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Analysis of the quench propagation along Nb₃Sn Rutherford cables with the THELMA code. Part I: Geometric and thermal models

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ABSTRACT

The paper describes the new lumped thermal model recently implemented in THELMA code for the coupled electromagnetic–thermal analysis of superconducting cables. A new geometrical model is also presented, which describes the Rutherford cables used for the accelerator magnets. A first validation of these models has been given by the analysis of the quench longitudinal propagation velocity in the Nb₃Sn prototype coil SMC3, built and tested in the frame of the EUCARD project for the development of high field magnets for LHC machine. This paper shows in detail the models, while their application to the quench propagation analysis is presented in a companion paper.

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1. Introduction

The new generation of superconducting magnets for thermonuclear fusion and for particle accelerators will store so huge magnetic energy and will be subjected to such severe loads that a detailed analysis of their working conditions and of the efficiency of their protection is fundamental to prevent possible irreversible damages and to permit an accurate magnet control.

Nominally, the conductors of both types of magnets have similar critical aspects regarding thermal stability, mechanical stresses and quench protection, however the different technical solutions adopted for their winding, cooling and the different environmental conditions cast these problems in quite different ways. As an example, Table 1 compares operational parameters of one ITER TF coil and MQXF, a prototype of new generation accelerator Low- β quadrupole magnet developed in the frame of the High Luminosity LHC project [1–5].

In the case of fusion magnets, the use of cable-in-conduit conductor (CICC) should give satisfying mechanical properties and guarantee an efficient conductor cooling even in the presence of plasma perturbations [6], however the impact of the resistive joints thermal load and possible termination-induced current imbalances on the cable stability are still an open issue. On the other side, the new generation of accelerator magnets will make

use of Rutherford cables made with Nb₃Sn conductors, capable of generating a magnetic field of 16 T [7], and potentially up to 20 T [5,8]. Compared with fusion magnets, accelerator magnets are characterised by much stricter geometrical constraints, lower energy density and larger current density and number of turns, as shown in Table 1. For them, no He forced-flow cooling is possible therefore, in the case of a quench, thermal effects may have easily wasteful consequences even in the presence of lower energy densities, if no proper quench detection and triggering system were adopted.

To improve the manufacturing technology of both these types of new generation magnets and to permit the validation of numerical analysis codes, prototypes have been and are still being manufactured and tested in several laboratories.

The current distribution in the Rutherford cables has been extensively analysed from both the theoretical and numerical points of view, in order to predict field errors, AC losses and stability margins (as a non exhausting bibliography, see e.g. [9–14]). As regards the numerical models, both lumped and distributed parameter electrical circuits have been adopted.

The THELMA code, originally developed for the analysis of superconducting CICC for fusion magnets, has been recently empowered with a brand-new thermal model, to permit the analysis of coupled electromagnetic–thermal problems under more general hypotheses. This model has already been used for a predictive analysis of the NAFASSY magnet, a large-bore magnet with a field up to 6.7 T, to be employed for the test of superconducting

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Table 1
Data for one ITER TF coil and High Luminosity LHC Low- β quadrupole MQXF.

	ITER TF coil	MQXF
Stored energy	2.3 GJ	12 MJ
Energy density in the coils (J/mm ³)	0.67	0.12
No. of turns	134	200
Cable current (kA)	68	17.5
N. of SC cable conductors	900	40
Eng. current density (A/mm ²)	143	771
Non Cu current density (A/mm ²)	286	1411
Operating temperature (K)	5	1.9

devices and cables, suitable for applications to controlled thermonuclear fusion reactors and accelerators [15]. To this purpose, new code modules have been added, to describe the material non-linear thermal properties and to model the thermal conduction in transient regime in terms of a non-linear lumped network.

Furthermore, to tackle the analysis of the new generation accelerator magnets, a new geometrical model which describes the Rutherford cable geometry has also been implemented. Thanks to its flexibility, THELMA can analyse the thermal and electromagnetic behaviour of Rutherford cables and CICC's at the strand level, modelling both the electrical and thermal contact resistances between strands and enabling the analysis of the effects of local hot spots and possible quench heaters.

