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TARGET SPACE DUALITY, SUPERSYMMETRY BREAKING AND THE STABILITY OF CLASSICAL STRING VACUA

M. Cvetič¹⁾, A. Font²⁾, L.E. Ibáñez³⁾, D. Lüüst³⁾ and F. Quevedo⁴⁾

1) *Dept. of Physics, University of Pennsylvania, PA 19104-6396*

2) *Depto. de Física, Univ. Central de Venezuela,
Aptdo. 20513, Caracas 1020-A, Venezuela*

3) *CERN, CH 1211 Geneva 23, Switzerland*

4) *Theory Div., T-8, MS B285, Los Alamos N.L., USA*

ABSTRACT

Target space duality symmetries are present in many static string vacua studied up to now. They are conjectured to remain unbroken even after non-perturbative string effects are taken into account. If that is the case, one can have a handle on the non-perturbative string effects by requiring explicit duality invariance of the low-energy effective Lagrangian. As an explicit example we present a general discussion of the effective four-dimensional supergravity action consistent with the target space modular symmetry $SL(2, \mathbf{Z})$. The non-perturbative superpotential severely distorts the classical, four-dimensional string solutions. Specifically, it lifts the vacuum degeneracy with respect to the compactification scale and leads in general to spontaneous supersymmetry breaking with negative vacuum energy, but we also find particular solutions with vanishing cosmological constant. Charged massless matter scalars get negative $(mass)^2$ signalling an additional instability of the vacuum, further gauge symmetry breaking and the blowing up of orbifold singularities.

1. Introduction

Non-perturbative effects are believed to play a crucial role in low-energy string theory. They are expected to provide the mechanism for supersymmetry breaking, to solve the dilaton problem by generating a non-trivial potential for that field, to lift the vacuum degeneracy existent in string theories to all orders of perturbation theory and possibly also to give some light in solving the cosmological constant problem. Unfortunately, despite many efforts, a non-perturbative formulation of string theory is not available at the moment and these fundamental questions cannot yet be addressed from first principles.

Nevertheless, it is possible to use the known symmetries in perturbation theory to obtain general information about how the non-perturbative effects could appear in the low-energy effective action. Recently, under the assumption that target space duality ($R \rightarrow \alpha'/R$) [1-8] and its generalization (the modular group $SL(2, \mathbb{Z})$) [9-12]) is an exact symmetry of string theory, strong constraints have been found [13] on the general form of the non-perturbative four-dimensional effective supergravity action, providing a link between the superpotential and the theory of modular functions. It has also been realized [14-17] that very interesting consequences appear after reconciling the well-known gaugino condensation mechanism [18-21] for supersymmetry breaking with the requirement of target space modular invariance. As a general result [14],[15], the effective potential for the modulus field, representing the “size” of the internal space, is such that the theory dynamically prefers to be compactified, therefore lifting the vacuum degeneracy existent for this field in string perturbation theory.

In the present article we generalize the previous results in several directions. Guided by the general theory of $SL(2, \mathbb{Z})$ automorphic functions, we can extend the analysis to the most general, modular invariant superpotential, independent of the non-perturbative source that generates it. This turns out to be highly restrictive due to the limited possibilities for automorphic forms of a given modular weight. Since the superpotential must have negative modular weight, it necessarily possesses poles, either at finite values of the T -field (finite radii) or at $T = \infty$ (weak coupling limit of the σ -model). We will mainly concentrate on superpotentials which are regular inside the fundamental domain. Then the corresponding scalar potential necessarily blows up at $T = \infty$ and there is always a minimum inside the fundamental domain forcing the theory to be compactified. This minimum is either found at the self-dual points and therefore leaving space-time supersymmetry unbroken (at least in the T -field sector) or, more generically, corresponds to supersymmetry breaking vacua with negative cosmological constant. However it is remarkable that one

can also find special modular invariant potentials with supersymmetry breaking minima but zero cosmological constant. The scale of supersymmetry breaking depends essentially on the non-perturbatively generated dilaton potential which is independent of the duality requirements.

We will briefly compare this analysis, based on the duality symmetries alone, with an explicit calculation of threshold corrections to the gauge coupling constant which is relevant for the gaugino condensation mechanism. In this context, the exponential divergence of the superpotential at $T = \infty$ is due to the presence of an infinite number of Kaluza-Klein states which become massless in the infinite radius limit and produce a linear divergence in the threshold correction.

As a second way to generalize the previous results, we consider the inclusion of “matter fields”. In the case of orbifold compactifications they correspond to twisted moduli (blowing-up modes), untwisted and twisted matter. For simplicity they were set to their vanishing vacuum expectation values (v_{ev}/s) in previous analysis. Because of limitations of the present knowledge about their effective couplings, it is only possible to investigate the effective action perturbatively around their zero v_{ev} . Even though these fields transform under $SL(2, \mathbb{Z})$ as modular functions of different weights, we have found that the classical vacuum is actually a maximum in those field directions and is generically unstable. Therefore the theory is forced to a vacuum with spontaneously broken gauge symmetries, which is phenomenologically desirable. In particular a non-vanishing vacuum expectation value of the blowing-up modes implies that a smooth manifold point is preferred over the singular orbifold point. This result is independent of the general form of the modulus (T -field) superpotential.

The paper is organized as follows. In the next section we discuss the form of $N = 1$ supersymmetric low-energy effective string actions which are invariant under target space duality symmetries. Mentioning briefly general target space duality groups (not restricted to the case of $SL(2, \mathbb{Z})$) and the connection to the “special” Kähler geometry of $(2, 2)$ heterotic string compactifications we mainly concentrate in the following on the target space modular group $SL(2, \mathbb{Z})$. We present the most general non-perturbative effective supergravity action for the single modulus T , invariant under this symmetry, and discuss its vacuum structure which generically breaks space-time supersymmetry with and (for particular choices of the superpotential) without creating a cosmological constant. We briefly extend this analysis to the case of three moduli. In section three we compare the results of the last section with the gaugino condensation mechanism which serves as an explicit non-perturbative example, although field-theoretical, to obtain a non-vanishing

superpotential for the T -field. In section four, the matter fields are introduced as fields transforming as modular functions of a given weight and show that their $(mass)^2$ is negative signalling that the vacuum is unstable, as long as their modular weight belongs to a given range. The twisted matter fields in orbifold models have weights in such a range, so those vacua are unstable. Finally we present our conclusions mentioning again the main assumptions made in the analysis of this paper and some open problems. The appendix provides a collection of properties of automorphic functions of the modular group $SL(2, \mathbb{Z})$.

2. Supersymmetry Breaking and the Vacuum Structure in Duality Invariant String Compactifications

2.1 TARGET SPACE DUALITY SYMMETRIES

The particle spectrum of string theory consists of a finite number of massless (or almost massless) fields ϕ_0 and an infinite tower of massive excitations ϕ_H . In general, if a field theory contains a set of fields ϕ_0 with small masses and another set ϕ_H of much larger mass, the effective action $\tilde{S}(\phi_0)$ of the light fields may be obtained from the action $S(\phi_0, \phi_H)$ of the full theory by a functional integral which integrates out the massive fields:

$$\exp[i\tilde{S}(\phi_0)] = \int d\phi_H \exp[iS(\phi_0, \phi_H)]. \quad (2.1)$$

In classical approximation this corresponds to eliminating the heavy fields from the equations of motion. The complete path integral also includes diagrams involving loops of the massive fields plus non-perturbative effects in the string coupling constant. Of course, $\tilde{S}$ will contain an infinite number of non-renormalizable interactions suppressed by powers of E/M where E is the characteristic scale of a reaction and M the mass of the heavy fields.

Here we will consider some general features of effective $N = 1$ supergravity actions in four dimensions arising from $(2, 2)$ compactification [22] of the ten-dimensional heterotic string [23], although the essential points do also apply to generic $(0, 2)$ models. Among other massless modes (like the graviton, gravitino, gauge bosons etc.) the spectrum contains massless chiral superfields $M_i(x^\mu)$ ($\mu = 0, \dots, 3$) whose vev 's correspond to the moduli of the underlying $c = 9$ $(2, 2)$ superconformal field theory (see for example [24]). Different but continuously connected compactifications are characterized by different points in the moduli space $\mathcal{M}$, $M_i \in \mathcal{M}$. The kinetic energy of the moduli fields is determined by the metric $\mathcal{M}_{ij}$ of $\mathcal{M}$, $S_{\text{kin}} \sim \mathcal{M}_{ij} \partial_\mu M_i \partial^\mu M_j$. At the level of string perturbation theory,

the moduli fields have a completely flat potential in the corresponding four-dimensional effective action. As is true for many known compactification schemes we assume that the superconformal field theory is invariant under the duality symmetries which are discrete automorphisms $M_i \rightarrow F(M_i)$. Since $\mathcal{M}$ is the target space of the four-dimensional σ -model for the moduli, the discrete duality transformations are also called target space duality transformations. These discrete transformations act in general non-trivially on the massive string spectrum; for example, the well-known duality symmetry $R \rightarrow 1/R$ for the case of toroidal compactifications exchanges momentum (Kaluza-Klein) modes with winding states. It follows that the complete action $S(M_i, \phi_H)$ including all massive fields must be invariant under the target space duality transformations. However one expects that $S(M_i, \phi_H)$ is not invariant term by term but only after transforming the infinite number of massive fields into each other. Therefore $S(M_i, \phi_H)$ will not be expressed in terms of smooth duality invariant functions of M_i . On the other hand, if one integrates out all massive fields, in particular all momentum and winding states, the effective action $\tilde{S}(M_i)$ will now contain smooth duality invariant functions of the moduli fields M_i . This has been explicitly shown in the case of tree-level orbifold Yukawa couplings involving twisted fields [25–31] and the one-loop correction to the orbifold gauge kinetic function [32].

Let us first recall some known facts (see [33] for a recent review on some material presented in this subsection) on how to construct the kinetic energy of the moduli fields, $\tilde{S}_{\text{kin}}(M_i)$, as it comes out from string tree-level S-matrix elements. The Kählerian moduli space $\mathcal{M}$ has to be a product space, $\mathcal{M} = \mathcal{M}^{(1,1)} \times \mathcal{M}^{(2,1)}$, with moduli fields $M_{i_1}^{(1,1)} \in \mathcal{M}^{(1,1)}$ ($i_1 = 1, \dots, h_{(1,1)}$), corresponding to the deformations of the Kähler form J of the compact six-space, and moduli fields $M_{i_2}^{(2,1)} \in \mathcal{M}^{(2,1)}$ ($i_2 = 1, \dots, h_{(2,1)}$), related to the deformations of the complex structure. In the language of the superconformal field theory the different types of moduli correspond to the two distinct rings of chiral and anti-chiral primary fields, respectively [34]. It follows that the discrete target space duality symmetries obey this product structure (except the mirror symmetry discussed below) $\sim M_{i_1}^{(1,1)}$ and $M_{i_2}^{(2,1)}$ are not mixed under the duality transformations. More specifically, the discrete duality transformations acting on $M_{i_2}^{(2,1)}$ have a geometrical interpretation where the duality symmetries involving the fields $M_{i_1}^{(1,1)}$ are of purely stringy nature and are not expected from the field theory point of view. Especially, non-perturbative effects on the world-sheet [35] (σ -model instantons) are crucial to understand the later discrete symmetries. Therefore we will refer to them as *quantum* duality symmetries. However many compactifications appear in identical mirror pairs; the so-called mirror symmetry [36–38] is the invariance under the exchange of the $M_{i_1}^{(1,1)}$ and $M_{i_2}^{(2,1)}$ moduli fields. Thus

the presence of these symmetries dissolves the conceptual difference between the two kinds of duality symmetries as expected from the conformal field theory point of view. In fact, using known mirror pair Calabi-Yau spaces it is possible to determine the quantum duality symmetries by pure geometrical means [12], [39-42].

