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CONSEQUENCES OF SUPERPOTENTIAL RENORMALISATION

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Abstract

Recently, the non-renormalisation theorem for supersymmetric theories has been shown to be invalid when massless fields are present. Nevertheless we show that if supersymmetry is unbroken in the tree approximation then it cannot be broken by radiative corrections. We also discuss whether spontaneously broken supersymmetry might be restored by radiative corrections.

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The non-renormalisation (NR) theorem has been of considerable interest in the development of supersymmetry [1,2]. The fact that the superpotential is not renormalised by quantum corrections is important because otherwise it might have been expected that quantum corrections break supersymmetry [3]. Additionally, the NR-theorem has been used to justify the fine tuning of superpotentials in model building because terms omitted in the tree level superpotential will not be introduced by quantum corrections.

The NR theorem states that contributions arising to the effective action are of the form

$$A_G = \int d^4\theta d^4x_1 \cdots d^4x_n G(x_1, \dots, x_n, \theta, \bar{\theta}) \quad (1)$$

with no contributions arising of the form of the superpotential,

$$A_W = \int d^2\theta d^4x W(x, \theta) + \text{c.c.} \quad (2)$$

For a renormalisable theory, the superpotential, W , should be a polynomial in the fields of at most cubic order. Recently [4,5,6] it has been shown that the NR-theorem is not valid in the presence of massless fields. In particular, in ref. [6] it was shown that for the simple one-field Wess-Zumino (WZ) model [7], the graph shown in fig. 1 gives rise to a correction to the superpotential, if the field is massless. Simple power counting shows that the correction is finite, but its existence is sufficient to invalidate the usual statements of the NR-theorem. If the field is massive, then it is easily demonstrated that the graph vanishes in the limit that the external momenta are set equal to zero, so that W is unrenormalised, in accordance with the NR-theorem.

Let us turn now to multi-field generalisations of the WZ model. Clearly, if not all the fields are massive, then there may arise corrections to W from, for example, fig. 1, when some or all the propagators are massless. These corrections may generate interactions omitted from the original W , if such interactions are consistent with the symmetries of the theory. As an example, consider a two field Wess-Zumino model with chiral superfields X and Y . If the superpotential is

$$W(X, Y) = \lambda_1 X^3 + \lambda_2 Y^3 \quad (3)$$

then the X^3 and Y^3 vertices will be renormalised. The two loop contribution to λ_1 is

$$\frac{3\zeta(3)}{(16\pi^2)^2} \lambda_1^5 \quad (4)$$

and similarly for λ_2 . (Although in ref. [6] this calculation was presented using component fields, D-algebra experts will have no difficulty obtaining the same results using superfields.)

Of course X and Y are trivially decoupled in this case so no new interactions are generated. If the superpotential was of the form

$$W(X, Y) = \lambda_3 X^2 Y + \lambda_4 X Y^2 \quad (5)$$

then X^3 and Y^3 interactions are generated via fig. 1. All four cubic interactions are consistent with an R -symmetry, with $X \rightarrow e^{i\alpha} X$, $Y \rightarrow e^{i\alpha} Y$, where α is a constant.

Thus unless the theory is natural (in the sense that all interactions consistent with a set of symmetries have been included) new cubic interactions can be generated in perturbation theory. Of course they could be reinserted in the tree approximation, and the resulting total contribution to the effective action tuned to zero, [8], but we submit that such a procedure would be decidedly unnatural, as opposed to “technically natural” (a term formerly applied to the omission from W of terms not forbidden by a symmetry).

Let us now consider the general situation: a many-field Wess-Zumino model with linear, quadratic and cubic terms. Clearly, if some of the superfields are massless, then there may be renormalisation of the cubic terms, as described above. But what of the linear and quadratic terms? In fact it is straightforward to see that these *do not* suffer finite renormalisations. Consider, for example, the quadratic terms. Any graph potentially contributing necessarily has at least one chirality-flipping propagator (in superspace). This has the form $mD^2/(k^2(k^2 + m^2))$, where D is the usual covariant spinor derivative. For large m , this propagator behaves as $1/mk$. If we make this replacement, however, straightforward power counting shows that the graph becomes at most logarithmically divergent. From this it is straightforward to see that the factor m^2/p^2 (where p is an external momentum), necessary to generate a finite renormalisation of the quadratic term, cannot appear. Similar arguments may be used to preclude renormalisation of the linear term.