This paper describes in detail the new geometrical model of the Rutherford cable and the new thermal model, showing in detail the two models assumptions. The results of the code application to a quench propagation problem are presented in a companion paper, where the computed quench propagation velocity is compared with the experimental results taken from the Short Model Coil 3 prototype, developed in the frame of a CEA-CERN-RAL-LBNL cooperation, which is part of the EUCARD project. The coil was tested in 2011 in the new vertical test facility of CERN building SM18 [16,17]. A good agreement has been found between computed and measured values, giving a first validation of the new THELMA modules.

2. The THELMA model of Rutherford cables

2.1. Geometrical model

In THELMA, the cable (CICC or Rutherford) is a 3D object characterised by its curvilinear axis, around which the individual strand geometry is built by means of a local reference frame defined at each point of the cable axis:

$$\mathbf{OP}_s(s) = \mathbf{OP}_a(s) + x_2(s)\mathbf{u}_2(s) + x_3(s)\mathbf{u}_3(s), \quad (1)$$

where $\mathbf{OP}_s$ and $\mathbf{OP}_a$ are the position vectors of the strand and the cable axis at its curvilinear coordinate s and $\mathbf{u}_2, \mathbf{u}_3$ are the unit vectors of the local reference frame. In the case of a Rutherford cable (Fig. 1) coordinates $x_2(s), x_3(s)$ correspond to the cartesian coordinates of a rectilinear cable with equivalent cross-section and transposition pitch, having its axis parallel to coordinate $x_1 = s$. The geometry of the strand axis in this rectilinear cable is described analytically in terms of a set of straight and circumference arc segments, so that no sharp corners exist along the axis and a continuous local reference frame can be considered along the strand axis. This feature is important as regards the numerical calculation of the magnetic field generated by the strand. When the rectilinear cable is mapped onto the curvilinear one, the strand final curvature and curvilinear coordinate are suitably computed starting from the strand final axis points coordinates.

The geometrical parameters of the cable are described in Fig. 2. Keystoning can be taken into account, and this may involve a geo-

Fig. 1. Sketch of the Rutherford cable geometry generation. Left, above: cable curvilinear axis with outlines of its cross-sections. Left, below: cable with rectilinear axis. Right: final geometry.

Fig. 2. Above: sketch of the Rutherford cable cross-section. Below: longitudinal view of a set of strand axes. The keystoning angle α is exaggerated and the strand axes are not smoothed for clarity purposes.

metrical interference of the strands in correspondence to the cross-section narrow edge. The twisting angle φ , in our model used for both the in-plane and the out-of-plane transpositions, is:

$$\varphi \approx \tan^{-1} \left(\frac{l_{tr}}{2 \left(h_{med} + \frac{w}{\cos \alpha/2} \right)} \right) \quad (2)$$

where l_{tr} is the transposition pitch, w the cable width and $h_{med} = (h_{max} + h_{min})/2$ is the cable average thickness. A pictorial view of a curved cable segment is given in Fig. 3.

2.2. Electromagnetic model

To model the cable from the electromagnetic (EM) point of view, the THELMA distributed parameter cable model has been used [18,19]. In the case of the Rutherford cables, all the cable strands are individually represented. Along the cable, each strand is divided into N_{el} longitudinal strand elements. The N_{el} value is suitably set according to the level of space resolution of the electrical and thermal phenomena investigated. Along each strand element the current is supposed to vary linearly.

Fig. 3. Pictorial view of a curved segment of a 14 strands non-keystoned Rutherford cable.

These strand elements are inductively coupled with each other and inter-strand current transfer can take place through distributed conductances, whose values are computed as described below. The model considers a non linear distributed network and its unknowns are the strand current imbalances with respect to the cable transport current uniformly distributed. Different strand scaling laws are available according to the superconductor type.