So let us restrict our discussion to the $M^{(1,1)}$ moduli fields. Moreover, we consider the most simple case of freezing all fields except one single complex modulus $T = R^2 + iB$ which corresponds to the radius of the compact six space plus the internal axion field. (In section 2.4 we briefly consider the case of three different radii.) Then the Kähler potential for this field takes the form

$$K(T, T^*) = -\log Y(T, T^*) = -\log((T + T^*)^3 + Y_{\text{instanton}}). \quad (2.2)$$

The first, model-independent term [43] dominates in the large radius limit and the asymptotic isometries of the two-dimensional Kähler space $\mathcal{M}^{(1,1)}$ are uniquely given by the group $SL(2, \mathbf{R})$ acting on T as $T \rightarrow \frac{aT-b}{icT+d}$ ($a, b, c, d \in \mathbf{R}$, $ad - bc = 1$). On the other hand, for small values of the T -field the world-sheet instantons may give exponential, model-dependent contributions to the Kähler potential which in general destroy all continuous isometries. However it is crucial to recognize that for many (if not all) compactifications like, on orbifolds [44], [45] or Calabi-Yau spaces [22], the non-compact isometry group is not completely broken by the world-sheet instantons, but the theory still possesses the discrete quantum duality symmetry Γ . Γ is a discrete, infinite subgroup of $SL(2, \mathbf{R})$ acting on the T -field as

$$\Gamma : T \rightarrow T' = \frac{aT - ib}{icT + d}, \quad ad - bc = 1 \quad (2.3)$$

where a, b, c, d is an infinite, but discrete set of real parameters. Since the duality transformations leave the kinetic energy invariant, Γ acts as a Kähler transformation on K :

$$K \rightarrow K' = K + f(T) + f^*(T^*), \quad f(T) = \log(icT + d)^3 + f_{\text{instanton}}(T) \quad (2.4)$$

It is very important to emphasize that Γ is in general different from the modular group $SL(2, \mathbf{Z})$ which has integer coefficients a, b, c, d . Only for compactifications which do not lead to instanton contributions to the Kähler potential, i.e. the Kähler potential is identical to its large radius limit $K = -3\log(T + T^*)$ and the geometry of T -field moduli space $\mathcal{M}^{(1,1)}$ is described by the coset $SU(1,1)/U(1)$, is the quantum duality group given by the target space modular group $SL(2, \mathbf{Z})$. This is true for flat backgrounds like covariant lattice constructions [46] fermionic models [47], [48] and orbifold compactifications.

To emphasize even more the model dependence of the target space duality transformations one can use the fact that for all $(2, 2)$ heterotic string compactifications the moduli spaces $\mathcal{M}^{(1,1)}$ and $\mathcal{M}^{(2,1)}$ are “special” Kähler spaces [49-52] and the Kähler potential can be written as

$$Y = \bar{z}_A F_A + z_A \bar{F}_A \quad (2.5)$$

where the variables z_A ($A = 0, \dots, h_{(1,1)}(h_{(2,1)})$) are homogeneous coordinates, being related in a “special” coordinate system to the projective coordinates as $M_i = -iz_i/z_0$. The F_A are the first derivatives of a holomorphic prepotential $F(z_A)$ of homogeneous degree two. Then it is not difficult to show that all discrete isometries, i.e. duality transformations, are just given by particular symplectic $Sp(2h_{(1,1)} + 2, \mathbf{Z})$ ($Sp(2h_{(2,1)} + 2, \mathbf{Z})$) transformations acting on the $2h_{(1,1)} + 2$ ($2h_{(2,1)} + 2$) dimensional vector $(z_A, \frac{1}{2}iF_A)$. (Note that not any $Sp(2h + 2, \mathbf{Z})$ transformation corresponds to a quantum duality transformation.)

E.g., for the case of a single modulus $T = -iz_1/z_0$ the holomorphic prepotential takes the form

$$F(z_0, z_1) = i \frac{z_1^3}{z_0} + F_{\text{instanton}}(z_0, z_1). \quad (2.6)$$

Then the specific form of the $Sp(4, \mathbf{Z}) \subset SL(2, \mathbf{R})$ symplectic matrices, which are related to quantum duality transformations, depends again on the model-dependent instanton contributions to $F(z_0, z_1)$. For flat backgrounds with vanishing instanton contributions the two generators S, T of the modular group $SL(2, \mathbf{Z})$ are related to the following two elements of $Sp(4, \mathbf{Z})$ [53]:

$$S = \begin{pmatrix} 0 & 0 & 12 & 0 \\ 0 & 0 & 0 & -4 \\ 3 & 0 & 0 & 0 \\ 0 & -9 & 0 & 0 \end{pmatrix}, \quad T = \begin{pmatrix} 2 & 0 & 0 & 0 \\ 2 & 2 & 0 & 0 \\ 1 & 3 & 2 & -2 \\ -3 & -6 & 0 & 2 \end{pmatrix}. \quad (2.7)$$

For non-flat backgrounds like Calabi-Yau compactification the generators of the duality transformations correspond to different symplectic $Sp(4, \mathbf{Z})$ matrices, reflecting the fact that the quantum duality groups are distinct.

After discussing the modular structure of the Kähler potential let us now include a superpotential $W(T)$. Since in any order of string perturbation theory $N = 1$, space-time supersymmetry will be unbroken, and the T -field has a totally flat potential, the purely T -dependent superpotential $W(T)$ can only be due to non-perturbative (infinite genus) string

effects. We will use the discrete quantum duality symmetries to strongly constrain the possible form of $W(T)$ regardless of what actual non-perturbative mechanism is responsible for the emergence of the superpotential. (In the next chapter we will discuss the gaugino condensation as a particular example. Wormholes as a different non-perturbative source of $W(T)$ were discussed in [54] [55].) For non-vanishing superpotential the $N = 1$ supergravity action is described [56] by the function

$$G(T, T^*) = K(T, T^*) + \log |W(T)|^2. \quad (2.8)$$

We now assume that the string tree-level transformation behaviour eq.(2.4) of $K(T, T^*)$ is not changed, including the non-perturbative contribution to the superpotential. This assumption is well justified in the weak coupling limit with small but non-vanishing $W(T)$. Then the duality invariance of the effective action requires $G(T, T^*)$ to be duality invariant, as well, thus implying that $W(T)$ must transform under Γ up to a phase as

$$W(T) \rightarrow W(T)e^{-f(T)}. \quad (2.9)$$

This means that $W(T)$ is related to the automorphic functions of the discrete duality group Γ . Unfortunately, not many explicit results about automorphic functions of general discrete subgroups of $SL(2, \mathbb{R})$ are known in the mathematical literature (see for example [57]). Therefore, and also for reasons of simplicity, we restrict ourselves to discussing the case $\Gamma = SL(2, \mathbb{Z})$ corresponding to flat backgrounds. However we believe that the following results concerning supersymmetry breaking and lifting the vacuum degeneracy of the moduli fields do not essentially depend on this technical simplification but are also true for general duality groups Γ as it reflects the presence of non-trivial stringy symmetries.

2.2 THE $PSL(2, \mathbb{Z})$ -INVARIANT EFFECTIVE SUPERGRAVITY LAGRANGIAN

For $\Gamma = SL(2, \mathbb{Z})$, the superpotential has to transform under target space modular transformations as [13]

$$W(T) \rightarrow e^{i\delta(a,b,c,d)} \frac{W(T)}{(icT + d)^3}. \quad (2.10)$$

Thus, $W(T)$ has to be a modular function of modular weight -3 . The field independent phase $\delta(a, b, c, d)$ is the so-called multiplier system. (See the appendix for a general discussion of the modular group $SL(2, \mathbb{Z})$ and its automorphic functions.) $W(T)$ can be written

in the following form:

$$W(T) = \frac{H(T)}{\eta(T)^6}. \quad (2.11)$$

Here $\eta(T)$ is the Dedekind function and $H(T)$, being an arbitrary modular invariant function with in general non-vanishing multiplier system, is a rational function of the absolute modular invariant $j(T)$. Note that eq.(2.11) implies that $W(T)$ has an expansion in $e^{T/\alpha'}$ (for large T) after reintroducing the dimensionful slope parameter α' . This form is obviously non-perturbative in the two-dimensional σ -model coupling constant α'/R^2 and clearly shows the relevance of the contribution of the world-sheet instantons to $W(T)$. (The non-perturbative character of W in the string coupling constant will be mentioned in the following.)

The behaviour of $W(T)$ will largely determine the vacuum structure of the $N = 1$ supergravity action. In particular, the scalar potential will inherit its singularities. In fact, since $W(T)$ has negative modular weight from formula (A5) in the appendix it follows that $W(T)$ will in general have singularities at ∞ and/or at finite values of T .

According to a mathematical theorem [58] quoted in the appendix, if we want $W(T)$ to avoid singularities inside the fundamental domain we must take $H(T)$ of the form

$$H(T) = \left(\frac{G_6(T)}{\eta(T)^{12}} \right)^m \left(\frac{G_4(T)}{\eta(T)^8} \right)^n \mathcal{P}(j) \quad (2.12)$$

or equivalently (up to a constant) as

$$H(T) = (j - 1728)^{m/3} j^{n/3} \mathcal{P}(j). \quad (2.13)$$

Here m, n are positive integers and $\mathcal{P}(j)$ is a polynomial of $j(T)$. $G_4(T)$ and $G_6(T)$ are the Eisenstein functions of modular weight 4 and 6 respectively (see the appendix for more details). Without loss of generality the zeroes of $\mathcal{P}(j)$ are distinct from the fixed points $T = 1, T = \rho \equiv e^{i\pi/6}$. This means that H vanishes at $T = \rho$ only if $n \geq 1$ and at $T = 1$ only if $m \geq 1$. Notice that superpotentials of this type always diverge at the cusp point $T \rightarrow \infty$ and at the modular transformed point $T = 0$.

So far we have only discussed the dependence of the superpotential on the internal T -field modulus. To discuss the issue of supersymmetry breaking in string theory one has also to include the S -field, $S = \frac{1}{g^2} + i\theta$, whose *vev* determines the string coupling constant

g and the space-time axion field θ . We take for the S -field Kähler potential its tree-level form which is at least justified for small couplings:^{*}

$$K(S, S^*, T, T^*) = -\log(S + S^*) - 3\log(T + T^*). \quad (2.14)$$

The non-perturbative effects involving the S -field are parametrized by the superpotential:

$$W(S, T) = \frac{\Omega(S)H(T)}{\eta(T)^6}. \quad (2.15)$$

Here $\Omega(S)$ is an unspecified function of the S -field; particular examples will be discussed in the next section. For the sake of simplicity we assume here that the dependence on the dilaton field S factorizes. Note however since the superpotential (2.15) is due to non-perturbative string effects, $\Omega(S)$ should have an expansion in terms of powers of e^S .

