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Ranges of Interaction in $p\bar{p}$ Scattering at low Energies

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Abstract:: The $p\bar{p}$ cross sections at low energies are analysed in the framework of the optical model. It is shown that the annihilation range is closely related to the size of the proton as measured in electromagnetic and hadronic interactions. At larger distances the $p\bar{p}$ interaction has the similar character as the NN one.

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1 Introduction

In the papers on the $p\bar{p}$ annihilation[1] and elastic scattering[2] we have presented the data on the total cross section, the annihilation cross section and the differential elastic cross section obtained with the $\bar{p}$ beam in the momentum range between 180 and 600 MeV/c. One of the aims of this experiment was to compare the $p\bar{p}$ cross sections with the nucleon-nucleon ones. The $p\bar{p}$ and NN interactions, excepting annihilation, are related by a symmetry transformation, and therefore, this comparison may reveal new aspects of the nuclear interaction. In this way it may add further constraints to the present models of nuclear force.

In the present paper we will summarize the analysis of the $p\bar{p}$ data in terms of the optical model potential. Parts of this analysis have been already presented in short communications[3,4]. In the optical model potential the annihilation is simulated by the imaginary part of the potential. As the $p\bar{p}$ interaction is dominated by the annihilation we expect that the analysis of the $p\bar{p}$ cross sections will teach us about this part of the interaction. We will show that the low energy annihilation can be well characterized by two parameters: the depth of the annihilation potential and a radius which one can interpret as the annihilation range. On the other hand, the real part of the $p\bar{p}$ optical potential is very likely closely related to the NN potential. Unfortunately, the low energy $p\bar{p}$ data, in particular the spin averaged cross sections, do not allow a detailed analysis of this part of the $p\bar{p}$ interaction. We will only show that the range of the $p\bar{p}$ interaction represented by the real part of the optical potential is consistent with that of the NN interaction.

Several theoretical papers[5] comparing the $p\bar{p}$ and NN interactions have been published in the last years. Most of these papers present a particular model of NN interaction to which an ad hoc annihilation potential is added in the case of $N\bar{N}$ interaction. The model calculations are compatible with the existing data. Our aim is, however, to analyse the low energy $p\bar{p}$ data in terms of the ranges of the interaction, an analysis which we feel is less model dependent than other approaches. The reason for our approach is to point out that the annihilation range in $p\bar{p}$ is closely related to the well known size of proton as measured in electron scattering and the high energy pp interaction. In this way the annihilation

range can be understood as the distance where the nucleons start to overlap, the size of the nucleon being approximately the same in electromagnetic and strong interactions.

2 Optical potential and Interaction Ranges

2.1 Optical potential

We have investigated the behavior of various potentials in the context of an optical model. By varying the shape, width, and depth of the real and imaginary parts of the potential, one can determine the limits of parameters which give acceptable fits to the data. If the potential is non-local, as is the case in many annihilation models at low energies, a different potential is required for each energy. However, we have realized that a single set of potential parameters can give good fits to all the $p\bar{p}$ cross sections from 180 to 600 MeV/c, indicating that a local potential is adequate to describe the $p\bar{p}$ data at low energies.

We have solved the Schrödinger equation with the potential:

$$U = V + iW \tag{1}$$

where the real part, V , is a Woods-Saxon type

$$V = -\frac{V_0}{1 + \exp(\frac{r-r_0}{a_0})} \tag{2}$$

with the parameters allowed to vary. The shape of W was chosen to be either Woods-Saxon or Gaussian.