In the cable model, the strand cross-section is considered as the electrical parallel of the Cu stabilizer, with area A_{Cu} and resistivity $\rho_{Cu}(B, T)$, and the superconducting (SC) material, with area A_{SC} and governed by the power law (Fig. 4): The following equation holds:

$$I_{st} = I_{SC} + g_{Cu} E_c \left(\frac{I_{SC}}{I_c} \right)^n, \quad (3)$$

where I_{st} is the strand total current, I_{SC} and I_c are respectively the total current and the critical current in the superconducting part of the strand, $g_{Cu} = A_{Cu}/\rho_{Cu}$ is the stabiliser longitudinal conductance per unit of length. During the solution, this equation is numerically solved in terms of I_{SC} as a function of I_{st} .

Presently, the model does not take into account the possible presence of screening currents inside the strands, due to self and background magnetic fields. This makes the model less suitable to describe the cable behaviour in the presence of low transport currents, when instabilities like flux jumps may be more important. The implementation of models of the intra-strand screening currents is one of the possible models upgrades.

2.2.1. Inter-strand electrical contact resistances

To describe the inter-strand current transfer, the distributed parameter cable model considers the inter-strand per unit length contact conductances $\sigma_{ij}(s)$ (S/m) between the couples of strands i and j [19]. These parameters are automatically computed as a function of the cable curvilinear coordinate starting from the strands geometry and the contact electrical properties, described

Fig. 4. Model of an infinitesimal strand segment as the parallel of a linear resistor and a superconductive non linear resistor. For clarity purposes, all the SC filaments have been concentrated in the central part of the strand.

by the spot R_s (Ω) and the distributed contact resistances R_d (Ω m) [20], which can be individually assigned to the cable strands.

The spot contact resistance is present when two strand have a localised small contact area (Fig. 5(a)), while the distributed contact is present when the two strands are in continuous contact along their longitudinal direction (Fig. 5(b)).

Usually, to describe the electrical resistances of a Rutherford cable, the cross-over R_c and the adjacent R_a resistances are used [21,22]. The first one corresponds directly to the series of the THELMA spot resistances of the two involved strands, while the second is associated also to the length of the crossover between two strands $l_a \approx l_{Tr}/(2(N_{st} - 1) \sin \varphi)$, being N_{st} the number of cable strands. Therefore:

$$R_{c_{ij}} = R_{s_i} + R_{s_j} \quad (4)$$

$$R_{a_{ij}} = (R_{d_i} + R_{d_j})/l_a \quad (5)$$

For the numerical solution of the EM model, conductances $\sigma_{ij}(s)$ are replaced by a finite set of values $\sigma_{ij}(\tilde{s}_k)$, with $k = 1, \dots, N_{el}$, located at the middle length $\tilde{s}_k$ of each strand element. To this purpose, the cable geometry is scanned at a large number of axial locations and the single local conductances are computed. Then, for each i, j and $\tilde{s}_k$, $\sigma_{ij}(\tilde{s}_k)$ is obtained as the average conductance associated to l_k , as sketched in Fig. 6.

With this approach, the level of resolution for the description of the EM phenomena along the cable is not strictly linked to the cable transposition pitch or to the band length, as in [14], and can be selected according to the geometrical size of the phenomena considered.

2.3. Thermal model

A brand-new thermal (TH) analysis model has been implemented in the THELMA code, based on a lumped network approach. This model is alternative to the original TH module, which is focused on the CICC strand-helium heat exchange and can take into account phase and mass-flow rate changes along the cooling channel with time. The thermal network is made of a set of lumped components, each representing a given thermal property, namely the thermal capacitance of a given solid body, the thermal conductance between two bodies, the thermal exchange between solid and fluid. In this network, a set of N_{th} nodes is present, at which the temperatures T_k , $k = 1 \dots N_{th}$, are defined as impressed or unknown quantities. The network components connect these nodes, giving rise to a set of L_{th} branches, along which the heat exchange occurs, expressed in terms of a set of heat currents Φ_k , $k = 1 \dots L_{th}$. A general-purpose library of thermal components has been developed, including temperature and heat current generators, linear and non linear thermal conductances (or, as an alternative, resistances), and capacitances. All these components can be connected in whatever configuration, thus permitting a high flexibility. The generators are used to impose boundary conditions and to represent known heat sources both in steady-state

Fig. 5. Contact area A_c for strands in (a) spot and (b) distributed contact.

