEUTERIUM

G.Q. LIU⁽¹⁾

Institut des Sciences Nucléaires
Université Joseph Fourier—IN2P3—CNRS
F-38026 Grenoble Cedex

J.-M. RICHARD⁽²⁾

CERN, Theory Division
CH-1211 Genève 23

and

S. WYCECH

Institute for Nuclear Studies
Warsaw, Poland

Abstract

We use very simple potential models to study the methods one can use to accurately calculate the level shift, the width and the wave-function of the exotic atom consisting of an antiproton and a deuteron.

⁽¹⁾ Present address: Physics Department, University of Adelaide, Australia

⁽²⁾ Permanent address: Institut des Sciences Nucléaires, Grenoble, France

I. Introduction

The level shift and width, $\epsilon = \epsilon_R - i\Gamma/2$, of the protonium ($\bar{p}p$) atom gives a useful measurement of the nucleon-antinucleon ($\bar{N}N$) interaction at rest [1], supplementing the information provided by scattering data. In particular, the real-to-imaginary ratio $q(0) \equiv 2\epsilon_R/\Gamma$ has to be compared to its value $q(E)$ in flight extracted from the Coulomb-nuclear interference. To obtain a reliable value of ϵ for $\bar{p}p$ in a given model, it is sufficient, in most cases, to use the Trueman formula [2], i.e. to compute the scattering length at vanishing energy.

The physics of protonium is, however, not restricted to ϵ . The distortion of the wavefunction, in particular the admixture of an $n\bar{n}$ component, reflects the channel dependence of the NN forces and dramatically influences the analysis of the branching ratios for annihilation into various meson channels [3]. This is why accurate calculations of the $\bar{p}p$ wave-function have been performed in some specific models [4].

Another important problem for the $\bar{N}N$ interaction at rest is the effect of strong interactions in the antiproton-deuteron ($\bar{p}d$) atom. First, the $\bar{p}d$ atom involves not only $\bar{p}p$ but also $n\bar{p}$ interaction at rest, so when combined with the knowledge of the protonium, it provides information on the isospin dependence of the $\bar{N}N$ interaction. Secondly, $\bar{p}d$ is an intermediate between the simple $\bar{p}p$ system and the complex antiprotonic atoms $\bar{p}A$ involving heavy nuclei, for which optical model approaches have been used with some success [5]. Finally the knowledge of the $\bar{p}d$ wave-function is necessary to interpret the data of antiproton annihilation on the deuteron. For instance, if $\bar{p}d \rightarrow K^+K^-n$ were to occur more often than $\bar{p}d \rightarrow K^0K^+p$, one should not draw too immediate conclusions about the isospin dependence of the $\bar{N}N \rightarrow K\bar{K}$ process: one should first know whether, in $\bar{p}d$, the antiproton is more often close to the proton than to the neutron.

Performing an accurate computation of the $\bar{p}d$ energy and wave-function seems at first glance a formidable task: spin and isospin complications, presence of numerous annihilation channels, possibility of $\bar{p}N$ quasi-nuclear states (baryonia), mixture of long-range and short-range forces, etc... Everything, in fact, seems to conspire to make this three-body problem as difficult as possible [6]. We thus wish here to proceed by steps, using first an oversimplified nuclear dynamics, to test some possible methods and, hopefully, receive some feedback from the Few-Body community.

In the present letter, we report on an investigation made with scalar and isoscalar nucleons. First, we use a real potential for the $\bar{p}N$ interaction, chosen as being strongly repulsive, to mimic the effect of violent absorption at short distances. Then we study the effect of annihilation, as parametrized by an imaginary potential.

Our aim, at the present stage, consists of comparing different approaches, such as the

Born-Oppenheimer approximation or variational methods, to obtain (in the case of a real potential) upper and lower bounds on the binding energy and test the accuracy of these methods. In a forthcoming paper, we shall eventually incorporate all necessary complications such as spin dependence, isospin dependence, S-D orbital mixing, etc... Besides the energy shift, we wish to study how reliable and accurate the estimate of the antiproton density is. This is crucial in order to appreciate whether future calculations, with realistic potentials, could predict fine effects such as differences between the annihilation rates on neutron and on proton. This is an important issue, as pointed out by Klempt [3].

