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OBSERVATION AND CORRECTION OF INSTABILITIES IN CIRCULAR ACCELERATORS

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Abstract

The correction of beam instabilities in circular accelerators requires a good knowledge of some basic concepts which form the basis of a widely accepted Standard Model. These concepts are introduced here in a simple semi-empirical way, and their relevance to the observation and understanding of collective phenomena is illustrated by practical examples.

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TABLE OF CONTENTS

1. Introduction	1
2. Continuous beams	2
2.1 Transverse modes	2
2.2 Instability mechanisms and damping	5
2.3 The resistive wall instability	6
2.4 Longitudinal effects	7
3. Bunch modes	7
3.1 Longitudinal modes	7
3.2 Transverse modes	10
4. Stability of bunch modes	13
4.1 The Head-Tail instability	13
4.2 The longitudinal Head-Tail	18
4.3 The coupled bunch instability	19
5. Mode Coupling and Beam-Break-Up	22
5.1 Mode Coupling	23
5.1.1 Theoretical considerations	23
5.1.2 Observations of Mode Coupling	25
5.2 Beam Break Up	28
5.2.1 Theoretical considerations	28
5.2.2 Observation of Beam Break Up in the CERN SPS	30
5.3 Feedback damping of the Mode Coupling instability	32
6. References	

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1. Introduction

The knowledge we have at present of Particle Physics is well described by the Standard Model. Similarly, in the field of Accelerator Physics, a general model of beam instabilities has been progressively built up over the years, which allows us to describe almost every observation. Many individuals contributed to the construction of this model, from the first observations of an electron beam instability in 1962 in the Mk II accelerator of Stanford and the first theory of resistive wall instability in 1965 by Neil and Sessler, to the recent developments on mode coupling instabilities. A decisive step in the unification of the known phenomena was made around 1974 by F. Sacherer. This illuminating work had a profound influence on the author of this paper, who had the privilege of collaborating closely with F. Sacherer at this time. The aim of this lecture is to show how the Standard Model of Beam Instabilities, as understood by the author, can be used to describe, classify, understand and cure the different collective instabilities that we encounter in circular accelerators.

The only justification of this exercise is of course the correction of these phenomena, which hopefully leads to better performance of the machines. This can be obtained by suppressing the cause of the instability (coupling impedance, phase shift), by active feedback damping or by Landau damping. This very important final step is however trivial once the phenomenon has been properly understood. The most important phase is the process of understanding, which requires good observation skills coupled to a thorough knowledge of the theoretical models. This will be reflected in the organization of this course : emphasis will be put primarily on the description of the coherent modes of oscillation of the beams. The case of the continuous beam will be examined first, because it is simple and allows the introduction of many basic concepts. Then the description will be extended to bunched beams, for which the dynamics is dominated by the RF focusing. As long as the forces due to the beam-environment interaction, which are responsible for the instabilities, are small compared to the RF focusing force, destructive instabilities can arise only from multibunch effects (very similar to continuous beam instabilities, although with some differences) and Head-Tail effects. For strong enough interactions, the RF focusing is no longer able to hold the bunch together, and single bunch instabilities known as Mode Coupling, Microwave instability, or Beam Break Up can arise.

2. Continuous beams

2.1. Transverse modes

Let us examine the case of a continuous beam in a circular machine, as shown in Fig.1. Suppose that at time $t = 0$ a perturbation is imposed on the beam so that each azimuthal slice is displaced transversely by an amount $y_0(\theta)$. This pattern $y(\theta)$ is necessarily closed around the circumference and therefore it can be decomposed into a Fourier series:

$$y(t=0, \theta) = \sum_{n=-\infty}^{+\infty} y_n e^{-jn\theta} \quad (1)$$

so that it is only necessary to consider the evolution of these sinusoidal waves. The betatron oscillation of a particle with a tune Q and a revolution frequency ω_0 , and which was at azimuth θ_0 at time $t = 0$ is given by

$$y_{\theta_0}(t) = y_n e^{j(Q\omega_0 t - n\theta_0)} \quad (2)$$

but since for any particle the azimuth θ after a time t is given by $\theta = \theta_0 + \omega_0 t$ we can describe the position of the whole beam in azimuth and time as follows:

$$y(\theta, t) = y_n e^{j[(Q+n)\omega_0 t - n\theta]} \quad (3)$$

Fig. 1

With this formula it is easy to see that the initial wave pattern rotates with an angular velocity $\omega_n = (1 + \frac{Q}{n})\omega_0$. Thus different wave patterns rotate at different velocities; for positive values

of n the angular velocity of the wave is larger than the velocity of the particle, whereas for negative values of n the wave travels slower than the particles. For this reason the waves associated with positive orbital harmonics are called fast waves, and those associated with negative orbital harmonics are called slow waves. A small subset of the slow waves travel backwards with respect to the particles: these are associated with orbital harmonics of order $n < 0$ such that $|n| < Q$.

Although a snapshot of the whole beam at a given time will reveal exactly the same pattern for waves of opposite harmonic number n and $-n$ these waves are of fundamentally different nature. The slope of the wave at azimuth θ and time t is given by $\partial y / \partial \theta = -j n y$, while the slope of the particle trajectory at the same time and azimuth is $dy/dt = j Q \omega_0 y$. For $n > 0$ the amplitude of a particle increases although the associated wave has a negative slope at the same azimuth; this is possible because the wave has a phase velocity larger than the particle velocity. For negative values of n the wave has a positive slope and the particle amplitude can increase because the phase velocity of the wave is smaller than the particle velocity (see Fig.2.) This is very reminiscent of the evolutions of a surfer on ocean waves.

Fig. 2

The wake fields generated by such a beam displacement wave in the surrounding structure exert a force on the particles. For a fast wave this force has the right phase to produce a damping of the betatron wave. On the contrary slow waves generate wake fields which are capable of increasing the amplitude of the wave, leading to an instability. Backward waves cannot lead to instabilities.

Up to now we have described the propagation of betatron waves around the machine, and we have seen how it is related to the motion of the individual particles which constitute the beam. What are the observable properties of such a wave ? The signal generated in a transverse detector at a fixed azimuth is (see equation 3).

$$y(\theta_o, t) = y_n e^{j[(Q+n)\omega_o t - n\theta_o]} \quad (4)$$

It consists of a single line at frequency $(Q + n) \omega_0$ for a fast wave ($n > 0$), $(|n| - Q) \omega_0$ for a slow wave ($n < 0$ and $|n| > Q$) and $(Q - |n|) \omega_0$ for a backward wave ($n < 0$ and $|n| < Q$). The amplitude of this single line is proportional to the displacement y_n of the beam and to the number N of particles.