Fig. 6. Simplified view of a Rutherford cable, showing the discretisation into strand elements whose length is l_k . Outlined are the two strand zones considered for the contact conductance in the case of adjacent strands.

and in transient regime. All these generators are inserted between the generic network node and the mass node, corresponding to the reference zero temperature.

In the case of the non linear thermal capacitance and conductance, the network dipole laws are respectively expressed as:

$$\begin{aligned} \Phi_i &= C_{thi} \frac{dT_i}{dt}, \quad \text{with} \quad C_{thi} = \mathcal{V}_i \gamma(T_i) c(T_i, B_i), \\ \Phi_{ij} &= G_{thij} (T_i - T_j), \quad \text{with} \quad G_{thij} = \mathcal{G}_{ij} k_{th}(T_i, T_j), \end{aligned} \quad (6)$$

where $\mathcal{V}_i$ (m^3) and $\mathcal{G}_{ij}$ (m) are constant geometrical factors, γ is the mass density (kg/m^3), c the specific heat ($\text{J}/\text{kg K}$) and k_{th} the thermal conductivity ($\text{W}/\text{m K}$). All these material properties can be function of temperature, magnetic field and other state variables, according to the model adopted to describe the material [23]. During the system solution, at each timestep, these material properties and therefore the corresponding network parameters are updated whenever their state variables change exceed a given threshold.

The network equations are written in terms of the temperatures T_k of all the nodes and branch heat currents Φ_k for all the heat current-driven components using the *modified node approach* [24]. With this choice, no Kirchhoff's law over the network meshes needs to be written. The only needed Kirchhoff's equations come from the heat current balance at the nodes. Since each node corresponds to an unknown temperature it is preferable, whenever possible, to use temperature-driven components in place of heat current-driven components (e.g., using thermal conductances in place of thermal resistances).

The resulting system of algebraic–differential equations can be written as:

$$\begin{pmatrix} \mathbf{A}_r^T & \mathbf{R}_{th} \\ \mathbf{G}_{th} & \mathbf{A}_r \end{pmatrix} \cdot \begin{pmatrix} \mathbf{T} \\ \mathbf{\Phi} \end{pmatrix} + \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{C}_{th} & \mathbf{0} \end{pmatrix} \frac{d}{dt} \begin{pmatrix} \mathbf{T} \\ \mathbf{\Phi} \end{pmatrix} = \begin{pmatrix} \mathbf{T}^i \\ \mathbf{\Phi}^i \end{pmatrix}, \quad (7)$$

where $\mathbf{T}^i, \mathbf{\Phi}^i$ are the arrays of the impressed temperatures and heat currents, $\mathbf{A}_r$ is the thermal incidences reduced matrix [25], $\mathbf{R}_{th}, \mathbf{G}_{th}$ are the matrices of thermal resistances and conductances, $\mathbf{C}_{th}$ is the matrix of thermal capacitances and $\mathbf{T}, \mathbf{\Phi}$ are the arrays of the unknown temperatures and heat currents in the heat current-driven branches. To solve the system, equations are sorted by grouping the algebraic (a) and differential (d) equations. Correspondingly, the system unknowns are grouped into differential variables (d) (i.e., the temperatures of nodes connected to capacitances) or simple algebraic (a) unknown quantities (i.e., all the other unknown temperatures and the heat currents). The sorted form of the system is the following [26]:

