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COMPUTATION OF DESIGN PARAMETERS FOR A PULSED QUADRUPOLE IN
THE 2D-TRANSIENT SKIN EFFECT REGIME

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Two dimensional field analysis of a pulsed quadrupole having an aperture within a range of a few millimetres have been made in the transient skin effect regime with the finite element code ANSYS. A complete survey of relevant design parameters, like the average power consumption, magnetic stresses, temperature rises, field energy and peak voltage has been made for a range of conductor geometries and pulse durations. This allows to optimize the design and performance and in particular the focusing strength of the quadrupole.

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Computation of Design Parameters for a Pulsed Quadrupole in the 2D – Transient Skin Effect Regime

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Abstract

Two dimensional field analysis of a pulsed quadrupole having an aperture within a range of few millimetres, have been made in the transient skin effect regime with the finite element code Ansys. A complete survey of relevant design parameters, like the average power consumption, magnetic stresses, temperature rises, field energy and peak voltage has been made for a range of conductor geometries and pulse durations. This allows to optimize the design and performance and in particular the focusing strength of the quadrupole.

1. INTRODUCTION

For the focusing of Tev electron and positron beams at their collision points in linear e^-/e^+ colliders, presently under consideration [1], quadrupoles with gradients of well above 1000 T/m are required. Permanent magnet quadrupoles are being developed [2], which are however limited to apertures of about 1 mm, due to saturation of the material. Alternatively, quadrupoles consisting of four single axial conductors and powered in alternating direction have no such principal limit. In the following, a study has been carried out to provide design figures for such devices, like the magnetic stresses, required currents and dissipated power. In particular the latter will be minimized by powering the magnet with short half-sine like current pulses which are synchronized with the passage of the discrete particle bunches of the incident e^- and e^+ beams.

2. FORMULATION OF THE PROBLEM

The basic design of the quadrupole consist of four single axial conductors in vacuum, each of them arranged equidistantly around the centre on the horizontal and vertical axes of the magnet (see fig.1) representing a pure 2-dimensional problem. By powering the four conductors in alternating direction, an adequate quadrupolar field is achieved along the centre. Due to the 8-fold symmetry of the cross section it is sufficient to model only a sector between a horizontal axis and a diagonal. Since it is envisaged to pulse the magnet by discharging a capacitor bank, it is reasonable to assume half-sine current pulses $I(t) = I_0 \sin(\frac{\pi t}{\tau})$ (with $0 \leq t \leq \tau$, and τ : pulse duration), similar to a classical R-L load switched to a charged capacitor. From this, the time dependent gradient G at the center becomes:

$$G = \frac{\partial B_x}{\partial y} = \frac{\partial B_y}{\partial x} = \frac{4\mu_0 I}{2\pi D_{eff}^2};$$

D_{eff} : Distance between the magnet axis and the centre of gravity of the current distribution within the conductor, depending on τ , t and the geometric configuration of the quadrupole.

Fig.1 : Quadrupole structure

The current I can be introduced into the diffusion equation for the vector potential A , governing the transient skin effect regime, by setting:

$$I = I_s + I_e; \quad \text{with: } I_s : \text{Source current and } I_e : \text{Eddy current.}$$

Following A. Konrad [3] and D. Ostergaard [4], this leads to the integro – differential equation of the vector potential A :

$$\frac{1}{\mu_0} \nabla^2 A - \sigma \frac{\partial A}{\partial t} + \frac{\sigma}{f} \frac{\partial}{\partial t} \iint A df = - \frac{I}{f}; \quad \text{with } f : \text{conductor cross section}$$

where the material parameters are assumed to be constant. Applying the appropriate boundary conditions ($\frac{\partial A}{\partial t} = 0$ along the Y – axis and $A = 0$ along the diagonal direction), the above equation is solved in A , by means of the finite element code Ansys, from where all relevant parameters such as the magnetic field, the field gradient at centre, the current density, energy deposition density, adiabatic temperature rise, total dissipated energy/pulse, field energy and voltage between the conductors can be deduced.

3. COMPUTATIONS AND RESULTS

We consider round, infinitely long copper wires with the specific electrical conductivity $\sigma = 5.88 \cdot 10^7 \text{ S/m}$. The mesh of the structure is chosen in order to optimize the number of elements taking into account the maximum allowed number in a "coupled" analysis [Ref.4], and the expected distribution of current inside the wires (nearly axi – symmetrical). A total current $I(t)$ is imposed as input value, and a nodal map of the vector potential A and current density values J_i in the conductors is obtained as output of the analysis. The calculation of the total current density ($J_t = J_s + J_e$) requires an additional iteration in each calculation step. The magnetic field, total field energy, temperature rises and dissipated energy are calculated with post – analysis routines, involving A and J_i and their integrals over space and time. As an example fig.2 shows the magnetic flux density at the peak current, where $t = \tau/2$, for the geometry $Y = 1 \text{ m}$, $R = 0.5 \text{ m}$ and $I_0 = 1.5 \cdot 10^6 \text{ Ampere}$ (current which in this "unitary reference case" gives a maximum gradient of 1 T/m). Figs. 3 – 4 – 5 show, for these parameters, the distribution of B , J_i and ΔT along the y axis of the structure at different times during the pulse.

