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Gauge Invariance and CP-odd (un)observables at LEP

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ABSTRACT

We reexamine some CP-odd observables for the processes $e^+e^- \rightarrow f\bar{f}$ and $e^+e^- \rightarrow f\bar{f}\gamma$ for $f=e,\mu,\tau,q$ in light of an effective CP-odd lagrangian which respects the Standard Model symmetry. We find a relationship between some terms of this lagrangian and the well-known dipole moments. Both using this relation and by explicit computation we find that the gauge symmetry systematically induces cancellations which render the observables either difficult or impossible to observe.

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Almost all data gathered until now in particle accelerators seem to confirm the Standard Model of Particle Physics, based on an $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry group spontaneously broken around the 100 GeV scale to an $SU(3)_C \times U(1)_{em}$ group. Above this scale the complete gauge symmetry must be fully manifest. It might be that the symmetry we observe with our humble apparati is nothing but a low energy manifestation of a bigger one. If this is the case, the true theory will, at low energies, be describable by an effective lagrangian in which the heavy particles we cannot see are integrated out. However, the operators of this effective lagrangian, generated by the reader's favourite extension of the Standard Model, must respect $SU(3)_C \times SU(2)_L \times U(1)_Y$ invariance. This has dramatic consequences for LEP as we shall see.

An ideal place to look for New Physics is CP-violating processes in the fermionic sector. As the Standard Model provides us with a completely unobservable signal [1], all the CP violation found there (if any) can be blamed on some New Physics.

Our aim is to study the phenomenological consequences at LEP I of the most general possible CP-violating effective lagrangian. Such a lagrangian has been constructed in [2, 3]. There, the lowest possible dimension CP-violating operators respecting $SU(3)_C \times SU(2)_L \times U(1)_Y$ were written down. Operators of higher dimension were not considered because they are suppressed by powers of the New Physics scale. We recall here the lagrangian [3] after the spontaneous symmetry breaking; restricting ourselves to the neutral current content (which is the relevant one at LEP I):

$$\mathcal{L}_{CP} = \frac{i}{2m_Z} \delta_f^j \bar{\psi}_f \{\sigma_{\mu\nu} \gamma_5\} \psi_f \mathcal{F}_j^{\mu\nu} + \frac{i}{m_Z^2} \bar{\psi}_f \left\{ \gamma_\mu \vec{D}_\nu \right\} (\Sigma_f^j + \Delta_f^j \gamma_5) \psi_f \mathcal{F}_j^{\mu\nu}, \quad (1)$$

where

$$\mathcal{F}_j^{\mu\nu} \equiv \left(e F^{\mu\nu}, \frac{e}{\sin \theta_W \cos \theta_W} Z^{\mu\nu}, g_s G_a^{\mu\nu} \frac{\lambda^a}{2} \right) \quad (2)$$

gathers the ensemble of standard gauge field tensors in a single vector. The covariant derivative, which acts only on the fermionic fields, is

$$\vec{D}^\mu = \vec{\partial}^\mu + i e Q_f A^\mu + i e (v_f - a_f \gamma_5) Z^\mu + i g_s G_a^\mu \frac{\lambda^a}{2}, \quad (3)$$

the Z field tensor is

$$Z^{\mu\nu} = \partial^\mu Z^\nu - \partial^\nu Z^\mu, \quad (4)$$

and we have used the standard notation

$$v_f = \frac{(T_f^3 - 2 Q_f \sin^2 \theta_W)}{2 \sin \theta_W \cos \theta_W}, \quad (5)$$

$$a_f = \frac{T_f^3}{2 \sin \theta_W \cos \theta_W}. \quad (6)$$

The index j in the dimensionless parameters δ_f^j , Δ_f^j and Σ_f^j refers to a photonic, Z-onic or gluonic operator.