$$\begin{pmatrix} \mathbf{D}_{a,a}^{th} & \mathbf{D}_{a,d}^{th} \\ \mathbf{D}_{d,a}^{th} & \mathbf{D}_{d,d}^{th} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{X}_a \\ \mathbf{X}_d \end{pmatrix} + \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{F}_{d,d}^{th} & \mathbf{0} \end{pmatrix} \frac{d}{dt} \begin{pmatrix} \mathbf{X}_a \\ \mathbf{X}_d \end{pmatrix} = \begin{pmatrix} \mathbf{Y}_a \\ \mathbf{Y}_d \end{pmatrix}, \quad (8)$$

where $\mathbf{X}_a, \mathbf{X}_d$ are the sorted algebraic and differential unknowns, $\mathbf{Y}_a, \mathbf{Y}_d$ are the sorted algebraic and differential known terms, $\mathbf{D}_{a,a}^{th}$

to $\mathbf{F}_{d,d}^{th}$ are the sorted submatrices of the linear and non linear parameters. In order to get an efficient solution algorithm, each of these matrices is made of a linear part, constant with time, and a non linear part, straightforwardly updated when the material properties change. In the end, these two equation systems are obtained, which are solved at each iteration:

$$\begin{cases} \mathbf{X}_a = \mathbf{D}_{a,a}^{th}{}^{-1} \mathbf{Y}_a - \mathbf{U}_{a,d}^{th} \mathbf{X}_d \\ \frac{d}{dt} \mathbf{X}_d = \mathbf{F}_{d,d}^{th}{}^{-1} \mathbf{Y}_d - \mathbf{U}_{d,a}^{th} \mathbf{X}_a - \mathbf{U}_{d,d}^{th} \mathbf{X}_d \end{cases} \quad (9)$$

where $\mathbf{U}_{a,d}^{th} = \mathbf{D}_{a,a}^{th}{}^{-1} \mathbf{D}_{a,d}^{th}$, $\mathbf{U}_{d,a}^{th} = \mathbf{F}_{d,d}^{th}{}^{-1} \mathbf{D}_{d,a}^{th}$, $\mathbf{U}_{d,d}^{th} = \mathbf{F}_{d,d}^{th}{}^{-1} \mathbf{D}_{d,d}^{th}$.

Since all the thermal capacitances are inserted between the generic node and the mass node, matrix $\mathbf{F}_{d,d}^{th}$ is diagonal, which permits its straightforward inversion and therefore the direct computation of $\mathbf{U}_{d,a}^{th}$ and $\mathbf{U}_{d,d}^{th}$. In addition, in transient regime, the almost ubiquitous presence of thermal capacitances makes the large majority of the nodes of the differential type, therefore matrix $\mathbf{D}_{a,a}^{th}$ has a small size, since it corresponds only to the few nodes connected to temperature generators, which correspond to Dirichlet boundary conditions (i.e., impressed temperature or isothermal conditions), and few more conductances/resistances, if any. To solve the algebraic systems, an iterative approach is usually preferable, the use of LU decomposition being convenient only in the case of linear conductive properties, which give constant matrices $\mathbf{D}$. The system of ordinary differential equations is numerically solved by means of a Runge–Kutta method of the fifth order.

2.3.1. Thermal network of the Rutherford cable

The thermal network of the Rutherford cable is automatically created starting from the cable geometry and its material properties. In the cable, both distributed and concentrated heat currents are present, due to the strand longitudinal conduction, the inter-strand thermal contacts and, if any, the distributed exchange with helium or the impregnated insulating material. To properly describe these heat currents, a mixed distributed and lumped model should therefore be considered. Hence, the use of a lumped network requires a careful discretisation in order to achieve the maximum accuracy. As regards the strands, the *discrete formulation* of the thermal conduction has been adopted, which is directly written in terms of global variables, i.e., variables associated to volumes, surfaces, lines or nodes of two staggered, the *primal* and the *dual, complexes of cells*, which are space regions with finite size [27]. Some of the model variables are associated to the primal cell complex, the others to the dual complex: the temperature is associated to the primal nodes N_k located at the corners of primal cells, while the heat current is associated to the boundary surfaces $\tilde{S}_k$ between adjacent dual cells. It can be shown that this approach permits a better accuracy with respect to a corresponding first-order numerical method (like finite volumes method) based on a single mesh discretisation. In this way the thermal model differential equations are replaced by a set of balance and constitutive equations, which can be naturally mapped into the Kirchhoff's and dipole laws of a lumped network.

