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The Heavy Quark Potential in $SU(2)$ Gauge Theory at High Temperature

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ABSTRACT

We present a detailed investigation of the heavy quark potential in the deconfined phase of pure $SU(2)$ gauge theory at intermediate temperatures. The analysis is based on close comparison of numerical and analytical methods. The extent of the short distance range, where because of asymptotic freedom perturbation theory gives a good description of the interaction, is determined. Slightly beyond the perturbative horizon the potential becomes that of screened colour averaged charges. The screening length determined in that range appears to be surprisingly close to the value that derives from a partial resummation of the continuum perturbation series.

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1. Introduction.

One of the most interesting problems of the high temperature plasma phase in QCD is to clarify the structure of the heavy quark potential. Important physical quantities are the Debye screening length and the "perturbative horizon". The perturbative horizon determines the range where the interaction is well described by short distance physics governed by asymptotic freedom. The screening length determines the range of the heavy quark potential beyond the horizon and derives from the long distance tail of the interquark potential as the collective remnant of the complicated gluon dynamics. It plays an important role in models for J/ψ suppression.

There are persuasive arguments that screening occurs at least for sufficiently high temperature [1-4]. In the context of lattice gauge theory, it is well known that at low temperature the nontrivial center symmetry of the gauge group provides a zero expectation value $\mathcal{L} = \langle L \rangle$ of the Polyakov loop L [5], which is a Wilson line wrapping once around the world in temperature direction. This symmetry is spontaneously broken at high temperature T [6,7], which characterizes the phase of deconfinement $T > T_c$. Investigations of Weiss [8] for weak coupling, and of Polonyi and Szlachanyi [9] for strong coupling suggest that $SU(N_c)$ shares properties of Z_{N_c} spin models. Such models show exponential clustering. For gauge theories this would imply Debye screening:

$$| \langle L(\mathbf{x})L(0)^* \rangle - \langle L(0) \rangle^2 | \leq F(R) \exp(-\lambda(T)R) \quad (1)$$

for sufficiently large $R = |\mathbf{x}|$, where F is some bounded polynomial, implies that the heavy quark potential $V(\mathbf{x}, T)$ between colour averaged charges, given by

$$\exp\left(-\frac{V(\mathbf{x}, T)}{T}\right) = \frac{\langle L(\mathbf{x})L(0)^* \rangle}{|\langle L(0) \rangle|^2} > \quad (2)$$

screens, i.e.

$$\frac{V(\mathbf{x}, T)}{T} \leq F(R) \exp(-\lambda(T)R). \quad (3)$$

A more intricate question is the order of the magnitude of the Debye screening length, respectively its precise value, as well as the horizon beyond which screening sets in. Both quantities will, of course, depend on the temperature. The physical picture we expect is roughly the following.

1. Asymptotic freedom predicts that the short distance behaviour is perturbative. This range can, in principle, be investigated in detail by weak coupling expansion methods. The infrared (IR) problems of thermal perturbation theory can be cured by working in a finite (but nevertheless large) volume. Moreover, finite-size effects, such as zero momentum modes, are under control in the thermal situation [10].
2. At larger distances the weak coupling expansion becomes worse. This intermediate range can, in principle, develop arbitrary complex dynamics.
3. Beyond this range we are leaving the "gluon cloud". The gluon plasma screens the static charges. This is the range where the Debye mass (inverse screening length) should be measured.

The aim of our present work is to verify the aforementioned picture in the context of lattice gauge theory. Reliable results on the screening effect are difficult to obtain in the full theory, since it is computationally very time consuming and furthermore requires an accurate determination of correlations over large distances. The need to go to large separations in order to unambiguously establish the screening effect is emphasized by the recent study in [10]. There it is shown that a purely perturbative approach which carefully takes into account finite-size effects successfully describes Monte Carlo (MC) data at all distances probed on lattices of moderate size. Most available data up till now for the interquark potential and the Debye mass were, however, obtained using a phenomenological ansatz for the interaction essentially in that range [11-13]. Furthermore, in this paper we reemphasize the idea of staying in finite, but possibly large, volume as long as the renormalized gauge coupling g is small but non-zero.

