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Experimental Overview of Jet Physics and Tests of QCD

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ABSTRACT

Latest e^+e^- -annihilation and $p\bar{p}$ collider results on jet physics and tests of QCD are reviewed. Within the framework of QCD shower models, hadronic event shapes and their energy evolution which are observed in e^+e^- annihilation are well described by QCD scaling violations plus an energy independent parametrisation of hadronisation. The energy dependence of jet production rates, observed in e^+e^- annihilation at $\sqrt{s} = 29$ to 91 GeV, provides evidence for asymptotic freedom. Angular correlations within 4-jet final states of Z^0 decays further confirm the nonabelian nature of the strong interaction. A compilation of α_s measurements at LEP and SLC results in $\alpha_s(M_{Z^0}) = 0.118 \pm 0.008$ in $O(\alpha_s^2)$, where the error is dominated by theoretical uncertainties. Jet production at the $p\bar{p}$ colliders provides significant tests of leading and next-to-leading order QCD calculations.

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1. Introduction

Studies of jet production in hadronic final states of high energy $p\bar{p}$ collisions and e^+e^- annihilations have proven to be a significant testing ground for Quantum Chromodynamics (QCD), the nonabelian gauge theory of the strong interactions between quarks and gluons [1]. One of the basic ingredients of QCD is the principle of “asymptotic freedom”. It determines that the QCD coupling strength, α_s , decreases with increasing momentum transfer, such that the production of quarks and gluons in high energy, deep inelastic scattering processes can be calculated and described by means of perturbation theory. However, at larger distances or, equivalently, in regions of smaller momentum transfers, the outgoing quarks and gluons are “confined” in a colour force field of increasing strength. They are forced to dress themselves up into (more or less collimated) jets of colourless hadrons. Although it is not possible to describe the hadronisation process by QCD perturbation theory, hadron jets are believed to maintain the kinematic properties of the underlying quarks and gluons. Experimental studies of jets in high energy elementary particle reactions are therefore ideal tools to test the basic features and predictions of perturbative QCD calculations, and to determine the free parameters of the theory.

The aim of this article is to review the results of experimental jet studies in high energy e^+e^- and $p\bar{p}$ collisions. In view of the limited time and space available for this review, especially within a workshop which is devoted to the discussion of future projects and analyses, special emphasis will be given to the most recent results obtained from the e^+e^- colliders SLC at the Stanford Linear Accelerator Center (SLAC) and LEP at the European Organisation for Nuclear Research (CERN), and from the $p\bar{p}$ colliders *Tevatron* at Fermilab and *SPPS* at CERN. As an overview and reference to earlier results, a brief history of jet physics is given in chapter 2. Chapters 3 to 6 are devoted to the most recent results from hadronic decays of the Z^0 boson in e^+e^- annihilation, namely, studies of global hadronic event shapes (chapter 3), studies of jet production rates (chapter 4), a compilation of α_s -measurements (chapter 5) and tests of the gluon self coupling (chapter 6). In chapter 7, the most significant results from jets at the $p\bar{p}$ colliders are summarised. Chapter 8 completes this review with a summary and discussion of the results.

2. A Brief History of Jet Physics.

The first “Evidence for Jet Structure in Hadron Production by e^+e^- Annihilation” was reported back in the year 1975 [2]. The data, taken with the SLAC-LBL magnetic detector at the SPEAR storage ring, showed increasingly two-jet like event structures when the centre of mass energy, E_{cm} , was raised from 3 to 7.4 GeV. The jet structure manifested itself in a decrease of the mean sphericity, a measure of the global shape of hadronic events (ideal back-to-back two-jet events have sphericity values S of 0, while spherical events have $S = 1$). The jet axis angular distribution from the same

measurement provided evidence that the underlying partons must have spin $\frac{1}{2}$. These observations, which were further corroborated by similar measurements at $E_{cm} = 14$ to 34 GeV at the e^+e^- storage ring PETRA (at DESY, Hamburg) [3], confirmed the basic ideas of the quark-parton model, which relates the constituents of the static quark model with partons produced in deep inelastic scattering experiments.

In 1979 a small fraction of planar, well separated 3-jet events were observed by the PETRA experiments around $E_{cm} \approx 30$ GeV [4], which could convincingly be attributed to the emission of a third parton with zero electric charge and spin 1 [5], exactly as expected for the gluon bremsstrahlung process predicted by QCD. It is interesting to note that the first evidence for 3-jet like events came from visual scans of the energy flow within hadronic events, followed by more detailed statistical analyses of global event shapes like oblateness, planarity and others. At the end of 1979 the MARK-J collaboration reported the first measurement of α_s , in first order perturbative QCD, from the shape of the oblateness distribution [6]. The first analysis based on the definition and reconstruction of resolvable jets was published in 1980 by the PLUTO collaboration [7].

In 1981, the JADE collaboration found first evidence [8] that the string hadronisation model of the Lund group [9] provides a better description of the observed hadron flow in 3-jet events than does an independent jet hadronisation model [10]. Although it took several years until this observation was confirmed by other experiments [11], the string picture is nowadays the most commonly used hadronisation model.

The year 1982 brought first evidence for 4-jet like events, observed by JADE [12] at $E_{cm} = 33$ GeV, and the first determination of α_s in second order perturbation theory ($O(\alpha_s^2)$), again by JADE [13]. In 1982 it was also shown by UA2 that events with large transverse energies (E_t) in $p\bar{p}$ collisions predominantly exhibit 2-jet structures [14].

Many more detailed QCD studies in e^+e^- annihilation at PETRA, PEP (at SLAC) and TRISTAN (at KEK, Japan) emerged in the years to come, of which only a few of the pioneering results which are still of importance for our present-day studies shall be mentioned here. In 1986 JADE published the first detailed analysis of n -jet event production rates [15], introducing a jet finding mechanism which has since then been used in many other studies (see chapter 4). Interesting to note that the ratio of 4-jet over 3-jet event production rates was found to be significantly larger than predicted by $O(\alpha_s^2)$ QCD, an observation that motivated further studies of the influence of the choice of renormalisation scales in finite order perturbative QCD [16,17,18]. In 1988 it was demonstrated by JADE that the energy dependence of 3-jet event production rates gives evidence for the running of α_s [19]. First signs of the presence of the gluon self coupling have been observed in a study of 4-jet events by AMY [20] around $E_{cm} = 56$ GeV. More comprehensive reviews about jet physics in e^+e^- annihilations below the Z^0 resonance can be found, for example, in [21] (1984),

Fig. 1. Decay of a Z^0 into 4 jets of hadrons, as observed with the OPAL detector at LEP.

[22,23] (1987) and in [24] (1989).

The first significant determination of α_s in next-to-leading order QCD from $p\bar{p}$ collisions was reported by the UA2 collaboration in 1988 [25], obtained from W plus jet production. Inclusive jet cross sections at the highest $p\bar{p}$ collider energies of 1.8 TeV, measured with the CDF detector at Fermilab [26] and extending over more than 5 orders of magnitude, were published in 1989.

