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NON-PERTURBATIVE CONTRIBUTION TO TOTAL CROSS-SECTIONS IN NON-ABELIAN GAUGE THEORIES

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Abstract

We consider the instanton-induced contribution to the total cross-section of vector boson scattering in the $SU(2)$ gauge theory with and without Higgs mechanism. The exponential suppression of instanton-induced processes disappears asymptotically at energies far above the infrared cutoff for the instanton scale-size, which is provided, in the Higgs (confinement) phase, by the vacuum expectation value (virtuality of the gluons). Instanton-induced processes may therefore be important in close-to-mass-shell gluon collisions. Our finding also suggests that electroweak baryon- and lepton-number violation with the associated production of many W-bosons may become important at very high energies.

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The possibility [1-5] that the scattering of weakly-interacting particles becomes non-perturbative at energies above $\sim m_w/\alpha_w \sim 10$ TeV has recently gained a lot of attention. This possibility is suggested by the leading-order instanton calculations [1-3] for anomalous baryon- and lepton-number violating processes [6] in the electroweak theory, like

$$q + q \rightarrow 7\bar{q} + 3\bar{l} + n_h H + n_w W , \quad (1)$$

which yield [7-9] an exponential growth of the total cross-section, which is dominated by multi-W production,

$$\sigma_{\text{tot}}^{\Delta(B+L)} \sim \exp \left[\frac{4\pi}{\alpha_w} F \left(\frac{\sqrt{s}}{M_0} \right) \right] , \quad (2)$$

$$F(\epsilon) \simeq -1 + \left(\frac{9}{16\sqrt{8}} \right)^{2/3} \epsilon^{4/3} , \quad \epsilon \equiv \sqrt{s}/M_0 . \quad (3)$$

Here $M_0 = \pi m_w/\alpha_w$ is of the order of the sphaleron [10] energy, the minimum energy barrier between topologically inequivalent vacua [11] in the electroweak theory. Taking eqs. (2),(3) at face value, we would conclude that electroweak multi-particle production and (B+L)-violation becomes strong at $\sim M_0$. However, since the leading-order instanton calculations violate unitarity at high energies, the higher-order corrections must be substantial at $E \sim M_0$. It is expected [8,12-14] that the higher-order corrections (in the single-instanton sector) to the basic amplitudes for the processes (1) leave the form of the total cross-section, as indicated in eq. (2), unchanged. The fate of these non-perturbative processes therefore depends crucially on the behaviour of the function F : only if F approaches zero at high energies is there a chance that electroweak (B+L)-violation in association with multi-W production becomes relevant at future colliders. Unfortunately, we know F at present only at low energies, $\epsilon \ll 1$, where it is given by eq. (3) (the next-to-leading-order term in an expansion in terms of ϵ has been found in ref. [15]).

In this letter we want to discuss the general behaviour of the function F . We use the method outlined in ref. [15], namely we investigate the instanton/anti-instanton ($I\bar{I}$) contribution to the imaginary part of the forward elastic scattering amplitude by means of the valley method [16,17]. We consider the fundamental SU(2)-Higgs model in both phases, the confinement and Higgs phase. We are interested in the non-perturbative, instanton-induced contribution to the total cross-section for vector boson scattering. The dispersion relation (optical theorem) for the total cross-section reads

$$\sigma_{\text{tot}} \sim \frac{1}{s} \cdot \text{Im } \mathcal{T}(p_1, p_2, p_1, p_2) = \frac{1}{s} \cdot \text{Im} \left\{ (\text{LSZ}) \left(\langle T \prod_{i=1}^4 A(x_i) \rangle \right) \right\} . \quad (4)$$

Here $\mathcal{T}$ denotes the elastic scattering amplitude and “LSZ” stands for the usual LSZ procedure. The euclidean functional integral for the 4-point Green function entering in eq. (4),

$$G_E = N^{-1} \int \mathcal{D}A \mathcal{D}\Phi \prod_{i=1}^4 A(x_i) \exp(-S_E[A, \Phi]) , \quad (5)$$

receives, in general, dominant contributions from usual perturbation theory around the perturbative vacuum as well as contributions from non-trivial configurations of steepest descent which arise due to the existence of (anti-)instanton solutions [6,18,19].

