More Bunches in PEP: Single Particle Dynamics

W.J. Corbett
Storage Rings Division
Stanford Linear Accelerator Center
Stanford University, Stanford, California 94309

Abstract

A luminosity improvement in PEP may be possible by colliding up to 9 bunches in each beam. The beams would be separated using the existing separator plates with a trivial change in the bias polarities and a modification of the betatron functions. In this note, the bunch separation scheme and associated optics are reviewed. The beam separation distance, beam size and long-range beam-beam tune shift are then found as a function of separator plate voltage.

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Introduction

The first proposals to raise the luminosity in PEP by colliding 9x9 bunches were made by M.H. Donald and J.M. Patterson[11]. A similar horizontal 'pretzel' configuration has been very successful in CESR with 7x7 bunches[2,3], and preparations are underway to increase the number of bunches in LEP[4,5]. For PEP, studies of RF loading[6,7], vacuum performance[8], and control of collective effects[9] with more bunches and higher currents look favorable. Projections for the luminosity based on 3x3 bunch data indicate that given a current limitation of 90mA, colliding 6x6 bunches could lead to a peak luminosity of $L=9 \times 10^{32} cm^{-2} s^{-1}$[10].

The basic argument for more bunches is that higher luminosities can be reached while keeping the single bunch current below the tune shift and collective instability limits. Colliding 3x3 bunches in PEP, for instance, we find that the vertical tune shift normally saturates at $\Delta\nu_{v}=30$mA and the specific luminosity decreases at higher currents. Around 42mA, the horizontal tune shift reaches $\Delta\nu_{h}=0.05$ and the beams become unstable. By distributing higher currents into more bunches, considerable gains in luminosity could be possible before the horizontal tune shift limit is reached.

As shown in Fig. 1, colliding either 6x6 or 9x9 bunches in PEP requires that in addition to the normal interaction points, the beams must also be separated in the arcs. The 6x6 bunch mode is identical to the 9x9 mode with every third bunch missing, but has the advantage of operating closer to the vertical tune shift limit in a current limited regime. In either case, the $e^{+}\bar{e}^{-}$ orbit separations are created by biasing all of the existing separator plates (IRs 4-12) with the same polarity. This configuration makes the residual orbit distortions from the IR bumps add in phase in the arcs. The orbit distortions can also be forced to add in phase at the IRs by reflecting the longitudinal part of the nominal tunes ($\nu_{y}=18.2, \nu_{x}=21.3$) to values below the integer ($\nu_{y}=17.760, \nu_{x}=20.700$). Bipolar corrector plates are used to bring the beams into collision at IR2.

The potential hazards of the new configuration include aperture limitations, operating in a new region of the tune diagram, emittance and dispersion coupling in the sextupoles, and increased long-range beam-beam effects. The benefits include broadening the database for many-bunch colliders and, of course, luminosity.

In this note, the proposed method for separating the PEP beams and the optics modifications are reviewed. The closed orbit, Twiss parameters and beam size are then found as a function of the separator plate strength with OMEGA, a lattice simulation code[10]. Using these results, the long-range beam-beam tune shift is found. Implications of the increased beam size on luminosity are considered in Ref. [10].
Beam Separation Scheme

The most straightforward approach to collide more bunches in PEP is to use the present set of vertical separator plates\cite{12-15} and roughly the same optics in the insertions. Since the phase advance between the plates is about 187°, the phase mismatch produces vertical orbit distortions which then propagate like free betatron oscillations through the arcs. If all the separator plates are biased with the same polarity, the orbit distortions from each pair of plates can be made to constructively interfere in the arcs. Since the vertical betatron tune is ~18, the peak amplitude of the closed orbit is roughly coincident with the bunch crossing points in the arcs. Normally, the separator plates are biased with alternating polarity between each IR to cause the orbit errors to cancel.

