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QUANTIZATION OF THE FREEDMAN-TOWNSEND MODEL OF MASSIVE VECTOR MESONS

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ABSTRACT

We examine a model for massive vector mesons in four dimensions proposed by Freedman and Townsend, where the masses for non-Abelian vector gauge fields are generated without symmetry breaking through a gauge invariant coupling to anti-symmetric tensor fields. The model is quantized using the formalism of Batalin and Vilkovisky. While the Abelian version immediately gives a renormalizable model for massive vector fields, it is shown that in the non-Abelian case the addition of an extra gauge invariant term in the initial Lagrangian leads to an ultraviolet behaviour consistent with power-counting renormalizability.

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Whereas in two(1) and three(2) dimensions, there exist gauge invariant models of massive vector fields that generate consistent quantum field theories, in four dimensions spontaneous or dynamical symmetry breaking is widely held as being the only viable way of giving a mass to non-Abelian gauge fields. The Stueckelberg-Kunimasa-Goto mechanism⁽³⁾, although it gives rise to a gauge invariant theory of massive vector mesons, is not consistent with renormalizability(4). A variant of this approach introduced by Delbourgo and Thompson⁽⁵⁾ has been shown to violate unitarity⁽⁶⁾.

In the Higgs mechanism⁽⁷⁾, fundamental scalar fields are used to generate the longitudinal mode of the massive vector fields. In the standard model this technique also gives rise to a residual spin-zero particle in the physical states which so far has remained undetected in experiments. The advantage of this model is that it is consistent with both renormalizability and unitarity⁽⁸⁾.

In this paper we use the gauge invariant anti-symmetric second rank tensor formulation of a massless spin zero particle to provide the longitudinal mode of the massive vector meson⁽⁹⁾. In this way at the classical level the vector meson acquires mass, has the correct number of propagating modes, but is not accompanied by a spin zero state in the physical sector. In any case, an investigation of the anti-symmetric tensor field is interesting in its own right, as these fields arise naturally in String theories⁽¹⁰⁾.

After presenting a consistent quantization of such a model within the framework of manifestly unitary formalism of Batalin and Vilkovisky⁽¹¹⁾, we draw attention to the fact (which has escaped previous discussions) that the propagator of the massive vectors turns out to have a power-counting renormalizable form. While in the Abelian case this immediately yields a renormalizable theory of massive vector fields, we discuss the intriguing

possibility of a renormalizable formulation also in the non-Abelian case. Freedman and Townsend⁽¹²⁾ have considered the Lagrangian⁽¹³⁾

$$(1) \mathcal{L} = \frac{\mu^2}{2} \nabla_\mu V^a_\mu + \frac{\mu}{2\sqrt{2}} \epsilon^{\mu\nu\alpha\beta} \phi^a_{\mu\nu} \nabla_\alpha \nabla_\beta V^a_{\lambda\sigma} - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu a}(\lambda)$$

$$([D_\mu, D_\nu]^{ab} = f^{abc} F^c_{\mu\nu}).$$

This has the tensor gauge invariance

$$(2a) \delta \phi^a_{\mu\nu} = D^{ab}_{[\mu} (A \cdot V)_{\nu]} \quad \delta V^a_\mu = 0 = \delta A^a_\mu$$

as well as the Yang-Mills gauge invariance

$$(2b) \delta \phi^a_{\mu\nu} = f^{abc} \phi^b_{\mu\nu} \omega^c \quad \delta V^a_\mu = f^{abc} V^b_\mu \omega^c \quad \delta A^a_\mu = D^ab_{[\mu} (A)_{\nu]} \omega^b.$$