The different lines corresponding to the case of an accelerator with $Q=2.25$ are shown in Fig. 3. In reality, when observed on a spectrum analyzer the lines with a negative frequency are folded onto the positive frequency axis as indicated. Around each revolution harmonic $n'\omega_0$ one observes two lines at $(n'+q)\omega_0$ and $(n'-q)\omega_0$, where q is the fractional part of Q . However, the lines $n'+q$ and $n'-q$ are not associated with n' but with $n = n'-k$ and $n = -n'-k$ respectively (if k is the integer part of Q). Although the nature of these lines cannot be distinguished on a spectrum analyzer their associated phases contain the information on the sign of the wave harmonic n .

Fig. 3

Betatron lines for an accelerator with $Q = 2.25$

2.2. Instability mechanism and damping

Any interacting structure which presents a non zero resistive impedance at the frequency corresponding to any slow wave can drive the beam unstable. Since all the wave patterns of a continuous beam are independent, there can be no cancellation effects, contrary to what happens for more complicated systems that will be analyzed later on.

As a consequence the frequency range of a feedback system capable of stabilizing such a beam has to be as large as that of the coupling impedance itself, a requirement very difficult to meet in practical cases. In fact, the stability of continuous beams is assured by Landau damping, and feedback is only used for the few most unstable modes.

Landau damping is a very powerful damping mechanism in continuous beams. It arises from the spread in the betatron frequencies of the single particles which constitute the beam. The largest contribution to the frequency spread comes from the momentum dispersion, which induces both a spread of the revolution frequencies given by

$$\frac{\Delta\omega}{\omega_0} = -\eta \frac{\Delta P}{P_0} \quad (5)$$

and a spread in the betatron tunes given by

$$\frac{\Delta Q}{Q_0} = \xi \frac{\Delta P}{P_0} \quad (6)$$

where $\eta = \frac{1}{\gamma_{tr}^2} - \frac{1}{\gamma^2}$, with γ_{tr} the relativistic factor at transition energy, and ξ the machine chromaticity.

With these formulae the frequency dispersion on a betatron sideband at $(n+Q)\omega_0$ can be written:

$$\Delta(n+Q)\omega_0 = (Q_0\xi - n\eta) \frac{\Delta P}{P_0} \omega_0. \quad (7)$$

There is always a value of n for which the betatron frequency spread induced by momentum dispersion vanishes. In this case the stability can only be assured by an additional frequency spread induced by octupoles, but this term can be insufficient if the beam has small transverse dimensions. It is therefore imperative to choose the sign of ξ and η such that the cancellation in formula 7 only happens for a positive value of n (this corresponds to a fast wave, which is always stable). This means that above transition energy ($\eta > 0$) ξ must be positive, and that below transition ($\eta < 0$) ξ must be negative.

The frequency around which this cancellation occurs, $\omega_\xi = Q_o \omega_o \frac{\xi}{\eta}$, is called the chromatic frequency. As we will see later on, this quantity also plays a very important role in the stability of bunched beams.

2.3. The resistive wall instability

The transverse coupling impedance of a smooth beam pipe of radius b and resistivity ρ is given as a function of the frequency ω by:

$$Z_T = \frac{c^2}{\omega_o b^3} \sqrt{2\mu_o \rho / |\omega|} ((\text{sign}(\omega) + j)) \quad (8)$$

where c is the speed of light.

This function is sketched in Fig.4 together with the frequency lines of the coherent modes. If the tune value lies just below an integer, there is one slow wave close to the origin, where the value of the resistive wall impedance is large, leading to a large growth rate of the instability. When this phenomenon is dominant, as was the case in the CERN PS Booster during its commissioning, it is preferable to choose a tune value just above an integer, since this pushes the frequency of the most dangerous mode to higher values and reduces its growth rate.

Fig. 4

Resistive wall impedance and betatron lines for $Q = k + 0.75$.

One of the slow waves sees a large impedance.

2.4. Longitudinal effects

In addition to the transverse displacement waves analyzed above, a debunched beam can sustain longitudinal charge density waves. These generate single coherent lines at frequencies $n\omega_0$ corresponding to the number n of wavelengths of the perturbation around the machine.

3. Bunch modes

In bunched beams the particles describe synchrotron orbits around the synchronous particle as sketched in Fig.5. The time of arrival of a particle at a fixed azimuth varies like

$$\tau = \hat{\tau} \cos(\omega_s t + \psi_0) \quad (9)$$

where $\hat{\tau}$ is the amplitude and ψ_0 the phase of the synchrotron oscillation. As a result the spectral lines of this particle are split into synchrotron sidebands.

Fig. 5

The synchrotron orbit

3.1. Longitudinal modes

Let us treat first the case of the longitudinal motion, which is somewhat simpler than the transverse one.

In absence of RF focusing a particle generates spectral lines at each harmonic $n\omega_0$ of the revolution frequency ω_0 [1]. In presence of synchrotron motion these lines are split into an infinite series of sidebands of amplitude

$$S = J_m(n\omega_0 \hat{\tau}) e^{j[(n\omega_0 + m\omega_s)t + m\psi_0]} \quad (10)$$

where m is the sideband number and J_m is the Bessel function of order m .

For a bunch containing a large number N of particles the signal is obtained by adding for each sideband the contribution of the individual particles. If the motion of the particles are uncorrelated the phases ψ_0 are randomly distributed and the global signal is very small. This signal, which does not vanish completely because the number N of particles, although usually very large, is not infinite, constitutes the Schottky signal of the bunch. By introducing a correlation between the particles one can increase the signal on one of the sidebands. For instance it is easy to see that with a particle distribution of the form

$$\rho(\psi_0) = e^{-jm\psi_0} \quad (11)$$

the signal is maximum for the sideband of order m . We will define a normal mode m of the bunch as a special distribution of particle phases which enhances coherently the signal on sideband number m .

In a real bunch the particles are also distributed over a certain range of amplitudes $\hat{\tau}$. When the bunch is unperturbed the particles are evenly distributed in phase and the distribution can be written:

$$\rho_0(\hat{\tau}, \psi) = \rho_0(\hat{\tau}) \quad (12)$$

When applying a perturbation Liouville's theorem has to be satisfied, that is the bunch distribution in the two dimensional phase space with coordinates τ and $\hat{\tau}/\omega_s$ (Fig. 5) can be deformed provided this preserves the local density around individual particles. Such a deformation can be described by adding a term $\rho'(\hat{\tau}, \psi)$ to the unperturbed distribution so that

$$\rho_0(\hat{\tau}, \psi) = \rho_0(\hat{\tau}) + \rho'(\hat{\tau}, \psi) \quad (13)$$

and ρ' can be Fourier analyzed in azimuth ψ and written :

$$\rho' = \sum_{m=-\infty}^{+\infty} \rho_m(\hat{\tau}) e^{-jm\psi} \quad (14)$$

Isodensity contours in phase space relative to terms $m = 1$ to 3 are drawn on Fig. 6a while the corresponding line charge density appears in Fig. 6b. By subtracting the contribution of the unperturbed distribution, one obtains the signal induced in a sum pick-up

Fig. 6

Longitudinal dipole, quadrupole, and sextupole modes.

a) Isodensity contours in phase space, b) projection on the time axis, c) coherent signal

as a result of the perturbation. This signal is given by the projection on the time axis of the distribution ρ' . The exact form of this signal depends on the distribution $\rho'(\hat{\tau})$ but in any case for $m = 1$ it has one node in the middle, for $m = 2$ it has two nodes and so on. As a consequence, its Fourier Transform is peaked around a frequency Ω_m which is close to $\pi(m + 1)/\tau_L$ where τ_L is the bunch length.