In the paraxial approximation, the vertical closed orbit distortion ($y_{COP}$) produced by the separator plates is a linear superposition

$$y_{COP}(s) = \frac{\sqrt[2]{\beta_y(s)}}{2 \sin v_y} \cdot \sum_j \sqrt[2]{\beta_y} \cdot \Delta \Theta_j \cdot \cos v_y (\pi - 2 \pi \phi_j - \phi(s)) l_j$$

[1]

where ($\beta_j, \beta(s)$) and ($\phi_j, \phi(s)$) refer to the local betatron function ($m$) and betatron phase advance ($\phi$) at the beam deflection and beam observation points, respectively.

The reason for lowering the vertical tune to $v_y=17.76$ is contained in the denominator of Eqn. 1. That is, in order to make the vertical orbit 'errors' add in phase in the IR regions, the sign of $\sin v_y$ was reversed. As a consequence, the beams have about 20% more separation in the IRs than in the 3x3 mode. Designs for a vertically separated lattice with $v_y$ at the nominal value of $v_y=18.20$ proved unsuccessful because the residual orbit perturbations were out of phase with the local beam bumps in the IRs\cite{1}.

Since the residual orbits will add in phase in both the arcs and in the IRs, the voltage range on the compensating plates in IR2 (TPC-2Y) has been increased to bring the beams into collision. Vertical dispersion at IR2 is uncompensated. Analytically, the beams are 'brought into collision' by setting $y_{COP}(IR2)=0$ and solving for the kick required by the plates at IR2. To calculate the closed orbit with GEMINI (a lattice simulation code incorporating higher order modes\cite{11}) the deflector plate kicks can be modelled as 'higher order modes' of zeroth order. The corrector plate voltage at IR2 is found by solving for the $\beta$-functions associated with $y_{COP}(IR2)=0$, then successively finding the new closed orbit, betatron functions, etc, until convergence if found. Deviations from the linear formula (Eqn. 1) are small.

For a given beam energy and separator plate geometry, the transverse impulse seen by the beam is,

$$\Theta = \frac{\Delta p_x}{p} = \frac{eVl}{Bq}$$

[2]

At $E=15$GeV, $V=110$kV, $l=3.6$m and $g=11$cm, the beams are deflected by $\Theta=\pm0.24$mmrad.

In the following, the deflection angle of 0.24mmrad will be referred to as the 'nominal' kick that produces the vertical orbit plotted in Fig. 2. For this kick, the average vertical bunch separation distance is 4.8mm in the arcs and 2.6mm in IR4-12, respectively.

Sensitivity to Vertical Tune

Operating with tunes below the integer allows for the orbit perturbations to add in phase in the IRs. The horizontal component of the long-range force, however, will tend to push the $x$-tune up toward the integer. Simultaneously, the long-range interaction will decrease the vertical tune. This defocusing force may cause the separator plates to become less effective since the denominator in the closed orbit formula (Eqn. 1) is resonant only near the integer. The dependence of the beam separation on vertical tune is plotted in Fig. 3. In practice, if the vertical tune is raised, the potential instability may be tuned out with the main quads since the beams will tend to separate and the long-range interaction decreases. In summary, the long-range forces may limit the window of operation to large beam separation distances where the beam-beam interaction is minimized.

Optics

Two changes were made to the machine optics to accommodate 9 bunch colliding beams. First, the tunes were reflected below the integer so that the residual closed orbit would add in phase with the local beam bumps in the interaction regions. This simplifies the problem by eliminating the need for additional separator plates. The second change was to lower the betatron functions at IR4-12 by ~10% to decrease the sensitivity to long range beam-beam interactions\cite{11}. This change adds about two degrees vertical phase advance through the IRs from one separator plate to the next. It also enhances the residual orbit error (beam separation) and leads to larger peak vertical betatron values in the first insertion quad, Q1.

<table>
<thead>
<tr>
<th>Location</th>
<th>Nominal PEP</th>
<th>Low Tune Optics</th>
</tr>
</thead>
<tbody>
<tr>
<td>BetaY IR4-12</td>
<td>18cm</td>
<td>16cm</td>
</tr>
<tr>
<td>BetaX IR4-12</td>
<td>4.5m</td>
<td>4.0m</td>
</tr>
<tr>
<td>BetaY peak IR4-12</td>
<td>266</td>
<td>300</td>
</tr>
<tr>
<td>BetaX peak IR4-12</td>
<td>128</td>
<td>138</td>
</tr>
</tbody>
</table>

Table 1 Beta functions in the Insertions.
As shown in Fig. 4, the reduced phase advance through the arcs (to lower the tunes) is primarily localized in the short interval between the separator plates and the first FODO-cell magnet, QF1, by increasing \( \beta_y \) in that region. After passing through the matching in Cells 1 and 2, the betatron functions in the arcs are much like the nominal PEP lattice. The accumulated phase advance through the arcs (between separator plates) is nominally 4\( \pi \)+189\(^\circ\) and for the low tune lattice the phase advance is 4\( \pi \)+161\(^\circ\).