The $\bar{p}d$ atom was first studied in Ref. [7], whose authors presented straight away an approximate estimate of the effect of the entire interaction, including absorption. Our approach is somewhat complementary. We shall compare the results obtained using their approximation with those coming from other treatments of the three-body problem. More recently, a three-body calculation with realistic $\bar{N}N$ potentials has been undertaken [8]. The $\bar{p}d$ optical potential is first determined and then used to find the ground state of the two-body atomic system. The accuracy of this method has not been established.

II. Models

We wish to make extensive use of the variational principle and other similar inequalities for three-body systems, to obtain upper and lower bounds on the binding energy and thus test the accuracy of various approximations. Hence we adopt a real potential for the first set of calculations namely models (1) and (2). An imaginary component is introduced in models (3) and (4).

The masses are taken to be the same for all three particles, namely $m = m(p) = m(n) = 1 \text{ GeV}$ and the natural units where $\hbar = c = 1$ are used. We adopt the labelling $(\bar{p}, p, n) = (1, 2, 3)$, and introduce the Jacobi coordinates

$$\vec{x} = \vec{r}_2 - \vec{r}_3 \quad \vec{y} = \vec{r}_1 - \frac{\vec{r}_2 + \vec{r}_3}{2} \quad (1)$$

For the n - p interaction, we use a simple attractive potential

$$V_{np} = -g_0 \exp(-\beta x) \quad (2)$$

with $g_0 = 0.09403 \text{ GeV}$ and $\beta = 0.2 \text{ GeV} \simeq 1 \text{ fm}^{-1}$, which gives a binding energy $\epsilon_d = -2 \text{ MeV}$ for the deuteron. Again, we simply want to propose a prototype. More realistic potentials have been elaborated for the deuteron, and presented e.g. in Ref. [9]. What really matters in the present calculation is the scale associated to the decrease of the deuteron density, i.e. the inverse of $\kappa = (-mE)^{1/2}$ and its ratio to the Bohr radius or to the antiproton-nucleon scattering length.

Beside the Coulomb interaction $V_{pp}^c = -\alpha/|\vec{y} - \vec{x}/2|$ with $\alpha^{-1} = 137$, the antiproton feels the strong potentials V_{pp} and V_{pn} . We use different models, which are not intended to be realistic, but whose range and strength are of the right order of magnitude. In *model (1)*, we adopt a simple exponential shape similar to that of eq. (2)

$$V_{pn}^c(|\vec{y} \mp \vec{x}/2|) = g \exp(-\beta|\vec{y} \mp \vec{x}/2|) \quad (3)$$

with $g = 1$ GeV, and $\beta = 0.2$ GeV.

As for *model (2)*, we use instead a Yukawa type of interaction

$$V_{pn}^c(|\vec{y} \mp \vec{x}/2|) = g \frac{\exp(-\beta|\vec{y} \mp \vec{x}/2|)}{|\vec{y} \mp \vec{x}/2|} \quad (4)$$

with $g = 1$ and the same range $\beta = 0.2$ GeV as in *model (1)*.

Model (3) uses the exponential shape of eq.(3) and an imaginary strength $g = -ig'$, with $g' = 1$ GeV. Similarly, *model (4)* is a Yukawa potential (4) with $g = -i$. Absorptive potentials of this order of magnitude are used in current phenomenological models [10].

