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An Effective Lagrangian Approach to the Quark/Hadron Phase Boundary

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Abstract

We consider an effective Lagrangian approach to the quark-hadron phase transition at both finite temperature, T , and baryon chemical potential μ . In addition to the pressure equilibrium point (T_c, μ_c) , we find critical points (T_x, μ_x) in the phase diagram beyond which the hadron phase no longer exists. We find a substantial (model-dependent) region of super-heated hadron phase between (T_c, μ_c) and (T_x, μ_x) . This may have important consequences for searches for quark-matter in heavy-ion collisions.

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The phase boundary between hadronic matter and the quark-gluon plasma is the object of active theoretical research using many different approaches¹⁻⁴⁾, is being investigated intensively in accelerator experiments⁵⁾ at CERN and elsewhere, and is important for modelling condensed objects in astrophysics and the quark-hadron phase transition in the early Universe. The most reliable calculational approach to this phase boundary would appear to be on the lattice¹⁾. On present computers this approach can only deal with the case of vanishing chemical potential. However, the experimental and astrophysical problems of interest require an understanding of the behaviour of strongly-interacting nuclear matter with a non-vanishing chemical potential. A promising strategy for tackling this case is to use an analytical model that embodies our general knowledge of QCD and reproduces successfully the lattice results at zero chemical potential.

In previous papers⁴⁾ we have shown how an effective Lagrangian with approximate scale and chiral invariance can be used to reproduce the conventional wisdom about the quark-hadron phase transition at zero chemical potential. It strongly suggests simultaneous (possible weakly) first-order transitions for gluon and quark condensation and confinement. These results are consistent with many lattice studies of the quark-hadron phase transition at finite temperature T . The model can give a phenomenologically acceptable value of the critical temperature T_c , at least if the non-perturbative trace anomaly coefficient, related to the bag constant, is large enough. A crucial role in driving the simultaneous phase transitions is played by the onset of the exponentially-divergent Hagedorn spectrum of hadronic bound states.

In this paper we extend our previous analyses⁴⁾ to the general case of non-vanishing chemical potential μ , and attempt to delineate the quark-hadron phase boundary in the (T, μ) plane. As in the $\mu = 0$ case, for any fixed value

of $\mu \neq 0$ we find two critical temperatures: the lower one is $T_c(\mu)$ where the pressures in the quark and hadron phases are equal, and the higher one is $T_x(\mu)$ marking the disappearance of the local minimum of the effective potential that represents hadronic matter. In between these two temperatures, nuclear matter could in principle superheat temporarily in the hadronic phase, even though this is thermodynamically disfavored. Thus there is a band in the (T, μ) plane in which a relativistic heavy-ion collision (for example) might not create a quark-gluon plasma, despite it being thermodynamically favored. The precise location of this ambiguous band depends on parameters of the effective potential such as the bag constant B and hadronic coupling constants and interactions. We give detailed numerical results for the choice $B = 450 \text{ MeV/fm}^3$, and comment on the modifications entailed by other choices of B .

Before starting our discussion of the phase boundary in the (T, μ) plane, we first remind the reader of the effective Lagrangian approach that we have previously used together with B. Campbell to analyze the $\mu = 0$ case. We use an effective Lagrangian that embodies the approximate chiral and scale symmetries⁶⁾ of QCD, and therefore expresses accurately the dynamics of the light hadronic degrees of freedom, as well as the phenomena of quark and gluon condensation. The nine (pseudo-)Goldstone pseudoscalar mesons ϕ_i ($i=0,1,\dots,8$) are represented by a dimensionless meson field

$$U(x) = \exp \left(i \sum_{i=0}^8 \frac{\lambda_i \phi_i(x)}{f_\pi} \right) \quad (1)$$

where the λ_i are SU(3) octet representation matrices and $f_\pi \approx 93 \text{ MeV}$ is the pion decay constant. The conventional chiral Lagrangian constructed out of $U(x)$ alone does not have the same scaling properties as QCD. This defect can