The quadrupole strengths for the two lattices are compared in Table 1. From this table, it is clear that most of the change in IR4-12 comes from a reduction in the strength of Q3 and QF1. In IR2 the betafunction change is made in the lattice sequence Q2.3-Q3-QF1.

<table>
<thead>
<tr>
<th>Magnet</th>
<th>Nominal PEP</th>
<th>Low Tune</th>
<th>( \Delta K / K ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1H</td>
<td>K = -0.12477 K = -0.12353</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>Q2H</td>
<td>K = -0.10195 K = 0.10051</td>
<td>1.4</td>
<td></td>
</tr>
<tr>
<td>Q3</td>
<td>K = -0.10917 K = -0.18146</td>
<td>25.4</td>
<td></td>
</tr>
<tr>
<td>QF1</td>
<td>K = 0.14846 K = 0.12798</td>
<td>13.8</td>
<td></td>
</tr>
<tr>
<td>QD1</td>
<td>K = -0.21981 K = -0.22654</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>QD2</td>
<td>K = 0.16765 K = 0.16565</td>
<td>1.2</td>
<td></td>
</tr>
<tr>
<td>QF3</td>
<td>K = 0.21931 K = 0.22130</td>
<td>0.9</td>
<td></td>
</tr>
<tr>
<td>QF</td>
<td>K = 0.16723 K = 0.16608</td>
<td>0.7</td>
<td></td>
</tr>
<tr>
<td>QD</td>
<td>K = -0.18287 K = -0.18227</td>
<td>0.3</td>
<td></td>
</tr>
<tr>
<td>QD8</td>
<td>K = -0.18424 K = -0.18378</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>QPF</td>
<td>K = 0.12619 K = 0.12361</td>
<td>0.5</td>
<td></td>
</tr>
<tr>
<td>Q1HR2</td>
<td>K = -0.17289 K = -0.17188</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>Q2HR2</td>
<td>K = 0.12033 K = 0.11598</td>
<td>3.6</td>
<td></td>
</tr>
<tr>
<td>Q2SHR2</td>
<td>K = -0.08844 K = -0.00261</td>
<td>69.1</td>
<td></td>
</tr>
<tr>
<td>Q3R2</td>
<td>K = -0.11849 K = -0.07723</td>
<td>34.8</td>
<td></td>
</tr>
<tr>
<td>Q1R2</td>
<td>K = 0.13817 K = 0.10867</td>
<td>21.4</td>
<td></td>
</tr>
<tr>
<td>Q3R2</td>
<td>K = -0.19880 K = -0.20992</td>
<td>5.6</td>
<td></td>
</tr>
<tr>
<td>Q2R2</td>
<td>K = 0.17385 K = 0.17056</td>
<td>1.9</td>
<td></td>
</tr>
<tr>
<td>Q3R2</td>
<td>K = 0.21362 K = 0.21696</td>
<td>1.6</td>
<td></td>
</tr>
</tbody>
</table>

Table II Quadrupole strengths for nominal PEP lattice and low tune PEP configurations.

The resulting betatron functions for the matched low tune optics are plotted in Fig. 5. One penalty for this design is that the vertical betatron functions are about 30% higher in the RF sections (see Fig. 6). The increased betatron amplitude increases susceptibility to collective effects (most of the single resonance and broadband impedances are in the PEP cavities), HOM excitation from orbit errors (Eqn. 1) and synchrobetatron coupling.