Since we are interested in the non-perturbative, instanton-induced contribution to the total cross-section, we will consider here only the contribution of the $I\bar{I}$ configuration.

This contribution can be calculated systematically by semiclassical methods [15-17]. Let $\phi_{I(\bar{I})} = \{A_{I(\bar{I})}, \Phi_{I(\bar{I})}\}$ be the $I(\bar{I})$ solution [6,18,19]. It is characterized by a set of collective coordinates $\tau_{I(\bar{I})} = \{\rho_{I(\bar{I})}, x_{I(\bar{I})}, U_{I(\bar{I})}\}$, such as scale-size ρ , position x and isospin orientation U , which correspond to (approximate) zero modes of the fluctuation operator $\Delta(\phi) = \delta^2 S_E[\phi]/\delta\phi^2$. The sum of I and $\bar{I}$ is an approximate solution at large distance $R = x_I - x_{\bar{I}}$. At small distances, however, it ceases to be an approximate solution, because of the $I\bar{I}$ interactions. Therefore, to expand only around the sum of I and $\bar{I}$ is not reliable, since there are large linear terms in the exponent due to interactions. The valley method [16,17] gives a means to circumvent these problems. The idea is the following (for details see refs. [15-17]). Although there is no non-trivial *minimum* of the action in the zero-topological-charge sector, there is nevertheless a *valley* in this sector: there are almost *flat* directions in functional space corresponding to *quasizero* modes of the fluctuation operator. The $I\bar{I}$ valley trajectory (streamline) $\phi_\tau = \phi(x; \tau_I, \tau_{\bar{I}})$ satisfies the boundary condition¹⁾

$$\phi(x; \tau_{Ii}, \tau_{\bar{I}j})|_{R \rightarrow \infty} = \phi_I(x; \tau_{Ii}) + \phi_{\bar{I}}(x; \tau_{\bar{I}j}) . \quad (6)$$

To find the valley trajectory ϕ_τ , note that $\phi_\tau = \phi(x; \tau)$ for *fixed* τ is a minimal point of $S_E[\phi]$ within the subset of functions ϕ which satisfy

$$\left\langle (\phi - \phi_\tau), \frac{\partial \phi_\tau}{\partial \tau} \right\rangle_w = 0 , \quad (7)$$

where the scalar product reads $\langle f, g \rangle_w = \int d^4x g_\tau(x) w(x; \tau) f_\tau(x)$. In eq. (7), w means some weight function²⁾. The action minimization gives the following equation for the valley trajectory ϕ_τ [17]

$$\epsilon^2(\tau) w(x; \tau) \frac{\partial \phi_\tau}{\partial \tau} = \frac{\delta S_E[\phi]}{\delta \phi} \Big|_{\phi=\phi_\tau} , \quad (8)$$

where $\epsilon^2(\tau)$ are Lagrange multipliers. By means of the Faddeev-Popov technique one may now express the $I\bar{I}$ contribution to the Green function (5) in terms of collective coordinates,

$$G_E^{(I\bar{I})} = N^{-1} \int \prod_{i=1}^{2p} d\tau_i \Delta_{\text{FP}}(\tau) \int \mathcal{D}\phi \prod_{i=1}^{2p} \delta \left(\left\langle (\phi - \phi_\tau), \frac{\partial \phi_\tau}{\partial \tau_i} \right\rangle_w \right) \quad (9)$$

$$\times \left[\prod_{i=1}^4 \phi(x_i) \right] \times \exp \left[-S_E[\phi_\tau] - \frac{1}{2} \left\langle (\phi - \phi_\tau), \Delta(\phi_\tau)(\phi - \phi_\tau) \right\rangle_w - S_{\text{int}} \right],$$

where the collective coordinates are $\tau = \{\rho_I, \rho_{\bar{I}}, x_I, x_{\bar{I}}, U_I, U_{\bar{I}}\}$, and Δ_{FP} denotes the familiar Faddeev-Popov determinant and Jacobian. There are no linear terms in the exponents due to the presence of the δ -functions, provided ϕ_τ satisfies the valley equation