be corrected by introducing a flavour-singlet (pseudo-)dilatons field $\chi(x)$, whose vacuum expectation value $\langle \text{vev} \rangle \chi_0 = \langle 0|\chi|0 \rangle$ breaks scale invariance spontaneously⁷⁾. Our effective Lagrangian reads

$$L = \frac{1}{16} f_\pi^2 \left(\frac{\chi}{\chi_0} \right)^2 \text{Tr} (\partial_\mu U \partial^\mu U^\dagger) + \frac{1}{2} \partial_\mu \chi \partial^\mu \chi - c \left(\frac{\chi}{\chi_0} \right)^3 \text{Tr}(m_q(U+U^\dagger)) - \frac{1}{2} m_\phi^2 \phi_0^2 \left(\frac{\chi}{\chi_0} \right)^4 - V_\chi \quad (2)$$

where $m_q = \text{diag}(m_u, m_d, m_s)$ is the light quark mass matrix, c is a constant with mass dimension 3, $m_\phi^2 = 0(1/N_c)$ is an extra $U(1)$ -breaking mass term for the ninth pseudoscalar meson ϕ_0 , and V_χ is a potential term reflecting the QCD scale anomaly⁸⁾ $\partial^\mu S_\mu = (\beta/2g) F_{\mu\nu} F^{\mu\nu}$ that takes the general form⁹⁾

$$V_\chi = B [1 + 4 \left(\frac{\chi}{\chi_0} \right)^4 \ln(\chi/e^{1/4} \chi_0)] \quad (3)$$

We therefore relate χ_0 to the gluon condensate

$$\langle 0|\partial^\mu S_\mu|0 \rangle = \langle 0|(\beta/2g) F_{\mu\nu} F^{\mu\nu}|0 \rangle = -4B \quad (4)$$

where B is the conventionally-normalized bag constant and the quark condensate is given by

$$\langle 0|\bar{q}q|0 \rangle = c \left(\frac{\chi}{\chi_0} \right)^3 \langle 0|(U+U^\dagger)|0 \rangle \quad (5)$$

The particle related to the χ field can be regarded as the lightest scalar glueball. Heavy hadron multiplets have couplings to χ of magnitude

$$g_{\text{hh}\chi} = m_h(\chi/\chi_0) + 0(m_q) \quad (6)$$

reflecting the origin of the bulk of their masses in the dynamically-induced spontaneous breakdown of QCD scale invariance: $\chi_0 \neq 0$.

It is evident from Eq. (5) that the quarks can condense only if the gluons have already condensed (4). In view of the flat Coleman-Weinberg nature of V_χ (3), one would expect the $\langle 0|\chi|0 \rangle \neq 0$ gluon condensation transition and hence also the quark condensate formation to be generically first order^{*}, and these expectations have been borne out by dynamical calculations⁴⁾ using heavy hadronic couplings of the form (6). Comparing the hadronic pressure $P_H(T, \mu=0)$ - $-V_H(\phi, \chi, T)$ with the quark-gluon plasma pressure $P_Q(T, \mu=0)$ - $-V_g(B, N_g, N_f, T)$:

$$V_Q = B - \frac{1}{90} \pi^2 \left\{ 2N_g + \frac{21}{2} N_f \right\} T^4; \quad N_g=8, \quad N_f=3 \quad (7)$$

that is relevant for $\langle 0|\chi|0 \rangle = 0$, one finds the qualitative picture shown in Fig. 1. The hadronic phase with $\langle 0|\chi|0 \rangle = \chi_0$ is thermodynamically favoured when $0 \leq T < T_c$. At T_c the quark and hadronic phases have equal pressures, and this would normally be regarded as the critical temperature. However, if $T_c < T_x$, there is the possibility of superheating hadronic matter in the local minimum $\langle 0|\chi|0 \rangle \neq 0$ of the effective potential. At T_x the local hadronic minimum finally disappears, and only the quark phase exists. We show in Fig. 2a the quark and hadronic pressure curves for $T \neq 0$, $\mu = 0$, assuming $B = 450 \text{ MeV/fm}^3$, and we see that in this case $T_c = 170 \text{ MeV}$ and $T_x = 240 \text{ MeV}$.