We choose to work with pure $SU(2)$ gauge theory, which is considerably less time consuming to simulate than full QCD. This choice allows us to determine the potential with high precision and also to address the important question of finite-size effects within a reasonable amount of computer time. By comparing high-statistics numerical data with analytical results we investigate in detail the applicability range of weak coupling perturbation theory, that is we determine the perturbative horizon. We consider two values for the temperature in the range 3-6 times the critical temperature T_c , on lattices of various sizes. The perturbative horizon turns out to be about equal to the inverse temperature. It appears, however, that there is no clear separation of the intermediate and the large distance range, at least not for the temperatures considered here. Even

slightly above the horizon the interquark potential has a screening form.

This paper is organized as follows. In Sect.2 we give a general description of the applied weak coupling expansion. Sect.3 discusses details of the numerical work. Both investigations are combined in Sect.4 and a detailed analysis of the interquark potential is given. Concluding remarks are given in Sect.5. Technicalities are collected in the Appendices.

2. Short distance behaviour.

2.1. Notation.

First of all we introduce some commonly used notations. We work on a 4-dimensional periodic lattice Λ with lattice spacing a and of size $L_0 \times L_s^3$ in units of a . Temperature and spatial volume are then given by $T = (L_0 a)^{-1}$ and $L_s^3 = (L_s a)^3$, respectively. In the rest of section 2, a is set equal to one. To each bond $(x; \mu)$, $x = (x_0, \mathbf{x}) \in \Lambda$, $\mu = 0, \dots, 3$, we associate an element $U(x; \mu) \in \text{SU}(N_c)$. For any gauge invariant quantity $I(U)$ we define an expectation value by

$$\langle I \rangle = \frac{1}{Z} \int \prod_{x, \mu} dU(x; \mu) \cdot I(U) \cdot \exp(-S_W(U)), \quad (4)$$

$$Z = \int \prod_{x, \mu} dU(x; \mu) \cdot \exp(-S_W(U)), \quad (5)$$

where dU denotes the Haar measure on $\text{SU}(N_c)$ and $S_W(U)$ is the standard Wilson plaquette action with periodic boundary conditions,

$$S_W(U) = \beta \sum_{\substack{x \in \Lambda \\ \mu < \nu}} \left(1 - \frac{1}{N_c} \text{Re tr} \left[U(x; \mu) U(x + \hat{\mu}; \nu) U^{-1}(x + \hat{\nu}; \mu) U^{-1}(x; \nu) \right] \right), \quad (6)$$

where $\hat{\mu}$ denotes the unit vector in μ -direction, and $\beta = 2N_c/g_0^2$ with g_0 the bare coupling constant. The Polyakov loop is defined as the path-ordered product

$$L(\mathbf{x}) = \frac{1}{N_c} \text{tr} \prod_{x_0=1}^{L_0} U((x_0, \mathbf{x}); 0). \quad (7)$$

and we denote its spatial average by

$$\bar{L} = \frac{1}{L_s^3} \sum_{\mathbf{x}} L(\mathbf{x}). \quad (8)$$

On a finite lattice, the expectation value $\langle L(\mathbf{x}) \rangle$ vanishes at all couplings due to the global Z_{N_c} center symmetry. This has to be taken into account when defining the connected correlation function. One possibility is to add to the action an explicit symmetry breaking term $h\bar{L}$, which must then be removed after the thermodynamic limit has been taken. We choose, instead, to modify the measured operator, keeping $h = 0$, as usually done in MC calculations. We define the connected Polyakov loop correlation by

$$P(\mathbf{x}) = \langle L(\mathbf{x}) L^*(0) \rangle - \langle \exp(\frac{2\pi i \nu(\bar{L})}{N_c}) \bar{L} \rangle^2, \quad (9)$$

where $\nu(\bar{L}) \in \{0, 1, \dots, N_c - 1\}$ is defined such that $\exp(-\frac{2\pi i \nu(\bar{L})}{N_c})$ has minimal distance to $\bar{L}$ in the complex plane. By the center symmetry, this is equivalent to a restriction of phase space to one central sector. More precisely, if we let $\langle \cdot \rangle'$ denote an average over field configurations with $|\arg(\bar{L})| < \pi/N_c$, then Eq. (9) can be written as

$$P(\mathbf{x}) = \langle L(\mathbf{x}) L^*(0) \rangle' - \langle L(0) \rangle'^2. \quad (10)$$