- 1) The boundary condition (6) defines the simplest case, namely the single-instanton /single-anti-instanton valley.
- 2) The valley method exploits essentially the existence of a scalar product in functional space. The requirement of gauge and conformal invariance of the scalar product in the pure gauge theory uniquely fixes the transformation law of w [17]. Different choices for w will not change the full answer, but lead to a redistribution of classical and quantum effects.

(8). Evaluating the integral in the Gaussian approximation, i.e. neglecting the fluctuations in the prefactor and in the exponent (S_{int}), one has for (9)

$$G_E^{(I\bar{I})} = \int d\mu(\tau) \prod_{i=1}^4 \phi_\tau(x_i) \exp(-S_E[\phi_\tau]) , \quad (10)$$

where the measure is given by

$$d\mu(\tau) = N^{-1} \prod_{i=1}^{2p} d\tau_i \Delta_{\text{FP}}(\tau) \left[\det^{(2p)} \Delta(\phi_\tau) \right]^{-1/2} . \quad (11)$$

Here $\det^{(2p)} \Delta(\phi_\tau)$ means the determinant evaluated in the space of functions orthogonal to the valley trajectory. To proceed we have to find the valley configuration in the $I\bar{I}$ sector of our model.

Let us first consider our model in the confinement phase. In this case we should find the $I\bar{I}$ valley trajectory and $I\bar{I}$ interaction potential for the pure gauge theory. To this end we will follow the ideas of ref. [17], namely: (i) the general $I\bar{I}$ valley trajectory can be obtained from the valley with coinciding centres x_0 by means of the inversion coordinate transformation centred at $x = 0$; (ii) the $I\bar{I}$ valley trajectory with coinciding centres reduces to the kink/anti-kink valley trajectory in double-well quantum mechanics. Thus, one can obtain the $I\bar{I}$ valley for the pure gauge theory from the solution of the quantum mechanical valley equation. Indeed, let us try to find the $I\bar{I}$ valley with the same centres x_0 using the ansatz

$$A_\mu(x) = \frac{-i}{g} \frac{(\bar{\sigma}_\mu \sigma_\nu - \delta_{\mu\nu})}{(x - x_0)^2} (x - x_0)_\nu s((x - x_0)^2) . \quad (12)$$

where s is an arbitrary function. Note that the ansatz (12) is the general expression for the valley with coinciding centres in the case of maximal $I\bar{I}$ attraction. Indeed, the matrix form of eq. (12) is the same as the form of $\bar{I}$ in the singular gauge or I in the regular one. Thus, the $I\bar{I}$ valley, where I is in the regular and $\bar{I}$ is in the singular gauge, should have the form of eq. (12) if one neglects the relative $I\bar{I}$ gauge orientation. This corresponds to the general case of maximal $I\bar{I}$ attraction [17]. The $\text{SU}(2)$ gauge theory action S_E^{gauge} reduces on the ansatz (12) to quantum mechanics in the double-well potential

$$S_E^{\text{gauge}} = \frac{48\pi^2}{g^2} \int_{-\infty}^{+\infty} dt \left(\frac{1}{2} (d_t s)^2 + \frac{1}{2} \left(\left(s - \frac{1}{2} \right)^2 - \frac{1}{4} \right)^2 \right) = \frac{48\pi^2}{g^2} \cdot S_E^{\text{qm}} . \quad (13)$$

Here $t = \ln(x - x_0)^2$. Thus BPST instantons [18] in gauge theory correspond to kinks in quantum mechanics. In order to find the kink/anti-kink ($K\bar{K}$) valley trajectory (streamline) one has to find the solution of the valley equation in quantum mechanics. Let us consider simply the sum of kink and anti-kink,

$$s_\xi = \frac{1}{2} \left(\tanh\left(\frac{1}{2}(t - t_0 - \xi)\right) - \tanh\left(\frac{1}{2}(t - t_0 + \xi)\right) \right) , \quad (14)$$

where 2ξ is the $K\bar{K}$ separation and t_0 is the center of the $K\bar{K}$ pair. Equation (14) coincides with the exact streamline in the region of *large* and *small* separations ξ . For $\xi \rightarrow 0$ the r.h.s. of eq. (14) goes to the perturbative vacuum $s_\xi \rightarrow 0$, reflecting the $K\bar{K}$ collapse.