This model is classically equivalent to a massive vector field⁽³⁾, as can be seen by using the equation of motion of $\phi^a_{\mu\nu}$ to eliminate the auxiliary field V^a_μ from $\mathcal{L}$. In the Abelian case, a more direct demonstration of the dynamical content of $\mathcal{L}$ can be seen by following Hagen⁽¹⁴⁾. We first split the vector

$$A^i_\mu = \frac{1}{2} \epsilon^{ijk} \phi_{\mu\nu}{}^k \quad \text{and} \quad b^i = \phi^{0i} \quad \text{into longitudinal (L) and transverse (T)}$$

parts, and then perform field dependent gauge transformations with

$$\omega = -\nabla \cdot \vec{A}_T / \nabla^2, \quad \vec{A} = \nabla \vec{a}_T / \nabla^2 \quad \text{so } \vec{A}_L \text{ and } \vec{a}_T \text{ can be set to zero. In this case we}$$

find

$$(3) \mathcal{L} = -a_L^i (\partial^2 + \mu^2) a_L^i - \frac{1}{2} \lambda_T^i (\partial^2 + \mu^2) \lambda_T^i + \frac{1}{2} \mu^2 \left[V^0 - \frac{\sqrt{2}}{\mu} (\nabla \cdot \vec{a}_T) \right] + \frac{1}{2} \left[\nabla \lambda^0 + \sqrt{2} \mu \vec{a}_T^2 \right]^2$$

$$-\frac{1}{2}\mu^2\left[\vec{\psi}_L - \frac{\sqrt{2}}{\mu}\partial_\mu^2\right]^2 - \frac{1}{2}\mu^2\left[\vec{\psi}_T + \frac{\sqrt{2}}{\mu}\nabla\times\vec{B}_T\right]^2 + \left[\nabla\times\vec{B}_T - \frac{\mu}{\sqrt{2}}\vec{A}_T\right]^2$$

demonstrating that the only propagating degrees of freedom are a_L^i and A_T^i , which are appropriate to a massive spin one particle.

Quantization of $\mathcal{L}$ has been discussed in (12) and (15). We quantize using the procedure of Batalin and Vilkovisky⁽¹¹⁾, appropriate to a "first stage theory". The ghosts in our "minimal set" of fields associated with (2a) are c_μ^a and η^a ; those with (2b) are $\tilde{\gamma}^a$. We also have to the usual set of auxiliary ghosts and Lagrange multipliers (viz. $\bar{c}_\mu^a, \pi_\mu^a; \bar{\eta}^a, \lambda^a; d^a, \rho; \tilde{\gamma}^a, \sigma^a$). (The ghost, auxiliary ghost and Lagrange multiplier fields $\gamma^a, \tilde{\gamma}^a, c^a, \bar{c}^a, \lambda^a$ and ρ^a are Fermionic, all other fields are Bosonic.)

In terms of the fields given above, a solution to the "master equation"

$(S, S) = 0$ is

$$\begin{aligned} (4) \quad S = S_{c1} + \int d^4x \Big[& \phi^{* \mu \nu} \left(D_{[\mu}^a b_{\nu]} + f^{abc} b_{\mu\nu}^c \right) + v^{* a} f^{abc} b_{\mu\nu}^c \\ & + \lambda^{* a} D^{cab} b_{\mu\nu}^c + c^{* a \mu} \left(D_{\mu}^a \eta^b + f^{abc} b_{\mu}^c \right) \\ & + \frac{1}{2} \gamma^{* a} f^{abc} b_{\mu\nu}^c - \eta^{* a} f^{abc} b_{\mu\nu}^c \\ & - \frac{1}{2\sqrt{2}\mu} \phi^{* a} b_{\mu\nu}^a \phi_{\lambda\sigma}^b \epsilon^{\mu\nu\lambda\sigma} f^{abc} \epsilon^a{}_{\eta} \\ & - \tilde{\gamma}^{* a} \sigma^a - \bar{c}^{* a \mu} \pi_\mu^a + \bar{\eta}^{* a} \rho^a + d^{* a} \lambda^a \Big] \end{aligned}$$

$$\left(D^{ab} (A \cdot V) = D^{ab} \quad D^{ab} (A) = D^{cab} \right)$$

We choose the "gauge fixing Fermion"⁽¹¹⁾ to be

$$(5) \quad \bar{\psi} = -\tilde{\gamma}^a \left(\partial \cdot A^a + \frac{\alpha}{2} \sigma^a \right) - \bar{c}^{a\mu} \left(\partial \phi_{\mu\nu}^a + \frac{\beta}{4} \pi_\mu^a \right) - \bar{\eta}^a \left(\partial \cdot c^a + \frac{\lambda}{2} \lambda^a \right) + d^a \left(\partial \cdot \bar{c}^a + \frac{\rho}{2} \rho^a \right)$$