Finally, in 1989 the SLC and LEP came into operation. The large, resonant cross section of the reaction $e^+e^- \rightarrow Z^0 \rightarrow q\bar{q}$, which is factors of 100 to 400 larger than of $e^+e^- \rightarrow \gamma^* \rightarrow q\bar{q}$ at PETRA, PEP and TRISTAN, the higher energies and reduced hadronisation effects if compared to the lower energy machines, and the clean and background free hadronic event samples allow more detailed and precise studies to be made of jet physics and QCD in general. An example of an event of the type $e^+e^- \rightarrow Z^0 \rightarrow 4 \text{ jets}$, observed by the OPAL collaboration at LEP, is shown in Figure 1 (two of the jets contain a high energetic muon penetrating the outer detector layers, possibly indicating the semileptonic decays of heavy (charm or bottom) quarks). The QCD related studies published by the SLC and LEP experiments so far are summarised in Table 1. Results from studies of global event shapes, jet production rates, α_s determinations and the gluon self coupling will be further discussed in chapters 3 to

Topics	MARK2	ALEPH	DELPHI	L3	OPAL
Particle incl. distr.'s	⊗	⊗	⊗ ⊙	⊗	⊗
Intermittency			⊗ ⊙		
Global event shapes	⊗	⊗	⊗		⊗
Soft gluon Coherence				⊗	⊗
Jet production rates			⊗	⊗	⊗ ⊗
α_s from: jet rates	⊗	⊗	⊗	⊗	⊗ ⊙
event shapes		⊗			⊙
energy correl.'s		⊗	⊗	⊗	⊗
Gluon self coupling			⊗	⊗	⊗

Table 1. Topics of QCD related analyses published (⊗) by the experiments at SLC (i.e. MARK2) and LEP; ⊙ stands for second party analyses based on that experiment's data.

6, respectively. A summary of experimental and theoretical examinations of gluon coherence effects can be found in [27].

3. Global Event Shapes of Hadronic Z^0 Decays.

Global event shapes of hadronic final states of e^+e^- annihilation are largely determined by the jet-structure of the events and the relative proportions of multi-jet events in the event sample. The comparison of measured event shape distributions with QCD plus hadronisation models thus provides important information both on QCD (e.g. the value of α_s) and on the details of modelling the hadronisation process. While these are interesting studies in their own right, models with carefully tuned QCD- and hadronisation parameters are also indispensable for general studies of detector acceptance and resolutions, and also for searches for new phenomena where the physics of QCD and hadronisation is just a background that must be well understood.

Global event shapes of hadronic Z^0 decays have been studied by MARK2 [28], ALEPH [29], DELPHI [30] and OPAL [31]. MARK2, DELPHI and ALEPH compare their data with models using parameters tuned to lower energy data, while OPAL optimized the most important model parameters to their data (ALEPH prepares a second study with model parameters tuned to the Z^0 data, too). Models studied in these comparisons are the Jetset QCD shower plus string hadronisation model

Fig. 2. The Thrust distribution of hadronic Z^0 decays measured by DELPHI [30] compared to models with parameters tuned at lower c.m. energies (a); and the Aplanarity distribution measured by OPAL, compared to models with parameters optimised to the Z^0 data in distributions which do not explicitly constrain this observable [31]; the difference between data and models in χ^2 is shown in the small insert (b).

[32], the Herwig QCD shower plus cluster hadronisation model [33], the Jetset $O(\alpha_s^2)$ QCD model plus string hadronisation [32] and, in the case of OPAL, the Ariadne QCD colour dipole plus string hadronisation model [34]. QCD shower models are based on the development of quark and gluon cascades down to invariant parton masses of about 1 GeV, calculated in leading log approximations (LLA) of perturbative QCD. At Z^0 energies, these models result in partonic final states of typically 10 quarks and gluons. The main event characteristics are largely determined by the parton cascade and the available phase space for hadronisation is relatively small. $O(\alpha_s^2)$ QCD models are based on next-to-leading order calculations, which predict the production of up to 4-parton final states of massless quarks and gluons; hadronisation sets in at much larger energy scales (typically 10 GeV at Z^0 energies).

Event shape parameters studied are Thrust T , Aplanarity A and others which

Fig. 3. The Thrust distribution measured at $E_{cm} = 91, 35$ and 29 GeV, compared to the Jetset QCD shower model with parameters tuned at $E_{cm} = 91$ GeV [31].

all are sensitive to the difference between spherical, multijet like events ($T \rightarrow 0.5$, $A \rightarrow 0.5$) or planar, 3-jet like events ($T \approx \frac{2}{3}$, $A \approx 0$) and back-to-back two-jet like events ($T = 1$, $A = 0$); see e.g. [31] for further definitions. The T distribution, measured by DELPHI, and the A distribution, measured by OPAL, are shown in Figures 2a and 2b together with the various model calculations. From Fig. 2a and similar measurements one sees that the different models, with parameters as tuned at lower energies, describe the data at the higher energies almost equally well. $O(\alpha_s^2)$ models, however, fail to reproduce distributions which measure the momentum flow out of the event plane, as seen in Fig. 2b for the model labelled “ERT-E0” (specifying the $O(\alpha_s^2)$ QCD matrix element used [35]) - apparently even if the parameters are retuned to give the best overall description of data and if an optimised renormalisation scale is used in the QCD calculation [31], see also chapter 4.

OPAL compared their data and the models optimised at $E_{cm} = 91$ GeV with data from lower energies, as shown in Figure 3 for the Thrust distribution and the Jetset parton shower model. From such studies one concludes that parton shower models, with unchanged values of the QCD and hadronisation parameters, provide a good description of the general features of hadronic events in a large range of centre of

mass energies. The energy dependence of data distributions can be explained, within these models, by scaling violations of QCD (i.e. the running of α_s) plus an *energy independent* parametrisation of hadronisation. In contrast, $O(\alpha_s^2)$ QCD models - where the region of phase space which is described by hadronisation increases with increasing c.m. energy - need retuning of the hadronisation parameters at each energy and provide a less complete description of the higher energy data.