Finally we define, in analogy with Eq. (9), the colour averaged potential $V(\mathbf{x})$ between quarks and anti-quarks by

$$\exp\left(-\frac{1}{T} V(\mathbf{x})\right) = 1 + \frac{P(\mathbf{x})}{|\langle \exp(\frac{2\pi i \nu(\bar{L})}{N_c}) \bar{L} \rangle|^2}. \quad (11)$$

While different definitions of the connected correlation should agree in the thermodynamic limit, they may differ on finite lattices. The ambiguities on finite spatial lattices are due to the tunneling between the N_c degenerate minima of the free energy. Tunneling gives a contribution to the unsubtracted correlation which in the limit $L_s \rightarrow \infty$ becomes the constant subtracted in $P(\mathbf{x})$. For finite L_s , the unphysical tunneling contribution is not strictly a constant since the associated correlation length ξ_t is finite. It is therefore not immediately clear that the use of Eq. (9) is appropriate. To ensure this, we have to require that L_s is sufficiently large. However, such a condition, at fixed coupling and fixed L_0 , could in principle be incompatible with the IR bound on the weak coupling expansion, as will be discussed below. That both these conditions are, in fact, fulfilled in the systems under investigation will become clear from the results to be presented in the subsequent sections. We therefore take Eq. (9) as a common starting point for our MC and weak coupling calculations.

2.2. Infrared singularities.

The weak coupling expansion is one of the most powerful tools for the investigation of the short distance behaviour in asymptotically free field theories, such as non-Abelian Yang-Mills theories. At finite temperature T , this expansion reflects the infrared (IR) behaviour of three dimensional massless field theories, which suffer from severe IR singularities. Stated otherwise, in finite spatial volume L_s^3 the saddle point expansion about the classical (T -dependent) vacuum becomes non-uniform for very large L_s once T stays fixed. Of course, no IR singularities appear as long as L_s stays finite as well. It is natural that there is some critical volume $(L_s^c)^3$ beyond which the expansion breaks down. Actually, this critical volume depends on the specific subtraction scheme. Suppose, for instance, that at the temperature under consideration the interaction is described to the two leading orders of the (renormalized) gauge coupling g by zero mass excitations. That means all fields are massless at tree level, and they stay massless to one loop order, that is the corresponding renormalized vertex functions $\Gamma_R^{(2)}(p)$ satisfy

$$\Gamma_R^{(2)}(p) \Big|_{1\text{loop}} = O(p) \text{ as } p \rightarrow 0. \quad (12)$$

Then

$$L_s^c T \leq \frac{\text{const}}{g^\epsilon}, \quad (13)$$

with $0 < \epsilon < 2$, preserves the ordering of the perturbation series, although it is no longer an expansion in powers of the gauge coupling. This prescription was applied in [10] to the SU(3) gauge theory by imposing a lattice cutoff, to show that indeed in small volumes, i.e. on moderate sized lattices, the Polyakov loop correlation is well described by massless perturbation theory at all distances, at least for temperatures of about twice the transition temperature T_c .

In this article we do not impose the zero mass constraint Eq. (12), but consider the "mass of the gluons" to be a thermally generated quantity (i.e. by the collective interaction mediated by the heat bath). If we are not close to T_c , a renormalization procedure that takes this property into account works even better than that one given in [10]. Let us comment on this. Following conventional wisdom [14] the thermal field theory is considered as a branch of the zero temperature field theory (however cf. [15]). In particular, there is clear evidence that all ultraviolet (UV) singularities at any T are removed by the counterterms at zero temperature [16,17]. This way T -dependent properties emerge in a natural way,

being governed by the renormalization group equations. However, from a more general point of view one can object that there is a long way from zero temperature to the deconfined phase, say, crossing at least one phase transition. One cannot expect the renormalization group flow to be essentially perturbatively behaved in all the regimes, in particular not too close to the transition temperature T_c (at least not before a closer inspection). The picture can be refined by choosing a normalization temperature $T_0 \geq T_c$. This in turn determines T -dependent properties of observables once renormalized quantities at T_0 are given. Still, within the weak coupling expansion, they are related to the $T = 0$ quantities by finite renormalizations.