We have seen that new cubic terms can be generated by radiative corrections. Is it possible, therefore that supersymmetry breaking may be thereby generated? We can prove quite simply that this does not occur.

Consider a many-field WZ model with superpotential $W(\phi^i)$. Suppose there exists a supersymmetric ground state: that is the equations

$$W_i \equiv \partial W / \partial \phi^i = 0 \quad (6)$$

have a solution $\phi^i = \phi_0^i$. If we expand W about this vacuum, then we obtain quadratic and cubic terms,

$$W(\phi_0^i + \phi_q^i) = W(\phi_0^i) + W_{ij} \phi_q^i \phi_q^j + W_{ijk} \phi_q^i \phi_q^j \phi_q^k. \quad (7)$$

Note that there is no terms *linear* in the quantum fields. Now if there are massless fields, the cubic term may be renormalised, and new interactions generated. There will still be no *linear* term, however. Consequently, $\phi_q^i = 0$ remains a solution to equation (6) and a supersymmetric ground state; supersymmetry breaking cannot be induced by the quantum corrections. Additionally, since the mass spectrum only depended upon the quadratic terms the mass spectrum is also unaffected by the renormalisation.

Suppose, on the other hand, that there is F-type supersymmetry breaking: i.e. the equations $dW/d\phi = 0$ do not have a simultaneous solution. Might supersymmetry be restored by radiative corrections? Expanding the fields about their v.e.v.s as before, the resulting W has the structure [8]

$$W = -f\eta + a_{ij}\phi_i\phi_j + b_{ijk}\phi_i\phi_j\phi_k + c_{ij}\phi_i\phi_j\eta \quad (8)$$

Here η is the goldstino (super)field. Its v.e.v. is in fact arbitrary in the tree approximation, but gets fixed by radiative corrections, in general [9]. It appears possible (in principle) that a η^3 interaction could be generated via fig. 1, whereupon it is easy to see that there is a supersymmetric ground state. We have been unable to construct an explicit example, however: in O’Raifeartaigh’s original model for instance [10], the relevant fields are massive and hence the couplings are subject to the NR-theorem. It would be of considerable interest to know whether the possibility of restoration of supersymmetry could be realised.

We have for clarity limited ourselves to WZ models, but it is clear that most of the discussion, and in particular our proof of the impossibility of radiative corrections inducing F type breaking of supersymmetry, carries over to the case of a general renormalisable gauge theory. We have not considered D -type breaking, but believe that the same conclusions will hold in that case.

With regard to phenomenology, it is clear that, in general terms, the considerations of this paper do not adversely affect the attractions of low energy supersymmetry:- in particular *vis à vis* the hierarchy problem. Supersymmetry breaking is usually achieved by simply adding soft supersymmetry breaking terms, including a common mass for all the scalar fields (arising, perhaps, from the gravitational couplings to an otherwise decoupled “hidden” sector). Without massless scalars, chiral superfields would not, we believe, alone give rise to a renormalisation of W ; but such an effect remains possible in the presence of massless vectors. In fact, if there are both massless vector and chiral scalar superfields, it is easy to see that there is a one-loop graph providing a possible renormalisation of W .

To summarise, we have shown that although the presence of massless chiral superfields leads to violations of the non-renormalisation theorem, supersymmetry cannot be broken through radiative corrections -at least in perturbation theory. The question of

phenomenological consequences remains open: the exploitation of “technical naturalness” to omit undesirable terms from the superpotential is clearly best avoided, however.

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Figure 1. Leading potential contribution to the renormalisation of the superpotential in the Wess-Zumino model.