For the nominal beam separation, the Twiss parameters evaluated at the bunch crossing points are:

<table>
<thead>
<tr>
<th>S</th>
<th>( \beta_x )</th>
<th>( \beta_y )</th>
<th>( \gamma_x )</th>
<th>( \gamma_y )</th>
<th>( \Delta \beta_x )</th>
<th>( \Delta \beta_y )</th>
<th>COG (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>122.68</td>
<td>24.104</td>
<td>16.670</td>
<td>1.096</td>
<td>1.007</td>
<td>2.517</td>
<td>-1.686</td>
<td>1.275</td>
</tr>
<tr>
<td>244.70</td>
<td>22.056</td>
<td>18.252</td>
<td>2.359</td>
<td>1.959</td>
<td>1.354</td>
<td>-1.148</td>
<td>1.217</td>
</tr>
<tr>
<td>366.67</td>
<td>4.027</td>
<td>0.161</td>
<td>3.450</td>
<td>2.960</td>
<td>-0.001</td>
<td>-0.001</td>
<td>0.017</td>
</tr>
<tr>
<td>489.35</td>
<td>24.099</td>
<td>16.667</td>
<td>4.545</td>
<td>3.967</td>
<td>2.517</td>
<td>-1.686</td>
<td>1.272</td>
</tr>
<tr>
<td>611.37</td>
<td>22.046</td>
<td>16.256</td>
<td>5.809</td>
<td>4.919</td>
<td>1.354</td>
<td>-1.149</td>
<td>1.219</td>
</tr>
<tr>
<td>733.33</td>
<td>4.024</td>
<td>0.161</td>
<td>6.899</td>
<td>5.920</td>
<td>-0.001</td>
<td>-0.001</td>
<td>0.000</td>
</tr>
<tr>
<td>856.02</td>
<td>24.124</td>
<td>16.667</td>
<td>7.995</td>
<td>6.927</td>
<td>2.518</td>
<td>-1.686</td>
<td>1.275</td>
</tr>
<tr>
<td>978.04</td>
<td>22.043</td>
<td>18.227</td>
<td>9.259</td>
<td>7.879</td>
<td>1.355</td>
<td>-1.148</td>
<td>1.220</td>
</tr>
<tr>
<td>1100.00</td>
<td>1.006</td>
<td>0.040</td>
<td>10.35</td>
<td>8.879</td>
<td>0.000</td>
<td>0.000</td>
<td>0.008</td>
</tr>
<tr>
<td>1222.68</td>
<td>24.051</td>
<td>16.643</td>
<td>11.45</td>
<td>9.887</td>
<td>2.511</td>
<td>-1.682</td>
<td>1.276</td>
</tr>
<tr>
<td>1344.70</td>
<td>22.091</td>
<td>18.246</td>
<td>12.71</td>
<td>10.838</td>
<td>1.358</td>
<td>-1.147</td>
<td>1.220</td>
</tr>
<tr>
<td>1466.67</td>
<td>4.023</td>
<td>0.161</td>
<td>13.80</td>
<td>11.839</td>
<td>0.001</td>
<td>0.001</td>
<td>0.000</td>
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<tr>
<td>1589.35</td>
<td>24.055</td>
<td>16.657</td>
<td>14.89</td>
<td>12.846</td>
<td>2.511</td>
<td>-1.683</td>
<td>1.274</td>
</tr>
<tr>
<td>1711.37</td>
<td>22.087</td>
<td>18.224</td>
<td>16.16</td>
<td>13.798</td>
<td>1.356</td>
<td>-1.145</td>
<td>1.217</td>
</tr>
<tr>
<td>1833.33</td>
<td>4.026</td>
<td>0.161</td>
<td>17.25</td>
<td>14.800</td>
<td>0.001</td>
<td>0.001</td>
<td>0.000</td>
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<tr>
<td>1956.02</td>
<td>24.057</td>
<td>16.642</td>
<td>18.35</td>
<td>15.807</td>
<td>2.512</td>
<td>-1.683</td>
<td>1.273</td>
</tr>
<tr>
<td>2078.04</td>
<td>22.084</td>
<td>18.250</td>
<td>19.61</td>
<td>16.759</td>
<td>1.357</td>
<td>-1.148</td>
<td>1.220</td>
</tr>
<tr>
<td>2200.00</td>
<td>4.028</td>
<td>0.161</td>
<td>20.70</td>
<td>17.760</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table III Twiss parameters at beam crossing points.