Let us confine our attention to Yang-Mills theories again. As at zero temperature, the Ward identities which follow from BRS invariance of the effective (i.e. gauge fixed) theory insure that the vacuum polarisation tensor $\Pi_{\mu\nu}$ vanishes at zero momentum, $\Pi_{\mu\nu}(0) = 0$, to all orders of perturbation theory (in the infinite volume case, of course). This is no longer the case at finite T , in particular $\Pi_{00}(0) \neq 0$ already to one loop order. Nevertheless, this quantity stays UV finite. Of course, for non-Abelian gauge theories $\Pi_{00}(0)$ is not a gauge invariant object (although gauge-independent to one loop order). Normalization conditions are a priori gauge dependent because non-invariant quantities evolve ultraviolet singularities and need subtractions. These constraints should ultimately be converted to gauge invariant conditions.

In principle we can use the fact that, in the SU(2) gauge theory at the critical temperature, the correlation length becomes very large, and impose the constraint that $\Pi_{\mu\nu}(0) = 0$ at T_c . The temperature range we consider here is far off T_c in the sense that it makes only a small difference whether we normalize at T_c or at zero temperature[†]. We exploit this and normalize the mass at zero temperature, i.e., we do not apply any mass subtraction at all. As before, this would cause IR problems in infinite volume. But again, IR singularities can be avoided, by now imposing the volume constraint

$$L_s^c T \leq \frac{\text{const}}{g^\epsilon}, \quad 0 < \epsilon < 1, \quad (14)$$

[†] We expect that the bare coupling g_0 , Eq. (6), is related to the physical length scale $a(g_0)$ by the asymptotic scaling relation within an error of 10-15% (at least in the range $T \geq 3T_c$ considered here, cf. below). On the other hand, the change made by imposing zero T normalization is much smaller.

that is with somewhat smaller ϵ compared to Eq. (13). This constraint maintains the ordering of the weak coupling expansion. The proof is only slightly different from that of Eq. (13), as given in [10], and is outlined in Appendix B. Let us emphasize that this scheme does not mean resummation of the perturbation series [18,19], which so far lacks rigorous justification. Rather, we use it to describe the short distance region as alluded to in the introduction.

2.3. Zero momentum modes.

We work in a finite, periodic volume. So necessarily the field configurations constant in space-time pay their respects to us. In principle they evolve unduly complicated behaviour. The classical vacuum degenerates to the toron manifold [20,21], but only a finite number of those configurations are quantum mechanically stable, which gives rise to the division of the physical Hilbert space into orthogonal subspaces of superselection character, so called central sectors. The hamiltonian approach allows a detailed investigation of the low lying energy spectrum of (asymptotically free) field theories [22–24]. It appears that the central sectors show exact degeneracy to all orders of perturbation theory. From the point of view of perturbation theory as the saddle point expansion, zero mode contributions to local observables can be given explicitly only in special circumstances [25]. In the large volume limit these contributions become arbitrary small. What is left is the infinite volume perturbation theory which does not exhibit IR singularities [26].

At non-zero temperature T the situation is somewhat different [10]. We cannot take the limit $L_s \rightarrow \infty$ at fixed T . Zero modes are therefore always present. Fortunately, as pointed out in [10], if the spatial volume is not too small, the thermal zero modes (constant gauge fields pointing in T -direction) have Gaussian decay properties. This holds if

$$L_s T \geq \frac{\text{const}}{g^{2/3}}, \quad (15)$$

and if $L_s T$ is sufficiently large to insure that $\Pi_{00}(0) < 0$. For $SU(2)$ this is the case for $L_s > 2T^{-1}$. As long as we consider thermal observables and their correlations, i.e. those which depend explicitly only on the gauge field in T -direction, the leading zero mode contributions can be worked out explicitly. This is the case for the Polyakov loops.

Combining the above mentioned results, it follows that the weak coupling

expansion proposed here is valid in the range

$$\frac{c_1}{g^{2/3}} \leq L_s T \leq \frac{c_2}{g^{\epsilon}}, \quad \frac{2}{3} < \epsilon < 1. \quad (16)$$

For the gauge group $SU(2)$ the constants c_1 and c_2 assume values about 2 and 9, respectively. The ideal case appears to be $L_s T \approx 6$.