III. Comparison of methods

The first reference is the Bohr energy of an antiproton surrounding a point-like deuteron of mass $M = 2m$. It reads

$$\epsilon_0 = -\frac{m}{3} \alpha^2 = -17.76 \text{ keV} \quad (5)$$

A very rough estimate of the energy shift (complex for models 3 and 4) is obtained from the one-body operator

$$V_1(r) = V_{pn}^c(r) + V_{pp}(r) + V_{pn}(r) \quad (6)$$

with a reduced mass $\mu = 2m/3$. It corresponds to neglecting both the size and the distortion of the deuteron. The energy shift is simply computed as $\delta E = E - \epsilon_0$, where E is the binding energy in the potential V_1 . This method will be referred to as *point-like d.*

More realistic calculations should account for the n-p interaction and for the size of the deuteron. The energy shift has thus to be redefined as

$$\delta E = E - \epsilon_d - \epsilon_0 \quad (7)$$

where E is the actual binding of the ($\bar{p}pn$) system.

In the case of a real potential, we obtain an upper bound for E by a *variational* method. For instance, one can account for the size of the deuteron but keep its shape frozen by considering the trial wave-function

$$\Psi(\vec{x}, \vec{y}) = \phi_d(\vec{x})\varphi(\vec{y}) \quad (8)$$

The antiproton wave-function $\varphi(\vec{y})$ is then governed by the one-body operator

$$V_2(\vec{y}) = \int d^3\vec{x} |\phi_d(\vec{x})|^2 [V_{pp}(|\vec{y} - \vec{x}/2|) + V_{pp}^c(|\vec{y} - \vec{x}/2|) + V_{pn}(|\vec{y} + \vec{x}/2|)] \quad (9)$$

We also apply this method to complex potentials.

To calculate lower bounds, an *adiabatic* approach is used. There are several possible ways of splitting the Hamiltonian into two pieces. We shall try two of these. The first one is

$$H = H_z + H_y = \frac{\vec{p}_z^2}{m} + V_{np}(\vec{x}) + \frac{3\vec{p}_y^2}{4m} + V_{pp}(|\vec{y} - \vec{x}/2|) + V_{pp}^c(|\vec{y} - \vec{x}/2|) + V_{pn}(|\vec{y} + \vec{x}/2|) \quad (10)$$

Then one replaces H_y by its lowest eigenvalue $\eta(\vec{x})$ (for strong attractive potentials or complex potentials the eigenvalue closest to the Bohr energy), resulting in the Born-Oppenheimer potential

$$V_3(\vec{x}) = V_{np}(\vec{x}) + \eta(\vec{x}) \quad (11)$$

which overbinds (for real potentials without deeply bound states) since $H_y \geq \eta$ [11]. An alternative splitting of H is

$$H = \widetilde{H}_y + \widetilde{H}_x = \frac{3\vec{p}_y^2}{4m} + \frac{\vec{p}_x^2}{m} + V_{np}(\vec{x}) + V_{pp}(|\vec{y} - \vec{x}/2|) + V_{pp}^c(|\vec{y} - \vec{x}/2|) + V_{pn}(|\vec{y} + \vec{x}/2|) \quad (12)$$

It leads to the alternative Born-Oppenheimer potential

$$V_4(\vec{y}) = \widetilde{\eta}(\vec{y}) \quad (13)$$

where $\widetilde{\eta}(\vec{y})$ is the lowest eigenvalue of $\widetilde{H}_x$ (if V is complex, the eigenvalue with the lowest real part). To compute $\eta(\vec{x})$, one may use a simple partial-wave expansion of the eigenfunctions of H_y

$$f(\vec{x}, \vec{y}) = \sum_{l=0}^{\infty} \frac{u_l(\vec{x}, \vec{y})}{y} \mathcal{P}_l(\hat{x} \cdot \hat{y}) \quad (14)$$

which results in a set of coupled radial equations. We checked that the convergence is very satisfactory as the number of partial waves is increased. For the results displayed below, we include up to D-wave. If $V_{pn} = V_{pp} = 0$, one recovers the pure Coulomb case with shifted coordinates: checking that $\eta(\vec{x}) = \epsilon_0$ for any $\vec{x}$ provides a good test of the numerical algorithms. A similar procedure is used for $\widetilde{\eta}(\vec{y})$. The results for the real and complex shifts are given in Table 1.