4. Studies of Jet Production Rates at $E_{cm} = 91$ GeV.

4.1 Experimental and Theoretical Definition of Jets.

In $O(\alpha_s^2)$ QCD calculations, jet production rates in e^+e^- annihilations are predicted to be quadratic functions of $\alpha_s(\mu)$:

$$\begin{aligned} R_2 &\equiv \frac{\sigma_2}{\sigma_{tot}} = 1 + C_{2,1}(y_{cut}) \cdot \alpha_s(\mu) + C_{2,2}(y_{cut}, f) \cdot \alpha_s^2(\mu) \\ R_3 &\equiv \frac{\sigma_3}{\sigma_{tot}} = C_{3,1}(y_{cut}) \cdot \alpha_s(\mu) + C_{3,2}(y_{cut}, f) \cdot \alpha_s^2(\mu) \\ R_4 &\equiv \frac{\sigma_4}{\sigma_{tot}} = C_{4,2}(y_{cut}) \cdot \alpha_s^2(\mu), \end{aligned} \quad (1)$$

where σ_{tot} is the total hadronic cross section, σ_n are the cross sections for n -parton event production, μ is the renormalisation scale at which α_s is evaluated and $f = \mu^2/E_{cm}^2$ is the renormalisation scale factor. The k^{th} order QCD coefficients for n -jet production, $C_{n,k}$, depend on the jet resolution parameter y_{cut} ; in addition, the next-to-leading order coefficients $C_{2,2}$ and $C_{3,2}$ are recombination scheme dependent and exhibit an explicit dependence on the renormalisation scale factor f . The coupling constant $\alpha_s(\mu)$ can be written as a function of $\ln(\mu^2/\Lambda_{\overline{MS}}^2)$ [36], where $\Lambda_{\overline{MS}}$ is the QCD scale parameter which must be determined by experiment:

$$\alpha_s(\mu) = \frac{12\pi}{(33 - 2 \cdot N_f) \cdot \ln(\frac{\mu}{\Lambda_{\overline{MS}}})^2} \cdot \left(1 - 6 \cdot \frac{153 - 19 \cdot N_f}{(33 - 2 \cdot N_f)^2} \cdot \frac{\ln(\ln(\frac{\mu}{\Lambda_{\overline{MS}}})^2)}{\ln(\frac{\mu}{\Lambda_{\overline{MS}}})^2} \right), \quad (2)$$

with the number of active quark flavours N_f equal to 5.

The most commonly used algorithm to define and reconstruct jets of hadrons was introduced by the JADE collaboration [15,19]: the scaled pair mass of two resolvable jets i and j , $y_{ij} = M_{ij}^2/E_{vis}^2$, is required to exceed a threshold value y_{cut} ; E_{vis} is the total visible energy of the event. Jets with $y_{ij} < y_{cut}$ are combined into a single jet k . While this definition provides a means to compare experimental jet rates to the theoretical predictions, it also introduces ambiguities which are commonly called “recombination scheme uncertainties”: the results of the calculations depend on the detailed prescription to recombine two unresolvable jets into a single jet. The ambiguity is of theoretical nature: $O(\alpha_s^2)$ QCD calculations are carried out for

Scheme	M_{ij}^2	recombination	remarks
E	$(p_i + p_j)^2$	$p_k = p_i + p_j$	Lorentz invariant; exact
E0	$(p_i + p_j)^2$	$E_k = E_i + E_j;$ $\vec{p}_k = \frac{E_k}{ \vec{p}_i + \vec{p}_j }(\vec{p}_i + \vec{p}_j)$	conserves $\sum E$, but violates $\sum \vec{p}$
JADE	$2E_i E_j (1 - \cos\theta_{ij})$	$p_k = p_i + p_j$	conserves $\sum E, \sum \vec{p}$
p	$(p_i + p_j)^2$	$\vec{p}_k = \vec{p}_i + \vec{p}_j;$ $E_k = \vec{p}_k $	conserves $\sum \vec{p}$, but violates $\sum E$
p0	$(p_i + p_j)^2$	$\vec{p}_k = \vec{p}_i + \vec{p}_j;$ $E_k = \vec{p}_k $	as p-scheme, but E_{vis} up- dated after each recomb.

Table 2. Definition of recombination schemes ($\vec{p}_i$ denotes a 3-vector and $p_i \equiv (E_i, \vec{p}_i)$ is the corresponding 4-vector .

massless partons only; however when the four-vectors of two jets (or partons) are combined, the resulting jet is not massless. A number of recombination schemes have been introduced, which differ in the definition of the pair mass M_{ij}^2 and in the prescription of how to combine two unresolvable jets, as outlined in Table 2 (see also [37,38]).

Calculations of the coefficients $C_{n,k}$ in complete $O(\alpha_s^2)$ perturbation theory are available from Kramer and Lampe [39,16] and from Kunszt and Nason [38]; the latter are based on the work of Ellis, Ross and Terrano [35] and are available for the E, E0, p and p0 schemes (in $O(\alpha_s^2)$ the E0 and the original JADE scheme are equivalent).

4.2 Experimental Jet Rates and Comparison with QCD.

All experiments at the SLC and at LEP have studied jet production rates of hadronic Z^0 decays [40,41,42,43,37,44] using the JADE algorithm; OPAL, DELPHI and L3 have published the measured jet rates in detail, and OPAL has also studied jet production in the E-, p0- and p-schemes [37]. Figure 4 shows the jet rates, measured by OPAL and corrected for acceptance and resolution of the detector, as a function of y_{cut} for the four different recombination schemes. The absolute jet rates differ by up to a factor of two between the different schemes; the E-scheme yields the largest and

Fig. 4. Measured production rates of n -jet events (from OPAL), compared to QCD shower model calculations before and after the hadronisation process.

the p -scheme yields the lowest rates of multijet events for identical values of y_{cut} . The corresponding jet rates predicted by the Jetset QCD shower model, calculated both from partons at the end of the QCD shower and from particles after hadronisation, are also shown in Fig. 4. It can be seen that hadronisation corrections are large for the E -scheme, moderate for the p - and the $p0$ -scheme and almost negligible for the $E0$ -scheme. After hadronisation, however, the model describes the data well in all schemes, which indicates that it can be used to correct the data for hadronisation effects, e.g. in order to determine α_s (see chapter 5).

Measured jet production rates are also compared with models based on $O(\alpha_s^2)$ matrix elements plus string hadronisation, as shown in Fig. 5 for the ($E0$ -scheme) data of L3 [43]. The small, overall hadronisation effects for jets defined in the $E0$ -scheme (see Fig. 4) even allow to compare data directly with $O(\alpha_s^2)$ analytic calculations, as was previously done e.g. by OPAL [40], see Fig. 6. In both these cases, however, a good description of the data, especially in regions of $y_{\text{cut}} < 0.05$, requires to use rather small renormalisation scales $\mu \ll E_{\text{cm}}$ [40,42,43]. The influence of the choice of the renormalisation scale and of the recombination scheme on determinations of

Fig. 5. Jet production rates from L3, compared to $O(\alpha_s^2)$ plus string hadronisation model calculations.

Theory	C_F	N_C	T_R
QCD	4/3	3	$N_f/2$
Abelian	1	0	$3 \cdot N_f$

Table 3. Group constants of QCD and of the abelian vector gluon theory.

α_s is further discussed in chapter 5.