From Table III, we find the lattice for the low-tune optics contains vertical dispersion generated by the separator plates. Unfortunately, the dispersion waves propagate like forced betatron oscillations and the maxima occur near the crossing points. There is also vertical dispersion in the RF cavities, as shown in Fig. 6. With the additional orbit distortion generated by the separator plates, more emphasis must be placed on minimizing both the orbit and dispersion errors through the RF sections to suppress HOM generation and synchrobetatron coupling.

Perhaps even less controllable is proximity of the bunch crossing points to the maximum horizontal dispersion sections in the arcs. Here, synchrobetatron coupling can arise from the long-range force because individual particles see a different transverse kick depending on their momentum. The beams are less susceptible to synchrobetatron coupling in the IRs where \( \beta_y \) is less and \( \eta_y \) = 0 by design.

Most of the vertical emittance is generated directly from the vertical dispersion created by the vertical 'dipole' kicks from the separator plates, with only a small increase due to coupling in the sextupoles. Simulations indicate that the dynamic aperture of the separated-beam optics is roughly equal to the aperture of the nominal configuration. The sensitivity of the aperture to tune still needs to be investigated.
A comparison of the beam properties shows only a small change to the damping times as shown in Table IV. About 2.5 units of natural chromaticity in the vertical plane is generated from the reduced betafunctions in IR4-12.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Nominal PEP</th>
<th>Low Tune Optics</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tau_{\perp}$</td>
<td>10.7ms</td>
<td>9.0ms</td>
</tr>
<tr>
<td>$\xi_{\perp}$ (natural)</td>
<td>5.35m</td>
<td>4.5m</td>
</tr>
<tr>
<td>$\xi_{\parallel}$ (natural)</td>
<td>73.5</td>
<td>36.5</td>
</tr>
<tr>
<td>$\xi_{\parallel}$ (natural)</td>
<td>46.5</td>
<td>36.5</td>
</tr>
</tbody>
</table>

Table IV Time constants and natural chromaticities

Vertical Beam Size

The vertical size of the beam at IR2 can be expressed as the quadrature sum of the RMS betatron amplitude and dispersion terms,

$$
\sigma_y = \sqrt{\varepsilon_y \beta_y + \left( \eta_y \frac{\Delta p_y}{p} \right)^2}
$$

Here, the vertical emittance, $\varepsilon_y$, is the sum of the 'natural' emittance $\varepsilon_y$ generated by quantum effects, sextupole and error field coupling, and a current dependent emittance due to the beam-beam interaction, $\varepsilon_{yB}$. The term $\beta_y$ is the vertical beta function.

For completeness, we assume that some horizontal emittance is coupled into the vertical plane of the 6x6 (9x9) bunch collider. To be consistent with PEP-NOTE-384, 1.3x-nm-rad of 'stray' emittance will be added to the 'natural' emittance. Of course, as the magnitude of the vertical orbit distortion is increased, the dispersion and consequently the 'natural' emittance increase leading to a blow-up in the vertical beam size at IR2. For this study, the beam-beam component of the emittance, $\varepsilon_{B}$, is not considered.

PEP is normally operated with $\beta_y^* = 5.5$cm to enhance the injection efficiency. For the nominal separator plate voltage, we estimate for the 6x6 (9x9) collider $\varepsilon_y(\text{natural}) = 3.35 \times 10^{-3}$-nm-rad, $\eta_y^* = 8$mm, and $\sigma_y^* = 17.6$mm.

Using GEMINI, the vertical orbit, dispersion, beam emittance and beam size were calculated as a function of the separator plate kick and plotted in Figs 7-10. The amplitude of the dipole kick seen by the beam at the separator plates was chosen as the independent variable because there are two different beam separation distances (1) in the arcs and (2) in the IRs. In the orbit distortion plot, the two curves correspond to these positions for the nominal kick value of $\theta = \pm 0.24$rad. Note that the emittance blows-up quadratically, and has increased by almost a factor of 4 at $\theta = \pm 0.24$rad. The effect of $\eta_y^*$ is to increase the vertical beam size by about 10% at the design beam separation distance. The rapid increase of vertical beam size with separator strength limits efficient operation at large separation distances because PEP is currently limited by RF drive constraints.