This leads on the surface Σ where $\Phi_\Lambda^* = \partial\bar{\psi}/\partial\Phi_\Lambda$ to the quantum action S_Σ . The Batalin-Vilkovisky procedure ensures that the quantum action is BRST invariant, and is independent of the parameters α, β and λ .

We find that

$$\begin{aligned} (6) \quad S_\Sigma = S_{c1} + \int d^4x \Big[& \left(\partial \phi_{\mu\nu}^a \right) \left(D_{[\mu}^a b_{\nu]} + f^{abc} b_{\mu\nu}^c \right) \\ & + \partial \tilde{\gamma}^a D^{cab} b_{\mu\nu}^b \\ & + \left(\partial \eta^{a\mu} \right) \left(D_{\mu}^a b^b + f^{abc} b_{\mu}^c \right) \\ & - \frac{1}{2\sqrt{2}\mu} \left[\partial \phi_{\mu\nu}^a \right] \left[\partial \phi_{\lambda\sigma}^b \right] \epsilon^{\mu\nu\lambda\sigma} f^{abc} \epsilon^a{}_{\eta} \\ & + \left(\partial \cdot A^a + \frac{\alpha}{2} \sigma^a \right) \sigma^a + \left(\partial \phi_{\mu\nu}^a + \frac{\beta}{4} \pi_\mu^a + \partial d^a \right) \pi_\mu^a \\ & - \left(\partial \cdot c^a + \frac{\lambda}{2} \lambda^a \right) \rho^a + \left(\partial \cdot \bar{c}^a + \frac{\rho}{2} \rho^a \right) \lambda^a \Big] \end{aligned}$$

We have verified that the BRST transformation is nilpotent on shell.

Upon eliminating the Lagrange multipliers $\sigma^a, \pi_\mu^a, \lambda^a$, and ρ^a and discarding those vertices (namely the triple ghost couplings) that don't contribute to processes with no external ghost lines), S_Σ reduces to

$$[7] S_{\Sigma}^{\text{phys}} = S_{c1} + \int d^4x \left[-\frac{1}{2\alpha} (\partial \cdot A)^2 - \frac{1}{\beta} (\partial^\mu A_\mu)^2 \right]$$

$$- \frac{1}{2} \partial_\mu (\mu \cdot \nu) \left[D_{\mu}^{ab} \frac{b}{c} \right] - \frac{1}{\alpha} (\partial \cdot c^a) (\partial \cdot c^a) \\ - \bar{\eta}^a \partial_\mu D^{cab} \eta^b - \bar{\eta}^a \partial_\mu D^{ab} \eta^b \Big] .$$

Remarkably, with the above gauge fixed Lagrangian one finds that the propagator of the new massive vector gauge field is precisely the same as in the conventional Higgs model in the ordinary covariant gauge, (16)

$$[8] \langle \Delta \Delta \rangle = -i \left(\frac{g_{\mu\nu} - k_\mu k_\nu / k^2}{k^2 - \mu^2} + \alpha \frac{k_\mu k_\nu}{k^4} \right) .$$

The only propagator with bad ultraviolet behaviour is that of the auxiliary field V . Hence, in the Abelian case where V has no vertices with A , the Freedman-Townsend model gives a renormalizable theory for massive vector fields, as has been noted by the authors of ref. (17).