2.4. Polyakov Loop Correlations.

To start the perturbative framework, an admissible gauge fixing [27] must be imposed which divides out the unphysical gauge degrees of freedom. This allows us to restrict attention to a neighbourhood Ω of $U(x; \mu) \equiv 1$. As the appropriate gauge fixing in the thermal situation we choose the static, time averaged Landau gauge [28] (STALG) first proposed in Ref. [29]. We then parametrize Ω by

$$U(x; \mu) = \exp A_\mu(x), \quad x \in \Lambda, \quad \mu = 0, \dots, 3, \quad (17)$$

where the Lie algebra valued gauge field A is subject to the constraints

$$\begin{aligned} & \frac{1}{L_0} \sum_{x_0} \sum_{i=1}^3 \partial_i^* A_i(x_0, \mathbf{x}) \\ & \equiv \frac{1}{L_0} \sum_{x_0} \sum_{i=1}^3 (A_i(x_0, \mathbf{x}) - A_i(x_0, \mathbf{x} - \hat{i})) = 0, \end{aligned} \quad (18)$$

and

$$A_0(x_0, \mathbf{x}) \equiv A_0(\mathbf{x}). \quad (19)$$

The Polyakov loop L and the correlation $P(\mathbf{x})$ become

$$L(\mathbf{x}) = \frac{1}{N_c} \text{tr} \exp (L_0 A_0(\mathbf{x})) \quad (20)$$

and

$$\begin{aligned} P(\mathbf{x}) \approx & \frac{1}{N_c^2} \left[\frac{1}{4} L_0^4 < \text{tr} A_0^2(\mathbf{x}) \cdot \text{tr} A_0^2(0) >_{c,1} \right. \\ & - \frac{1}{36} L_0^6 < \text{tr} A_0^3(\mathbf{x}) \cdot \text{tr} A_0^3(0) >_{c,0} \\ & \left. + \frac{1}{24} L_0^6 < \text{tr} A_0^2(\mathbf{x}) \cdot \text{tr} A_0^4(0) >_{c,0} \right], \end{aligned} \quad (21)$$

where $\langle \cdot \rangle_{c,i}$ denotes the set of connected diagrams with at most i loops. The zero modes are separated off by

$$A_\mu(x) = gW_\mu(x) + B_\mu, \quad \sum_{x \in \Lambda} W_\mu(x) = 0. \quad (22)$$

g denotes the renormalized coupling constant. Let us write

$$P(R) = P_{lead}(R) + P_{1loop}(R) + P_{zm}(R). \quad (23)$$

We restrict attention to separation of the loops in the direction of one of the spatial lattice axes only, i.e. $\mathbf{x} = (R, 0, 0)$ and henceforth write $P(R)$. The resulting expression of $P(R)$ for a general number of colours N_c is given in Appendix A. There we also point out the differences in the renormalization prescription compared to the one given in [10]. Here we only stress some prominent features.

The leading order is given by

$$P_{lead}(R) = \frac{N_c^2 - 1}{8N_c^2} T^{-2} g^4 (\Delta(R))^2, \quad (24)$$

with the propagator (cf. Appendix A)

$$\begin{aligned} \Delta(R) &= \frac{1}{L_s^3} \sum_{\mathbf{k} \neq 0} \frac{\cos k_1 R}{\hat{\mathbf{k}}^2} \\ &= a_0(R) - \frac{0.22578}{L_s} + \frac{R^2 - 0.045}{6L_s^3} + O(L_s^{-5}), \end{aligned} \quad (25)$$

where

$$a_0(R) = \int_{-\pi}^{\pi} \frac{d^3 k}{(2\pi)^3} \frac{\cos k_1 R}{\hat{\mathbf{k}}^2} = \frac{1}{4\pi R} + O(R^{-n}), \quad (26)$$

for any positive integer n . $\Delta(R)$ has considerable finite-size dependence.

From the above discussion and Appendix B, it is clear that there is an IR singularity in P_{1loop} as $L_s \rightarrow \infty$, which comes from the one loop correction $g^2 \delta \Delta(R)$ to the propagator. Let us explain it in more detail. We write

$$\delta \Delta(R) = \frac{1}{L_s^3} \sum_{\mathbf{k} \neq 0} \frac{\cos k_1 R}{(\hat{\mathbf{k}}^2)^2} \Pi_{00}(\mathbf{k}), \quad (27)$$

where $g^2 \Pi_{\mu\nu}(\mathbf{k})$ now denotes the one loop part of the vacuum polarization. The singularity arises at $\mathbf{k} = 0$. For small momentum, $\Pi_{00}(\mathbf{k})$ admits the expansion

$$\Pi_{00}(\mathbf{k}) = \Pi_{00}(0) + c |\hat{\mathbf{k}}| + O(\mathbf{k}^2 \log |\mathbf{k}|). \quad (28)$$