4.3 Jet Production Rates as a Test of the Nonabelian Nature of QCD.

The nonabelian nature of QCD manifests itself in the process of gluon self coupling and in the specific energy dependence of α_s (i.e. asymptotic freedom). This leads to theoretical predictions of the relative n-jet production rates R_n that vary as a function of y_{cut} and of E_{cm} . In order to demonstrate these effects, the respective data distributions are also compared with an alternative, *abelian* gauge theory of the strong interaction which does not contain the process of gluon self coupling. Such a model can be obtained, in second order perturbation theory, from the corresponding $O(\alpha_s^2)$ QCD calculations by replacing the group constants of SU(3) with the ones of U(1), according to Table 3 [45].

The (E0-scheme) jet rates measured by OPAL as a function of y_{cut} [40] are com-

Fig. 6: Jet production rates from OPAL ($E_{cm} = 91$ GeV), compared with analytic second order QCD and abelian vector model calculations. The QCD parameters come from adjustments to the 29 GeV data shown in Fig. 7.

Fig. 7: Relative production rates of 2-, 3-, 4- and 5-jet events as a function of y_{cut} (E0-scheme), observed at e^+e^- centre of mass energies around 91 GeV and 29 GeV.

pared to second order analytic QCD calculations in Figure 6, for two different choices of μ , and to the abelian vector theory. The QCD parameters $\Lambda_{\overline{MS}}$ and μ are taken from a fit to the MARK2 data at $E_{cm} = 29$ GeV [18,46]; the coupling constant α_A and the corresponding scale μ_A for the abelian model were adjusted to describe the data at $y_{cut} = 0.04$. Since hadronisation effects are small for the E0-scheme and nothing is known about a hypothetical abelian hadronisation process, no hadronisation corrections are applied to the data. While QCD describes the data well, the abelian model cannot reproduce the observed y_{cut} dependence of the data.

A more direct test of the nonabelian nature of QCD is the energy dependence of jet production rates within a given recombination scheme and for given values of y_{cut} . Such a comparison is shown in Fig. 7 for the jet production rates observed by MARK2 at $E_{cm} = 29$ GeV and by OPAL around $E_{cm} = 91$ GeV, for jets defined in the E0 scheme. Significant scaling violations can be observed: for values of $y_{cut} \geq 0.03$, the ratio of 3-jet rates is

$$\frac{R_3(91 \text{ GeV})}{R_3(29 \text{ GeV})} = 0.77 \pm 0.03 \text{ (stat.)},$$

which in leading order QCD should be equal to $\alpha_s(91\text{GeV})/\alpha_s(29\text{GeV})$.

The latter relation, however, only holds if hadronisation effects do not produce an energy dependence of jet rates, too. From previous studies it is known that this is true, to a good approximation, for jets defined in the E0 scheme [19,46,40,37]. This

Fig. 8. The ratio r of 3-jet event rates, calculated from Jetset QCD shower model events before and after hadronisation, for four recombination schemes and at $y_{\text{cut}} = 0.08$, as a function of E_{cm} .

is demonstrated in Fig. 8, where from a model study with the Jetset QCD shower model the ratio r of 3-jet rates, analysed after hadronisation, and 3-jet rates derived from partons at the end of QCD shower are plotted as a function of the e^+e^- centre of mass energy¹, for the recombination schemes discussed above. As can be seen, only the E0 scheme provides an almost constant ratio r for E_{cm} above about 25 GeV; it gives an overall hadronisation correction of about 4% to 8% in the region between $E_{\text{cm}} = 25$ to 100 GeV. This result is largely independent of the actual choice of model parameters, and identical results are also obtained if e.g. the Herwig shower model is employed.

Thus, within a hadronisation uncertainty of about $\pm 2\%$, the energy dependence of jet production rates in the E0 scheme, if y_{cut} is kept constant, is strictly determined by the energy dependence of α_s (see Eq. 1). The energy dependence of measured jet production rates is therefore an ideal tool to test the energy dependence of α_s , without the need to actually determine α_s itself. The first analysis in this sense was done by JADE [19].

A compilation of the experimental results of R_3 , analysed with the JADE (E0) jet finder at different E_{cm} using $y_{\text{cut}} = 0.08$, is presented in Fig. 9. Besides the data from PETRA [19,47], PEP [46] and TRISTAN [20,48], all LEP experiments now contribute [37,42,43,44] at $E_{\text{cm}} \approx 91$ GeV. The data are compared with analytic $O(\alpha_s^2)$ QCD

¹The model parameters were kept constant for all E_{cm} , which is justified by the observation that this provides a good description of global event shapes measured in this energy range (see [31] and the discussion in chapter 3).

Fig. 9. Energy dependence of three-jet event production rates R_3 ($y_{\text{cut}} = 0.08$), compared with predictions of analytic $O(\alpha_s^2)$ QCD calculations, with the hypothesis of an energy independent α_s and with the abelian vector theory in $O(\alpha_A^2)$.

Theory	fit result	$\chi^2/\text{d.o.f.}$
QCD, $f = 0.0017$	$\Lambda_{\overline{\text{MS}}} = (111 \pm 5)\text{MeV}$	7.8 / 12
QCD, $f = 1.0$	$\Lambda_{\overline{\text{MS}}} = (255 \pm 11)\text{MeV}$	6.6 / 12
$\alpha_s = \text{const.}$	$\langle R_3 \rangle = 20.3 \pm 0.3$	83.2 / 12
Abelian theory	$\alpha_A(44\text{GeV}) = 0.26$	****

Table 4. Fit results for various assumptions about the energy dependence of 3-jet production rates, obtained for the data between $E_{\text{cm}} = 29$ GeV and 91 GeV shown in Fig. 9 (an additional relative systematic point-to-point error of 2% was included in the fit).

calculations [39,16], using two different renormalisation scale factors $f = 1$ and $f = 0.0017$, and with the hypothesis of an energy independent coupling constant (which is however not predicted by any gauge theory). Also shown are the predictions of the abelian vector theory in $O(\alpha_A^2)$, where α_A was adjusted such that the jet rates at $E_{cm} = 44$ GeV are reproduced. The energy dependence of α_A is then predicted by an analytic solution of the renormalisation group equation in $O(\alpha_A^2)$ [49]. For the QCD predictions and the hypothesis of $\alpha_s = \text{constant}$, the free parameters $\Lambda_{\overline{MS}}$ and a constant 3-jet rate $\langle R_3 \rangle$, respectively, were determined by minimising χ^2 for the data from $E_{cm} = 29$ to 91 GeV. The data points at $E_{cm} = 22$ GeV were not included in the fit since hadronisation effects may already bias the measurements at this energy, see Fig. 8. The results for these parameters and the corresponding values of χ^2 are listed in Table 4. In order to account for the small, energy dependent hadronisation effects as predicted by the model calculations shown in Fig. 8, a relative systematic point-to-point uncertainty of $\pm 2\%$ was included in the calculation of χ^2 .