We failed to find the exact streamline analytically, but in the case of quantum mechanics it was found numerically in ref. [20]³⁾. The idea is simply to use the definition of the valley. One should start at the remote initial point (well separated $K\bar{K}$ pair) and then just follow the direction of the gradient line (direction of the force $f(\xi) = \delta S_E^{\text{qm}}/\delta s$). It can be done iteratively, by calculating the force and making small steps in its direction, $s_{n+1}(\xi) = s_n(\xi) - \epsilon f_n(\xi)$, where ϵ is some small numerical parameter. It was shown in ref. [20] that the trial function (14) fits the streamline reasonably well for *any* values of separation ξ . Thus, we will use the trial function (14) to represent the valley trajectory.

In four-dimensional notations the $\bar{I}\bar{I}$ trajectory (12),(14), after the substitution

$$\xi = \ln z, t_0 = \ln \rho^2, \quad (15)$$

takes the form

$$A_\mu^{(\tau)} = -\frac{i}{g} (\bar{\sigma}_\mu \sigma_\nu - \delta_{\mu\nu})(x - x_0)_\nu \left(\frac{1}{(x - x_0)^2 + \rho^2/z} - \frac{1}{(x - x_0)^2 + \rho^2 z} \right). \quad (16)$$

Let us now perform, following ref. [17], the inversion transformation $x_\mu \rightarrow x_\mu \rho^2 z / x^2$ with the centre $x = 0$. This leaves the action invariant if the vector field potential transforms as [21]

$$A_\mu(x) = \frac{\rho^2 z}{x^2} \left(\delta_{\mu\alpha} - 2 \frac{x_\mu x_\alpha}{x^2} \right) \cdot A_\alpha \left(\frac{\rho^2 z}{x^2} x_\mu \right). \quad (17)$$

After the transformation (17) (accompanied by a gauge transformation) the $\bar{I}\bar{I}$ trajectory, expressed in terms of the new set of parameters $\{x_1, x_2, \rho_1, \rho_2\}$,

$$\begin{aligned} x_1 &= x_0 \frac{\rho^2 z}{x_0^2 + \rho^2/z}, \quad x_2 = x_0 \frac{\rho^2 z}{x_0^2 + \rho^2 z}, \\ \rho_1 &= \rho/\sqrt{z} \frac{\rho^2 z}{x_0^2 + \rho^2/z}, \quad \rho_2 = \rho\sqrt{z} \frac{\rho^2 z}{x_0^2 + \rho^2 z}, \end{aligned} \quad (18)$$

takes the form⁴⁾

$$\begin{aligned} A_\mu^{(\tau)} &= -\frac{i}{g} \cdot \left(\frac{y_2}{\sqrt{y_2^2}} \frac{\bar{x}}{\sqrt{x^2}} \frac{R}{\sqrt{R^2}} \frac{(\bar{\sigma}_\mu y_1 - y_{1\mu})}{y_1^2(y_1^2 + \rho_1^2)} \frac{\rho_1^2}{\sqrt{R^2}} \frac{\bar{R}}{\sqrt{x^2}} \frac{x}{\sqrt{y_2^2}} + \frac{(\sigma_\mu \bar{y}_2 - y_{2\mu})}{y_2^2(y_2^2 + \rho_2^2)} \frac{\rho_2^2}{\sqrt{R^2}} \frac{\bar{R}}{\sqrt{x^2}} \frac{\bar{y}_2}{\sqrt{y_2^2}} \right. \\ &\quad \left. + \frac{\rho_1 \rho_2}{z y_1^2(y_2^2 + \rho_2^2)} \left[\frac{y_2}{\sqrt{y_2^2}} (\bar{\sigma}_\mu y_1 - y_{1\mu}) \frac{y_2}{\sqrt{y_2^2}} - \frac{y_2}{\sqrt{y_2^2}} \frac{\bar{x}}{\sqrt{x^2}} \frac{R}{\sqrt{R^2}} (\bar{\sigma}_\mu y_1 - y_{1\mu}) \frac{\bar{R}}{\sqrt{R^2}} \frac{x}{\sqrt{x^2}} \frac{\bar{y}_2}{\sqrt{y_2^2}} \right] \right), \end{aligned} \quad (19)$$