approximation	potential	model (1)	model (2)	model (3)	model (4)
point-like d	eq.(6)	4.70	2.74	3.79 - i0.87	3.86 - i1.03
variational	eq.(9)	5.02	3.14	3.79 - i0.87	3.86 - i1.03
adiabatic 1	eq.(11)	4.85	2.93	3.99 - i0.86	4.03 - i1.03
adiabatic 2	eq.(13)	4.47	2.47	4.03 - i1.26	4.19 - i1.19
mult. scatt.	eq.(15)	4.95	3.99	5.10 - i0.28	4.37 - i0.78

Table 1. Energy shifts of the antiprotonic deuterium (in keV), for models (1-4), obtained from various methods.

The last row in Table 1 corresponds to the method of ref. [7], which is a *multiple scattering expansion* for the inverse level shifts. Some intermediate states in the $\bar{p}$ propagation are put on the mass shell, leading to a summation of the dominant contributions. The result has the form of a Trueman formula [2]

$$\Delta E - i\frac{\Gamma}{2} = \frac{2\pi}{\mu_{\text{pd}}} |\bar{\Psi}_A|^2 \{A\} = 3.89 \text{keV} \left\{ \frac{2a}{1 + 1.266a} \right\} \quad (15)$$

where $\bar{\Psi}_A$ is the atomic wave-function averaged over the deuteron density, while A is the $\bar{p}d$ scattering length. The numbers correspond to the deuteron wave-function generated by the n - p interaction (2). In eq. (15), a is the $\bar{p}$ -nucleon scattering length in fm. The four potentials we use give $a = 4.32, 1.65, 4.32 - i1.55, 1.56 - i1.13$ fm. To comply with Trueman's expansion, we use the Coulomb-corrected scattering length A_c given by a boundary condition model [12].

$$\frac{1}{A_c} = \frac{1}{A} - \frac{2}{B} \left[\ln \left(\frac{2R}{B} \right) + 2\gamma \right] \quad (16)$$

where B is the Bohr radius, $\gamma = 0.577$, and a strong radius $R = 3$ fm is used. This correction improves the results by about 10%. The multiple scattering expansion is expected to work well for real a . With complex a , the results are good as long as $|a|$ is smaller than the deuteron radius. Hence, the result for model (4) is consistent with other methods. The large a of model (3) makes the multiple scattering method questionable. We note that the two "adiabatic" methods give different results for model (3).

We now consider the wave-function. Fig. (1) shows the radial density $p(y) = |\varphi(y)|^2$ for the models (1) and (4), in the crude approximation where the deuteron is point-like, in the variational approximation of eq. (8), and in the first of our adiabatic approximations.

In the latter case, $p(y)$ is computed for several values of the internucleon distance x . The results are astonishingly convergent. If one were to include the usual y^2 factor in the radial density, the curves would be hardly distinguishable. For model (4), the densities obtained with a point-like deuteron, a variational approximation or an adiabatic calculation at small x almost coincide. When the deuteron is artificially inflated by freezing x at a larger value, the antiproton feels a stronger absorption and the density is further suppressed. With the real potential (1), the density is slightly more dependent on the approximation which is used. In short, we conclude that the antiproton density is well under control, at least for the models we have tested.

IV. Model dependence

So far, we have discussed the technical aspects. We now compare the results obtained from different antiproton potentials.

In Table 1, it is quite remarkable that potentials with rather different shapes and rather different scattering lengths induce very similar shifts for $\bar{p}d$. The reason is that in all these models, the antiproton feels a strong repulsion for distances below 2 fm and does not see the details of the deuteron structure and of the short-range $\bar{N}\bar{N}$ potential. Once the antiproton is kept outside the deuteron, adding some repulsion to the $\bar{p}$ - N interaction does not change the wave-function much.