This signal oscillates in time at the frequency $m\omega_s$ and this generates coherent synchrotron sidebands of order m around each orbital harmonic $n\omega_0$.

In summary any perturbation of the bunch in the longitudinal phase plane can be decomposed into an infinite number of normal modes m each characterized by an oscillation frequency $m\omega_s$. The envelope of the coherent synchrotron satellites generated by a mode m is peaked around $\pi(m + 1)/\tau_L$.

Up to now the density distribution in amplitude $\rho_m(\hat{\tau})$ has not been specified, and a smooth enough law has been assumed. For more complicated distributions the signal can have more nodes than specified by the azimuthal mode number m . For instance the dipole mode ($m = 1$) can be obtained by shifting the unperturbed distribution $\rho_0(\hat{\tau})$ in one direction and letting it rotate in phase space at frequency ω_s . But by displacing different parts of the bunch by

different amounts while retaining the symmetry of order 1 in ψ as sketched in fig. 7 it is

Fig. 7

Mode $m = 1$, $r = 3$

possible to generate a mode which oscillates at ω_s ($m = 1$) but induces a signal which has three nodes. Its Fourier transform will then be peaked at $\Omega_3 = \pi \times 4/\tau_L$ as for the mode $m = 3$. Therefore one must in fact consider a two-fold infinity of modes ρ_{mr} where m is the azimuthal and r the radial mode number. We will usually refer in the following to the basic modes for which $m = r$ and which are called dipole, quadrupole, sextupole etc. modes.

3.2 Transverse modes

As in the case of the longitudinal motion we are going to construct a coherent mode by arranging particle phases so that only one set of synchrotron satellites are coherently enhanced. The amplitudes of the synchrotron satellites of the betatron spectral lines of a single particle are given by (see for instance ref. [1]).

$$S_T = J_m(\Omega_n \hat{\tau}) e^{j(\omega_{nm} t + \varphi_0 + m\psi_0)} \quad (15)$$

where $\Omega_n = (n + Q) \omega_0$, $\omega_{nm} = (n + Q) \omega_0 + m\omega_s$ and φ_0 and ψ_0 are the initial phases of the betatron motion and of the synchrotron motion respectively.

Clearly the signal from all particles on the same synchrotron orbit will add up coherently on the synchrotron satellites of order m if $\varphi_0 + m\psi_0 = 0$. For instance the lines with $m = 0$ are coherent if the initial betatron phase φ_0 is the same for all the particles around the synchrotron

orbit. This is the case when all the particles are displaced at time $t = 0$ in the same direction, in other words the mode $m = 0$ corresponds to a rigid, parallel motion of the bunch. On the contrary for the $m = 1$ satellites to be coherent initial phases must be opposite in the head and the tail of the bunch. For $m = 2$ the head and the tail have equal phases but the center is displaced in the opposite direction, and so on. Modes of oscillation corresponding to these special arrangements of the betatron phase are called head-tail modes of order m .

Let us follow what happens to the particles in a bunch which executes a coherent head-tail mode. The case of the rigid bunch mode is straightforward : since all particles are in phase, the synchrotron motion, while shuffling them back and forth along the bunch, has no observable coherent effect. On the contrary it plays a very important role for all the other modes $m \neq 0$. Let us study in detail the case of the head-tail mode $m = 1$. This mode corresponds to a special arrangement of the betatron phases along the bunch, such that the head and the tail oscillate 180° out of phase. Figure 8 will help us to follow 4 particles which have the same maximum betatron amplitude and are evenly distributed around the synchrotron orbit.

At time $t = 0$ (fig. 8) particles 1 and 3 have maximum opposite displacement y , whereas particles 2 and 4 are centered. The small circles close to each particle show the betatron phase space while the large circle represents the synchrotron phase space.

Fig. 8
Head-tail mode $m = 1$

The resulting transverse dipole moment of the bunch is shown below. The particles have all the same frequency (chromaticity is zero) and therefore the situation after a quarter, respectively half a betatron period is described by fig. 8b and 8c : the extremities of the bunch oscillate but the dipole moment is always zero at the center. On a position pick-up the signal will always have a node in the middle : this is characteristic of head-tail mode 1. In general the signal corresponding to head-tail mode m has m nodes, and for this reason the envelope of its spectral lines peak at approximately $\omega_m = \pi (m + 1)/\tau_L$ for $m \neq 0$.

While executing their betatron oscillations our reference particles migrate slowly (ω_s is usually much smaller than ω_β) anticlockwise. After a quarter of a synchrotron period particle 1 has replaced particle 2 and so on. The betatron phases of the particles which occupy the reference positions have been advanced by a quarter of a betatron period as a consequence of the synchrotron motion. After a complete synchrotron period the coherent signal has made one betatron oscillation more than it would have in absence of synchrotron motion : we see that for an external observer the coherent signal of mode $m = 1$ oscillates at $\omega_\beta + \omega_s$. Generally, the coherent signal of mode m oscillates at $\omega_\beta + m\omega_s$.

Now let us come back to Fig. 8a : we have chosen the phase of particle 2 to be retarded by $\pi/2$ with respect to particle 1, and the phase of particle 4 to be advanced by $\pi/2$ with respect to particle 1. We could equally well have chosen the opposite : particle 2 advanced by $\pi/2$ and particle 4 retarded by $\pi/2$, and the aspect of the coherent mode would have been preserved. The very important difference is that in this last case the coherent frequency of oscillation turns out to be $\omega_\beta - \omega_s$: the mode $m = -1$ has been generated instead of mode $m = 1$. This can be generalized to all modes $m \neq 0$.