To build the two cell complexes of a strand, each strand segment is viewed as a 1D cell V_k of the primal cell complex. The primal mesh nodes N_k are located at the axial coordinates s_k corresponding to the boundaries between adjacent primal cells (Fig. 7). Each dual cell $\tilde{V}_k$ corresponds to a primal node, whose temperature T_k is associated to the whole dual cell and is used to identify its internal energy (Fig. 8(a)). The strand element length must therefore be so short that temperature differences along the dual cell can be neglected. Each dual boundary surface $\tilde{S}_k$ between adjacent dual cells corresponds to a dual node $\tilde{N}_k$ and to a segment between two primal nodes, whose temperature difference is used

Fig. 7. Sketch: (a) of the strand elements; (b) of the primal nodes N_k along the strand axis, (c) of the dual cells (volumes) $\tilde{V}_k$ and (d) of their dual surfaces $\tilde{S}_k$.

Fig. 8. Topological correspondence between (a) a primal node N_k and a dual volume $\tilde{V}_k$, and (b) between a primal line and a dual boundary surface $\tilde{S}_k$.

to express the heat current across the surface (Fig. 8(b)). In the case of non linearity, the relevant thermal conductivity is evaluated at the average temperature of the two nodes.

To build the lumped network, the i -th strand primal mesh nodes correspond to the network nodes N_k^i , each dual cell corresponds to a thermal capacitance C_k^i , connected between the node and ground, and each dual boundary surface corresponds to a longitudinal thermal resistance/conductance $G_{k-1,k}^{i,i}$, as shown in Fig. 9. The thermal power generated by the electrical resistances and cable losses is represented by heat current generators $P_k^{g,i}$, also connected between the nodes and ground.

The inter-strand thermal conductances are also taken into account through suitable thermal conductances $G_k^{i,j}$, which are considered variable with temperature according to a power law [28]:

$$G_k^{i,j}(T) = \alpha T^\beta, \quad (10)$$

with α, β are suitable constant determined through a numerical fit of experimental results.

Fig. 9. Sketch of the thermal network, showing the components connected to two strand nodes. For clarity purposes, only two strands are shown.

2.4. Coupling between the thermal and the electromagnetic models

The coupling between EM and the TH models is explicit. This means that the two systems of differential–algebraic equations are solved independently with internal independent time steps and solution methods. An implicit coupling, i.e. a combination of the two models into a single system of equations with a single timestep has not been considered, on the one hand being less modular and flexible, on the other hand being the EM and the TH models characterised by quite different characteristic time constants, with consequent possible useless computations and an increased problem stiffness in the case a single overall system of equation were to be solved. Since both the EM and TH modules can deal independently with their respective electric and thermal non linearities, the only missing detail is the strategy to take into account the non linear behaviour with temperature in the EM module. A coupling time step h_c is defined to this purpose. The following phases are suitably cycled during the coupled problems solution:

- The evolution of EM model during time step h_c is computed assuming the initial temperatures or those given by the TH model during the previous coupling time step.
- The EM losses are computed, averaged over h_c and used as input for a corresponding TH module integration, again over h_c .
- The temperatures are computed and the maximum temperature change ΔT_{max} is compared with a given threshold Θ_{max} , which is a model input parameter. If $\Delta T_{max} > \Theta_{max}$, h_c is reduced and the thermal module solution is repeated until $\Delta T_{max} \leq \Theta_{max}$.