The scalar potential resulting from eqs.(2.14),(2.15) is given by

$$V = |h^S|^2 G_{SS}^{-1} + |h^T|^2 G_{TT}^{-1} - 3\exp(G) \\ = \frac{1}{S_R T_R^3 |\eta|^{12}} \left\{ |S_R \Omega_S - \Omega|^2 |H|^2 + \frac{T_R^2}{3} |H'| + \frac{3}{2\pi} H \hat{G}_2 |\Omega|^2 - 3 |H|^2 |\Omega|^2 \right\} \quad (2.16)$$

where $T_R = T + T^*$, $S_R = S + S^*$, $H' = dH/dT$, $\Omega_S = d\Omega/dS$ and $\hat{G}_2$ is the non-holomorphic Eisenstein function of modular weight two (see the appendix). Space-time supersymmetry is spontaneously broken if at the minimum of V one of the auxiliary fields

$$h^i = \exp\left(\frac{1}{2}G\right) G^i = |W| \exp\left(\frac{1}{2}K\right) \left(K^i + \frac{W^i}{W}\right) \quad (2.17)$$

has a non-zero vacuum expectation value. We now turn to an analysis of the extrema of V .

For the derivative in the S direction we find

$$\frac{\partial V}{\partial S} = \frac{1}{S_R^2 T_R^3 |\eta|^{12}} \left\{ S_R^2 \Omega_{SS} (S_R \Omega_S - \Omega)^* |H|^2 \right. \\ \left. + \Omega^* (S_R \Omega_S - \Omega) \left(\frac{T_R^2}{3} |H'| + \frac{3}{2\pi} H \hat{G}_2 |^2 - 2 |H|^2 \right) \right\}. \quad (2.18)$$

The above shows that, independently of the value of T , $\partial V / \partial S = 0$ has a solution when

$$S_R \Omega_S - \Omega = 0. \quad (2.19)$$

In this case $h^S = 0$ at the extremum. If $h^S \neq 0$ there is another extremum candidate that

^{*} It is conceivable that one can redefine the S -field in any order of string perturbation theory such that the Kähler potential always takes the tree-level form.

we will discuss below.

Now let us concentrate on the vacuum structure with respect to the T -field. For $m, n \geq 0$ and $\mathcal{P}(j)$ a polynomial the only singularity of $V(T, T^*)$ is a pole at the cusp point $T \rightarrow \infty$. At this point the potential goes to $+\infty$ since for large T the term $|h^T|^2 G_{TT}^{-1}$ dominates over $-3|W|^2$ independently of the form of $H(T)$. Since V is modular invariant it must also have a pole at $T = 0$. Therefore we obtain the model-independent result that for this class of regular superpotentials the non-perturbative dynamics always fixes the vev of the modulus T . The theory is forced to be compactified since the minimum of V always occurs at finite radius. Furthermore, generically $V < 0$ at the global minimum since the potential is not positive definite.

Another generic property of V is the fact that the modular fixed points $T = 1, \rho$ must be extrema. This is because $\partial V / \partial T$ transforms with modular weight two and has no singularities inside the fundamental domain. This can be explicitly verified from

$$\frac{\partial V}{\partial T} = \frac{3}{2\pi} \hat{G}_2 V + \frac{1}{S_R T_R^3 |\eta|^{12}} \left\{ (|S_R \Omega_S - \Omega|^2 - 2|\Omega|^2) H^* H' \right. \\ \left. + |\Omega|^2 \left[\frac{2T_R}{3} |H'| + \frac{3}{2\pi} H \hat{G}_2 \right]^2 + \frac{3}{2\pi} |H|^2 \hat{G}_2 \right. \\ \left. + \frac{T_R^2}{3} (H' + \frac{3}{2\pi} H \hat{G}_2)^* (H'' + \frac{3}{2\pi} H' \hat{G}_2 + \frac{3}{2\pi} H \hat{G}_{2T}) \right\}. \quad (2.20)$$

From the above it is also obvious that $\partial V / \partial T = 0$ at other points where $H = H' = 0$, i.e. points where the polynomial $\mathcal{P}(j)$ has a multiple zero. We can show that these points are local minima with $V = 0$ and $h^T = 0$.

The self-dual points $T = 1, \rho$ are extrema but not necessarily local minima of V . To determine the exact position of the true minima it is very useful to observe that, besides modular invariance, the scalar potential possesses an additional Z_2 symmetry $T \rightarrow T^*$ which follows from the reality of V . This discrete symmetry has as fixed points the line $\text{Im}T = 0$. Therefore the potential is necessarily extremized on this curve in the direction of $\text{Im}T$ (assuming a "smooth behaviour" on this curve). Analogously, all points on the curve $\text{Im}T = \frac{1}{2}$ are fixed under the combined modular, Z_2 -symmetry $T \rightarrow T^* + i$, and all points on the unit circle $\text{Re}T^2 + \text{Im}T^2 = 1$ are fixed under the combined modular, Z_2 -symmetry $T \rightarrow \frac{T}{|T|^2}$. It follows that V is extremized on $\text{Im}T = \frac{1}{2}$ in the $\text{Im}T$ -direction and on all points of the unit circle in the direction normal to it.

In fact, the union of the curves $\text{Im}T = \frac{1}{2}$ ($\text{Re}T > \frac{\sqrt{3}}{2}$), $\text{Re}T^2 + \text{Im}T^2 = 1$ ($\text{Im}T \geq 0$), $\text{Im}T = 0$ ($\text{Re}T > 1$) just describes the boundary $\mathcal{B}$ of the moduli space of the combined

modular, $\text{Im}T$ sign flip transformations $PSL(2, \mathbb{Z}) \times \mathbb{Z}_2$ (see figure 1). Since all points of the boundary of this slightly generalized, half-sized moduli space are already extrema of V in at least one direction, we conjecture that all extrema of V entirely lie on $\mathcal{B}^*$. We have checked this conjecture in several examples.

To develop a better understanding of the shape of V it is useful to study its behaviour at the self-dual points. For example, if the self-dual point $T = 1$ is a maximum in the $\text{Re}T$ -direction and a minimum in the $\text{Im}T$ -direction we can expect to find a true (local) minimum for some value of $\text{Re}T > 1$, $\text{Im}T = 0$ since V has to grow for large T . The nature of the extrema at $T = 1, \rho$ can be analyzed in a straightforward manner taking into account that $\hat{G}_3, j^i$ vanish at both $T = 1, \rho$ whereas $j, j'', G_4, \hat{G}_{2T}, (G_6/\eta^{12})'$ vanish only at $T = \rho$ and $G_6, (G_4/\eta^8)'$ vanish only at $T = 1$. We now summarize the results.

Let us consider $T = \rho$ first. When $n \geq 2$, $T = \rho$ is a minimum with $V = 0$ and $h^T = 0$. When $n = 1$ it is a minimum if $Y(S) = |S_R \Omega_S - \Omega|^2 - \frac{2}{3}|\Omega|^2 > 0$ and a maximum if $Y(S) < 0$, in both cases $V > 0$. When $n = 0$ it is a minimum if $Z(S) = |S_R \Omega_S - \Omega|^2 - 2|\Omega|^2 > 0$ and a maximum if $Z(S) < 0$.

We now turn to $T = 1$. Again we distinguish several cases. When $m \geq 2$ it is a minimum with $V = 0$ and $h^T = 0$. When $m = 0, 1$ the situation is quite involved but can be greatly simplified assuming that $(S_R \Omega_S - \Omega) = 0$. In this case, define $B = H'''/H'$, $\beta = 3\hat{G}_{2T}/2\pi$, $A = (H''/H) + \beta$ with all functions evaluated at $T = 1$ ($\beta \sim 1.19$). Then for $m = 1$ $T = 1$ is a saddle point if $B < 1 - 3\beta$ or $B > 2 - 3\beta$ and a relative maximum if B is in between. For $m = 0$ $T = 1$ is a relative minimum when $|A| > \frac{3}{2}$, a relative maximum when $-\frac{3}{4} < A < \frac{3}{4}$ and a saddle point when $-\frac{3}{2} < A < -\frac{3}{4}$ (minimum in $\text{Re}T$ direction, maximum in $\text{Im}T$ direction) or when $\frac{3}{4} < A < \frac{3}{2}$ (maximum in $\text{Re}T$ direction, minimum in $\text{Im}T$ direction).

In all the above cases, the minima found at $T = \rho, 1$ have unbroken SUSY (at least in the T -sector). Notice that, although the minima are supersymmetric, the value of the scale of compactification is determined. This shows that target-space modular invariance allows for a situation in which non-perturbative effects lift the degeneracy of the compactification radius but do not break supersymmetry. Notice however that even though for $m, n \geq 1$, $T = 1, \rho$ are supersymmetric minima, they cannot be the global vacuum of the theory because at those points $V \geq 0$. The theory, then, generically prefers a vacuum such that $T \neq 1, \rho, \infty$ with $V < 0$ and supersymmetry broken.

* We furthermore believe that, considering more general target space duality groups Γ , the extrema of the scalar potential also lie on the boundary of the moduli space.

We have studied numerically several cases with $\mathcal{P} = 1$ and various values of m, n . In these examples we also assumed $(S_R \Omega_S - \Omega) = 0$. In all cases we verified that $V < 0$ at the global minimum which occurs at points on the boundary $\mathcal{B}$. Examples with $\mathcal{P} = 1$ and $(m, n) = (0, 0), (1, 1), (0, 3)$ are shown in figures 2, 3, 4. For $(m, n) = (0, 0)$, the case already shown in [14], the minimum lies on the real axis, $T_{\min} \sim 1.2$ and for $(m, n) = (1, 1)$ the minima lie on the unit circle, $\text{Re}T_{\min} \sim 0.97$, $\text{Im}T_{\min} \sim \pm 0.24$. For $(m, n) = (0, 3)$ there are two minima with unbroken supersymmetry, one with negative potential at the self-dual point $T_{\min} = 1$ and a second with zero potential at the self-dual point $T_{\min} = \rho$.

Before leaving this section let us briefly comment on the situation with respect to the S -field. Since target-space modular invariance does not constrain the S -dependence of the potential, it is difficult to state general results. For a generic S -dependence $\Omega(S)$, eq.(2.18) leads to two types of extrema:

$$1) \quad S_R \Omega_S - \Omega = 0 \quad (2.21)$$

$$2) \quad S_R \Omega_S - \Omega \neq 0 ; \quad S_R^2 \Omega_{SS} |H|^2 = E e^{2i\gamma} \Omega^* \quad (2.22)$$

where $\gamma = \arg(S_R \Omega_S - \Omega)$ and

$$E \equiv 2|H|^2 - \frac{T_R^2}{3}|H'| + \frac{3}{2\pi}H\hat{G}_2|^2.$$

Computing the second derivatives as a function of E one finds that the conditions for having a minimum for large S_R (weak coupling) are easy to fulfill for the first of those types of extrema. An example is the choice of ref.[21], $\Omega = c + he^{-\alpha S}$, in which a solution at weak coupling is easily allowed for small c and $S_R \Omega_S - \Omega = 0$. For the second class of extrema in eq.(2.22), although appropriate minima may exist (depending on E) they tend to occur at strong coupling.

2.3 THE VANISHING OF THE COSMOLOGICAL CONSTANT

As we have seen in the last section, for general choices of the modular invariant function $H(T)$, including $H(T) = \text{const}$, the scalar potential takes a negative value at its supersymmetry breaking minimum. Thus we are dealing in general with a negative cosmological constant after supersymmetry breaking; four-dimensional space-time is described by an anti-de Sitter space. (From the conformal field theory point of view the negative cosmological constant can be regarded as a shift of the central charge $\delta c < 0$, such that we are dealing with a non-critical string after taking into account the non-perturbative effects.)

Obviously, it is a very interesting question whether there exist special choices for $H(T)$ with broken space-time supersymmetry and vanishing scalar potential at some (possibly local) minimum $T = P$.