The operators in the first term of (1), dipole moments, are descendants of dimension six operators with a Yukawa-like coupling to a scalar doublet Φ needed to render them gauge invariant. This mechanism is independent of the detailed structure of the scalar

field: elementary, effective, multiple, *et cetera*. This scalar acquires a vacuum expectation value, $\Phi \rightarrow v + H_0$, and it is the constant term that generates what would seem by naive dimension counting to be dimension five dipole moments. Thus the first lesson we learn from imposing gauge invariance is that dipole moments are no longer the only CP-violating operators with the lowest possible dimension, sharing this honour (or shall we say heavy burden?) with polyps (so dubbed in [3]), second term in (1). There are two kinds of polyps: axial (proportional to Δ_f^j) and vector (proportional to Σ_f^j).

A similar program was followed by Bernreuther *et al.*[4]. Their lagrangian was not $SU(2)_L$ gauge invariant because they did not want to exclude models where the Z and W bosons are composite objects (possibility that we are not contemplating). However, even in their framework, their lagrangian was not complete. The reason for their missing operators was their inappropriate use of the free field (on-shell) equations of motion for fermions where it was not suitable. Due to these differences, our respective conclusions will be very different.

When the operators in (1) are inserted in a convergent (or at least, regularized) Feynman graph, the use of field equations is allowed. For LEP we are interested in tree-level graphs, so it is valid to use the equations of motion for fermions, to wit:

$$\left\{ \gamma_\mu \vec{D}^\mu + im_f \right\} \psi_f = 0. \quad (7)$$

In this equation, we do not include the effective operators of dimension bigger than four, as their effects are suppressed by higher powers of the New Physics scale. This equation implies the following equalities between terms in the lagrangian :

$$\frac{1}{m_Z^2} \bar{\psi}_f \left\{ \gamma_\mu \vec{D}_\nu \right\} \gamma_5 \psi_f \mathcal{F}_j^{\mu\nu} = \left(\frac{2m_f}{m_Z} \right) \times \frac{1}{2m_Z} \bar{\psi}_f \{ \sigma_{\mu\nu} \gamma_5 \} \psi_f \mathcal{F}_j^{\mu\nu}, \quad (8)$$

$$\frac{1}{m_Z^2} \bar{\psi}_f \left\{ \gamma_\mu \vec{D}_\nu \right\} \psi_f \mathcal{F}_j^{\mu\nu} = 0. \quad (9)$$

This does not mean, as a potential naive reader might think, that a polyp is nothing more than a dipole moment in a bizarre disguise. We stress that (8) and (9) are always true for convergent diagrams, but not for divergent ones. Regularizing the latter *à la* Pauli-Villars with a fermionic regulator mass M , we see that if the counterterm does not vanish in the limit $M \rightarrow \infty$, then anomalous terms not completely unlike the celebrated triangle anomaly of Adler, Jackiw and Bell appear. An example of an anomalous relation of this ilk between the gluonic polyp, the gluonic dipole moment and an effective three-gluon operator induced by the counterterm is given in the appendix of [3]. But returning to tree-level processes, our potential naive reader would be safe; vector polyps vanish and axial polyps can be traded for dipole moments, their coefficients related by

$$\delta_f^j = \frac{2m_f}{m_Z} \Delta_f^j. \quad (10)$$

Equipped with our operators, we now face their phenomenological consequences. The first trial will be the simplest possible process at LEP: $e^+e^- \rightarrow ff$, where $f = e, \mu, \tau$ or a quark. The contributing diagrams are depicted in fig.1a for any final fermion; fig.1b refers only to Bhabha scattering. The heavy dots represent the CP-violating vertices. Had we not used the field equations to demonstrate the equivalence between polyps and dipole moments,

we could try to calculate the amplitude of the process mediated by the polyp. The only term in the operator that can participate here is the one in which the partial derivative is singled out from the covariant derivative, the interchanged boson coming from its strength tensor. An explicit computation of the amplitude in fig.1 shows that, when the heavy dot represents the only relevant term in the polyp, we obtain the same amplitude as when the heavy dot represents the dipole moment, provided their coefficients are related as in (10). This is a check of the result obtained with the field equations.

To lowest order, the leading CP-violating effects arise from the interference between terms in our effective lagrangian and terms in the Standard Model lagrangian. When these effects are calculated for unpolarized fermions in $e^+e^- \rightarrow f\bar{f}$, the result is zero for any CP-odd correlation, because the interference term vanishes. To circumvent this snag, we can either use polarized electron beams, or observe the final fermion polarizations, feasible only for τ leptons, whose decay products are strongly correlated with its spin [5].