where $y_1 = x - x_1$, $y_2 = x - x_2$, $R = x_1 - x_2$, $x = x_\mu \sigma_\mu$, $\bar{x} = x_\mu \bar{\sigma}_\mu$.

Note that the valley configuration (19) is the sum of anti-instanton $\bar{I}(x_1, \rho_1)$ and instanton $I(x_2, \rho_2)$, both in the singular gauge, for the case of large R/ρ , up to $\mathcal{O}(1/R^3)$ corrections. Generally speaking, it is possible to view the valley trajectory (19) as the sum of *distorted* [3,22] instanton and *distorted* anti-instanton, both in the singular gauge. Despite the fact that in quantum mechanics the valley was represented by kink plus anti-kink, the corresponding valley in gauge theory is not just the sum of instanton plus anti-instanton.

3) We are grateful to E.V. Shuryak for supplying us with this reference.

4) Equation (19) can be obtained from eq. (3.21) of ref. [17].

The valley trajectory (19) is conformally invariant. This means that after a conformal transformation it reduces to the original form (19), with redefined collective coordinates. Due to the conformal invariance of the pure gauge theory, the action on the valley (19) coincides with the action on the valley (16). This action can be found analytically taking into account eqs. (13)-(15),

$$S_{\text{gauge}}^{(\tau)} = \frac{48\pi^2}{g^2} \cdot \left[\frac{6z^2 - 14}{(z - 1/z)^2} - \frac{17}{3} - \ln(z) \left(\frac{(z - 5/z)(z + 1/z)^2}{(z - 1/z)^3} - 1 \right) \right]. \quad (20)$$

The valley action, being an invariant of the conformal group, is a function of the conformally invariant parameter z . Solving eqs. (18) we arrive at the following *general* expression for z in terms of $I\bar{I}$ collective coordinates,

$$z = \frac{R^2 + \rho_1^2 + \rho_2^2 + \sqrt{(R^2 + \rho_1^2 + \rho_2^2)^2 - 4\rho_1^2\rho_2^2}}{2\rho_1\rho_2}. \quad (21)$$

It is easy to see that in the limit of large $I\bar{I}$ separation, $R \rightarrow \infty$,

$$z \rightarrow \frac{R^2 + \rho_1^2 + \rho_2^2}{\rho_1\rho_2}, \quad S_{\text{gauge}}^{(\tau)} \rightarrow \frac{16\pi^2}{g^2} - \frac{96\pi^2}{g^2} \cdot \frac{\rho_1^2\rho_2^2}{(R^2 + \rho_1^2 + \rho_2^2)^2}, \quad (22)$$

which coincides with Yung's formula [17]. For the case $R \rightarrow 0$, $\rho_1 = \rho_2$, one has

$$z \rightarrow 1, \quad S_{\text{gauge}}^{(\tau)} \rightarrow 0, \quad (23)$$

which reflects the $I\bar{I}$ collapse into the perturbative vacuum. These two limits, (22) and (23), reflect the general behaviour of the action on the valley trajectory⁵⁾. Due to the numerical arguments from ref. [20], which justify our approximation of the valley trajectory by the trial function (14), we claim that we know the action on the valley trajectory, eq. (20), and the valley parameter, eq. (21), for the *pure* gauge theory to a sufficient accuracy.