*. Possibly only weakly first order.

We now extend the previous analysis to $\mu \neq 0$, and analyze the loci of $T_c(\mu)$ and $T_x(\mu)$.

In our previous analysis at finite temperature⁴⁾, the simplest hadron model we considered was that of a free hadron gas. We found that at T_x , the pressure in the hadron phase $P_H(T_x)$ was less than the corresponding pressure in the quark-gluon phase $P_Q(T_x)$. In other words, T_x is sufficiently low that the multitude of hadronic states do not yet dominate P_H . In our present treatment of the phase transition at finite baryon density, we begin by examining free hadrons as well, though, as we will see shortly, with less favorable results.

The pressure for a free hadron gas at finite temperature and chemical potential can be written as

$$P_H = \mp \frac{T}{(2\pi)^3} \sum_i [g_i \int d^n q_i (1 \pm n_{q_i})^3 d^3 q_i] - V_x$$

$$- \sum_i \frac{g_i}{2\pi^2} \int_0^{q_i} \frac{n_{q_i} d q_i}{3\epsilon_{q_i}} - V_x \quad (8)$$

where

$$n_{q_i} = \frac{(\epsilon_{q_i} - \mu_i)/T}{(e^{\epsilon_{q_i} - \mu_i/T} \pm 1)} \quad (9)$$

$$\epsilon_{q_i} = (q_i^2 + m_i^2)^{1/2} \quad (10)$$

and $\pm$ refers to fermions or bosons. The sum in Eq. (8) runs over all known hadronic states. For the present calculations we have included 158 hadronic

mass states making up a total of 2898 spin states. According to our previous arguments regarding scale symmetry, the hadron masses m_i are scaled by the gluon condensate

$$m_i = \hat{m}_i (X/X_0) \quad (11)$$

where $\hat{m}_i$ is the measured mass in the low T/μ vacuum ($X \approx X_0$). We have also assumed the chiral limit and have taken the masses of the lowest lying meson octet to vanish.

A net baryon density is accounted for by a non-zero baryon chemical potential. In general, there are two independent chemical potentials corresponding to the conserved quantities baryon number and electric charge. In the hadron phase, the chemical potential of the i 'th hadron is assigned simply by

$$\mu_i = B_i \mu_B + Q_i \mu_Q \quad (12)$$

where B_i and Q_i are the baryon number and electric charge. Eq. (12) also applies in the quark phase and to leptons which have been included. Strangeness conservation has not been imposed. To guarantee the absence of any net electric charge, μ_Q is adjusted for each fixed μ_B . These assumptions are applicable to astrophysical and cosmological conditions, but not to relativistic heavy-ion collisions, where we expect similar conclusions.

In the quark phase, we consider a gas of non-interacting u , d , s and c quarks with masses (0, 0, 150, 1500) MeV respectively. (Higher order

perturbative corrections have not been included here *.) The free gas pressure P_Q is calculated analogously as in Eq. (8) with the substitution of the potential V_X with the bag constant, B , and the gluonic pressure at finite temperature, $(8\pi^2/45)T^4$.

Results for the free hadron case at finite temperature and zero net baryon density were given in Ref. (4). In Fig. 2b, we show the corresponding phase diagram for the case with T=0 and finite baryon density. The pressure P/μ_B^4 is plotted vs. μ_B . The quark pressures are labeled by the value of the bag constant chosen in units of MeV/fm^3 . The thick line represents the hadron pressure. The X's on this curve represent the critical points in the hadron phase where the $\chi \neq 0$ minimum of the free energy (maximum of the pressure) ceases to exist, for $B = 50 \text{ MeV}/\text{fm}^3$ (left X) and $B = 450 \text{ MeV}/\text{fm}^3$ (right X). We see in Fig. 2b that a sensible transition only occurs when $B = 50 \text{ MeV}/\text{fm}^3$. In fact, for $B \gtrsim 100 \text{ MeV}/\text{fm}^3$, we find no transition at all. However, the transition for $B = 50 \text{ MeV}/\text{fm}^3$, occurs at extremely low densities $n_c \ll n_0$, where $n_0 = 0.153 \text{ fm}^{-3}$ is the density of normal nuclear matter. The sharp increase in P is understandable as it occurs at $\mu_B \approx 1 \text{ GeV}$, and the bulk of the hadronic states begin to contribute.