From Fig. 9 and from Table 4 it is evident that the measured jet production rates significantly decrease with increasing energy, excluding the possibility of an energy independent coupling with a significance of 8 standard deviations (3.1 standard deviations without the LEP data). The abelian theory with an increasing coupling strength is completely ruled out; the differences between data and the abelian theory are too large to be explained by any reasonable assumption about a hypothetical abelian hadronisation model. The $O(\alpha_s^2)$ QCD calculations describe well the energy dependence of R_3 , and thus of α_s , seen in the data. The predicted degree of energy dependence is largely insensitive to the renormalisation scale chosen (however the actual value of $\Lambda_{\overline{MS}}$ and of α_s do strongly depend on the scale, see chapter 5). Thus the data provide convincing evidence for the “running” of α_s .

Another way to demonstrate that jet production rates are in good agreement with asymptotic freedom, is to plot R_3 as a function of $1/\ln(E_{cm})$, as shown in Fig. 10. The dashed line drawn in Fig. 10 indicates the leading order QCD prediction, namely

$$R_3 \propto \alpha_s \propto \frac{1}{\ln E_{cm}}$$

or, equivalently, Asymptotic Freedom. The corresponding prediction in $O(\alpha_s^2)$ is also shown, indicating that higher order terms affect the energy dependence of R_3 only slightly. At infinite energy ($1/\ln(E_{cm}) \rightarrow 0$) R_3 and α_s are expected to vanish; an assumption which apparently is in good agreement with the data.

5. A Summary of α_s Measurements at LEP and SLC.

The determination of α_s has always been one of the key analyses of hadronic final states in e^+e^- annihilation. Comprehensive summaries of α_s measurements before the era of LEP and SLC can be found e.g. in [22,23] (1987) and in [24] (1989). Based

Fig. 10. The same data as shown in Fig. 9, as a function of $1/\ln(E_{cm})$, compared to the prediction of Asymptotic Freedom.

in part on the experience gained in this field from PETRA, PEP and TRISTAN, the LEP and SLC experiments were able to contribute many significant and precise determinations of α_s within a rather short time; see Table 1 for a list of measurements. Most of these new results are very precise due to the large event samples available at LEP, but also in terms of detailed studies of systematic, both experimental and theoretical uncertainties.

5.1. Determination of $\alpha_s(M_{Z^0})$ from the Hadronic Partial Width of the Z^0 .

A clean way to determine $\alpha_s(M_{Z^0})$ is from a measurement of

$$\delta_{QCD} = \left(\frac{\Gamma_{had}}{\Gamma_{lept}} \right)_{exp.} \cdot \left(\frac{\Gamma_{lept}}{\Gamma_{had}^0} \right)_{theo.} - 1,$$

where Γ_{had} and Γ_{lept} are the hadronic and leptonic partial widths of the Z^0 , and Γ_{had}^0 is the hadronic width without QCD corrections. The QCD correction δ_{QCD} , which is only about 5%, is precisely predicted in $O(\alpha_s^2)$ QCD, and no hadronisation uncertainties are expected to influence the measurement. However, the precision of α_s from a measurement of $R \equiv (\Gamma_{had}/\Gamma_{lept})$ is given, to a good approximation, by

$$\frac{\Delta\alpha_s}{\alpha_s} \approx \frac{\Delta R}{R}.$$

The latest compilation of LEP data on R indicate an overall error of $\Delta R/R = 1\%$ [50], and thus the value of α_s determined from R is [51]

$$\alpha_s(M_{Z^0}) = 0.16 \pm 0.03.$$

Within its rather large uncertainty, this value is compatible with other measurements of $\alpha_s(M_{Z^0})$ which are summarised in the next chapters.

It should be noted that at the time where this report is written (February 1991), newer but still not official results of $\alpha_s(M_{Z^0})$ from measurements of the Z^0 lineshape and of lepton pair asymmetries and based on higher event statistics, result in smaller uncertainties ($\Delta\alpha_s \approx 0.025$ from one experiment only) while the value of $\alpha_s(M_{Z^0})$ tends to decrease to about 0.14. Updates of these analyses are expected to be published soon.

5.2. Determinations of $\alpha_s(M_{Z^0})$ from Jet Production Rates.

Determination of $\alpha_s(M_{Z^0})$ from jet production rates, based on the JADE jet algorithm, are provided by all experiments [41,40,42,43,37,44]. Except for the analysis of L3 [43], where α_s is determined from jet rates measured at a fixed value of y_{cut} , the experiments apply a method which was first presented by OPAL [40] and MARK2 [41]: the QCD parameter $\Lambda_{\overline{MS}}$ (and sometimes μ) are determined from the measured, differential distribution $D_2(y)$, which is defined by ($y \equiv y_{\text{cut}}$)

$$D_2(y) = \frac{R_2(y) - R_2(y - \Delta y)}{\Delta y}. \quad (3)$$

It measures the distribution of y_{cut} values for which the classification of events changes from 3-jet to a 2-jet configuration.

The experiments differ in many details of the analyses. For instance, the analyses are based on either charged particles or electromagnetic energy clusters alone, or on both charged and neutral particles. While the data are always corrected for detector acceptance and resolution, hadronisation effects are applied either to the data or to the $O(\alpha_s^2)$ QCD calculations. In most cases the calculations of Kunszt and Nason [38], based on the original work of Ellis, Ross and Terrano [35], are used, and some analyses are based on the $O(\alpha_s^2)$ matrix elements of Kramer and Lampe [39,16]. Due to the multitude of different studies only a few important features can be mentioned here.

MARK2 is the only experiment which covers data taken with the same detector at two different centre of mass energies (29 GeV and 91 GeV) [41]. Although their event statistic is rather limited at the Z^0 peak, the final results of $\alpha_s(29 \text{ GeV}) = 0.149 \pm 0.002 \pm 0.007$ and $\alpha_s(91 \text{ GeV}) = 0.123 \pm 0.009 \pm 0.007$ are consistent with the QCD prediction of a running α_s , see Fig. 11.

Fig. 11. Results of α_s from jet production rates measured by MARK2, at two different E_{cm} . The lines are extrapolations from the measurement at $E_{cm} = 29$ GeV for a constant α_s and for the prediction of QCD.