The case of polarized beams is studied in [6]. Their most optimistic expectation is a sensitivity to δ_e^γ of order 10^{-3} . But we know from an experiment on ^{250}Tl that $\delta_e^\gamma < 8.9 \times 10^{-11}$ [7]. Although this bound comes from low-energy data, we do not expect dipole moments to increase by so many orders of magnitude between that energy and the LEP energy if the New Physics scale is beyond m_Z . From the bound for δ_e^γ , δ_e^Z must be smaller than 10^{-8} , otherwise quantum loops would drive δ_e^γ too high [3].

The possibility of observing final fermion polarizations is considered in [6, 8]. The conclusion is that by using the decay $\tau \rightarrow \pi\nu_\tau$ and determining the τ -spin from the π -momentum, LEP could be sensitive to $\delta_\tau^\gamma > 3$ [6] and $\delta_\tau^Z > 2 \times 10^{-2}$ [8] at the 2σ confidence level. This last figure comes from an analysis which does not take experimental efficiency into account. Let us estimate how this can affect δ_τ^Z . First of all, due to the geometry of the electromagnetic calorimeter, 30% of the τ s are lost. Secondly, only about 50% of the decays in $\pi\nu_\tau$ of the remaining τ s are reconstructed [9]. Since the sensitivity to any observable is inversely proportional to the square root of the number of events, we have to divide the previous value of δ_τ^Z by the factor $0.5 \times \sqrt{0.7}$. The result is a sensitivity to $\delta_\tau^Z > 5 \times 10^{-2}$. At the same 2σ confidence level, we know from the measurement of the cross section $e^+e^- \rightarrow \tau^+\tau^-$ at Petra that $\delta_\tau^\gamma < 0.46$ [10], and from the measurement of the Z width that $\delta_\tau^Z < 8 \times 10^{-2}$ [3].

To sum up the situation for the process $e^+e^- \rightarrow f\bar{f}$, we have seen that of all possible δ s, the only one that has any chance of being observed is δ_τ^Z (and consequently Δ_τ^Z), provided one looks for τ -polarization. Experimentally, $\delta_\tau^Z < 0.08$, but it can be observed only if $\delta_\tau^Z > 0.05$. This narrow window is expected to be shifted towards smaller values of δ_τ^Z ; on one side, more precise measurements of the Z width will decrease the experimental upper bound on the δ_τ^Z , but on the other side, to consider additional τ -decay channels like ρ or a_1 [11] will increase the experimental sensitivity to δ_τ^Z .

We turn now our attention to the process $e^+e^- \rightarrow f\bar{f}\gamma$. Here we will only consider CP-violating vertices for final fermions other than electrons since the afore-mentioned bounds on the electronic operators ($\delta_e^\gamma < 10^{-11}$, $\delta_e^Z < 10^{-8}$) exclude any observable effect. As before, our use of the field equations allows us to identify the polyps and the dipole moments. But, as before, without using them we can compute the contribution from polyps explicitly as an exercise. On the resonance, the Z channel dominates over the γ channel due to the enhancement factor $\frac{m_Z}{\Gamma_Z}$ of the resonance, unless δ_f^γ is unnaturally larger than δ_f^Z . Because of this, in what follows we will consider only the Z channel. The corresponding diagrams

are shown in fig.2 for the photonic polyp and in fig.3 for the Z-onic one. Figures 2a and 3a represent the terms in which we single out the required boson from the covariant derivative. In 2b and 3b we single out the partial derivative from the covariant one and we insert another boson in the Standard Model way. Here the internal fermion, which propagates from the CP-violating vertex to the Standard Model one, is not on shell. If we cannot use the free field equations, then the operator $\frac{1}{m_Z^2} \bar{\psi}_f \left\{ \gamma_\mu \vec{\partial}_\nu \right\} \gamma_5 \psi_f \mathcal{F}_j^{\mu\nu}$ is independent from $\frac{1}{2m_Z} \bar{\psi}_f \{ \sigma_{\mu\nu} \gamma_5 \} \psi_f \mathcal{F}_j^{\mu\nu}$ and has its own right to appear in the lagrangian. This does not happen in [4], where the authors do not consider the first operator, thus spoiling their alleged claim of completeness of their lagrangian. And this is not trifling; the consequences are far reaching if $SU(2)_L \times U(1)_{em}$ gauge invariance holds as a guiding principle.