Values of $\Pi_{00}(0)$ and of c are given in Table 1, for the gauge group $SU(2)$ and for various lattice sizes. Appropriate methods to derive large volume expansions of finite momentum sums are developed in Ref. [30]. Working out the singular term of $\delta \Delta(R)$, we obtain

$$\begin{aligned} \delta_1 \Delta(R) &\equiv \frac{1}{L_s^3} \sum_{\mathbf{k} \neq 0} \frac{\cos k_1 R}{(\hat{\mathbf{k}}^2)^2} \Pi_{00}(0) = L_s \frac{1.675}{16\pi^2} \Pi_{00}(0) + O(1) \\ \delta_2 \Delta(R) &\equiv \frac{1}{L_s^3} \sum_{\mathbf{k} \neq 0} \frac{\cos k_1 R}{(\hat{\mathbf{k}}^2)^2} c |\hat{\mathbf{k}}| = \frac{1}{4\pi^2} c \log L_s^2 + O(1) \end{aligned} \quad (29)$$

as $L_s \rightarrow \infty$, and

$$\delta \Delta(R) - \delta_1 \Delta(R) - \delta_2 \Delta(R) = O(1) \quad (30)$$

converges in that limit. We now note the following. For values of L_s larger than 30, the linear divergence starts to be the dominant contribution of the one loop correction to the propagator $\Delta(R)$. We take this as the signal that L_s has reached its critical value $L_s^c(T)$. This is not the case for smaller volumes.

As stated before, for sufficiently asymmetric volumes $L_s > T^{-1}$, the effective zero mode action favours Gaussian behaviour of the thermal modes. The corresponding quadratic term is given by

$$S_2(B_0) = \frac{1}{2} L_s^3 T^{-1} \Pi_{00}(0) \text{tr } B_0^2. \quad (31)$$

From the first term in Eq. (21) we obtain the leading zero mode contribution to $P(R)$,

$$P_{zm}(R) = \frac{N_c^2 - 1}{8N_c^2} T^{-2} \left\{ 2g^2 \Delta(R) L_s^{-3} (-\Pi_{00}(0))^{-1} + [L_s^{-3} (-\Pi_{00}(0))^{-1}]^2 \right\}. \quad (32)$$

Corrections are down by $O(g L_s^{-9/2})$, with the same distance behaviour as the first term in Eq. (32). At small distances R the first term dominates, whereas the second one is small. At larger R , however, the second term starts to dominate because of its R -independence.

Let us remark at this point that $\Pi_{00}(0)$, being a zero momentum quantity, is most sensitive to the spatial as well as the thermal size of the system. Both finite-size corrections work in the opposite directions, as can be seen from Table 2 and Fig. 1. In the infinite spatial volume continuum limit, $-\Pi_{00}(0)$ converges to $(N_c/3)T^2$, which in lattice units means

$$\lim_{L_0 \rightarrow \infty} \lim_{L_s \rightarrow \infty} \Omega(L_0, L_s) = 1, \quad (33)$$

with

$$\Omega(L_0, L_s) = \frac{3}{N_c} L_0^2 (-\Pi_{00}(0)).$$

It is remarkable that particularly on the 4×24^3 lattice the latter quantity is very close to 1.

3. Monte Carlo calculations

To simulate SU(2) gauge theory we employ an $\omega = 2$ overrelaxation algorithm [31], supplemented with heatbath updates [32] in order to ensure ergodicity. Following Ref. [33], we use three overrelaxation sweeps per heatbath sweep. This overrelaxation algorithm is expected to decorrelate configurations more efficiently compared to the standard heatbath method. The CPU time required per sweep is, moreover, shorter in the overrelaxation method. In our calculations, performed at temperatures well into the deconfined phase, we expect the gain in decorrelation time to be moderate; a test run on a 4×8^3 lattice revealed a net efficiency gain for the overrelaxation update by a factor of around two.