Fig. 9

A few head-tail modes. For the signal d several consecutive traces are drawn to show the nodes

Up to now we have considered particles evenly distributed in phase around a single synchrotron orbit of amplitude $\hat{\tau}$. In this case a transverse coherent motion was created with all the particles having the same betatron amplitude $\hat{y}$, and their betatron phases arranged in different patterns corresponding to head-tail modes m . In fact particles are distributed over a range of synchrotron amplitudes from zero to $\tau_L/2$, and we must specify the betatron amplitude on each synchrotron orbit. If the distribution of betatron amplitudes as a function of synchrotron amplitudes is smooth, almost uniform, as in fig. 9a, the behavior of the modes is not changed very much. However, as for the longitudinal modes one can find distributions which create additional nodes in the coherent signal. For instance with a distribution function $\hat{y}(\hat{\tau})$ which goes to zero at some intermediate amplitude like in fig. 9b, the coherent signal has necessarily two nodes in the case of the mode $m = 0$. In fact each head-tail mode m is split into an infinity of radial modes m, r which induce signals in a transverse pickup with $r = m, m + 2, m + 4$, etc... nodes. The signal of mode $m = 0, r = 2$ which is displayed as an example in Fig. 9b looks very much like the signal of the basic head-tail mode with azimuthal mode number $m = 2, r = 2$. The difference is that $m = 2$ oscillates at $\omega_\beta + 2\omega_s$ whereas mode $m = 0, r = 2$ is a head-tail mode zero and oscillates at ω_β . On a spectrum analyzer the overall image of their signal will look the same, but a closer look at any one of the spectral lines will show the difference of $2\omega_s$.

In general modes with high radial mode numbers are more difficult to drive, and it is often sufficient to consider only the basic modes with $r = m$.

4. Stability of bunch modes

The normal modes defined in the preceding chapter are standing wave patterns. Two different points arbitrarily chosen along the bunch coherent signal are either in phase or 180° out of phase. The resistive wake fields generated by such modes in the surrounding structures are proportional to the signal and therefore cannot drive these modes unstable : for this to happen a 90° phase shift between the perturbing force and the displacement is necessary.

However, such a phase shift can arise from different mechanisms. When the value of the chromaticity is different from zero a phase shift is introduced between the head and the tail of the bunch. This phenomenon is responsible for the so-called Head-Tail instability, a single bunch effect. In the case of a multibunch beam, the coupling between successive bunches provides the necessary 90° component of the force.

4.1 The Head-Tail instability

When chromaticity is not zero the instantaneous betatron frequency of a particle varies along its synchrotron orbit. Let us consider the case of $\xi > 0, \eta > 0$ and follow a particle as it performs a phase oscillation (Fig. 10). It starts at point 1 with a betatron phase $\varnothing = 0$. Relative to the phase it would have maintained if held at point 1, $\varnothing$ is constantly increasing as the particle migrates along the top of the synchrotron orbit, where ΔP is positive (see equation 6).

At point 2 the particle has returned to the nominal momentum and thereafter, along the bottom part of the orbit, its betatron phase will start to decrease because now the betatron frequency is smaller than nominal, and at point 1 it will have recovered its initial phase $\varnothing = 0$. This phase excursion can easily be calculated (see for instance ref. [1]).

Fig. 10

Head-tail phase shift due to chromaticity

Its linear variation as a function of the position along the bunch is represented on Fig. 10 : we see that at a certain position $\Delta\tau$ the phase is the same for the particles which are on the upper part of the orbit and for those which are on the lower part. The maximum phase excursion from the head to tail of the bunch is $\chi = \tau_L \omega_\xi$, where τ_L is the bunch length in time units, and $\omega_\xi = Q_o \omega_o \frac{\xi}{\eta}$ is the chromatic frequency already defined in paragraph 2.

If we now distribute particles along the synchrotron orbit with betatron phases just matching the above curve, they will trade places due to the synchrotron motion but always arrive at a given position with the phase appropriate to that point. Therefore a coherent mode can still be created with a finite chromaticity but at the expense of introducing a phase slewing $\varnothing = \omega_\xi \Delta\tau$ along the bunch. The appearance of the head-tail modes is modified by this phase shift. As an example Fig. 11 shows the deformation of modes $m = 0$ and $m = 1$ due to a phase shift $\chi = \pi$ between the head and the tail. The mode $m = 0$ does no longer correspond to a rigid displacement, whereas mode $m = 1$ shows a non negligible center of mass motion. In the frequency domain the spectra of all modes are shifted by the chromatic frequency ω_ξ . For this reason high order modes can have spectral lines of large amplitude at low frequency (this is the case of mode 1 in Fig. 11b).

Fig. 11

Head-tail modes for a phase shift $\chi = 0$ and $\chi = \pi$

Signals from high order head-tail modes were first recorded in the CERN PS Booster around 1974^[2]. Typical patterns are shown in Fig. 12 for different values of χ . Several successive traces have been superimposed to clearly show the existence of the nodes for $m \neq 0$ and the phase slewing for $\chi \neq 0$.

As far as stability is concerned the phase shift from the head to the tail of a bunch due to the finite chromaticity has important consequences : it allows an interaction between these two parts (mediated by wake fields) with the right quadrature component of the phase so that energy transfer and therefore instability is possible.

In the frequency domain the situation is sketched in Fig. 13. For a zero value of the chromaticity the power spectra of the head-tail modes are symmetric around the origin and their interaction with the resistive part of the coupling impedance, which is antisymmetric, vanishes (the broad-band, $Q = 1$ resonator model is assumed here). For a non-zero chromaticity the mode spectra are shifted in frequency by an amount ω_ξ , so that the interaction no longer vanishes. It is easy to see that the most dangerous mode $m = 0$ is maximally unstable if $\omega_\xi = -\omega_r$ (ω_r is the resonant frequency of the broad band resonator). This is reminiscent of a result already found in continuous beams : although the theoretical treatment is very different for a single bunch and a continuous beam, it is natural to find similar properties in both cases, since by reducing adiabatically the RF voltage to zero it is possible to go from one state to the other.

Fig. 12

Head-tail modes 0,1 and 2 observed in the CERN PS Booster
on a transverse position pick-up electrode

a) $m = 0, \chi = 0$; b) $m = 0, \chi = \pi$; c) $m = 1$; d) $m = 2$

Fig. 13

Chromatic frequency shift

In accelerators in which the transition energy is crossed the head-tail instability is a serious problem. Close to transition the value of η becomes very small and as a consequence the chromatic frequency ω_ξ assumes large values even for a well corrected chromaticity. This allows a strong coupling of the $m = 0$ bunch mode to the maximum of the resistive part of the broad-band coupling impedance which occurs near to the beam pipe cut-off frequency around a few GHz. Fig. 14 illustrates the difficulty of accelerating dense proton bunches through transition in the CERN SPS. In Fig. 14a the chromaticity is slightly positive and a fast head-tail instability occurs before transition. The signature of this phenomenon is a fast increase of the signal detected on a high frequency vertical monitor, followed by partial beam loss. When the value of ξ is reduced the instability occurs closer to transition (Fig. 14b) and when the sign of ξ changes to negative the instability occurs after transition. In such a situation the head-tail instability can only be cured by swiftly changing the sign of the chromaticity at transition energy. Such a ξ jump has been successfully implemented in the CERN PS, where the same phenomena occurred. Together with a fast γ_{tr} jump it allowed to accelerate dense bunches through transition. In the SPS, such schemes were too delicate and costly and it was decided instead to increase the injection energy well above transition when the machine was upgraded into a proton antiproton collider.