OPAL has studied in detail the recombination scheme dependence of jet rates and of α_s , for the schemes defined in Table 2 [37]. For each recombination scheme, the measured jet rates were corrected for hadronisation effects (as studied in Fig. 4) and $\Lambda_{\overline{MS}}$ was determined using $O(\alpha_s^2)$ QCD calculations corresponding to that scheme [38]. Also the renormalisation scale dependence was separately studied within each scheme, allowing the scale to vary between $\mu^2 = E_{cm}^2$ and the best fit result for μ . As an example, the corrected distributions of $D_2(y)$ for the E0 and the E recombination scheme are given in Figure 12, together with the fit results of $\Lambda_{\overline{MS}}$ for the two choices of μ mentioned above. The arrows indicate the regions of fits, which are determined by the demand $R_5 < 1\%$ if both $\Lambda_{\overline{MS}}$ and μ are fitted (5-jet events are only predicted in higher than second order QCD), and by the demand $R_4 < 1\%$ if $\Lambda_{\overline{MS}}$ is fitted for $\mu = E_{cm}$ (with this scale, data cannot be described in regions where a sizable fraction of 4-jet events is observed). In agreement with previous observations [18,20,40] it is found that $\Lambda_{\overline{MS}}$ largely depends on the choice of μ , and that the fit value of $f = \mu^2/E_{cm}^2$ is very small in the E0 scheme. Further conclusions are that the results for f , the overall sensitivity of $\Lambda_{\overline{MS}}$ and $\alpha_s(M_{Z^0})$ on variations of f and the dependence on the parton virtuality Q_0 to which the data are corrected, are different for each recombination scheme, as summarised in Table 5. Within the overall uncertainties, however, the results from the four different recombination schemes agree with each other, resulting in an overall $\alpha_s(M_{Z^0}) = 0.118 \pm 0.008$ from OPAL.

Fig. 12. Measured distributions of $D_2(y)$, corrected for detector acceptance and hadronisation effects, together with the corresponding analytic $O(\alpha_s^2)$ QCD calculations.

Scheme	$\alpha_s(M_{Z^0})$	$\Delta\alpha_s$ (exp)	$\Delta\alpha_s$ (had)	$\Delta\alpha_s$ (Q_0)	$\Delta\alpha_s$ (scale)	$\Delta\alpha_s$ (tot)
E0	0.118	± 0.003	± 0.003	± 0.003	± 0.007	± 0.009
E	0.126	± 0.003	± 0.003	± 0.003	± 0.013	± 0.014
p0	0.118	± 0.003	± 0.003	± 0.005	± 0.004	± 0.008
p	0.118	± 0.003	± 0.003	± 0.006	± 0.003	± 0.008

Table 5. Final results of $\alpha_s(M_{Z^0})$ for different jet recombination schemes, from OPAL.

A summary of measurements of $\alpha_s(M_{Z^0})$ from jet production rates at $E_{cm} \approx 91$ GeV is given in Figure 13. Note that each experiment quotes the central value of $\alpha_s(M_{Z^0})$ for $\Lambda_{\overline{MS}}$ determined at different scale μ ; however - with the exception of MARK2 - scale uncertainties are considered in the respective error. Within their overall uncertainties, the measurements agree well with each other. Some of the systematic uncertainties are common to all experiments, explaining why the results scatter less than expected from their errors. The average of all results, weighted by the overall uncertainty of each measurement, is

$$\alpha_s(M_{Z^0}) = 0.119 \pm 0.008,$$

where the experimental and theoretical errors of the average are added in quadrature.

Fig. 13. Results of $\alpha_s(M_{Z^0})$ from measurements of jet production rates at $E_{cm} \approx 91$ GeV.

5.3. Measurements of $\alpha_s(M_{Z^0})$ from Energy Correlations.

Energy-energy correlations (EEC) between hadrons are basically a histogram of the angles between any pair of hadrons in a hadronic event, weighted by the normalised product of their energies and summed over all events. The EEC and its asymmetry (AEEC) were introduced by Basham et al. [52] as well suited measures of α_s ; in fact for many years it was the most favoured observable used for α_s determinations at PETRA and PEP.

Studies of energy correlations at LEP are available from all LEP experiments [53,54,55,56]. Their results, listed separately for the EEC and AEEC since systematic uncertainties are rather different, are summarised in Figure 14. After correcting the data for detector resolution and hadronisation effects, OPAL [54] and L3 [55] compare the integrals of both EEC and AEEC, in angular regions which are sensitive to hard gluon radiation, to several analytic calculations in $O(\alpha_s^2)$ QCD [57,38]. They consistently find that there are differences between these different sets of calculations, leading to a theoretical uncertainty of $\Delta\alpha_s(M_{Z^0}) \approx \pm 0.006$. The additional renormalisation scale uncertainty is even larger for the EEC, but it is small and almost negligible for the AEEC. DELPHI have only compared the measured AEEC to a specific $O(\alpha_s^2)$ QCD plus string hadronisation model; since the scale uncertainty for that observable is so small and no other sources of uncertainties have been studied by DELPHI, they quote a smaller overall error on α_s [53] than the other experiments.

ALEPH has followed a new idea in their analysis [56]: hadrons are first preclus-

Fig. 14. Results of $\alpha_s(M_{Z^0})$ from measurements of energy correlations at $E_{cm} \approx 91$ GeV.

tered into jets, using the JADE jet algorithm. The so-called “cluster-EEC” (CEEC) is calculated from those jets instead of from the hadrons. It is shown that this method significantly reduces hadronisation uncertainties in the measurement of energy correlations; however at the same time it becomes more subject to the scale uncertainties known from the studies of jet production rates. Besides this interesting trade of systematic uncertainties, this method complements the previously discussed studies of jet production, since now α_s is determined not only from jet production rates, but also from their energy and angular distributions. Consistent results of $\alpha_s(M_{Z^0})$ from these different methods thus constitute another significant test of QCD. The CEEC distributions, measured for two different jet resolutions y_{cut} , together with the corresponding correction factors for hadronisation as predicted by two different model calculations, are shown in Figure 15. The theoretical calculations, corrected for hadronisation effects, are derived from the $O(\alpha_s^2)$ ERT matrix elements [35] with methods developed by Kunszt and Nason [38].

From Fig. 14 it is evident that all the measurements of $\alpha_s(M_{Z^0})$ are in good agreement with each other. The overall average results are

$$\alpha_s(M_{Z^0}) = 0.121 \pm 0.010 \text{ [(C)EEC]}$$

$$\alpha_s(M_{Z^0}) = 0.113 \pm 0.008 \text{ [AEEC]},$$

where the largest contributions to the overall errors of (C)EEC come from scale and other theoretical uncertainties, while experimental uncertainties (phrased “sys”

Fig. 15. Measured CEEC distributions for two values of y_{cut} compared to $O(\alpha_s^2)$ QCD (top), and the hadronisation correction factors together with the angular range selected to determine $\alpha_s(M_{Z^0})$ (bottom).

in Fig. 14) are dominant in case of the AEEC [54,55]. The AEEC has thus the potential to provide more significant results in the near future.

5.4. Determinations of $\alpha_s(M_{Z^0})$ from Global Event Shapes.

In principle, α_s can be determined by almost any event shape observable which is sensitive to the emission of hard gluons, i.e. to the kinematic features of planar 3-jet or spherical multi-jet events. In practice, however, analyses are restricted to “infrared safe” quantities like Thrust T , Acoplanarity A , Oblateness O , C-parameter C and others, which can be reliably calculated in $O(\alpha_s^2)$ perturbation theory (for a definition of these observables, see e.g. [31]).