To test this explanation further, we have computed the shift of $\bar{p}d$ for potentials with smaller range and weaker strength. Consider for instance a series of variants of model (1), given by eq. (3), where a smaller range is adopted, namely $\beta = 0.4$ GeV instead of $\beta = 0.2$ GeV, and a variable strength g . One obtains the results shown in Table 2.

g (GeV)	adiabatic 1	variational
1.0	2.87	3.02
0.5	2.43	2.59
0.1	1.36	1.39

Table 2. Energy shifts of the antiprotonic deuterium (in keV), for the modified model (1), with shorter range and variable strength.

We still get a good agreement between the various methods, i.e. accurate bounds for the energy shifts. The results now depend rather sensitively on the strength of the interaction.

We have investigated with simple prototypes to what extent it is feasible to treat the antiprotonic deuteron as a genuine three-body problem. In fact, the energy shift and the width are well determined by means of simple approximations which are reminiscent of the variational or adiabatic methods one uses with real potentials. We also checked that the multiple scattering expansion is reliable, provided the antiproton-nucleon scattering length is not too large. The antiproton density turns out to be very stable as one switches from one method to another.

So far, we have carefully avoided any pathological situation. For instance, if the $\bar{p}N$ interaction is attractive and produces a quasi-nuclear bound state very close to the threshold (baryonium), the wave-function will have at least two components, one with an antiproton orbiting around the deuteron, and another where a nucleon is linked to a baryonium. As a result, most of the approximation schemes would run into difficulties.

We hope to present in the future a full calculation of the antiprotonic deuteron with realistic potentials, including their spin- and isospin-dependent components.

Acknowledgements

We would like to thank T.E.O. Ericson and A.M. Green for useful discussions and for comments on the manuscript.

- [1] For a recent review, see C.J. Batty, *Rep. Progr. Phys.* **52** (1989) 1165.
- [2] T.L. Trueman, *Nucl. Phys.* **26** (1961) 57;
for a review, see J. Carbonell, S. Wycech and J.-M. Richard, *in preparation*.
- [3] E. Klempt, *Phys. Lett.* **B244** (1990) 122; **B245** (1990) 129.
- [4] J. Carbonell, G. Ihle and J.-M. Richard, *Z. Phys.* **A334** (1989) 329.
- [5] See, e.g., T.M. Jørgensen *et al.*, in *Proc. 9th European Antiproton Symposium*, Mainz, 1988, ed. E. Klempt *et al.*, Nucl. Phys. B (Proc. suppl.) **vol. 8** (1989).
- [6] C. Gignoux, A.M. Green, S.P. Mercuriev and M.E. Sainio, *private communications*.
- [7] S. Wycech, A.M. Green and J.A. Niskanen, *Phys. Lett.* **152B** (1985) 308.
- [8] C.P. Latta and P.C. Tandy, *Phys. Rev.* **C42** (1990) R1207.
- [9] J.M. Blatt and V.F. Weisskopf, *Theoretical Nuclear Physics* (Wiley, N.Y., 1952).
- [10] C.B. Dover and J.-M. Richard, *Phys. Rev.* **C21**, 1466 (1980);
C.B. Dover and J.-M. Richard, *Phys. Rev.* **C25**, 1952 (1982);
J.-M. Richard and M.E. Sainio, *Phys. Lett.* **110B**, 349 (1982);
M. Kohno and W. Weise, *Nucl. Phys.* **A454**, 429 (1986).
- [11] See, e.g., W. Thirring in *A Course in Mathematical Physics*, vol. 3: *Quantum Mechanics of Atoms and Molecules*, Springer-Verlag, 1979.
- [12] R.H. Dalitz and S.F. Tuan, *Ann. Phys. (N.Y.)* **10** (1960) 307.

Figure caption : Figure 1

Comparison of the radial densities $q(y) = 10^3 p(y)$ computed for the models (1) and (4)

- i) in the crude approximation of a point-like deuteron, as per eq. (6)
- ii) in the variational approximation of eqs.(8-9)
- iii) in the Born-Oppenheimer approximation (BOA), for 3 values of the neutron-proton separation x .

Units are fm for the distance y and fm^{-3} for the density $p(y)$.