Fig. 14

Head-tail instability around transition in the SPS for $\xi > 0$ (Fig. 13a and b) and $\xi < 0$ (Fig. 13c) trace 1 shows the evolution of the bunch intensity and trace 2 the signal from a wide band vertical monitor. The arrows indicate transition crossing.

4.2 Longitudinal Head-Tail

An effect similar to the transverse Head-Tail instability but concerning the longitudinal motion was conjectured long ago by H. Hereward [3]. This is a very weak instability because there is no chromatic effect in the longitudinal plane to first order ; only second order effects can provide a variation of the synchrotron frequency with momentum. Johnsen [4] described this effect in the following formula

$$\frac{\Delta L}{L} = \alpha_1 \frac{\Delta P}{P} (1 + \alpha_2 \frac{\Delta P}{P}) \quad (16)$$

where $\Delta L/L$ is the relative change of the orbit length of a particle with a relative momentum deviation $\Delta P/P$. Here α_1 is the usual term from which the transition energy is obtained ($\alpha_1 = 1/\gamma_{tr}^2$) while α_2 gives the variation of the transition energy with momentum :

$$\frac{\gamma_{tr} - \gamma_{tr_0}}{\gamma_{tr_0}} = - (0.5 + \alpha_2) \frac{\Delta P}{P} \quad (17)$$

The term α_2 which depends on the sextupoles used to correct chromaticity can be measured by recording the change in transition energy for different radial positions of the beam. In the CERN SPS $\alpha_2 = -0.7$ in normal operating conditions. The synchrotron frequency depends on the transition energy and therefore the instantaneous synchrotron frequency varies along the synchrotron orbit due to the second order effect described above. This effect is relatively stronger close to transition energy.

In the CERN SPS collider transition occurs at 21 GeV/c. Dense proton bunches are sequentially injected at 26 GeV/c and circulate at this energy for up to 28 seconds. A slow longitudinal dipole instability with an e - folding time of the order of a few seconds is observed, as shown in Fig. 15. This is a single bunch effect which has been attributed to a longitudinal Head-Tail instability [5].

Fig. 15

Slow single bunch dipole instability at 26 GeV/c in the SPS
(output of phase detector, 20 s full horizontal scale)

This very weak instability can be observed in the SPS because Landau damping is completely suppressed as a result of the interaction of the dense bunches with the inductive part of the broad-band coupling impedance : the frequency shift of the coherent mode is larger than the spread of the incoherent particle frequencies within the bunch. This instability is cured by feedback on individual bunches.

4.3 The coupled bunch instability

In the case of a continuous unbunched beam, a mode corresponds to a closed sinusoidal pattern with n wavelengths around the machine circumference. The signal is a sinusoid of frequency $\omega_n = n\omega_0$ (for longitudinal density waves) or $\omega_0 = (n + Q)\omega_0$ (for transverse displacement). It seems legitimate to assume that gathering the beam particles into a large number N of closely spaced bunches should not dramatically change the coherent behavior of the beam. This is indeed true, although some important differences appear which distinguish a bunched from an unbunched beam, as we shall see. For short, well separated bunches it is relatively easy to visualize a transverse coupled bunch mode : we start from the closed sinusoidal pattern of a continuous beam mode with its n wavelengths around the circumference, and we distribute the bunches like beads on top of this wave. For N equispaced bunches the betatron phase shift $\Delta\phi$ from bunch to bunch must therefore be $2\pi n/N$. If $\Delta\phi$ is different from zero or π , which always happen if the number of bunches N is larger than 2, the resistive wake fields have components in quadrature with displacement, and an instability can occur.

When bunches are long compared to the wavelength of the continuous beam pattern, we can still play this game but now the bunches must be bent to closely follow the sinusoidal curve : this is possible if either we make use of high order Head-Tail modes, or if we introduce a non zero chromaticity which, as we know, allows different parts of the bunch to oscillate with different phases even for mode $m = 0$. In the frequency domain this means that the frequency of the coupled bunch mode pattern we want to create corresponds to some part of the Head-Tail mode spectrum of a single bunch shifted by ω_ξ (Fig. 16).

Fig. 16 Multibunch modes

a) short bunches

b) long bunches

This can be illustrated by the case of the transverse coupled bunch instability induced by the resistive wall in the CERN PS Booster. Fig. 17 shows the low frequency wave pattern generated around the ring by the displacement of the 5 bunches and the exponential growth of the instability. The single bunch modes which contribute to this signal for different values of the chromatic frequency shift χ have already been shown in Fig. 12.

Fig. 17

Coupled bunch mode pattern a) and growth rate b) of the resistive wall instability in the CERN PS Booster

In the longitudinal case it is the phase of the bunches relative to the synchronous particles which is distributed according to a sinusoidal law. Again for short bunches such a mode pattern can be created by a rigid displacement of the bunches (dipole mode $m = 1$). But for long bunches the phase displacement has to vary within the bunch to fit the sinusoidal density variation of a continuous beam mode : in this case high order single bunch modes (quadrupole, sextupole ...) come into play, in very much the same way as Head-Tail modes in the transverse case (with the difference that here chromaticity effects are negligible).

In the case of a continuous unbunched beam, a transverse mode n corresponds to one single frequency $\omega_n = (Q + n)\omega_0$. In this situation what matters for the determination of the instability growth rate or frequency shift is the value of the coupling impedance at that frequency.

In bunched beams the spectra of coupled bunch modes contain aliased frequencies $\omega = k N\omega_0 + \omega_n$ in addition to the basic mode frequency ω_n . This is because, as the bunch occupies a very short fraction of the ring circumference, its spectrum extends to frequencies much higher than the revolution frequency, whereas its position is sampled only once per revolution. Therefore, many wave patterns of different frequencies can be invoked to account for the measured signal as can be seen easily from Fig. 18.

Fig. 18
Aliasing effect

The stability of this mode is therefore influenced by the value of the impedance at all aliased frequencies on which consequently a summation has to be performed. These spectral lines do not extend to infinite frequencies : they fall off at a frequency roughly equal to the inverse bunch length expressed in seconds.