At any particular energy, a number of combinations were found which gave a good fit to the data. It was realized that the fit was insensitive to the exact shape or depth of the imaginary potential W , but that it always had a depth of about 100 MeV at 1 fm whenever it gave a good fit. The size of the real potential is not unique, as is also the case for low energy NN scattering. However, we found that only the real potential with a range of between about 1.5 and 2.2 fm was able to reproduce the data. It was not possible to get a good fit if the real part of the potential had a shorter range than the imaginary part. The

real potential which gave the best fit over the entire energy range had a Woods-Saxon type with $V_0 = 46$ MeV, $r_0 = 1.9$ fm and $a_0 = 0.2$ fm. The fit obtained with these parameters for the elastic differential cross sections is shown in Fig. 1. The real potential giving the best fits is about the same which is used for the NN potential. It should be noted that the spin averaged potential is only weakly affected by the G-parity transformation because the $p\bar{p}$ system has a less restricted symmetry than the NN one.

We have just shown that the optical potential consisting of the short-range imaginary potential and a long-range real potential can reproduce the $p\bar{p}$ cross sections. However, the potentials themselves do not provide a direct physical insight. This is one of the reasons why the concept of scattering length was introduced in the early days of nucleon-nucleon scattering. For interpretations of the present results, as will be shown in next sections, it seems appropriate to define the ranges of particular interactions.

2.2 Annihilation Range

We now investigate the physical significance of our findings about the annihilation potential. For this, we use the standard expansion for the scattering of a plane wave. The incident plane wave, ψ , and the solution of the Schrödinger equation, Ψ , are expanded, respectively, as:

$$\psi = e^{ikz} = \sum_{\ell} (2\ell + 1) i^{\ell} j_{\ell}(kr) P_{\ell}(\cos\theta) \quad (3)$$

$$\Psi = \sum_{\ell} (2\ell + 1) i^{\ell} R_{\ell}(r) P_{\ell}(\cos\theta) \quad (4)$$

where j_{ℓ} is the spherical Bessel function and P_{ℓ} is the Legendre polynomial. The annihilation cross section can be expressed in terms of the function Ψ [6]:

$$\sigma_{ann} = -\frac{2}{hv} \int W |\Psi|^2 d\Omega \quad (5)$$

where v is the relative velocity. The quantity

$$S(r) = W |\Psi|^2 \quad (6)$$

can be interpreted as the probability for annihilation at a distance r . The form of $S(r)r^2$, as shown in Fig. 2, is typical for all good fits to the experimental data. We can use this relation to define the annihilation radius r_a as the point where half the absorption has taken place, that is

$$\int_0^{r_a} W |\Psi|^2 r^2 dr = \int_{r_a}^{\infty} W |\Psi|^2 r^2 dr \quad (7)$$

The sharp peak seen in Fig.2 appears at the point where the strength of the imaginary potential is about 100 MeV, nearly independent of the exact shape of the potential. Nearly the full absorption has taken place outside half fm, so the sensitivity to the shape or magnitude of the real or imaginary potential is small at very short distances. It is interesting to note that at the distance where the attraction is strongest in the NN potential, the $p\bar{p}$ potential is dominated by the annihilation.

2.3 Interaction Range

No good fit of the $p\bar{p}$ cross section can be obtained without the real part of the potential, the range of which was always larger in our fit than that of the imaginary one. The range of a potential, however, is not a very well defined quantity and one needs some discussion in order to make a connection to the actual shape of the potential.

The depth and the range of the real potential can be varied over a certain range. This is the same situation as in low energy pp scattering where s-wave scattering length and p-wave effective volume can be produced by a large variety of potential[7].

The feature of the real part of the potential can be further confirmed by using a G-parity transformed One-Pion-Exchange Potential (OPEP)[4]. The whole potential now consists of the real and imaginary Woods-Saxon potential and OPEP:

$$U = (1 + i)V + V_{OPEP} \quad (8)$$

where V is a Woods-Saxon form. The fitted parameters of the Woods-Saxon form were $V_0 = 500$ MeV, $r_0 = 0.74$ fm and $a_0 = 0.2$ fm. The fits are shown in Fig.3 for the elastic

channel. As with our earlier investigations, the imaginary potential has a depth of about 100 MeV at a distance of 1 fm. The OPEP has nearly the same range ($r \approx 2$ fm) as the real part of the potential discussed in the previous section although the tail of OPEP extends further out.