In this way any strong change of material properties with temperatures, e.g. in the proximity of the SC material resistive transition, can be taken into account. Both in the case of rejection and acceptance, the coupling time step h_c is adjusted according to this expression, that permits a smooth reduction of the time step as well as its increase:

$$h_c^{new} = 0.9 h_c \sqrt[4]{\frac{\Theta_{max}}{\Delta T_{max}}} \quad (11)$$

This expression is based on a similar formula for step-size control taken from [29].

3. Applications

For a first model validation of both the Rutherford geometric and the thermal models, the tests results from the sub-scale prototype coil Short Model Coil 3 (SMC3) have been selected [16]. SMC3 is formed by two double pancake coils, with a Rutherford cable made of 14 Nb₃Sn strands, with a diameter of 1.25 mm and a Cu/non Cu ratio 1.25. The short sample critical current at 11 T is 15,400 A at 4.2 K and 18,500 A at 1.9 K. The coil is inserted in an iron yoke suitably shaped to achieve the maximum magnetic field in the central straight part of the coil, which is 150 mm long. In the tests, SMC3 was fed with ramps of current until quench occurred. A plateau quench current of about 14 kA and a magnetic field up to 12.5 T in the conductor were reached.

The details of the coil THELMA model and its validation are reported in [17]. Here some additional results of this model are presented, to show the model capability to describe the cable local behaviour.

The analyses have been carried out assuming $R_c = 1$ mΩ and $R_a = 9.4$ μΩ as cable contact resistances [30], since no specific measurement on the coil cable were available. In the model of SMC3, all the Rutherford cable strands are individually represented and a 150 mm cable straight segment, corresponding to part of the coil

Fig. 10. SMC3 coil quench model: computed adimensional current distribution among the strands at $T_0 = 4.2$ K, $I_t = 14$ kA and $B = 12.5$ T. Above: 0.3 ms, middle: 1 ms, below: 2.5 ms after the disturbance.

innermost turn is modelled. The model longitudinal resolution, corresponding to a strand element length, has been set to about 1 mm. This value has been determined from a convergence analysis (not shown) carried out considering decreasing longitudinal resolutions, until the model results were almost independent on this parameter. Furthermore, the resolution adopted is in agreement with the geometrical size of the smallest detail represented in the model, i.e. the inter-strand spot contact.

Initially, the 14 cable strands carry a balanced constant current and a quench is triggered by applying a short heat pulse at the middle length of one of them. Fig. 10, reports the strand adimensional current, i.e. referred to a uniform distribution, as a function of the cable axial coordinate for different time values. At the very beginning of the quench, the heated strand shows a strong current reduction, taken over mainly by the two adjacent strands, which carry almost equal currents. Soon after, the transition propagates mainly in the cable cross-section through the adjacent thermal conductances, then the longitudinal propagation takes place and the strand current imbalances are strongly reduced in the strand quenched zone. The strand current imbalances propagate along the cable according to the temperature propagation.

4. Conclusions

The THELMA code, originally developed for the study of cable-in-conduit conductors, has been endowed with a new geometrical

model of Rutherford cables and a new thermal analysis module, coupled with the existing electromagnetic modules. In this way new applications of the THELMA code are possible, which include the study of stability of the new generation accelerator magnets and the analysis of the thermal conduction problems in cable-in-conduit resistive joints. A first validation of the code has been achieved with the quench longitudinal propagation velocity analysis on the Nb₃Sn prototype Short Model Coil 3, tested by CERN in 2011.

Further model developments are going to be implemented, among them is the thermal model of insulating layers and cable-in-conduit jackets. In this way it will be possible to study the quench propagation in Rutherford magnets also in transverse direction and the efficiency of the cable heaters as regards the quench protection. Another interesting subject will be the analysis of the temperature distribution in cable-in-conduit jackets, to evaluate possible temperature non uniformities affecting the measurements carried out on the jacket external surface.

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