Future large hadron colliders will use a large number N of bunches. In this case the aliased frequencies are far apart and it is often true that a given structure only influences the beam on one of its spectral lines : this is the case for the resistive wall and for high Q resonators. In this limit the bunched beam behaves very much like a continuous beam, in particular the growth rates are the same in both cases for the same total beam intensity. However, in a realistic environment particular features emerge for a bunched beam, which we are now going to study.

As we have already mentioned, the resistive wall instability is very important in large hadron colliders in particular because of the existence of transverse modes at very low frequency. However, it is easy to build powerful active feedback systems covering all modes up to the MHz range to damp this instability. At higher frequencies the growth rate of the resistive wall instability decreases like $\omega^{-1/2}$ and one could hope that a moderate octupole would be sufficient to Landau damp all the modes outside the bandwidth of the low frequency damper. Unfortunately, this is not true because the short bunches also interact strongly with the broadband coupling impedance. This interaction produces a real frequency shift which tends to suppress Landau damping : as a consequence, even a very small growth rate can lead to the eventual destruction of the beam. In such a situation the damping system must cover all the

possible collective modes, which is equivalent to saying that each bunch must be individually damped.

As an example, in the CERN Large Hadron Collider now under design the bunches are typically 30 cm long and are regularly spaced at about 5 m intervals. This means that the bunch spectrum extends over 1.25 GHz, and in this range a collective mode n produces of the order of 20 aliased frequency lines. All these lines sample the inductive part of the broad-band coupling impedance, which is constant in this frequency range, whereas the resistive-wall impedance mostly affects the line with the lowest frequency. Although it is not responsible for the growth rate, the broad-band coupling impedance thus dominates the stability properties of a large number of the coupled bunch modes.

5. Mode Coupling and Beam Break Up

We are now going to study the stability of single dense bunches interacting with short range wake fields (broad band coupling impedance). We have already seen that due to the symmetry of the power spectra of the bunch modes, one expects single bunches to be stable when chromaticity is suppressed. However, even in this case one observes destructive phenomena in both the longitudinal and transverse phase planes at high bunch densities. In the longitudinal case the microwave instability manifests itself by bursts of microwave signals followed by an increase of the energy spread. In the transverse case a fast increase of the transverse emittance followed by beam losses is observed.

Two complementary approaches are used to explain these phenomena. Historically the first idea was applied to the longitudinal motion : in the case where the wake fields are so strong that they dominate the RF focusing force it is legitimate to ignore the synchrotron motion and treat the bunch as a piece of coasting beam. This is the Hereward-Boussard approach [6] which leads to the prediction of the longitudinal microwave threshold by applying the Keil-Schnell criterion for coasting beams [7] to single dense bunches. In the same spirit the theory of the transverse Beam Break Up developed long ago for Linear Accelerators can be applied to circular machines if the synchrotron motion is so slow compared to the instability that it can be ignored.

Another approach of the same phenomena analyses the evolution of the bunch modes as the wake fields are increased, until the very concept of modes breaks down : this is the Mode Coupling theory [8].

We will successively describe the Mode Coupling and the Beam Break Up approaches, and illustrate with examples their very good predictive power. The Mode Coupling theory is to be used below the instability threshold, while the Beam-Break Up theory describes best phenomena occurring above threshold.

5.1 Mode Coupling

5.1.1 Theoretical considerations

We have defined a normal mode in absence of interaction as a distribution of particle phases and amplitudes which ensures global coherence on one synchrotron satellite. Any bunch oscillation can then be described by using an infinite set of orthogonal functions and their Fourier transforms $g_m(\omega)$, each corresponding to a mode m .

However in presence of wake fields these modes are no longer independent since the wake field generated by each mode acts on all the others : in this case the real, observable modes must be calculated self-consistently. The eigenvalues of the interaction matrix

$$M_n^m = C \ i \sum_{p=-\infty}^{+\infty} i^{m-n} g_m(p) Z(p) g_n(p) \quad (18)$$

give the new, perturbed frequencies, from which the beam stability can be evaluated. In this expression C is a real quantity, $Z(p)$ is the coupling impedance and p is the frequency normalized to the revolution frequency. The coefficient M_n^m represents the action of the wake field induced by mode m (which is expressed by $g_m Z$) on mode n . Fig. 19 shows the different mode spectra g_m superimposed on the broad band coupling impedance represented by the usual $Q = 1$ resonator model. It is clear that the interaction term between two modes of the same parity contains only a reactive component, which can be responsible for a frequency shift but cannot lead to instability. On the contrary two modes of different parity couple through the resistive part of the impedance, and this can produce an instability.

Fig. 19

Head-tail mode spectra superimposed on the broad-band coupling impedance
($Q = 1$ resonator model)

An important feature whose demonstration will not be given here but can be found elsewhere [9] is that the interaction matrix is real and has the property :

$$M_n^m = (-1)^{m-n} M_m^n \quad (19)$$

Two adjacent modes m and $m+1$ oscillate at different frequencies separated by ω_s . Therefore for small interactions one can usually neglect the off-diagonal terms which couple different mode numbers. In this case no single bunch instability can happen if the value of the chromaticity is zero. However this is no longer true for strong interactions. As an illustration, let us consider the case of two coupled modes, described by the following matrix and its eigenvalues λ :

$$\begin{vmatrix} \lambda - m - M_m^m & -M_n^m \\ -M_m^n & \lambda - n - M_n^n \end{vmatrix} \quad (20)$$

$$\lambda = \frac{1}{2} (m + M_m^m + n + M_n^n) \pm \sqrt{(m + M_m^m - n - M_n^n)^2 - 4 M_n^m M_m^n} \quad (21)$$

Here m and n are the mode frequencies in absence of interaction divided by ω_s .

Two different behaviors can be observed, depending on the relative parity of m and n . Using formula (19) one sees that for modes of different parities $M_n^m = M_m^n$ and the eigenvalues can acquire an imaginary part if the coupling terms become comparable to the frequency difference between the two modes. The corresponding evolution of the eigenvalues as the beam intensity is increased is sketched in Fig. 20a. On the contrary for two modes of the same parity $M_n^m = -M_m^n$ and the eigenvalues remain real : one sees on Fig. 20b how the two modes repel each other, whereas on Fig. 20a they are attracted towards each other.

Fig. 20

Eigenvalues as a function of bunch intensity for the cases $(m-n)$ odd a) and $(m-n)$ even b).

The manifestation of the Mode Coupling mechanism depends very much on the overlap of the bunch spectrum (related to the bunch length) with the broad band coupling impedance.