In the non-Abelian case, however, the non-renormalizable propagators participate. Still, the following observation gives rise to the hope that a renormalizable version of the non-Abelian Freedman-Townsend model exists. Without spoiling any gauge symmetries, we can also insert $-\frac{1}{2\gamma} (D^{ab} \cdot V^b)^2$ into the classical Lagrangian [1], and consequently into [7]. For finite values of γ , the occurrence of this term alters the number of dynamical degrees of freedom as V_0^a is clearly a dynamical field if this term is present. However, sending $\gamma \rightarrow \infty$ at the end, we can regard this term as a "regulator" in the sense of Lee and Yang (18), as for finite values of γ the theory is power-counting renormalizable. To see this we note that the part of S_{Σ}^{phys} that is bilinear in the fields $\{\phi_{\mu\nu}^a, V_\mu^a, \lambda_\mu^a\}$ involves a matrix whose inverse is

$$[9] M^{-1} = \delta^{ab} \begin{pmatrix} -\frac{g_{\alpha\beta} \lambda_\alpha}{\partial^2 + \mu^2} + \left(\frac{1}{\partial^2 + \mu^2} - \frac{1}{\partial^2 + \mu^2} \right) \frac{L_{\alpha\beta} \lambda_\alpha}{\partial^2} & \frac{1}{\sqrt{2}\mu} \frac{1}{\partial^2 + \mu^2} \epsilon_{\alpha\beta} \cdot V & \frac{1}{\sqrt{2}\mu} \frac{\mu^2}{\partial^2 + \mu^2} \epsilon_{\alpha\beta} \cdot V \\ -\frac{1}{\sqrt{2}\mu} \frac{1}{\partial^2 + \mu^2} \epsilon_{\lambda\sigma} \cdot \mu & \frac{(g_{\mu\nu} - (1-\gamma) \partial_\mu \partial_\nu / (\partial^2 + \gamma \mu^2))}{\partial^2 + \mu^2} & \frac{(g_{\mu\nu} - \partial_\mu \partial_\nu / \partial^2)}{\partial^2 + \mu^2} \\ -\frac{1}{\sqrt{2}\mu} \frac{\mu^2}{\partial^2 + \mu^2} \frac{\lambda_{\alpha\mu}}{\partial^2} & \frac{(g_{\mu\nu} - \partial_\mu \partial_\nu / \partial^2)}{\partial^2 + \mu^2} & \frac{(g_{\mu\nu} - \partial_\mu \partial_\nu / \partial^2)}{\partial^2 + \mu^2} + \frac{\partial_\mu \partial_\nu}{\partial^4} \end{pmatrix}$$

$$g_{\alpha\beta, \lambda\sigma} = \frac{1}{2} (g_{\alpha\lambda} g_{\beta\sigma} - g_{\alpha\sigma} g_{\beta\lambda}) \quad \epsilon_{\alpha\beta, \gamma} = \epsilon_{\alpha\beta} k_\gamma \\ L_{\alpha\beta, \lambda\sigma} = \frac{1}{2} \left(g_{\alpha\lambda} \partial_\beta^2 + g_{\beta\sigma} \partial_\alpha^2 - g_{\alpha\sigma} \partial_\beta^2 - g_{\beta\lambda} \partial_\alpha^2 \right) .$$

For finite values of γ , all propagators fall off as $1/k^2$ as $k \rightarrow \infty$, save for $\langle \phi_{\lambda\sigma}^a V_\mu^b \rangle$, which falls off as $1/k$ but always connects to a super-

renormalizable vertex proportional to μ . Consequently, the model is power-counting renormalizable when quantized using the above procedure. We are currently investigating whether the BRST structure of this model can be used to establish unitarity and renormalizability both for finite γ and in the limit $\gamma \rightarrow \infty$. A crucial question is the problem of stability under renormalization, as there exist further candidates for counter terms to the original Lagrangian (e.g. $(V^a \cdot V^a)^2$) that are consistent with both the symmetries of the theory and power-counting renormalizability. Nevertheless, it appears as if there is a renormalizable alternative to the Higgs mechanism for giving mass to vector gauge fields.

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