The authors of [4] propose two observables and compute their expected values with their lagrangian. In order to calculate the expectation value of any observable, one has to follow three steps. First, to compute the amplitudes both for the effective lagrangian and the Standard Model; second, to make them interfere; and third, to integrate the observable over the phase space, weighted by the interference term. When this is done for the case of fig.2a (which is the only one considered in [4]) one finds no chiral suppression because no individual term of the polyp changes chirality. Consequently the expected values obtained in [4] fall within the realm of observability.

Let us consider what happens when fig.2b is also included. First, we find that the amplitude for fig.2b is such that when we impose gauge invariance, relating the until-then independent figs 2a and 2b, the leading (chirally unsuppressed) contributions from these figures exactly cancel out. What is left is proportional to $(\frac{2m_f}{m_Z}) \Delta_f^\gamma$. Moreover, we find that the interference of fig.2 with the Standard Model amplitude (shown in fig. 4) is zero. Therefore the contribution of this operator to any conceivable observable is zero¹. Now let us consider the Z-onic polyp, which was not contemplated in [4]. In this case we find as before that the amplitude is proportional to $(\frac{2m_f}{m_Z}) \Delta_f^Z$ and also to the amplitude corresponding to the Z-onic dipole. Unlike last time, the interference of this operator with the Standard Model does not vanish. Instead, it picks up another suppression factor $(\frac{m_f}{m_Z})$ because dipole moments flip chirality (which is faithfully preserved by the Standard Model vertices) yielding an overall factor $(\frac{m_f}{m_Z})^2$. It follows that this perverse factor will propagate to any observable, making it difficult to observe.

Let us see what happens when we examine the two observables suggested in [4]. For their tensor observable, we find that when we perform the integration over the phase space, the result is identically zero. For the other observable, the asymmetry in the distribution of the cross-product of the final fermion trimomenta, the result of fig. 3a and 3b for the Z-onic polyp is automatically obtained when one computes fig 3b but with the heavy dot representing the Z-onic dipole moment, because as we have said before, at tree level both operators are interchangeable with the replacement (10). We have calculated the expression for this asymmetry before integration over phase space and we agree with [4]. Their Monte-Carlo simulations indicate that for light fermions LEP is not sensitive enough to their CP-violating interactions, and for heavy fermions (τ, c and b) any hope of observing them is ruled out by the bounds that can be set on them from the observed Z width.

¹If one considers also the photon-mediated process, the situation is analogous to the one we will find for the Z-onic polyp. To see this analogy it is enough to replace the Z in fig.3 by a photon.

More possibilities can be considered, but none of them will change our conclusions. For example, τ -polarization in $\tau^+\tau^-\gamma$ would be further suppressed by an extra α_{em} factor compared with $\tau^+\tau^-$, making the signal unobservable. For $e^+e^- \rightarrow q\bar{q}g \rightarrow 3 \text{ jets}$ the situation is analogous to $e^+e^- \rightarrow f\bar{f}\gamma$, what is not zero is too small to be detected.

To conclude, the only CP-violating operators that could be espied at LEP are the Z-onic dipole moment of the τ and its partner the Z-onic polyp of the τ , and this only if τ -polarization is looked for. However this possibility is seriously jeopardized by the current bounds on the coefficients of both operators.

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Figure captions

Fig.1: CP-violating diagrams contributing to $e^+e^- \rightarrow f\bar{f}$. [a] applies to any final fermion and [b] to Bhabha scattering.

Fig.2: Photonic polyp (heavy dot) contributions to $e^+e^- \rightarrow f\bar{f}\gamma$.

Fig.3: Z-onic polyp (heavy dot) contributions to $e^+e^- \rightarrow f\bar{f}\gamma$.

Fig.4: Standard Model contributions to $e^+e^- \rightarrow f\bar{f}\gamma$.

Fig. 1

Fig. 2

Fig. 3

Fig. 4