A sensible phase transition at finite density is recovered when we include interactions among the hadrons^{3,11}. Here, as before, we consider interactions based on a mean field approach. The hadronic pressure P_H is now calculated by first computing the hadron number density

*. These may be important since they tend to lower the pressure and could cause an inconsistency in the phase diagram. Such effects were found in Ref. (10).

$$n_H = \sum_i \frac{g_i}{2\pi^2} \int n_{q_i}^2 dq_i \quad (13)$$

and the energy per particle ϵ_{q_i} in Eq. (10) is modified by^{3,11,12}

$$\epsilon_{q_i} = (q_i^2 + m_i^2)^{1/2} + U_i(n_H) \quad (14)$$

$$U_i(n_H) = K n_H (m_i/m_N)^2 \quad (15)$$

where the constant in the potential energy is $K \approx 660 \text{ MeV fm}^3$, as implied by the Reid potential and the $\pi - \pi$ effective interaction. The factor m_i/m_N applies only for $m_i > m_N$, the nuclear mass. Because of the dependence on n_H in the potential energy, (13) must be solved self-consistently. Given $U(q_i)$ for a particular value of μ_B , P_H is computed using Eqs. (8, 9, 14, 15) and the total pressure is given by

$$P_H = \sum_i \frac{g_i}{2\pi^2} \int \frac{q_i^4}{3\epsilon_i} n_{q_i} dq_i + \frac{1}{2} K n_H^2 - V_X \quad (16)$$

(The middle term in (16) is due to the presence of interactions and is given in an approximate but numerically accurate form.)

At this point we need to comment on the possible χ dependence of the mean-field potential U . The constant K derived from the Reid potential is a sum of terms proportional to g_a^2/m_a^2 where m_a is the mass of the exchanged vector or scalar particle contributing to the $N - N$ interaction, and g_a is its coupling to nucleons. In principle because one would expect the mass m_a to scale with χ , there should at least be a factor of $(X_0/\chi)^2$ in Eq. (15).

However, one would also expect the coupling g_a to scale, though perhaps in a non-trivial way. Previously¹³⁾, we have considered the density dependence of the $\pi - N$ coupling. If we assume that the couplings g_a scale in the same way, the Goldberger-Treiman relation gives us $g_a \propto m_N g_A / f_\pi$. Estimates for the density dependence of the axial-vector coupling constant g_A and f_π were given in Ref. (14). If we further assume that f_π scales with X , we find

$$g_a/g_a(X=X_0) = (1-g'_0) + g'_0 (X/X_0) \quad (17)$$

where g'_0 is the Landau-Migdal parameter. For $g'_0 = 0$, there is no scaling of g_a and $U \propto (X/X_0)^2$. For $g'_0 = 1$, $g_a \sim (X/X_0)$ and U is independent of X . Accepted values for g'_0 are between 0.5 - 0.7. We have adopted $g'_0 = 1$, which is also the simplest case to calculate, and ignore the X -dependence of the mean field potential. Aside from its obvious simplicity, any other value of g'_0 results in a potential which grows as $(X/X_0)^2$ for small X . We would expect our description of the mean field to break down in this case and an undetermined modification of U would need to be derived. One should be aware however, that the X dependence can change the qualitative as well as quantitative results we are describing. Indeed in Refs. (10, 15) the effective choice $g'_0 = 0$, was made and no phase transition to quark-gluon matter was observed, or occurred at large values of (X/X_0) .