In the pure gauge theory (or confinement phase of the SU(2)-Higgs model) we should consider the scattering of *off-shell* gluons, since, due to confinement, the gluons are not the correct asymptotic states. We will consider here gluons with Minkowski-four-momenta $k_{1/2} = (E/2, \pm \vec{k})$, $k^2 \equiv k_{1/2}^2 < 0$. The $I\bar{I}$ -induced contribution to the Green function for the process gluon (k_1) + gluon (k_2) $\rightarrow$ *anything* is obtained by inserting the valley trajectory (19) and the valley action (20) into eq. (10). After a rotation to Minkowski space and applying the standard LSZ procedure we obtain for the instanton-induced contribution to the total cross-section [15]

$$\sigma_{\text{tot}}^{(I)} \sim \text{Im} \int d\mu(\tau) f^{(\tau)}(k_1, k_2) \exp[ER - S_{\text{gauge}}^{(\tau)} - 2\sqrt{-k^2}(\rho_1 + \rho_2)], \quad (24)$$

where $R = \sqrt{R^2}$. The first and third term in the exponent arise after Fourier transformation of the four-point Green function (the third term, which provides an infrared cutoff for the scale-size, was absent in ref. [15], where the external particles were taken on-shell). Since we are not interested in the pre-exponent of the total cross-section, we do not need to give the explicit form of the function $f^{(\tau)}$, which may be obtained by using the valley trajectory (19).

5) The collapse into the perturbative vacuum is inevitable [17] due to the absence of a non-trivial solution in the zero-topological-charge sector.

The final integral over the collective coordinates can be done in a saddle-point approximation. A non-trivial saddle-point (with one negative mode) arises due to the existence of the infrared cutoff for the scale-size, provided by the virtuality $-k^2$ of the gluons. The saddle-point will be at the symmetric value, $\rho_1 = \rho_2 \equiv \rho$. The total cross-section is given by $\sigma_{\text{tot}}^{(I)} \sim \exp \Gamma_*(\epsilon)$, where $\epsilon \equiv E/\sqrt{-k^2}$ and the function $\Gamma_*(\epsilon)$ denotes ($\chi \equiv R/\rho$)

$$\Gamma(E, k^2; \chi, \rho) \equiv E\rho\chi - S_{\text{gauge}}(\chi) - 4\sqrt{-k^2}\rho, \quad (25)$$

taken at the saddle-point. We find

$$\chi_* = 4\epsilon^{-1}, \quad \rho_* = \frac{1}{E} \frac{\partial S_{\text{gauge}}}{\partial \chi} \Big|_{\chi_*}, \quad \Gamma_* = -S_{\text{gauge}}(\chi_*). \quad (26)$$

From our knowledge of the valley action, eq. (20), we infer that Γ_* is a monotonically increasing function of ϵ and has the asymptotics

$$\Gamma_*(\epsilon) \sim -\frac{16\pi^2}{g^2} \begin{cases} 1 - O(\epsilon^4), & \text{for } \epsilon \rightarrow 0; \\ \epsilon^{-2}, & \text{for } \epsilon \rightarrow \infty; \end{cases} \quad (27)$$

The transition between the asymptotics occurs at $\epsilon \sim 4$. Instanton-induced cross-sections are exponentially suppressed for high virtualities, but become important for close-to-mass-shell gluon collisions. It should be noted that the instanton size is always bounded. In particular, we obtain, for soft gluon collisions, $\rho_* \sim (16\pi^2/g^2)(\sqrt{-k^2}/E^2)$: at very high energies, but small virtualities, the dominant instantons are very small.