Magnoli, Nason and Rattazzi [58] have published a detailed analysis of the event shape distributions measured by OPAL [31]. They determine α_s in both leading and next-to-leading order QCD and find that $O(\alpha_s^2)$ corrections are in general larger than hadronisation effects at $E_{\text{cm}} \approx 91$ GeV. It is thus concluded that α_s determinations from global event shapes are much more reliable at LEP than at the lower PETRA and PEP energies, and that higher order QCD corrections play an increasingly important

Fig. 16. Results of $\alpha_s(M_{Z^0})$ from measurements of global event shapes at $E_{cm} \approx 91$ GeV.

rôle at the higher energies.

ALEPH has determined α_s from their measured event shape distributions [44]. Their results and those of Magnoli et al. (all in $O(\alpha_s^2)$) are summarised in Figure 16, where the error bars include uncertainties due to scale variations from $f = \frac{1}{16}$ to $f = 1$. The different results agree well with each other and with the overall value of

$$\alpha_s(M_{Z^0}) = 0.126 \pm 0.016.$$

This is the unweighted mean of all results excluding those from the Oblateness O , since O appears to be most sensitive on higher order QCD corrections, on scale variations and on hadronisation corrections. The overall error on $\alpha_s(M_{Z^0})$ is an estimate derived from the typical experimental and theoretical uncertainties and from the scattering of the individual results.

5.5. Final Summary of $\alpha_s(M_{Z^0})$.

In order to obtain an overall value of $\alpha_s(M_{Z^0})$, the results from measurements of jet production rates, of the (C)EEC and AEEC and of global event shapes, as described in the previous chapters, are summarised in Figure 17. The results were divided into these four different classes according to the somewhat different nature of their experimental and theoretical systematic uncertainties. In a wider sense, however, also jet rates and energy correlations are global event shape parameters, such that such a separation is to a certain degree arbitrary. As can be seen, the different measurements agree well with each other. It is therefore feasible to calculate and quote an average value of $\alpha_s(M_{Z^0})$ from these results. There is, however, no unique method to summarise and average different experimental results and, especially, their

Fig. 17. Final compilation of results on $\alpha_s(M_{Z^0})$.

systematic uncertainties. The mean values of $\alpha_s(M_{Z^0})$ and their overall errors quoted in this report therefore depend, to a small extent, on the procedure chosen to average the results.

Taking the weighted average of the four α_s values shown in Figure 17 and assigning the error from the most significant measurements, namely jet rates and the AEEC, as the overall systematic uncertainty, one derives

$$\alpha_s(M_{Z^0}) = 0.118 \pm 0.008$$

as the final result from LEP and SLC. Since theoretical systematic uncertainties dominate in most of the measurements, the precision of the overall result is not better than that of a typical single measurement. The important conclusion of this compilation is, however, that $\alpha_s(M_{Z^0})$ is consistently measured to within 7% accuracy, and that none of the single measurements deviates from the average by more than 2 standard deviations.

6. Tests of the Gluon Self Coupling.

Several experimental observables have been proposed which should be sensitive to the specific spin structure of the triple gluon vertex within 4-jet events in e^+e^- annihilation. The basic Feynman diagrams of 4-jet final states, consisting of $q\bar{q}q\bar{q}$ events and of $q\bar{q}gg$ events from either double gluon bremsstrahlung or the triple gluon vertex, are shown in Figure 18. In leading order (i.e. $O(\alpha_s^2)$) QCD predicts that about 4.7% of all 4-jet events defined with $y_{\text{cut}} = 0.01$ are $q\bar{q}q\bar{q}$ final states, while

Fig. 18. Generic Feynman diagrams for the process $e^+e^- \rightarrow Z^0 \rightarrow 4 \text{ jets}$.

in the alternative abelian gauge theory, due to the absence of the gluon self coupling, this number reads 31.3% [59].

Two angles χ_{BZ} [60] and θ_{NR}^* [61,62] defined by the momentum axes of the partons (see Figure 19) were demonstrated to be sensitive to the difference between QCD and the abelian model, even if quark and gluon jets cannot be identified on an event by event basis but are statistically separated by ordering the jet energies E_j according to $E_1 \geq E_2 \geq E_3 \geq E_4$ [59]. The difference between QCD and the abelian model in this case, however, is mainly due to the characteristic angular structure of $q\bar{q}q\bar{q}$ events rather than to the triple gluon vertex itself. Since the relative admixture of such events is definitely given by the specific gauge structure of the theory, a measurement of these angles is suited to provide further evidence for the nonabelian structure of QCD.

Both OPAL [63] and L3 [64] studied samples of about 4000 4-jet events in terms of these angles. The measured angular distributions are corrected for detector acceptance and for hadronisation effects. They are then compared to various predictions of perturbative QCD and of the abelian theory, both calculated using second order perturbation theory and QCD parton shower models, which partly include higher than second order contributions. The differences between these predictions were treated as theoretical uncertainties within the QCD and abelian model predictions. The final distributions are shown in Figure 20. As can be seen, the data are in good agreement with QCD and significantly rule out the abelian hypothesis.

Fig. 19. Definition of χ_{BZ} and θ_{NR}^* for energy ordered axes of 4-jet events.

The DELPHI collaboration has followed another strategy of analysis [65]. They analyse the two-dimensional distribution of $\cos\theta_{NR}^*$ and α_{34} , which is the angle between the two least energetic jets within 4-jet events. While θ_{NR}^* has a good discrimination power between four-quark and $q\bar{q}gg$ final states but cannot distinguish double gluon bremsstrahlung from triple gluon vertex events, the distribution of α_{34} is expected to discriminate between the latter. DELPHI determines the group constants N_C/C_F and T_R/C_F in a fit of an $O(\alpha_s^2)$ 4-jet event generator to the measured two-dimensional distribution, thereby separating the model events into 5 classes of event types which are ascribed to different combinations of the group constants C_F , N_C and T_R . The result is

$$\frac{N_C}{C_F} = 2.55 \pm 0.55 \text{ (stat.)} \pm 0.4 \text{ (hadr.)} \pm 0.2 \text{ (det. sim.)},$$

which is in agreement with the expectation of QCD ($9/4 = 2.25$) but not with the abelian theory ($N_C = 0$).

7. Jets at $p\bar{p}$ Colliders.

By far the highest energetic hadron jets which are currently observed at particle accelerators come from the $p\bar{p}$ colliders at Fermilab and at CERN: the transverse jet energies E_t , measured w.r.t. the beam axis, extend to more than 400 GeV at Fermilab ($\sqrt{s} = 1800$ GeV) and more than 200 GeV ($\sqrt{s} = 630$ GeV) at CERN. New measurements of inclusive jet production cross sections were recently reported by the CDF [66] and the UA2 [67] collaborations.