The fact that the differential cross sections for $p\bar{p}$ scattering can be well reproduced by the different potentials but with the similar ranges gives a strong indication that the range of the real potential represents a common feature which can be interpreted as the interaction range.

3 Discussion

The most important result of the present analysis is that the $p\bar{p}$ interaction can be satisfactorily reproduced by an optical potential with the real part having the same range as OPEP and an imaginary part leading to an annihilation range of about 1 fm. The spin averaged cross section do not give sufficient information to determine the exact shape of the potential. The short range NN potential, when transformed to $p\bar{p}$ system, cannot be identified as it is strongly masked by annihilation. This fact explains why quite different models of the NN interaction and annihilation can reproduce the general features of the $p\bar{p}$ cross sections.

The best studied cases of annihilation of quarks are the weak decays of vector mesons into leptons. The major problem of these calculations is the fact that the hadron generally is a highly relativistic many quark system which does not allow a simple description in terms of the nonrelativistic wave functions. Different approaches have been successful in circumventing this problem. We will consider the one which starts by the use of the nonrelativistic constituent quark wave function in the calculation of the rates taking into account the relativistic corrections, as has been discussed by several authors[8]. The major correction in the calculation of the decay rates by the use of the nonrelativistic wave function is due to the fact that the quark bound in the hadron is not localised to better than its Compton wave length. Therefore, the $q\bar{q}$ annihilation switches on at the relative distances of the quark Compton wave length which is of the order of ≈ 0.7 fm. Let us translate this

experience in the language of $p\bar{p}$ annihilation. In the optical model the p and $\bar{p}$ are treated as point like objects and the interaction is simulated by the optical potential. The annihilation will begin at latest when the trajectories of the two particles approach to a distance comparable to the Compton wave length of the light quark. In fact the annihilation range is about twice the Compton wave length of the light quark.

The size of the proton and the antiproton is not given only by the Compton wave length. In addition one has to consider the internal motion of quarks which coincides with the value of the radius of the non-relativistic quark model. This second contribution, which is the smaller one[9], is perfectly taken into account if we replace the Compton wave length by the proton and antiproton radius. Assuming that the strength of the $q\bar{q}$ annihilation is of the same order of magnitude as that of the qq interaction at high energies we would obtain

$$\langle r_{ann}^2 \rangle = 2 \langle r_p^2 \rangle \quad (9)$$

in excellent agreement with our analysis.

It is worthwhile to stress once again that the proton (antiproton) radius is to a large extent independent of the method used to determine it. The value of the radius obtained with electromagnetic probes, that observed in proton-proton and proton-antiproton scattering at high energies, as well as that deduced from the analysis of $p\bar{p}$ interaction within the optical model potential are consistent within the statistical errors.

4 Conclusions

Concluding we would like to point out the following results of the present analysis: The optical model analysis supplies us with an annihilation range which is compatible with the assumption of overlapping nucleons of the radius as measured in electromagnetic and hadronic interactions. In the quark picture of hadrons the annihilation process implies a coloured interaction between quarks and antiquarks. Therefore, precautions are required when applying the meson exchange model for the nuclear forces at distances of 1.2 fm and less. For distances larger than about 1.2 fm the $p\bar{p}$ interaction seems to have the same character as the NN interaction.

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Figure Captions

Fig.1 A fit to the $p\bar{p}$ differential elastic cross section (closed circles) with an imaginary Woods-Saxon potential and a real potential. The open circles are the pp cross section shown for comparison.

Fig.2 $P_\ell^{abs}(r) = S_\ell(r)r^2 = W |\Psi_\ell|^2 r^2$ as a function of r at 287 MeV/c.

Fig.3 Fits to the differential elastic cross sections with an imaginary and a real Woods-Saxon potentials and OPEP at (a)505, (b)287 and (c)181 MeV/c.

The deviations at small angles are due to the Coulomb scattering.

Fig.1

Fig.2

Fig.3