The bunch separation distances normalized to the horizontal beam size are shown in Fig. 11. At each bunch crossing position, horizontal beam size is calculated from,

$$
\sigma_x(s) = \sqrt{\varepsilon_x \beta_x(s) + \left( \eta_x(s) \frac{\Delta p_x}{p} \right)^2}
$$

and the separation distance, $<2 \varepsilon \cos \theta / \sigma_x>$ is the average of values taken over the arc and in IR4-12 crossing points, respectively. In the arcs, the nominal separation is $\approx 2.5 \times 10^{-3}$, and in the IRs the separation is $\approx 3.4 \sigma_x$ for the design orbit. As a general rule, beams crossing with a vertical displacement should be separated by $\approx 3.0 \sigma_x$ or more, and vertical crossings are less damaging than horizontal crossings from both an emittance growth and tune-shift point of view[16].

Long range beam-beam tune shifts

The expression for tune shift caused by a gradient error,

$$
\delta \nu = -\frac{1}{4 \pi} \int 5K \beta ds
$$

can be used to evaluate the effect of the long-range interaction on the tunes. Here, $\delta K$ is the linear component of the Taylor expansion for force evaluated at position 's' and normalized to the beam energy,

$$
\delta K(s) = \frac{\nu F(s)}{\nu m c^2}
$$

In the case of a highly relativistic test charge passing near a bunch with charge distribution $\rho(x,y)$, the test charge sees an instantaneous force $F = 2eE_{\text{test}}$ where $E_{\text{test}}$ refers to the electric field in the rest frame of the bunch. For a Gaussian bunch, the charge distribution, potential distribution and field gradients in the rest frame are:

$$
\rho(x,y) = \frac{N}{2\pi \sigma_x \sigma_y} \exp \left[ -\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} \right]
$$

$$
\Phi(x,y) = \frac{N e}{4 \pi \sigma_y} \int \exp \left\{ -\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2} \right\} dq
$$
\[
\frac{\partial E_x(x,y)}{\partial x} = \frac{N_e e}{2\pi \sigma_0} \int_0^{\infty} \left(1 - \frac{2x^2}{2\sigma_x^2 + q} \right) \exp \left\{ \frac{-x^2}{2\sigma_x^2 + q} - \frac{y^2}{2\sigma_y^2 + q} \right\} \frac{dq}{(2\sigma_x^2 + q)^{3/2} \sqrt{2\sigma_y^2 + q}}
\]

and similarly for \( \frac{\partial E_y}{\partial y} \). The quadrupole coefficients are then

\[
\delta K_x = -\frac{2\sigma_x}{\eta m_e c^2} \frac{\partial E_x}{\partial x} \quad \text{and} \quad \delta K_y = -\frac{2\sigma_y}{\eta m_e c^2} \frac{\partial E_y}{\partial y}.
\]

Note that from \( V+e = 0 \), the \( x \) and \( y \) field gradients will have opposite sign for beams sufficiently separated. Vertical separation will cause the horizontal tune to increase, and the vertical tune to go down. Substituting the field gradient errors into the tune-shift formula, integrating over the bunch length, and evaluating at a vertical separation \( y_0 \) between bunch centers we find for the long-range tune shifts:

\[
\delta v_x = \frac{\beta_x N_{b0}}{2\pi \gamma} \int_0^{\infty} \left(1 - \frac{2y_0^2}{2\sigma_y^2 + q} \right) \exp \left\{ \frac{-y_0^2}{2\sigma_y^2 + q} \right\} \frac{dq}{(2\sigma_y^2 + q)^{3/2} \sqrt{2\sigma_y^2 + q}} \quad [5]
\]

\[
\delta v_y = \frac{\beta_y N_{b0}}{2\pi \gamma} \int_0^{\infty} \left(1 - \frac{2y_0^2}{2\sigma_x^2 + q} \right) \exp \left\{ \frac{-y_0^2}{2\sigma_x^2 + q} \right\} \frac{dq}{(2\sigma_x^2 + q)^{3/2} \sqrt{2\sigma_x^2 + q}} \quad [6]
\]

where \( N_{b0} \) is the number of charges per bunch and \( r_0 \) is the classical electron radius, \( \frac{e^2}{4\pi \varepsilon_0 m_0 c^2} \).