The equation $V(P) = 0$ (see eq.(2.16)) has two solutions ($k(S) = |S_R \Omega_S / \Omega - 1|^2$):

$$\begin{aligned} \text{(i)} \quad H'(P) &= -\frac{3}{2\pi} G_2(P) H(P) + \frac{3}{P+P^*} H(P) (1 - \sqrt{1 - k(S)/3}), \\ \text{(ii)} \quad H'(P) &= -\frac{3}{2\pi} G_2(P) H(P) + \frac{3}{P+P^*} H(P) (1 + \sqrt{1 - k(S)/3}). \end{aligned} \quad (2.23)$$

To obtain non-trivial solutions of the above equations it is useful to observe that equations (i) and (ii) are just the duality transformed of each other. More precisely, if $T = P$ satisfies (i) then $T' = \frac{1}{P}$ automatically satisfies (ii) and vice versa. Therefore we can restrict ourselves, without loss of any generality, to discussing the solutions of equation (i). In order for P to be an extremum of V it has to satisfy $\frac{\partial V}{\partial T}(P) = 0$ (see eq.(2.20)) which leads to the following condition (we assume in the following that P is real):

$$\begin{aligned} H''(P) &= 3H(P) \left(\frac{3}{4\pi^2} G_2(P)^2 - \frac{1}{2\pi} G_{2T}(P) \right. \\ &\quad \left. + \frac{1}{P^2} (1 - \sqrt{1 - k(S)/3} - \frac{3}{4} k(S)) + \frac{3}{2\pi P} (\sqrt{1 - k(S)/3} - 1) \right). \end{aligned} \quad (2.24)$$

To decide whether this solution corresponds to a minimum one has to calculate the second derivatives and one obtains

$$\begin{aligned} \frac{\partial^2 V}{\partial T^2}(P) &= \frac{|\Omega(S)|^2}{S_R P^3 |\eta(P)|^2} H(P)^* \left(-2\sqrt{1 - k(S)/3} P H''(P) \right. \\ &\quad - \frac{27}{4\pi^3} \sqrt{1 - k(S)/3} P G_2(P)^3 H(P) \\ &\quad + \frac{27}{2\pi^2} \sqrt{1 - k(S)/3} P G_2(P) G_{2T}(P) H(P) - \frac{3}{\pi} \sqrt{1 - k(S)/3} G_{2T}(P) H(P) \\ &\quad + \frac{27}{4\pi^2} (3\sqrt{1 - k(S)/3} + k(S) - 3) G_2(P)^2 H(P) \\ &\quad + \frac{9}{4\pi} (12 - 4k(S) - 3(3 - k(S))\sqrt{1 - k(S)/3} - 3\sqrt{1 - k(S)/3}) \frac{1}{P} G_2(P) H(P) \\ &\quad + \frac{9}{2\pi} (3 - k(S) - 3\sqrt{1 - k(S)/3}) G_{2T}(P) H(P) \\ &\quad \left. + \frac{3}{4} (5k(S) - k(S)^2 - 8 + 8\sqrt{1 - k(S)/3} - k(S)\sqrt{1 - k(S)/3}) \frac{H(P)}{P^2} \right), \\ \frac{\partial^2 V}{\partial T \partial T^*}(P) &= \frac{|\Omega(S)|^2}{S_R P^3 |\eta(P)|^2} \frac{|H(P)|^2}{2P^2} k(S). \end{aligned} \quad (2.25)$$

To proceed let us first assume that $k(S)$ is non-vanishing, i.e. also the S -field sector breaks

supersymmetry. It is convenient to switch to real variables $T = x + iy$. Then the second derivatives take the form $\frac{\partial^2 V}{\partial x^2} = 2(\frac{\partial^2 V}{\partial T \partial T^*} + \text{Re} \frac{\partial^2 V}{\partial T^2})$, $\frac{\partial^2 V}{\partial y^2} = 2(\frac{\partial^2 V}{\partial T \partial T^*} - \text{Re} \frac{\partial^2 V}{\partial T^2})$ and $\frac{\partial^2 V}{\partial x \partial y} = -2(\text{Im} \frac{\partial^2 V}{\partial T^2})$. The point $T = P$ is a minimum of V if $\frac{\partial^2 V}{\partial x^2}(P)$, $\frac{\partial^2 V}{\partial y^2}(P)$, $\frac{\partial^2 V}{\partial x^2}(P)$, $\frac{\partial^2 V}{\partial y^2}(P)$ and $\frac{\partial^2 V}{\partial x \partial y}(P)^2 > 0$. Since $\frac{\partial^2 V}{\partial T \partial T^*}(P) > 0$ these conditions can be met by appropriately choosing $H''(P)$. (For example, $T = P$ is a minimum of V if $\frac{\partial^2 V}{\partial T^2}(P) = 0$.) Summarizing this analysis, a minimum of V is obtained if the first three derivatives of $H(T)$ satisfy suitable conditions at $T = P$. A class of modular invariant functions $H(T)$ with these properties can be easily constructed by a third-order polynomial in $j(T)$:

$$H(T) = j(T)^3 + aj(T)^2 + bj(T) + c. \quad (2.26)$$

The coefficients a, b, c are uniquely determined by a system of three linear equations.

The above analysis is valid for $k \neq 3$. When $k = 3$, condition (2.23) for $V(P) = 0$ (now implying that $H'(P) = 0$) automatically gives $\frac{\partial V}{\partial T}(P) = 0$ so that $H''(P)$ needs not be fixed as in (2.24). Evaluating the second derivatives shows that P is again a minimum.

The case $k(S) = 0$ is special. Then $\frac{\partial^2 V}{\partial T \partial T^*}(P)$ is zero. Thus to have the chance of finding a minimum at $T = P$ we have to demand that $\frac{\partial^2 V}{\partial T^2} = 0$. To decide whether we can obtain a minimum one has to further calculate the derivatives of V up to fourth order. One can do that (an algebraic computer program is recommended) to find the following result: All mixed derivatives like $\frac{\partial^3 V}{\partial T^2 \partial T^*}(P)$ or $\frac{\partial^3 V}{\partial T \partial T^* \partial T}(P)$ vanish. As a consequence of this, one now finds that $T = P$ is a saddle point (it is a minimum in the x and y directions but a maximum in the $x = \pm y$ directions). Thus for $k(S)$ going to zero the quadratic term of the potential around $T = P$ vanishes and $T = P$ becomes an unstable saddle point.

In summary, we have found a special class of superpotentials which leads to a supersymmetry breaking minimum of the corresponding scalar potential with vanishing cosmological constant. In order to find this minimum the auxiliary field t^S must be non-zero. Apart from this zero energy minimum the solution eq.(2.26) leads to additional minima with broken supersymmetry and negative cosmological constant. For example, at the self-dual points $T = 1, \rho, h^T = 0$, and the vacuum energy is negative (for $c \neq 0$, cfr. eq.(2.26)). Whether these two points actually correspond to minima of the potential depends on the definite values of the parameters a, b, c in eq.(2.26) and on the value of $k(S)$ (see the analysis in the previous section). Supersymmetry is unbroken at these points if $k(S) = 0$. Therefore the theory exhibits two phases, one with anti-de Sitter space-time and the other one with flat Minkowski space-time. It is an interesting question what kinds of specific non-perturbative string effects lead to a superpotential with these kinds of properties.

2.4 DIFFERENT RADII FOR TWO-TORI

Now one would like to study the case with the three-moduli T_i ($i = 1, 2, 3$) corresponding to the radii of the three two-tori of the internal space. The connection with the overall radius T is the following:

$$(T + T^*)^3 = \prod_{i=1}^3 (T_i + T_i^*). \quad (2.27)$$

We would like to establish that when the potential is symmetric with respect to the exchange of T_i the preferred minimum occurs when $T_1 = T_2 = T_3 = T$. This would in turn give support to studying only one modulus T . Although such a conclusion seems to be obvious starting from a Lagrangian which is completely symmetric under the exchange of the three T_i , there are examples of spontaneous breaking of similar discrete symmetries within the context of gauge theories beyond the standard model. An exemplary case is the spontaneous breaking of the left-right gauge symmetry [59], [60]. It is thus important to verify that such a spontaneous symmetry breaking does not happen in our case. The study we will present is not exhaustive and does not apply to the most general superpotential as well as the most general set of solutions for the extrema. However, even the simplest case yields the encouraging result that the minimum occurs for the symmetric point with $T_1 = T_2 = T_3$.

We take the standard form for the Kähler potential and assume that the superpotential is separable:

$$K = -\ln(S + S^*) - \sum_{i=1}^3 \ln(T_i + T_i^*), \quad W = \Omega(S) \prod_{i=1}^3 w(T_i). \quad (2.28)$$

For simplicity one puts $D_S \Omega = 0$ which has been shown to correspond to a minimum.* It is obvious that when $T_1 = T_2 = T_3 = T$ the potential reduces to the one for the T field only. In the above case the potential has the form:

$$V = e^G \left\{ \sum_{i=1}^3 |\sigma_i|^2 - 3 \right\}, \quad (2.29)$$

* Note that in the case when $\Omega(S) = w(S)$, i.e., there is duality in the strong-weak coupling regime as well [61], one could view the value of S as the fourth modulus. With notation $S = T_4$, there is then a discrete symmetry in the effective action under the exchange of T_1, T_2, T_3 and T_4 .

where

$$e^G = \frac{|\Omega|^2}{S R} \prod_{i=1}^3 \frac{|w(T_i)|^2}{T_{iR}}, \quad \sigma_i = T_{iR} \frac{w(T_i) T_i}{w(T_i)} - 1. \quad (2.30)$$

We shall study the case of extrema, where the first m moduli have one value T_α and the next n moduli have another value T_β , such that $T_\alpha \neq T_\beta$. Also note that $m + n = 3$. Then the constraints on extrema are:

$$\frac{\partial V}{\partial T_\alpha} = e^G m \frac{1}{T_{\alpha R}} \left\{ \sigma_\alpha \left[(m-1) |\sigma_\alpha|^2 + n |\sigma_\beta|^2 - 2 \right] + X_\alpha \sigma_\alpha^* \right\} = 0, \quad (2.31)$$

$$\frac{\partial V}{\partial T_\beta} = e^G n \frac{1}{T_{\beta R}} \left\{ \sigma_\beta \left[(n-1) |\sigma_\beta|^2 + m |\sigma_\alpha|^2 - 2 \right] + X_\beta \sigma_\beta^* \right\} = 0. \quad (2.32)$$

Here σ_α is defined analogously to eq.(2.30) and $X_\alpha = T_\alpha^2 \frac{w(T_\alpha)}{\sigma_\alpha w(T_\alpha)}$; similarly for σ_β and X_β . Eqs.(2.31), (2.32) can be satisfied by taking:

$$\sigma_\alpha = 0, \quad w(T_\alpha) \neq 0, \quad \sigma_\beta \left[(n-1) |\sigma_\beta|^2 - 2 \right] + X_\beta \sigma_\beta^* = 0. \quad (2.33)$$

Directions along T_α preserve supersymmetry while those along T_β do not. Note that we have chosen a solution of eqs.(2.31) and (2.32) which need not be the most general one. However, it is enough to spell out our main points.

In order to find the constraints for the minimum one has to evaluate the second derivatives. From the constraints for the positive mass square for the $T_{\alpha,\beta}$ fields one obtains the following necessary constraints:

$$1 - |\sigma_\beta|^2 > 0, \quad |X_\alpha|^2 > (2 - n |\sigma_\beta|^2) \quad (2.34)$$

$$\left[|X_\alpha| \pm (n |\sigma_\beta|^2 - 2) \right] \left[|X_\alpha| \pm 1 \right] > 0$$

where the $\pm$ sign refers to the case when $1 - n |\sigma_\beta|^2 \gtrless 0$. One additional constraint imposes a bound on $\partial_{T_\beta}^2 w(T_\beta)$. The simplest choice for $w(T_i) = \eta^{-2} T_i$. In this case $\sigma_\alpha = 0$ implies $T_\alpha = \{1, \rho\}$ and then $|X_\alpha| = |T_{\alpha R}|^2 \left| \frac{\partial^2 w}{\partial T_\alpha^2} \right| = \{0, 0.38\}$, respectively. Since in this case $|X_\alpha| < 1$, eqs.(2.34) for the local minimum cannot be satisfied for $n < 3$. Thus, all the cases with $n < 3$ correspond to saddle points and only the case $T_1 = T_2 = T_3 = T_\beta$ can correspond to the minimum.