The most unfavorable situation occurs in the case shown in Fig. 21 : the bunch length is such that two modes (here the $m = 0$ and $m = 1$ modes) sample mainly, one the inductive part of the broad band impedance, the other the capacitive part. As a consequence their frequencies move in opposite directions and become closer with increasing intensity. In addition both modes sample an important resistive term. At a certain point the two frequencies are close enough for the resistive coupling term to dominate : the two modes lose their identity, each becoming a mixture of $m = 0$ and $m = 1$ mode. At a slightly higher intensity there is only one coherent frequency : maximum stable coupling has been reached. Above this threshold, one of the mode frequencies acquires a positive imaginary part while the other acquires a negative imaginary part and the mode becomes unstable.

Fig. 21

Mode coupling for short bunches

The situation described above occurs when the wake field decays in about a bunch length, and this leads to the lowest threshold. It is characteristic of short bunches in electron colliders.

A rule of thumb which is often used to predict this instability threshold is the following : a mode coupling instability can arise when the frequency shift of the lowest order mode, extrapolated from measurements made at low intensity, becomes comparable to the synchrotron frequency. In the worst case described above this criterion is too optimistic : the true threshold

intensity is about half that predicted by this criterion. On the contrary, as we shall now see, for long proton bunches it is too pessimistic.

The case of long proton bunches is described in Fig. 22. The spectra of many of the low order modes lie in the region where the broad-band impedance is mainly inductive : their frequencies move in the same direction when the intensity is increased, although by different amounts since higher order modes are fundamentally more difficult to drive. Eventually, different mode frequencies merge, but the resistive coupling terms are always small compared to the reactive terms. For this reason, at an intensity slightly above threshold, the modes are again decoupled, and no important growth rates can be reached.

Fig. 22

Mode coupling for long bunches

The CERN SPS is well suited to illustrate the difference between long and short bunches, since both hadrons and leptons are accelerated in this machine. Proton bunches containing up to $1.5 \cdot 10^{11}$ particles and injected at 26 GeV/c with an r.m.s. length $\sigma_s = 30$ cm are stable, while positron bunches containing 10^{10} particles and injected at 3.5 GeV with $\sigma_s = 16$ cm are unstable.

The reason for this different behavior is explained in Figs. 23 and 24. For protons (Fig. 23) many modes cross as the intensity is increased but the growth rates acquired remain small. For positrons (Fig. 24) modes 0 and -1 cross at about $0.4 \cdot 10^{10}$ particles, but again the growth rates are limited in this region. It is only when modes -2 and -3 cross at about $0.8 \cdot 10^{10}$ that the instability is observed. The reason is that the spectra of these two modes overlap the resistive peak of the coupling impedance around 1.3 GHz.

Fig. 23

Mode frequencies versus number of particles for proton bunches in the SPS
at 26 GeV/c ($\sigma_s = 30$ cm)

Fig. 24

Mode frequencies versus number of particles for positron bunches in the SPS
at 3.5 GeV/c ($\sigma_s = 16$ cm)

5.2 Beam Break Up

5.2.1 Theoretical considerations

In linear accelerators the head of the bunch of particles induces wake fields in the accelerating structure which in turn deflect the tail of the bunch. This effect accumulates along the machine length and is more severe for long structures. When the amplitude of the bunch tail reaches the iris aperture part of the bunch is lost, hence the name of Beam Break Up instability given to this phenomenon which was observed long ago [10].

Since the particles go only once through the structure a simpler theoretical approach than for circular machines can be used. It is interesting to extend this approach to circular accelerators and this is possible whenever the wake fields decay completely in one turn, or between successive bunches : in this case a circular accelerator of radius R is equivalent to a linear machine of length $L = 2 \pi R n$, where n is the number of turns over which the instability develops. However a fundamental property of circular accelerators limits the number of turns over which this comparison with Linacs can be made : this is the existence of a bunch stretching mechanism described by formula (5) which can be rewritten

$$\frac{dT}{T} = \eta \frac{dP}{P} \quad (22)$$

where T is the revolution period.

This provides a natural damping mechanism: a Beam Break Up instability can only be observed in a circular accelerator if it is faster than the bunch stretching effect.

Whereas the beam environment interaction is treated in the frequency domain in circular machines, the use of the time domain is more natural in Linear structures. Considering our usual broad band impedance model described by a low Q resonator with a resonant frequency ω_r and a transverse maximum coupling resistance R_T the wake field generated by a point charge passing in the structure at time zero is

$$W_T = \omega_r \frac{R_T}{Q} e^{-\epsilon t} \sin S \omega_r t \quad (23)$$

with $\epsilon = \omega_r/2Q$ and $S = (1 - 1/4Q^2)^{1/2}$.

For bunches which are short compared to the oscillation period of the wake field it is instructive to use a two particles model, as sketched in Fig. 25 : half of the bunch charge is gathered in particle 1 (the bunch head) and half of the charge in particle 2 (the tail). If the head particle enters the structure with a transverse displacement δ (injection error, misalignment) the

maximum force exerted on the tail through the transverse wake field is

$$f = e \frac{Ne}{2} \delta W_1 \quad (24)$$

provided particle 2 arrives just at the maximum W_1 of the wake field. If the betatron oscillation of particle 1 is given by $y_1 = \delta \cos \omega_\beta t$ the oscillation of particle 2 is given by the equation :

$$\ddot{y}_2 + \omega_\beta^2 y_2 = \frac{Ne^2 W_1}{2m_o \gamma} y_1 \quad (25)$$

with the result :

$$y_2(t) = \delta \left[\cos \omega_\beta t + \frac{Ne^2 W_1}{4\omega_\beta m_o \gamma} t \sin \omega_\beta t \right] \quad (26)$$

At the extremity of a structure of length L the amplitude of the tail reaches :

$$y_2 = \delta \frac{Ne^2 W_1 L}{4\omega_\beta m_o \gamma c} \quad (27)$$

where c is the speed of light.

Fig. 25

Transverse wake field and "two particles" model

This simple treatment cannot be applied in the case of the CERN SPS because the bunches are longer than the oscillation period of the wake fields. A more sophisticated theory due to Yokoya [11] is interesting in this case : the bunch is split into a large number m of bunchlets, each one generating its own wake field. The effect of all these individual wake fields is summed up along the bunch and this results in an amplitude growth of the last bunchlet proportional to L^{m-1} . If m is very large the amplitude growth is given by :

$$y(n) = \frac{\delta c}{2\sqrt{2\pi}\omega_r\ell} \left(\frac{\Omega\ell}{c} \right)^{1/4} \exp \left[-\epsilon \frac{\ell}{c} + \sqrt{\frac{\Omega\ell}{c}} \right] \quad (28)$$

where ℓ is the bunch length ($4\sigma_s$) and $\frac{\Omega\ell}{c} = \frac{Nec}{\omega_p E/e} \frac{\omega_r R_T}{Q} n$.