The use of effective mean field potentials in studying the phase transition at finite temperature has been considered previously^{3,4,12)}. The study of the critical point T_x was examined recently within this approach using a Hagedorn exponential mass spectrum. In Fig. 2b we show the analogous result for the present case employing known hadrons. In Fig. 2c we show the results of the $T = 0$, finite baryon density calculations. The critical density

n_{B_c} using a mean potential was studied in Ref. (11). Here we concentrate on the calculation of n_x (or more particularly μ_x). Results are shown in Fig. 2c for five values of the bag constant B . Only two of the five $B = 50$ and 450 MeV/fm³ show sensible phase transitions. For the other three $P_H(\mu_x) > P_Q(\mu_x)$. Furthermore, when $B = 50$ MeV/fm³, $\mu_x \approx 920$ MeV and $n_x \ll n_0$. This implies that only the upper values of the bag constant $B > 400$ MeV/fm³ give rise to meaningful phase transitions. For $B = 450$ MeV/fm³, $\mu_x \approx 1650$ MeV and $n_x \approx 6.8 n_0$. The transition density in this case is $\mu_c \approx 1600$ MeV and $n_c \approx 6n_0$, not unreasonable values.

We have also extended these $T = 0$ calculations to finite temperature and density. The phase diagram in the $T - \mu$ plane is shown in Fig. 3 where both the transition curve ($P_H(T, \mu_c) = P_Q(T, \mu_c)$), and the critical curve (T_x, μ_x) beyond which the hadron phase no longer persists. The intermediate region $T_c < T < T_x$ and $\mu_c < \mu < \mu_x$ corresponds to a superheated hadron phase.

We have seen in this paper that our previous effective Lagrangian model can be used to map out the quark/hadron phase boundary in the (T, μ) plane. Including low-mass hadrons and repulsive interactions we obtained a reasonable picture for a bag constant $B \gtrsim 400$ MeVfm⁻³. This choice gives a realistic critical temperature at $\mu = 0$, and a plausible critical density $n_c \gtrsim 6n_0$. We found that the hadronic phase could exist in an unstable superheated state up to a second critical temperature T_x (chemical potential μ_x), somewhat larger than $T_c(\mu)$. It would in principle be possible for nuclear matter to remain in the unstable hadronic phase even when heated and compressed into the domain between these two critical curves in the phase diagram shown in Fig. 3.

More calculations are required to see whether such superheating occurs to a significant extent in heavy ion collisions. Our equation of state should be

modified to exclude leptons and to conserve strangeness, and it would be desirable to include the effects of bulk motion. The transition rates between the superheated hadron and thermodynamically-favored quark phases should be computed and compared with the expansion rate. The possible existence of a mixed phase should be studied. We plan to address these questions in future work, but feel that we have already demonstrated in this paper that our effective Lagrangian model can usefully be applied to conditions of experimental interest.

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Figure Captions

Figure 1: Sketch of the behavior of the effective potential for the gluon condensate parameter χ as the temperature T is increased. (The behavior for the chemical potential is similar.) At T_c there are two competing vacua, one with $\chi = 0$ and one with $\chi \neq 0$. At T_x , there ceases to be a minimum at $\chi \neq 0$.

Figure 2a: The pressure P/T^4 vs. temperature for the hadron phase (H) using known particles and mean field potentials and for the quark-gluon phase (Q) with $B = 450 \text{ MeV/fm}^3$. The χ indicates the critical point T_x in which the hadron phase ceases to exist.

2b: The pressure P/μ_B^4 vs. chemical potential for the hadron phase (H) using known particles in the absence of interactions and for the quark-gluon phase with values of the bag constant as indicated.

2c: As in Fig. 2b but including mean field potentials for the hadrons.

Figure 3: The phase diagram in the $T - \mu_B$ plane. The inside contour shows the points (T_c, μ_c) , while the outer contour shows (T_x, μ_x) . In interior region corresponds to a superheated hadron phase with $\chi \neq 0$.

Figure 1

Figure 2a

Figure 2b

Figure 2c

Figure 3