It would be instructive to study more general forms of separable superpotentials symmetric under the exchange of T_i 's in order to establish that the minimum with $T_1 = T_2 = T_3$ is generic. On the other hand it should also be pointed out that for a general string compactification, e. g., Z_N (blown-up) orbifolds with $N \neq 3, 7$, the discrete symmetry between T_i 's need not be present in the effective Lagrangian anymore. In this case it is natural to expect that the vev 's of T_i 's are different as well.

3. Gaugino Condensation and Singularities in Low-energy Effective String Actions

In the last section we analyzed general $PSL(2, \mathbb{Z})$ invariant effective potentials for the T -field without specifying the non-perturbative string dynamics being responsible for them. Now we want to consider the gaugino condensation mechanism as an explicit example of non-perturbative supersymmetry breaking. We also discuss the physics underlying the presence of singular points of the non-perturbative superpotential generated by the gaugino condensate.

In general, the effective string action will contain singularities if some of the "heavy" fields, ϕ'_H , become massless at some point in the T -field moduli space. These divergences will in particular show up in contributions to $\tilde{S}(\phi_0)$ where the light fields ϕ'_H are now circulating in loop diagrams. (Of course there are also divergences in $\tilde{S}(\phi_0)$ due to loop diagrams involving fields ϕ_0 , but these are usually "subtracted" leading to renormalized quantities and effects like the running of coupling constants.) Since we are dealing with a compactification from ten to four dimensions we immediately know one point in the T -moduli space where the effective action should diverge, namely at its boundary $\text{Re}T \rightarrow p = \infty$. This is the decompactification limit where the infinite tower of Kaluza-Klein states becomes light. (Of course, at the duality transformed point $T = 0$, the winding states are massless.) In addition to this singular point at infinity, there may be possible other points in the moduli space where a finite number of states, in particular states with non-vanishing momentum and winding number, become light. Again the effective action will diverge at the finite points $T = p$. Of course, one could avoid these singularities by explicitly keeping in $\tilde{S}$ the finite number of those states ϕ'_H which potentially become massless [62], [63]. This was described in [62] for the case of the stringy Higgs effect.

To be more specific we now discuss the one-loop contribution to the gauge coupling constant, the so-called threshold effects [64] which is relevant for the gaugino condensation. The one-loop threshold corrections describe the difference between the inverse string

coupling constant, as determined by the vev of the dilaton field S , and the inverse gauge coupling constant at the renormalization point $M_{GUT} \sim (T + T^*)^{-1/2}$ due to the integration over the massive states

$$\Delta_a = \frac{1}{g_a^2(M_{GUT})} - \frac{1}{2}(S + \bar{S}). \quad (3.1)$$

Δ_a was calculated for four-dimensional heterotic string models without tachyons in [64]:

$$\Delta_a = \int_{\Gamma} \frac{d^2\tau}{\tau_2} (B_a(\tau, \bar{\tau}) - b_a) \quad (3.2)$$

with

$$B_a(\tau, \bar{\tau}) \sim \text{Tr}(\mathcal{Q}_a^2(-1)^{F_2} F^H q^H \bar{q}^H). \quad (3.3)$$

($q = e^{2\pi i \tau}$ where τ is the one-loop world sheet modular parameter.) The trace in (3.3) runs over all charged states of the string spectrum, massless as well as massive. Thus the integral $\int_{\Gamma} \frac{d^2\tau}{\tau_2} B_a(\tau, \bar{\tau})$ is divergent – the trace in (3.3) is dominated, in the limit $\tau_2 \rightarrow \infty$, by the massless states that have $H = \bar{H} = 0$. Indeed, string theory and the effective field theory have the same kind of infra-red divergences giving rise to the running coupling constants [65], [66] with β -function coefficients $b_a = \lim_{\tau_2 \rightarrow \infty} B_a(\tau, \bar{\tau})$. It follows that Δ_a is finite since b_a is subtracted in (3.2) in such a way that the massless states do not contribute to the integral. However this last statement is only true if the trace over the left-over states does not contain any state ϕ'_H which becomes massless for some value of the T -field.

Let us now compute the divergence which arises due to the appearance of the light Kaluza-Klein modes as $T \rightarrow \infty$. (See also a similar discussion by [67], [68].) The masses of the Kaluza-Klein states are model-independently given by $M^2(T) = n^2/R^2$ ($T + T^* = 2R^2$). Then we obtain for Δ_a as $R^2 \rightarrow \infty$:

$$\Delta_a(T, T^*) \rightarrow b_a \int_{\Gamma} \frac{d^2\tau}{\tau_2} \sum_n e^{-\tau_2 n^2/R^2}. \quad (3.4)$$

After Poisson resummation Δ_a looks like

$$\Delta_a \rightarrow b_a \int_{\Gamma} \frac{d^2\tau}{\tau_2} R^2 \sum_n e^{-\tau_2 R^2 n^2} \rightarrow b_a R^2 \frac{\pi}{3}. \quad (3.5)$$

Thus the Kaluza-Klein states produce a linearly divergent threshold correction. (This agrees with the results obtained in ref. [69] from a truncation of the ten-dimensional anomaly cancelling terms.)

For some values of the moduli fields there are possibly additional charged massless states appearing in the spectrum. We would like to argue now that additional (but weaker) singularities are then expected in this case. Consider the contribution of a finite number of states possibly becoming massless at some finite value $T = p$ inside the fundamental domain. For T around p these states ϕ'_H in principal give an additional contribution to the running coupling constant, i.e. to b_a :

$$b_a^{\text{tot}} = b_a + b'_a. \quad (3.6)$$

Thus the additional states ϕ' may either enhance the asymptotic freedom of the gauge coupling constant (assuming that b_a is already negative) if $b'_a < 0$ or lead to a slower increase or even decrease of the coupling constant if $b'_a > 0$ as going to lower momentum scales. However we want to follow our strategy also integrating out the states ϕ'_H which will clearly cause the appearance of poles in $\Delta_a(T)$ as $T \rightarrow p$. Then the most dominant contribution to Δ_a looks like:

$$\Delta_a(T, T^*) \rightarrow b'_a \int_{\Gamma} \frac{d^2\tau}{\tau_2} e^{-M^2(T)\tau_2} \rightarrow -b'_a \log M^2(T). \quad (3.7)$$

Thus Δ_a is logarithmically divergent as $T \rightarrow p$. Assuming that ϕ'_H is a state with non-vanishing momentum and winding number its mass is generically given as $M^2(T) \sim |T-p|^2$ for $T \sim p$. Then the asymptotic behaviour of the threshold correction takes the simple form

$$\Delta_a \rightarrow -b'_a \log |T-p|^2. \quad (3.8)$$

Compared to eq.(3.5), the divergence is now less severe because of the contribution of an only finite number of almost massless states. Further comparing with eq.(3.1) we recognize that $\alpha_a^2(M_{GUT})$ is increased due to the contribution of Δ_a if $b'_a < 0$ but decreases if $b'_a > 0$. Thus one encounters, at least very qualitatively, the same situation compared to the case in which the ϕ'_H directly contribute to the running coupling constant.

So far we have only considered two asymptotic limits of Δ_a . In order to get the full functional dependence of Δ_a on T we seek for target space modular invariant functions which exactly reproduce these kinds of divergences. At this point we want to stress again that there exist many models in which there are no states which become massless at some finite points $T = p$, or only gauge singlet states become massless and therefore do not contribute to Δ_a . For example, this situation is realized for the Z_3 orbifold (and also for

other orbifold compactifications) considering the threshold corrections to the E_6 coupling constant. In spite of having states which are massless at $T = \exp(i\pi/6)$ which lead to an enhanced $U(1)^6$ gauge symmetry at this point (see [70],[62]), these states are singlets under E_6 and therefore neither contribute to the E_6 β -function nor to the threshold effects at the one loop level. Thus, for these types of models Δ_a is uniquely determined by a modular invariant function with the only linear singularity at $T = \infty$ [14], [71],[15],[16]:

$$\Delta_a(T, T^*) = -b_a \log[|\eta(T)|^4 (T + T^*)] \quad (3.9)$$

However there are conceivably other string compactifications with gauge non-singlet states becoming massless at finite values $T = p$. (However, note that we are not aware of any specific orbifold compactification of this kind.) Now, we have to look for modular functions which reproduce both types of singularities eqs.(3.8),(3.5). The most general ansatz for $\Delta_a(T, T^*)$ looks like :

$$\Delta_a(T, T^*) = -b_a \log[|\eta(T)|^4 (T + T^*)] + \frac{b_a}{3} \log |H(T)|^2. \quad (3.10)$$

Here $H(T)$ is a modular invariant function which is therefore a rational function of $j(T)$. In order to reproduce the correct singular behaviour at $T \rightarrow \infty$, eq.(3.5), we have to require that $H(T)$ has no zeroes or poles at $T = \infty$. This implies (cfr. eq.(A5)) that the difference between the number of zeroes and poles of $H(T)$ inside the fundamental region is zero: $\sum_{p \neq 1, \infty} \nu_p + \frac{1}{2}\nu_1 + \frac{1}{2}\nu_\rho = 0$. More specifically, if $H(T)$ has a zero ($\nu_p > 0$) or a pole ($\nu_p < 0$) it means in this context of gaugino condensation that some states become massless at $T = p$ contributing to the β -function with $b'_a = -\frac{b_a}{3}\nu_p$ (compare with eq.(3.8)). Thus a zero of $H(T)$ corresponds to a non-asymptotic free contribution to the β -function (assuming that $b_a < 0$) whereas a pole hints at additional massless states with an asymptotic free contribution.

After investigating the structure of the one loop gauge coupling constant we are now ready to discuss the gaugino condensation mechanism. This will also enable us to give a proper interpretation of the singularities appearing in the effective superpotential and their implications on the vacuum structure of the theory within this specific dynamical scenario. The effects of the gaugino condensation can be properly described by an effective superpotential induced by $\langle \lambda_a \lambda_a \rangle \neq 0$ [14],[15],[17]. Within this approach one can either

* Writing $\Delta_a(T, T^*) = \frac{1}{2}(\Delta_a(T) + \bar{\Delta}_a(T^*)) - b_a \log(T + T^*)$ the non-holomorphic part can be viewed as the radius dependent one-loop renormalization point.

treat the composite gaugino bilinear field as an independent, dynamical variable [15],[17] or directly substitute the gaugino condensate by its S and T dependent vacuum expectation value [14]. The relation between both methods was discussed in [72], [73] showing that they lead to the same results for weak coupling, i.e. for large S fields. Here we will follow [14]. Then the effective superpotential takes the form (we are dealing with the pure Yang-Mills case):

$$W_a = \left(\frac{\lambda_a \lambda_a}{S_0^3} \right) = (e^{-K/2} \delta^{3/2} \lambda_a \lambda_a) = (T + T^*)^3 / 2 \delta^{3/2} e^{\frac{3}{2} \frac{1}{2\pi} f_a(S,T)} \quad (3.11)$$

Here $S_0 = e^{K/6} \delta^{-1/2}$ is the chiral compensator [56] (δ is some phase factor) acting as a Weyl rotation on the gaugino fields (see [71],[14],[15] for details). b_a is the one-loop $N = 1$ β -function coefficient and $f(S, T)$ the one-loop gauge kinetic function

$$\begin{aligned} f_a(S, T) &= S + \Delta_a(T) - b_a \log(T + T^*) \delta \\ &= S - b_a \log((\eta(T))^4 (T + T^*) \delta) + \frac{b_a}{3} \log H(T)^2. \end{aligned} \quad (3.12)$$

Inserting (3.12) into eq.(3.11) one finally obtains

$$W = \frac{\Omega(S)H(T)}{\eta(T)^6}. \quad (3.13)$$

The form of the S -dependent function $\Omega(S)$ depends on the specific gaugino condensation dynamics assumed. It could be of the simple form $\Omega(S) = c + e^{\frac{3S}{2\pi}}$ as in ref.[21] or could describe the existence of several gaugino condensates [72] [74-76] or else could be a function with explicit duality in the variable S [61]. From eq.(3.13) we observe that the gaugino condensation effect can provide the required dynamics to generate the class of modular invariant superpotentials described in the previous section.