In the SPS a series of measurements performed over many years in different beam conditions have been used to adjust the parameters of the broad band impedance model : the best fit is obtained with a $Q = 1$ resonator with $R_T = 23 \text{ M}\Omega\text{m}^{-1}$ and $\omega_r/2\pi = 1.3 \text{ GHz}$. Using this model and applying formula (28) with a bunch length $\ell = 64 \text{ cm}$ and a number $N = 4.10^{10}$ particles per bunch gives $y(n)/\delta = 20$ for $n = 15$ turns. From this we conclude that a bunch of leptons with the above characteristics injected 1 mm off centre would be lost to the vertical aperture of about 20 mm after 15 turns. As we shall see later on a loss is observed after 11 turns, in reasonably good agreement with the above prediction.

Another approximate prediction which can be made using very simple arguments concerns the intensity threshold below which the Beam-Break up instability should not be observed. Using formula (22) with $\eta = 1.85 \cdot 10^{-3}$ in the SPS one sees that the length of a bunch injected with $\ell = 64 \text{ cm}$ and a total energy spread $\Delta E/E = 2.4 \cdot 10^{-3}$ doubles in about 40 turns. This defines an approximate threshold for the instability : if the initial growth rate is not large enough the competing effect of bunch stretching eventually wins and the amplitude does not grow significantly. Experimentally as the bunch intensity is decreased the losses become erratic between 20 and 40 turns after injection and are never observed after 40 turns. The threshold corresponds to a bunch population of about 10^{10} particles.

5.2.2 Observations of Beam Break Up in the SPS [12]

Fig. 26 shows that a bunch of 4.10^{10} positrons is partially lost soon after injection. Fig. 27 displays the signal of a wide band vertical monitor filtered between 1.4 and 1.7 GHz : the signal grows exponentially after injection for about 10 turns and then decays (particles are lost, see Fig. 26). Subsequently the signal grows again until turn number 30, where additional losses are visible in Fig. 26.

Fig. 26

Beam loss after injection as a function of the number of turns

Fig. 27

Signal from a vertical wide band beam position monitor filtered between 1.4 and 1.7 GHz

If the SPS coupling impedance can be represented by a broad band resonator peaked at 1.3 GHz as represented in Fig. 28, one expects the corresponding wake fields to distort the 64 cm long bunch as indicated. The signal induced by such a wriggling bunch has a wide spectrum centred around the peak of the resonator impedance. On the contrary the frequency spectrum of the bunch just after injection extends only to about 500 MHz and cannot be seen by the 1.4 to 1.7 GHz receiver : this is why the signal grows from zero after injection.

Fig. 28

- a) The resistive component R_T of the SPS broad band coupling impedance model and the bunch spectrum S at injection. The frequencies of the filters used, F_1 (460 MHz) and F_2 (1.4 - 1.7 GHz) are indicated.
- b) The wiggling bunch after a few turns in the SPS

By tuning the receiver around 460 MHz, one observes a large signal at injection, because this frequency lies within the bunch spectrum. This signal, which is caused by the injected bunch passing off centre in the monitor, does not grow on subsequent turns.

The synchrotron period at injection in the SPS is 70 turns, and therefore one expects no influence of the RF system on this fast instability above threshold : indeed switching the RF on or off does not change the loss pattern.

5.3 Feedback damping of the Mode Coupling Instability

The Mode Coupling instability develops when the frequency shift of the fundamental mode becomes comparable to the synchrotron frequency. As a consequence the growth rate just above the threshold can reach values as high as the synchrotron frequency ω_s ; in other words,

since Q_s is often of the order of 0.1 in electron machines, the e-folding time can be as small as a few turns, and a very fast feedback system has to be used. In the case of the SPS described previously, a somewhat slower feedback would suffice since Q_s is smaller. However, with long bunches there is no sizeable component of the oscillating bunch signal at low frequency, so that a high frequency wide band feedback is needed.

In any case it seems difficult to fight the Mode Coupling Instability with a classical resistive feedback. For this reason another approach has been proposed : it consists in holding the frequency of the mode $m = 0$ constant with a low frequency, reactive feedback. However, this works only for short bunches (for which the mode $m = 0$ is involved in the instability) and one cannot hope to increase the threshold in this way by a factor larger than about two, because in the end the resistive interaction, which is of the same order as the reactive one, dominates. Such a feedback has been tried in the storage ring PEP at STANFORD [13].

References

- [1] J. Gareyte, Beam observation and the nature of instabilities, Physics of Particle Accelerators, Fermilab summer school 1987, AIP Conference proceedings 184, p. 343,
- [2] J. Gareyte, F. Sacherer, Head-Tail type instabilities in the CERN PS and Booster, Proc. 9th Int. Conf. on High Energy Accelerators, Stanford 1974, P. 341,
- [3] H. G. Hereward, Longitudinal Head-Tail effect by the bunch length variation method, RL-74-062, EPIC/MC/48 Rutherford Lab. April 1974,
- [4] K. Johnsen, Effects of non linearities on phase transition, CERN Symposium on High Energy Accelerators, Geneva 1956, Vol. 1, p. 106,
- [5] D. Boussard, T. Linnecar, Longitudinal Head-Tail instability in the CERN-SPS Collider, 2nd EPAC, Nice, June 12-16, 1990,
- [6] D. Boussard, CERN Lab. II/RF/Int. 75-2 (1975),
H.G. Hereward, ISR performance report ISR/DI/HGH/amb (1975),
- [7] E. Keil, W. Schnell, CERN-TH-RF/69-48 (1969),
- [8] F. Sacherer, Bunch lengthening and microwave instability, IEEE Trans. Nucl. Sci. Vol. NS-24 N°3 (1977), p. 1393,
R. D. Kohaupt, Transverse instabilities in Petra, Proc. 11th Int. Conf. on High Energy Accelerators, CERN (1980) EXS Vol. 40, Birkhäuser Verlag, p. 562,
- [9] Y. H. Chin, Transverse mode coupling instabilities in the SPS, CERN/SPS/85-2 (DI-MST)
- [10] R. Helm, G. Loew, Beam Break Up, Linear Accelerators (eds P.M. Lapostolle and A.L. Septier) North Holland Book Co, Amsterdam, 1970, chapter B,
- [11] K. Yokoya, DESY 86-084,
- [12] D. Brandt, J. Gareyte, Fast instability of positron bunches in the SPS, 1st EPAC, Rome 7-11 June 1998, World Scientific Publishing Co. Pte. Ltd, Singapore, p. 690,
- [13] S. Myers, Stabilization of the fast Head-Tail instability by feedback, 1987 IEEE Particle Accelerator Conference, March 16-19, Washington, DC. 87CH2387-9, p. 503.