Jets are defined by clustering the energy measured in segmented electromagnetic and hadronic calorimeters. CDF uses a cone algorithm for jet identification, where the radius R of the cone is defined by $R = \sqrt{\Delta\eta^2 + \Delta\phi^2}$; $\eta = -\ln(\tan\frac{\theta}{2})$ is the

Fig. 20. Measured distributions of χ_{BZ} (OPAL) and $\cos\theta_{NR}^*$ (L3), together with the predictions of QCD and abelian vector gluon models.

pseudo-rapidity, θ is the angle of the calorimeter cell w.r.t. the beam axis and ϕ is the azimuthal angle around the beam. Energy clusters with $E_t > 100$ MeV within a cone of typically $R = 0.7$ are added to form a single jet. Within the jet algorithm of UA2, the energies deposited in adjacent calorimeter cells with $E_t > 400$ MeV are joined into clusters. Clusters within a cone radius of $R = 1.3$ are then merged into jets.

Sizeable uncertainties in the absolute normalisation of measured jet cross sections and in the theoretical predictions limit the accuracy with which the measurements can be compared with QCD. For example, in case of UA2 the overall systematic cross-section scale uncertainty is 32% [67]. It is attributed to the model dependence of the acceptance corrections (25%), uncertainties in the analysis parameters and the jet algorithm (15%) and in the absolute calorimeter energy scale (11%) and the integrated luminosity (5%). Leading order (LO) QCD calculations predominantly suffer from large renormalisation scale uncertainties, which amount to about 30% if the scale is varied between $E_t/2$ and E_t . The scale uncertainty is much reduced within next-to-leading, $O(\alpha_s^3)$, calculations (NLO) which recently became available [68], namely to about 5% over the same range. Additional theoretical uncertainties of about 20% are due to different proton structure functions used in the calculations.

The latest measurements of inclusive jet cross sections by UA2 and CDF are shown in Figure 21. The UA2 data, separated into various bins of η , are compared to the predictions of LO QCD with the scale chosen as $\mu^2 = p_t^2/4$ (the NLO calculations do not exactly match the jet definition of UA2 and can therefore not directly be

Fig. 21. Inclusive jet cross sections measured by UA2 and CDF in $p\bar{p}$ collisions at $\sqrt{s} = 630$ and 1800 GeV, respectively, compared to the predictions of LO (UA2) and NLO QCD (CDF).

compared with the data)[67]. The agreement is good for central values of η but marginal for increasing values of $|\eta|$. However, the increasing deviation at large $|\eta|$ is in the range of differences predicted from different sets of structure functions. The CDF data are compared to the NLO calculations of Ellis, Kunszt and Soper [68], which describe the data well over impressive seven orders of magnitude [66]. From the overall agreement of LO and NLO QCD with the measured jet (and di-jet mass) cross sections, possible deviations from standard QCD expectations can be limited by quoting lower limits on the quark compositeness scale of $\Lambda_c > 825$ GeV (UA2) and $\Lambda_c > 950$ GeV (CDF), with 95% confidence level..

While comparisons with LO calculations are suited to demonstrate the compatibility of data with QCD, more meaningful tests of QCD can be performed with the available NLO order calculations. For example, the cone size dependence of jet cross sections is only predicted in NLO, while in LO they do not depend on cone size. The cone size dependence of the cross sections for 100 GeV jets, measured by CDF and predicted by NLO QCD, are shown in Figure 22. The data exhibit a steeper cone size dependence than the theory, possibly due to higher than NLO effects, but the improvement in reduction of theoretical ambiguities when NLO terms are included is obvious.

Fig. 22. Cone size dependence of the cross section for 100 GeV jets, measured by CDF and compared to the NLO predictions.

8. Summary and Conclusions.

The multitude of studies of hadronic jets in high energy e^+e^- annihilation and $p\bar{p}$ collisions provides detailed and comprehensive tests of the basic structure of the strong interaction and its underlying theory, QCD. Especially the latest results obtained from hadronic Z^0 decays at the SLC and LEP and from large E_t jets observed at the Tevatron and the $SP\bar{P}S$ significantly increased our understanding of and confidence in this theory:

- Global event structures of hadronic final states in e^+e^- annihilations are well described by QCD (shower) models, and their observed energy dependence is understood in terms of QCD scaling violations plus an energy independent parametrisation of hadronisation.
- Jet Production Rates in e^+e^- annihilation significantly test the validity of QCD and its nonabelian structure: α_s undoubtedly runs as expected by asymptotic freedom.
- $\alpha_s(M_{Z^0})$ is consistently measured, by many experiments and from a variety of different observables, as

$$\alpha_s(M_{Z^0}) = 0.118 \pm 0.008$$

in $O(\alpha_s^2)$, where the error includes all experimental and known theoretical uncertainties. This corresponds to $\Lambda_{\overline{MS}}^{(N_f=5)} = 225_{-85}^{+120}$ MeV.

- Angular correlations within 4-jet events in e^+e^- annihilations provide further evidence for the nonabelian nature of QCD.

- Inclusive jet cross sections measured in $p\bar{p}$ collisions up to transverse jet energies of more than 400 GeV are in good agreement with leading order and next-to-leading order QCD predictions over a range of up to seven orders of magnitude.

The final result of $\alpha_s(M_{Z^0}) = 0.118 \pm 0.008$ is in remarkably good agreement with predictions from measurements at lower energies, namely with $\alpha_s(M_{Z^0}) = 0.11 \pm 0.01$ predicted by Altarelli in 1989 [69], with $\alpha_s(M_{Z^0}) = 0.109^{+0.004}_{-0.005}$ predicted from a recent analysis of deep inelastic scattering (DIS) and prompt photon data [70] and with $\alpha_s(M_{Z^0}) = 0.112 \pm 0.003$ from an analysis of structure functions in DIS by the BCDMS collaboration [71]. Comparing the values of $\alpha_s(M_{Z^0})$ and the size of their overall errors quoted from e^+e^- annihilation and from DIS, however, it should be noted that the latter are obtained for μ^2 fixed to Q^2 and no investigation of the scale uncertainty (which is likely to increase the overall error on α_s from DIS) is available yet.

Overall, the remarkably good agreement between many different experiments and between theory and experiment in a wide range of energies provides the most conclusive evidence that QCD is the correct theory of the strong interaction.

Finally, as a constructive stimulation for future studies, it should be mentioned that most of the recent results are no longer limited by statistics, by experimental systematics or by hadronisation uncertainties, but instead by theoretical uncertainties like the renormalisation scale ambiguity or, equivalently, unknown higher order contributions. Many of these studies therefore demonstrate the need for higher order QCD calculations, either in next-next-to-leading fixed order perturbation theory or in next-to-leading logarithmic approximations, which can be linked to experimental measurements. It is also necessary to clarify discrepancies between different theoretical calculations of the same observable in a given order, as for energy correlations in $O(\alpha_s^2)$. Should these discrepancies be resolved, a determination of $\alpha_s(M_{Z^0})$ from measurements of the AEEC will be possible with a precision of better than 5% in the near future.

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