Deriving this holomorphic superpotential it is essential that the b_a -coefficients in eqs.(3.11) and (3.12) agree, i.e. that the threshold correction $\Delta_a(T, T^*)$ contains the $N = 1$ β -function coefficient $b_a^{N=1}$. However explicit calculations [32] of Δ_a for some orbifold compactifications find the coefficient in eqs.(3.5)(3.9) to be different from the $N = 1$ coefficient $b_a^{N=1}$ but given by the $N = 2$ β -function coefficient $b_a^{N=2}$ or even completely vanishing. This comes from the fact that the untwisted (Kaluza-Klein) states with T -dependent masses form $N = 2$ respectively $N = 4$ subsectors of the total Hilbert space. However one has to keep in mind that using formula (3.2), as ref.[32] does, one actually calculates only the difference between the threshold contributions of two or more different gauge groups such that there is an additional T -dependent contribution to Δ_a . Therefore

we believe that any string compactification (including also the Z_3 , Z_7 orbifolds) leads to a universal T -dependent large radius limit of the one-loop gauge coupling constant determined by the $N = 1$ coefficient $b_a^{N=1}$ in eqs.(3.5)(3.9)(3.12).

All poles and zeroes of W are now inherited from the singular points of the threshold correction $\Delta_a(T)$. Thus for $T \rightarrow \infty$ the superpotential is exponentially divergent* :

$$W \rightarrow \exp \frac{\pi T}{2}. \quad (3.14)$$

This singularity originates from the occurrence of the light Kaluza-Klein states for large radii. Analogously we can argue that the possible presence of zeroes and poles of $W(T)$ inside the fundamental region is due to some states becoming massless at these points.

Let us finally discuss the implications of these singularities and zeroes of the superpotential on the vacuum structure of the compactified heterotic string theory. For large T the scalar potential V blows up exponentially (due to the term $V \sim |W|^2$). Thus one is always dynamically repelled from this singular point. This feature can be easily understood by remembering that for large T more and more Kaluza-Klein states become light and effectively contribute to the β -function such that the effective b_a -coefficient becomes more and more negative. This implies that the corresponding value of the gaugino condensate, as being proportional to the condensation scale, increases - a situation which is energetically not preferred. (Modulo supergravity corrections, the lowest possible condensation scale is energetically preferred.) The behaviour of the scalar potential at the finite singular points respectively zeroes $T = p$ has a similar interpretation. For $b'_a < 0$, the potential again blows up at this point such that the dynamics completely "dislike" $T \rightarrow p$, since the β -function becomes more negative and the condensation scale is increased. On the other hand, if $b'_a > 0$ the superpotential has a zero which will most likely lead to a minimum of the associated scalar potential. This comes from the fact that the condensation scale is now lowered, an energetically preferred situation.

* A superpotential which goes exponentially to zero for large T seems impossible in the context of gaugino condensation in pure Yang-Mills gauge theories. However this becomes possible when one considers the loss of asymptotic freedom due to the contribution of matter fields to the β -function [73].

4. Matter Fields

4.1 MODULAR INVARIANT POTENTIAL

So far we have been consistently neglecting the matter fields, i.e. we have been setting them to the naive vacuum state where all of them vanish. In orbifold compactifications, these fields come in three classes: the untwisted matter (A) corresponding to the Wilson lines of toroidal compactifications which survive the orbifold twist; some of them are actually moduli since they may have flat potential also; the twisted moduli or blowing-up modes (B) present on $(2, 2)$ (and some $(0, 2)$) models, whose vuv determines if we are at the orbifold point ($B = 0$) or at any other smooth manifold point, and finally we have the twisted matter fields (C), which like untwisted ones may or may not have a flat potential. They have been the ones responsible for breaking an original gauge group G to the Standard Model one in the quasi-realistic orbifold models constructed so far [77].

Since all of these fields are massless to start with, the original (perturbative) potential is such that their mass terms in the scalar potential vanish. It is then interesting to investigate if the non-perturbative induced potentials change this situation by providing a $(mass)^2$ to these fields which, if it is negative, will prove that the original vacuum was a maximum in these directions. The main reason we did not consider these fields before (besides simplicity) is that their Kähler potential is not completely understood even at tree-level. It takes the following form [50],[29], [78]:[†]

$$K = -\log\{(T + T^* - AA^*)^3 - BB^* - (T + T^*)CC^*\}. \quad (4.1)$$

With respect to the twisted fields B and C , eqn.(4.1) provides only the first term in a perturbative expansion of K around zero.^{*} This however will be enough for our purposes in this chapter since we will only check if the naive vacuum is stable under small perturbations. What we will not be able to find is some other extrema for larger values of the fields. It is known that all of the matter fields transform non-trivially under the $SL(2, \mathbb{Z})$ modular group. In particular the untwisted matter field transformations can be found directly from the generalized torus duality of references [9],[10]. We can see that they transform as

[†] This equation applies if all three $U(1)^3$ charges η_i (see ref.[50]) are non-vanishing. If this is not the case, there are further twisted moduli, besides B , which for our purposes will behave like the field A above so we do not consider them explicitly.

^{*} For the twisted moduli B higher order corrections to e^{-K} were discussed in ref.[53].

modular forms of weight 1 (up to a phase):

$$A \rightarrow (icT + d)^{-1} A \quad (4.2)$$

For the twisted fields B and C , we can find their behaviour under modular transformations by demanding the Kähler potential to transform adequately [29]:

$$\begin{aligned} B &\rightarrow (icT + d)^{-3} B, \\ C &\rightarrow (icT + d)^{-2} C. \end{aligned} \quad (4.3)$$

up to a T -dependent phase (multiplier system). Therefore A , B and C transform like modular forms of weight -1, -3 and -2 respectively. As we mentioned above, in order to analyze the stability of the $\langle A \rangle = \langle B \rangle = \langle C \rangle = 0$ vacuum, it is enough to find the scalar potential in an expansion around that point. Therefore, for simplicity, we will consider a generic field Z transforming as a modular form of weight $-n$ and expand the potential around $\langle Z \rangle = 0$. We will then consider the following Kähler potential:

$$K(S, S^*, T, T^*, Z, Z^*) = -\log(S + S^*) + K_0(T, T^*) + K_1(T, T^*)ZZ^* + \dots \quad (4.4)$$

Notice from (4.1) that for A , B and C we have $K_0 = -3\log(T + T^*)$ and $K_1 = \alpha_n(T + T^*)^{-n}$, $\alpha_n \geq 0$ which is consistent with the positivity of the kinetic energy terms. We will also consider the following expansion for the superpotential:

$$W(S, T, Z) = \Omega(S) W_0(T) + \Omega'(S) W_2(T) Z^2 + \dots \quad (4.5)$$

where we have allowed for the possible generation of a non-perturbative trilinear scalar superpotential ($\Omega' \neq 1$). The $W_i(T)$ have modular weight $n_i - 3$. Thus for the untwisted matter fields W_3 has to be modular invariant; at string tree level W_3 is just a constant for these fields. However notice that tree-level world-sheet instanton effects may also generate (for twisted fields) a non-trivial $W_3(T)$. Our aim is then to evaluate the scalar potential V in a neighbourhood of $Z = 0$

$$V = V_0(S, S^*, T, T^*) + V_1(S, S^*, T, T^*)ZZ^* + \dots \quad (4.6)$$

and find the explicit signature of V_1 in order to determine the stability of the vacuum.

The general form of the potential is (we neglect D -terms since, being of order Z^4 , they are not relevant for the stability analysis):

$$V = e^K \{ |D_S W|^2 K_{SS}^{-1} + |D_T W|^2 K_{TT}^{-1} + |D_Z W|^2 K_{ZZ}^{-1} + [(D_T W)(D_Z W)^* K_{\bar{T}\bar{Z}}^{-1} + h.c.] - 3|W|^2 \} \quad (4.7)$$

where as usual $D_\Phi W \equiv W_\Phi + K_\Phi W$ is the Kähler derivative of the superpotential and letter subscripts represent derivatives with respect to the corresponding field. Expanding V up to $\mathcal{O}(Z^2)$ and comparing with (4.6) we obtain

$$\begin{aligned} V_0 &= \frac{e^{K_0}}{(S+S^*)} \{ (S+S^*)^2 |D_S \Omega|^2 |W_0|^2 + K_{0T\bar{T}}^{-1} |D_T W_0|^2 |\Omega|^2 - 3|\Omega|^2 |W|^2 \}, \\ V_1 &= \frac{e^{K_0} K_1}{(S+S^*)} \left\{ (S+S^*)^2 |D_S \Omega|^2 |W_0|^2 + K_{0T\bar{T}}^{-1} |D_T W_0|^2 |\Omega|^2 \times \right. \\ &\quad \left[1 - \frac{K_{1T\bar{T}}}{K_{0T\bar{T}} K_1} + \frac{K_{1T} K_{1T^*}}{K_1^2 K_{0T\bar{T}}} \right] - 2|W_0 \Omega|^2 \right\}. \end{aligned} \quad (4.8)$$

Before analyzing V_1 , a few comments are in order. It is clear from (4.8) that the sign of (4.8) does not critically depend on the explicit form of W_0 , i.e. the explicit choice of the non-perturbative potential for T . However physical conclusions depend drastically on the value of $|D_S \Omega|^2$ as well as the explicit expressions for K_0 and K_1 . On the other hand, as expected, V_1 is independent of the form for W_3 .