Let us now consider our model in the Higgs phase. According to our ideology we should find the complete valley trajectory in this phase. Unfortunately, the solution of the valley equations in the Higgs phase is very complicated. However, let us assume that the dominant contribution to the instanton-induced total cross-section of (on-shell) WW scattering comes from the multiple production of transverse W's (as it is the case at low energies). In this case it seems reasonable to consider the situation where I and $\bar{I}$ interact only via the transverse W's, but do not interact via longitudinal W's and Higgses. Let us therefore omit the Higgs-induced part of the interaction. Analogously to eq. (24) we obtain now for the instanton-induced contribution to the total WW cross-section,

$$\sigma_{\text{tot}}^{(I)} \sim \text{Im} \int d\mu(\tau) f^{(\tau)}(k_1, k_2) \exp[ER - S_{\text{gauge}}^{(\tau)} - \pi^2 v^2(\rho_1^2 + \rho_2^2)]. \quad (28)$$

It is convenient to define the variables $\chi \equiv R/\rho$, $\xi \equiv m_w \rho$. In terms of these we have $\sigma_{\text{tot}}^{(I)} \sim \exp[(16\pi^2/g^2)F_*(\epsilon)]$, where now $\epsilon \equiv E/M_0$, $M_0 \equiv \pi m_w/\alpha_w$. The function $F_*(\epsilon)$ denotes

$$F(\epsilon; \chi, \xi) \equiv \frac{1}{4}\epsilon\xi\chi - \frac{g^2}{16\pi^2}S_{\text{gauge}}(\chi) - \frac{1}{2}\xi^2, \quad (29)$$

taken at the saddle-point. We have determined the stationary point of the function F . This stationary point has only one negative mode and it gives rise to the imaginary part of the forward elastic scattering amplitude. The solid line in Fig. 1 shows the function $F_*(\epsilon)$. At low energies the total cross-section rises exponentially, in agreement with eqs. (2),(3). At high energies F approaches zero asymptotically - there seems to be no problem with unitarity. The dashed line in Fig. 1 shows the instanton scale-size at the saddle point. At low energies the instanton size grows, reaching a maximum size $\xi_*^{\text{max}} \simeq 0.67$ at $\epsilon \simeq 2.7$. For higher energies it decreases, asymptotically approaching zero. The parameter χ_* decreases

Figure 1: The logarithm of the total cross-section, $F_* \sim (g^2/(16\pi^2)) \log \sigma_{\text{tot}}^{(I)}$ (solid line), and the instanton scale-size, ξ_* (dashed line), both as a function of $\epsilon \equiv E/M_0$.

monotonically with increasing ϵ ($\chi_* = 1$ at $\epsilon \simeq 2.7$) and goes to zero asymptotically: at asymptotic energies the saddle-point configuration approaches the perturbative vacuum.

Before we close this letter, let us make some comments on the region of validity of our results. Generally speaking, the total cross-section cannot be divided into a perturbative and a non-perturbative piece. But in the saddle-point approximation one may consider the general answer as the sum of contributions from different saddle-points, as long as they are isolated. We found that for finite energies there is an instanton-induced saddle-point, which is well separated from the trivial one (perturbative vacuum). In the present approach we neglected the fluctuations around the valley trajectory. This may be justified as long as the valley trajectory is much larger than the fluctuations, i.e. as long as the action is of the order of $1/g^2$. In our second example we have $S_* \geq 1/g^2$ for $\epsilon \leq 6$, whereas nevertheless $(16\pi^2/g^2)(F_*/S_*) \sim 10^{-2}$, for $\epsilon \sim 6$. This demonstrates that in spite of the fact that the asymptotic region of F_* is outside the range of validity of our investigation, since the saddle-point configuration approaches the perturbative vacuum at asymptotic energies, there may remain some energy region where our calculations are still valid and instanton-induced processes are essentially unsuppressed.

In summary, we have shown that (distorted) instantons may give an important contribution to total cross-sections of vector boson scattering at energies far above the infrared cutoff, $E \gg \sqrt{-k^2}(v)$, in both the confinement and Higgs phase of the SU(2) gauge theory. The phenomenological consequences of our result for close-to-mass-shell gluon collisions need a careful study. The question of whether electroweak multi-W production and (B+L)-violation becomes strong at high energies may be answered by a careful examination of the $I\bar{I}$ valley in the Higgs phase.

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