Another important aspect is the quadrupolar geometry, which for a fixed magnetic field gradient in the bore, minimizes power supply quantities like power loss through Joule effect, voltage and peak current. The variables considered are: the half – aperture Y , the wire radius R resp. the ratio R/Y , the peak current I_0 and the pulse duration τ resp. the skin depth defined as $d = \sqrt{\frac{2\rho\tau}{\mu_0\pi}}$. For this reason various different cases are analyzed changing geometries and current pulse parameters. Fig.6 shows the energy loss per meter length of the magnet and per pulse, versus the ratio R/Y , for 3 different pulse durations and a "no skin effect case" for which a current density uniform in space but proportional to $I(t)$ is assumed. Fig.7 shows the peak voltage required for a one meter long quadrupole with the wires connected in series, versus the ratio R/Y , for 3 different pulse durations. Fig.8 shows the maximum magnetic field at the surface of the conductors, versus the ratio R/Y , for 2 different pulse durations. In general it results that a quadrupole with $R/Y \approx 0.5$ represents an optimum choice. Sets of design parameters are given in table 1 for e^-/e^+ – focusing quadrupoles with half – apertures Y of 1 mm and 2 mm , for various gradients G and pulse durations τ resp. skin depths d . All these values have been obtained by appropriately scaling the results from the "unitary reference case" with $Y = 1 \text{ m}$, and $G = 1 \text{ T/m}$.

Fig. 3 : Magnetic field along one axis (in the bore and in the wire)

Fig. 4 : Total current density along one axial diameter

Fig. 5 : Temperature increase along one axial diameter

Fig. 2 : Magnetic flux lines at $t = \tau/2$

Quadrupole half – aperture Y: 1 mm, R/Y ratio: 0.5, length L: 1 m.

$G(B)$	d	I_{peak}	E_{loss}	T_{max}	B_{max}	S_{mag}	E_{sto}	$Volt$
1000	0.05	2.60e3	0.40	0.41	1.28	2.50	3.00	2.42e4
1000	0.15	2.66e3	1.40	0.50	1.25	0.98	3.39	3.10e3
1000	0.25	2.72e3	2.68	0.56	1.22	0.67	3.76	1.20e3
2500	0.05	6.50e3	2.52	2.58	3.03	15.60	18.76	6.04e4
2500	0.15	6.65e3	8.76	3.12	3.07	6.14	21.16	7.76e3
2500	0.25	6.80e3	16.76	3.53	3.06	4.18	23.52	3.01e3
5000	0.05	1.30e4	10.08	10.32	6.06	62.40	75.04	1.21e5
5000	0.15	1.33e4	35.04	12.48	6.24	24.60	84.64	1.55e4
5000	0.25	1.36e4	67.04	14.12	6.12	16.70	94.08	6.02e3

Quadrupole half – aperture Y: 2 mm, R/Y ratio: 0.5, length L: 1 m.

$G(B)$	d	I_{peak}	E_{loss}	T_{max}	B_{max}	S_{mag}	E_{sto}	$Volt$
1000	0.05	1.04e4	6.45	1.65	2.42	4.99	48.03	2.42e4
1000	0.15	1.06e4	22.26	1.98	2.49	1.95	53.76	3.09e3
1000	0.25	1.09e4	43.06	2.27	2.45	1.34	60.43	1.20e3
2500	0.05	2.60e4	40.30	10.32	6.06	31.20	300.16	6.04e4
2500	0.15	2.66e4	140.16	12.48	6.24	12.30	338.56	7.76e3
2500	0.25	2.72e4	268.16	14.12	6.12	8.36	376.32	3.01e3

Table 1

$G(B)$: magnetic field gradient [T/m], d : skin depth [mm],

I_{peak} : peak current in the pulse [A], E_{loss} : energy loss per pulse [J],

T_{max} : temp. rise per pulse [K], B_{max} : max. magn. field on y axis [T],

S_{mag} : max. magnetic stress [dN/mm²], E_{sto} : field energy at peak current time [J], $Volt$: max. voltage required [V]

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