In order to get a better feeling for V_1 , let us plug in the explicit expressions for $K_0 = -3 \ln(T+T^*)$ as well as K_1 :

$$K_1 = \frac{\alpha_n}{(T+T^*)^n}, \quad \alpha_n > 0. \quad (4.9)$$

E.g., $n = 1, 2, 3$, for untwisted matter fields (A), twisted matter fields (C) and blowing-up modes (B), respectively. Then eqs. (4.8) become:

$$\begin{aligned} V_0 &= \frac{1}{(S+S^*)(T+T^*)^3} \left\{ (S+S^*)^2 |D_S \Omega|^2 |W_0|^2 + \frac{(T+T^*)^3}{3} |D_T W_0|^2 |\Omega|^2 - 3|W_0 \Omega|^2 \right\}, \\ V_1 &= \frac{Z Z^* \alpha_n}{(T+T^*)^{n+3} (S+S^*)} \left\{ (S+S^*)^2 |W_0|^2 |D_S \Omega|^2 + \right. \\ &\quad \left. \frac{(T+T^*)^2}{3} |\Omega|^2 |D_T W_0|^2 \left(1 - \frac{n}{3} \right) - 2|\Omega W_0|^2 \right\}. \end{aligned} \quad (4.10)$$

For the minimum with $D_S \Omega \equiv 0$, which has been shown in the previous section to be the absolute minimum for $\Omega = c + h e^{(-\alpha S)}$ one sees from eqs.(4.10) that as long as $n \geq 1$,

$V_1 < 0^*$. Thus, m^2 terms are negative for all the three types of the “matter” fields. This result is independent of the explicit form for the non-perturbative supersymmetric potential W_0 and it depends only on the structure of the Kähler potential. One can also check that for $V_0 \rightarrow 0$ (limit of vanishing cosmological constant) the untwisted matter fields become massless. Our conclusion is less general if there exists a vacuum with $|D_S \Omega|^2 \neq 0$, i. e. when supersymmetry is broken due to the non-zero v_{ev} of the S fields. V_1 is now negative if[†]

$$(S+S^*)^2 |W_0|^2 |D_S \Omega|^2 + \frac{(T+T^*)^2}{3} |\Omega|^2 |D_T W_0|^2 (1-n) < 0. \quad (4.11)$$

However, as studied before (in chapter 2) one notes that the absolute minimum with $|D_S \Omega|^2 \neq 0$ is less likely to form.

It is interesting to note that the physical conclusions are novel and different from those of ref.[72], where the stable vacuum corresponds to the one of the most symmetric point in the moduli space. In our case moduli for the twisted sector necessarily acquire non-zero v_{ev} 's, i.e. $< B > \neq 0$. Thus, the preferred vacuum is not the orbifold limit but the one of the smooth Calabi-Yau manifolds, i.e. blown-up orbifold. Also the gauge symmetry is broken, since the twisted matter fields acquire non-zero v_{ev} as well. This is an intriguing scenario which enables one to break gauge symmetry as a consequence of non-perturbative string effects. Note also that when more than one field is included, at least one direction will have $m^2 < 0$.

An important question to be addressed is the existence of a “stable” vacuum near $Z \ll 1$, i.e. expansion around $Z = 0$ is justifiable. For that purpose an expansion up to $\mathcal{O}(Z^3)$ is necessary and thus the next term $\Omega'(S)W_3(T)Z^3$ in the superpotential (4.5) is relevant as well as the cubic terms in the Kähler potential (4.4).

We also evaluated the trilinear soft supersymmetry-breaking term which is present in the scalar potential once supersymmetry is broken. It can be usually parametrized as $(m_{3/2} \mathcal{A} Z^{*3} + h.c.)$, where $\mathcal{A}$ is a dimensionless parameter. We evaluate $\mathcal{A}$ at the original minimum (the one with $Z = 0$). Using the same assumptions as above and after some algebra one finds the following result for the supersymmetry-breaking soft parameter $\mathcal{A}$:

$$\mathcal{A} = \frac{1}{(S_R T_R^3)^{1/2}} \left[\left(\frac{W_{0T}}{W_0} - \frac{3}{T_R} \right) \left(\frac{T_R^3}{3} W_{3T} + T_R(n-1)W_3 \right) - W_3(S_R \frac{\Omega_S}{\Omega} - 1) \right]. \quad (4.12)$$

Here we have just considered the simplest tree-level case $\Omega' = 1$. Notice that then for

* Note, that for $n = 1, 2$ we took into account $V_0 < 0$.

† Note that again we took into account that $V_0 < 0$.

the untwisted matter case we have $n = 1$ and $W_{JT} = 0$ so that one has $\mathcal{A}_{untwisted} = 0$ (assuming that $S_R \Omega_S / \Omega - 1 = 0$). This is not in general the case for the twisted fields. Let us remark that the $\mathcal{A}$ -parameter in eq.(4.12) has been computed around the extrema with $Z = 0$. Since we have just shown that those minima are unstable this $\mathcal{A}$ value will in general be different from the one computed at the physical minimum. However, if the physical minimum is nearby (as it should be if one still wants to use the approximate Kähler potential in eq.(4.4)), eq.(4.12) should be a good approximation.

Analogously one can evaluate the corresponding trilinear term arising from the higher order terms in the Kähler potential (4.4). However, the general functional dependence of the trilinear terms in (4.4) is more complicated, i. e., there are terms proportional to $Z^3, Z^2 Z^*, ZZ^{*2}$ as well as Z^3 and their T dependence is not fully understood.

Another interesting issue to study is whether the above results persist for a generic string vacuum with a non-perturbative potential for T fields. Our conclusions for the unstable vacuum crucially depend on the fact that there is no quadratic term in the Z fields in the superpotential. This is the case for all the (blown-up) orbifolds. Namely, for the Z_N orbifolds with N an odd prime number, the Z_N discrete symmetry prevents such terms. On the other hand for N even, the invariance under the enhanced gauge group (additional $U(1)$'s) prevents such terms in the superpotential. Generalization of the above conclusions for other string vacua would be important.

4.2 STABILITY IN ANTI-DE SITTER BACKGROUNDS

There is a subtle issue associated with the stability of the anti-de Sitter vacuum, relevant for most of our cases with $V_0 < 0$. Namely, the negative m^2 does not automatically imply that the vacuum is not stable. In flat space-time, the criterion for stability of a static field configuration is well known to be parametrized by having positive m^2 . In curved backgrounds, the situation is less clear since there are gravitational corrections to the energy. For the anti-De Sitter space the condition for the stability of the vacuum has been discussed in the past [79], and the criterion is the following. For a massive field Φ in D dimensions satisfying

$$(\square + M^2)\Phi = 0, \quad (4.13)$$

the configuration is stable if

$$M^2 > -\frac{1}{4}(D-1)^2\kappa^2 \quad (4.14)$$

where κ is the constant determining the curvature of the space $R_{\mu\nu\lambda\rho} = \kappa^2(g_{\mu\rho}g_{\nu\lambda} - g_{\nu\rho}g_{\mu\lambda})$.

From Einstein's equations we can see that the cosmological constant $\Lambda \equiv V_0 = -\frac{\kappa^2}{2}(D-1)(D-2)$. In four dimensions $\Lambda = -3\kappa^2$, so the stability criterion can be expressed as

$$M^2 > \frac{3}{4}\Lambda. \quad (4.15)$$

In the present case, we have a Lagrangian

$$\alpha T_R^{-n} \partial Z \partial Z^* + V_1 Z Z^* \quad (4.16)$$

so $M^2 = \frac{V_1}{\alpha T_R^{-n}}$. For a field of modular weight $-n = -1$, we know that $V_1 = \frac{2}{3}\alpha T_R^{-n} V_0$ so $M^2 = \frac{2}{3}V_0 > \frac{3}{4}V_0$ and this maximum is then stable. For more negative weights however the V_1 term is "more negative" and they will be generically unstable (as can be easily checked for particular cases on the blowing-up modes and twisted matter fields).

We have seen then that the above constraint is barely not satisfied for the untwisted matter fields (A), however it is possible to satisfy it for the twisted matter (C) and the blowing-up modes (B). Actually, for the realistic case when the cosmological constant is extremely small, i.e. $V \rightarrow 0$, B and C fields have strictly negative m^2 , while the mass of A fields goes to zero. Thus our conclusion that the blowing-up modes and the twisted matter fields destabilize the vacuum is generic.

5. Conclusions

We have seen how powerful the constraint of $SL(2, \mathbb{Z})$ modular invariance is for the low-energy string effective actions. The most general invariant superpotential that we considered still has a large amount of degrees of freedom which parametrize our ignorance of the sources of non-perturbative string effects. Nevertheless, we have been able to extract general properties which will hold for any modular invariant string action. Probably the most striking one is that due to the fact that the superpotential is a modular form of negative weight, then it has to have singularities either inside the fundamental domain or at infinity. The scalar potential will in general inherit these singularities and we are then faced with a very interesting situation that was not expected in string theory before. The scalar potential was assumed to vanish at infinite radius (weak sigma model and uncompactified domain) and no poles were expected at finite radius.

In chapter 3, we presented our interpretation of the appearance of poles at infinite radius when the non-perturbative source is the condensation of gauginos of a hidden gauge

group. It is the point where an infinite number of Kaluza-Klein modes become massless. It would certainly be interesting to find a similar “physical” explanation for the existence of poles in general and understand if there is a criterion for avoiding poles inside the fundamental domain or if both kind of singularities may be present. This would constrain further the classes of allowed superpotentials.

It is very encouraging that the class of non-perturbative superpotentials that we considered are actually suitable for approaching the outstanding problems in string theory which could not be treated perturbatively, namely supersymmetry breaking and lifting the continuous degeneracy of string vacua. Our results on the latter appear to be more interesting than expected, since as we discussed in chapter 4, the naive classical vacuum with vanishing expectation values for the matter fields is not actually a minimum of the potential and is usually unstable. This implies that a standard Higgs mechanism is at work and the energetically preferred vacuum will have a relatively small gauge group (remember that the canonical gauge group obtained in string constructions has rank ≥ 16). For the cases with an originally flat direction, this mechanism will choose one out of a continuum of classical vacua, lifting the original degeneracy. When the scalar field was massless but with non-flat potential, inducing a negative $mass^2$ will indicate that the global minimum corresponds to a value of the field which was not classically allowed i.e. it did not correspond to a conformal field theory. This could lead to interesting phenomenological consequences and may need further study. For $(2, 2)$ orbifold models, the more concrete example of “matter fields” are the blowing-up modes which, by having a non-vanishing v_{ev} , lead to a nice geometrical interpretation where a smooth Calabi-Yau point is preferred over the singular orbifold. Notice that unlike the untwisted fields a negative $mass^2$ for the twisted modes is guaranteed independent of the value of the cosmological constant. That is, even in the cases with vanishing vacuum energy, the potential has a relative maximum in those directions.

Another interesting consequence that can be extracted from this analysis refers to string models with the gauge group coming from Kac-Moody algebras with higher values of the “level” k . The relevance of knowing the level of a given gauge group is because it gives information about the spectrum of the model (higher level implies higher representations allowed) and also on the gauge coupling ($g^2 = g_{string}^2/k$) at the compactification scale. A systematic way of constructing higher level string models was developed in [80]. They are basically obtained from flat directions of a level $k = 1$ model. In this class of models our present analysis would imply that the higher level models (non-vanishing v_{ev}) are preferred over the level-one models. Notice however that many level-one models are not

continuously connected to higher level ones.

As for the breakdown of supersymmetry we have seen that it is very generic and far from trivial that the global minimum breaks supersymmetry. The scale of breaking is actually unknown since it depends mostly on dilaton dependence of the superpotential which we cannot restrict. We have also seen in chapter 2, that there is enough freedom in the superpotential to have a non-supersymmetric minimum with vanishing cosmological constant which is very encouraging. On the other hand there is no criterion based on modular invariance alone to prefer those superpotentials to the ones leading to anti-de Sitter space. A similar comment applies to the question of obtaining a natural hierarchy from a vacuum with large radius which would induce the hierarchy from the exponentially suppressed couplings. There is enough freedom to obtain minima with relatively large values of the radius but the symmetry is not restrictive enough as to decide if they are preferred.

Let us now mention the most important limitations of the present work which may indicate the direction of future generalizations. First, we could not have obtained any constraint on the dilaton dependence of the superpotential due to the fact that S is inert under modular transformations. In [61], we proposed a dilaton duality which could be combined with the $SL(2, \mathbb{Z})$ symmetry of the T field to restrict the form of the superpotential. We preferred not to invoke this duality in the present paper in favour of generality. Still, the assumption that W factorizes in S and T (or the three moduli of chapter two) does not represent the most general case, and it is conceivable that the arbitrary coefficients determining the function H in eq.(2.15) for instance could be actually functions of S or other moduli. A generalization in this direction is beyond the scope of this paper.