To calculate the long-range tune shifts, the transverse beam dimensions \( \sigma_{x,y} \) evaluated at the crossing points in the arcs and in the IRs can be used in conjunction with the beam separation distances. Substituting the values found in the last section, (Figs. 7 & 10 with \( \theta = 0.24 \text{mrad} \)) into Eqs. 5, 6, the mean long-range beam-beam tune shift can be evaluated numerically and summed over all crossing points. The results are plotted in Figs. 12 and 13 for the 6x6 and 9x9 bunch colliders with a total beam current of 85mA. In the vertical case, \( \delta v_y \), the tune shift is the opposite of the value shown on the plots (\( V+e = 0 \)). Virtually no vertical tune shift is predicted from bunch crossings in the IRs (not shown).

The largest tune shift is in the horizontal plane, with the greatest component coming from bunch crossings in the arcs. For the 6x6 case, the horizontal total tune shift, \( \delta v_x \approx 0.04 \) is comparable to the tune shift for the 3x3 bunch configuration operating at 45mA. The differences stem primarily from the different \( \beta^* \) in the two configurations (sensitivity) and the charge per bunch. With a horizontal tune of 20.70, there appears to be no immediate danger of pushing \( v_x \) out to the integer even at high currents.

Finally, we note that these calculations are not self-consistent because (1) the actual beam-beam interaction includes a dipole kick which will modify the closed orbit (bunch separation) and (2) no emittance blow-up is included from either the head-on collisions at IR2(10) or from non-linear components in the long-range force.

**Discussion**

In this paper, we have reviewed a beam separation scheme and optics that would permit up to 9 bunches in each beam to be collided with a minimum of hardware changes. In fact, since the voltage range for the compensating plates at IR2 has already been upgraded, the experiment can in principle proceed. Operationally, injection must first be established for the new 'low tune' lattice, followed by general characterization of the configuration, and finally the beams brought into collision. It is unknown at this time how PEP will behave in the unexplored region of the tune diagram, and in particular, how the dynamic aperture will evolve.

The calculations outlined here indicate that the vertical emittance will increase considerably blow-up due to quantum emission in regions of vertical dispersion through the arcs. The beam blow-up, of course, demands more current to produce luminosities in the range of interest. To counter the increased beam size, one could collide 6x6 bunches to operate closer to the tune shift limit. In a companion paper(10), it is estimated that a luminosity of \( 9 \times 10^{34} \text{cm}^{-2} \text{s}^{-1} \) may be possible by colliding 6x6 bunches with ~85mA in PEP. Considerations derived from this paper indicate that the long-range tune shift should not be a problem, but the issues of x-y coupling via the beam-beam interaction and in the sextupoles remains open. Considering both the coupling mechanisms and the potential loss of aperture for large beam separation distances, the maximum average luminosity will probably be found with the minimum stable beam separation distance.

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List of Figures
1. Plan view of PEP showing bunch crossing points for 9x9 operation.
2. Vertical closed orbit distortions for design beam separation.
3. Sensitivity of closed orbit to vertical tune.
4. Phase advance for nominal PEP and low-tune lattices.
5. Betanfunctions for low-tune lattice.
6. Vertical closed orbit and dispersion functions in the RF straight.
7. Closed orbit amplitude vs. separator strength.
8. Vertical Dispersion at IR2 vs. separator strength.
9. Vertical emittance vs. separator strength.
10. Vertical beam size vs. separator strength.
11. Beam separation normalized to horizontal beam size vs. separator strength.
12. Estimates for the accumulated long range tune shift colliding 6x6 bunches.
13. Estimates for the accumulated long range tune shift colliding 9x9 bunches.

References
Fig. 5

Fig. 4

Nominal FEC lattice
Low Time Lattece