Secondly, even though we extended the one-parameter modular group by briefly considering three different moduli, corresponding to an $SL(2, \mathbb{Z})^3$ group, in most cases the full modular group is larger and in many models it appears not even to be related to $SL(2, \mathbb{Z})$, as is the case with some classes of Calabi-Yau manifolds. The automorphic functions and forms for these groups are not as well understood as the ones for $SL(2, \mathbb{Z})$ but probably general results like the existence of singularities and extrema of the potential lying at the boundary of moduli space may also hold for those groups.

Another point which gives room for improvement corresponds to loop corrections to the T-field Kähler potential. We have only worked with the well-known tree-level expression for it as the first non-vanishing term in a weak coupling expansion, whereas we used a non-perturbative expression for the superpotential since it vanishes perturbatively. The

ideal situation would include all perturbative and non-perturbative corrections to both, but unfortunately this seems out of reach with the present understanding of string effective actions. The modular invariance constraints on real functions like K are less restrictive than for holomorphic functions like W . Finally we have to remember that the non-perturbative status of target space modular invariance is not firmly established and is the main assumption of the present paper. It would be extremely interesting to verify if this is indeed a symmetry of the full string theory. (For a recent discussion on related topics see [81], [82].)

All in all we believe that this effective Lagrangian approach based on discrete modular symmetries, $SL(2, \mathbb{Z})$ or otherwise, offers a very effective and elegant way to deal with non-perturbative string effects, and so far it is the only avenue to approach the outstanding problems string theories face to eventually make contact with low-energy physics.

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APPENDIX

Modular Group and Modular Functions

The modular group acting on the complex modulus T is defined as the group of linear fractional transformations

$$T \rightarrow \frac{aT - ib}{icT + d}, \quad a, b, c, d \in \mathbb{Z} \quad ad - bc = 1. \quad (A1)$$

Since changing the sign of all the parameters simultaneously does not change (A1), the relevant group is actually $PSL(2, \mathbb{Z}) = SL(2, \mathbb{Z})/\mathbb{Z}_2$.

The transformations (A1) divide the half complex plane ($\text{Re}T > 0$) into equivalence classes or fundamental domains for which two interior points are not mapped into each other under (A1). The standard fundamental domain ($\mathcal{D}$) is presented in figure 1. The group $PSL(2, \mathbb{Z})$ has fixed points in $\mathcal{D}$, namely $T = 1$, $T = \rho \equiv e^{i\pi/6}$, which are fixed points of elliptic transformations ($|a + d| < 2$, $c \neq 0$) of order two and three respectively and the cusp at ∞ which is a parabolic fixed point ($c = 0$).

Holomorphic functions defined on $\mathcal{D}$ can be studied according to how they transform under $PSL(2, \mathbb{Z})$. A modular form of weight r is a meromorphic function with the transformation property

$$F\left(\frac{aT - ib}{icT + d}\right) = (icT + d)^r F(T). \quad (A2)$$

More generally, the transformation law can include the so-called multiplier system $\exp(i\delta(a, b, c, d))$.

A modular form F is meromorphic in $\mathcal{D}$ in the following sense. At all interior points p F has a Laurent expansion and the order of F at p , denoted ν_p , is defined as the lowest power in this expansion, i.e. ν_p is negative (positive) if F has a pole (zero) at p . At the fixed points F has expansions in terms of uniformizing variables. For instance, at $T \rightarrow \infty$

$$F(T) = q^{\nu_\infty} \sum_{n=0}^{\infty} a_n q^n, \quad q \equiv e^{-2\pi T} \quad (A3)$$

where ν_∞ is the order of F at ∞ . At $T = 1$ the uniformizing variable is $t_1 = \left(\frac{T-1}{T+1}\right)^2$ and

around this point F has an expansion

$$(T+1)F(T) = t_1^{r/2} \sum_{n=0}^{\infty} b_n t_1^n \quad (\text{A4})$$

where ν_1 is the order of F at $T = 1$. At $T = \rho$ there is a similar expansion with leading power $t_\rho^{\nu_\rho/3}$, where $t_\rho = \left(\frac{T-\rho}{T+\rho}\right)^3$. Modular functions with non-trivial multiplier systems can have non-integer orders $\nu_\infty, \nu_1, \nu_\rho$. One example is provided by the Dedekind $\eta(T)$ function (see below). It can be proven that the orders are related to the modular weight r by the useful formula

$$\sum_{p \neq 1, \rho, \infty} \nu_p + \nu_\infty + \frac{1}{2}\nu_1 + \frac{1}{3}\nu_\rho = \frac{r}{12} \quad (\text{A5})$$

This shows that functions of negative dimension must have singularities in the fundamental region.

The space of (holomorphic) modular forms of a given weight is a finite dimensional vector space. A suitable basis is obtained by means of the Eisenstein series

$$G_{2k}(T) = \sum_{n_1, n_2 \in \mathbb{Z}} (in_1 T + n_2)^{-2k} \quad (\text{A6})$$

For $k > 1$ G_{2k} is a modular form of weight $2k$. For $k = 1$ the series does not converge unless the order of summation is specified. In this way one arrives at $G_2(T)$, holomorphic but transforming as a connection $G_2 \rightarrow (icT + d)^2 G_2 - 2\pi ic(icT + d)$. On the other hand

$$\hat{G}_2(T, T^*) = G_2(T) - \frac{\pi}{ReT}$$

transforms as a weight two form but it is not holomorphic.

The Eisenstein functions ($k > 1$) have no poles and their zeroes are at $T = 1$ for k odd and at $T = \rho$ for $k \neq 0 \bmod 3$. $\hat{G}_2$ has no poles and its only zeroes are at the self-dual points $T = 1, \rho$. In general, a modular form of weight r and unit multiplier system vanishes at $T = 1$ if $r \neq 0 \bmod 4$ and at $T = \rho$ if $r \neq 0 \bmod 3$. In particular, the derivative of any modular invariant function (holomorphic or not) vanishes at the self-dual points that are then extrema of the function.

A modular form which vanishes at the cusps is called a cusp form. There are no cusp forms with weight $r < 12$. For $r = 12$ there exists a cusp form

$$\Delta(T) = \frac{675}{256\pi^{12}} [20G_4^3 - 49G_6^2] \quad (\text{A7})$$

The Dedekind function $\eta(T)$ is related to $\Delta(T)$ by

$$\eta(T) = \Delta(T)^{1/24} \quad (\text{A8})$$

The derivative of the Dedekind function satisfies the equation

$$\eta'(T) = -4\pi\eta(T)G_2(T) \quad (\text{A9})$$

The absolute modular invariant $j(T)$ can also be written in terms of the Eisenstein series as

$$j(T) = \frac{3^6 5^3 G_4^3}{\pi^{12} \Delta} \quad (\text{A10})$$

$j(T)$ is a one-to-one and onto function from $\mathcal{D}$ to the entire complex plane. It can be shown that any modular invariant function is a rational function of $j(T)$. More generally, any modular form of weight r (possibly including a non-trivial multiplier system), regular in $\mathcal{D}$, can be written as

$$F(T) = \eta^{2r} \left(\frac{G_6}{\eta^{12}} \right)^m \left(\frac{G_4}{\eta^8} \right)^n P(j) \quad (\text{A11})$$

where $P(j)$ is a polynomial of j and m, n are positive integers.

A "covariant" derivative on weight r modular forms can be defined by

$$D_T F(T) = F'(T) + \frac{r}{2\pi} G_2(T) F(T) \quad (\text{A12})$$

It can be shown that $D_T F$ is a modular form with weight $r + 2$. In particular, for the Eisenstein functions we find ($k > 1$)

$$G'_{2k} + \frac{k}{\pi} G_2 G_{2k} = C(k) G_{2k+2} \quad (\text{A13})$$

where $C(k) = \pi k \zeta(2k) / 3 \zeta(2k + 2)$. For G_2 we instead find

$$G'_2 + \frac{1}{2\pi} G_2 G_2 = \frac{5}{2\pi} G_4 \quad (\text{A14})$$

Using (A9) and (A13) we obtain the useful results

$$\begin{aligned} \left(\frac{G_4}{\eta^8}\right)' &= \frac{7}{\pi} \frac{G_6}{\eta^8} \\ \left(\frac{G_6}{\eta^{12}}\right)' &= \frac{10}{\pi} \frac{G_8}{\eta^{12}} = \frac{30}{7\pi} \frac{G_4}{\eta^{12}} \\ j' &= \frac{21}{\pi} j \frac{G_6}{G_4} \end{aligned} \quad (A15)$$

Since modular forms with unit multiplier system have the periodicity property $F(T + i) = F(T)$ they can be expanded in a Fourier series with parameter $q \equiv \exp(-2\pi T)$. In particular, it has been shown that for $k > 1$

$$G_{2k}(T) = 2\zeta(2k) \left[1 - \frac{4k}{B_{2k}} \sum_{n=1}^{\infty} \sigma_{2k-1}(n) q^n \right] \quad (A16)$$

where ζ is Riemann's Zeta function, B_{2k} are the Bernoulli numbers and $\sigma_p(n)$ is the sum of the p -powers of all divisors of n . The G_2 expansion is instead

$$G_2(T) = \frac{\pi^2}{3} \left[1 - 24 \sum_{n=1}^{\infty} \sigma_1(n) q^n \right] \quad (A17)$$

For reference we now give the first terms in some expansions.

$$\begin{aligned} G_2(T) &= \frac{\pi^2}{3} [1 - 24q - 72q^2 - 96q^3 - 168q^4 - 144q^5 - \dots] \\ G_4(T) &= \frac{\pi^4}{45} [1 + 240q + 2160q^2 + 6720q^3 + 17520q^4 + 30240q^5 + \dots] \\ G_6(T) &= \frac{2\pi^6}{945} [1 - 504q - 16632q^2 - 122976q^3 - 532728q^4 - 1575504q^5 - \dots] \\ j(T) &= \frac{1}{q} [744 + 196884q + 21493760q^2 + 864299970q^3 \\ &\quad + 20245856256q^4 + 333202640600q^5 + \dots] \\ \eta^{-1/24}(T) &= q^{-1/24} [1 + q + 2q^2 + 3q^3 + 5q^4 + 7q^5 + \dots] \end{aligned} \quad (A18)$$

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Figure captions

Fig.1. The standard fundamental domain $\mathcal{D}$ for the T -field. The boundary of the moduli space of the $PSL(2, \mathbb{Z}) \times \mathbb{Z}_2$ transformations is denoted by $\mathcal{B}$ where all minima of the scalar potential are expected to lie.

Fig.2. The scalar potential V as a function of $(\text{Re}T, \text{Im}T)$ for $\mathcal{P} = 1$ and $(m, n) = (0, 0)$. The negative energy minimum lies on the real axis at $T \sim 1.2$ (and at the modular transformed images).

Fig.3. The scalar potential V as a function of $(\text{Re}T, \text{Im}T)$ for $\mathcal{P} = 1$ and $(m, n) = (1, 1)$. The negative energy minima lie on the unit circle $\text{Re}T \sim 0.97$, $\text{Im}T \sim \pm 0.24$.

Fig.4. The scalar potential V as a function of $(\text{Re}T, \text{Im}T)$ for $\mathcal{P} = 1$ and $(m, n) = (0, 3)$. The negative energy minimum is at the self-dual point $T = 1$ and the zero energy minimum is at the self-dual point $T = \rho$.

Figure 2

Figure 1

Figure 3

Figure 4