In order to reduce the variance in our measurements we implement the multihit technique proposed in Ref. [34]. For SU(2) this method entails replacing link variables U in measured operators by

$$\bar{U} = \frac{\int dU U \exp(-S_W)}{\int dU \exp(-S_W)} = X \frac{I_2(\lambda)}{\lambda I_1(\lambda)}, \quad (34)$$

with $U X^\dagger = \sum_p U_p$ the sum of plaquettes which share the link U . U_p is the ordered product of link variables belonging to the plaquette p , with U ordered to the left. Furthermore, $\lambda = \beta \det(X)^{1/2}$ [35]. The integration is over one link variable U and I_1 and I_2 denote modified Bessel functions. In computing Polyakov loop correlations, this replacement can be done whenever the separation between the two loops is larger than one lattice unit. The effectiveness of the multihit method we expect to depend upon the operator studied. To test its usefulness for the present problem, we measured the correlations both with and without multihit links on a 4×16^3 lattice. For 4×10^5 sweeps, the error bars for the connected Polyakov loop correlations were reduced by about a factor of two, with a slightly larger reduction for larger separations. Without multihit links we were in fact not able to obtain reasonable results for $R \geq 7$. The multihit technique is therefore an important ingredient in our study of the long distance behaviour. We also checked that mean values, obtained from the two different methods, were consistent.

Using these techniques, we performed simulations on $4 \times L_s^3$ lattices for different values of L_s at the couplings $\beta = 2.80$ and 3.00 , which correspond to $T/T_c \simeq 3.5$ and 6 , respectively [36]. For $L_s = 16$ we carried out 8×10^5 and 4×10^5 sweeps at $\beta = 2.80$ and 3.00 , respectively, measuring every 20th iteration. We also performed calculations for $L_s = 24$ at $\beta = 2.80$ and for $L_s = 32$ at $\beta = 3.00$. On these larger lattices we carried out 1.5×10^5 sweeps, measuring every 25th iteration. For thermalization purposes we discard at least 25,000 sweeps. Statistical uncertainties quoted later are Jackknife errors [37], obtained using typically 16 Jackknife bins. We checked that the results were reasonably stable with respect to changes of the number of Jackknife bins used.

In Table 3 we collect the MC data for the unsubtracted Polyakov loop correlations $c(R)$, calculated as

$$c(R) = \frac{1}{3L_s^3} \sum_{i=1}^3 \sum_{\mathbf{x}} < L(\mathbf{x}) L(\mathbf{x} + i\hat{R}) >, \quad (35)$$

for the various lattice sizes and β values studied. From $c(R)$ we determine the connected correlation function $P(\mathbf{x})$ according to Eq. (9), which for gauge group SU(2) reads

$$P(\mathbf{x}) = c(R) - < |\vec{L}| >^2. \quad (36)$$

As discussed in Sect. 2.1, the use of this definition is justified only if the tunneling correlation length is much larger than the spatial extent of the lattice, $\xi_t \gg L_s$. We therefore performed some measurements of ξ_t on smaller lattices. We found that $\xi_t \simeq 100$ and $\xi_t > 500$ on $4 \times 6^2 \times 24$ and $4 \times 8^2 \times 32$ lattices, respectively. Since the tunneling correlation length is expected to grow exponentially with increasing cross-sectional area, we therefore estimate that the tunneling contribution indeed should be effectively a constant on $4 \times L_s^3$ lattices with $L_s \geq 16$. This will be confirmed by the results for the functional form of $c(R)$, to be presented in the next section.

4. Results and analysis.

Let us reintroduce the notation a for the lattice spacing. If we increase β on a given periodic $L_0 \times L_s^3$ lattice, the physical volume $\mathcal{L}_s^3$ of the torus shrinks and the temperature T is increased at the same time, whereas the bare quantities $L_0 = aT^{-1}$ and $L_s = \mathcal{L}_s a^{-1}$ stay fixed. That provides scaling laws which differ from the infinite volume case at zero temperature.

In order to combine the MC data with the weak coupling results, we choose as the appropriate scale μ where the bare quantities are mapped onto renormalized ones proportional to T . The wave function renormalization for the electric gluons (the only one that matters here) is done at spatial momentum

$$\kappa_0 = \pi T (1, 1, 1), \quad (37)$$

that is

$$\mu = |\kappa_0| = 3^{1/2} \pi T = \frac{3^{1/2}}{2L_0} \Lambda, \quad (38)$$

with the UV cutoff $\Lambda = 2\pi a^{-1}$. The renormalized coupling constant g is determined by adjusting the weak coupling result for the connected correlation function $P(R)$ to the numerical data at some fixed distance R_0 . We choose the lowest possible R_0 for the β 's we have,

$$R_0 = 1a = \frac{3^{1/2} \pi}{L_0} \mu^{-1}. \quad (39)$$

This is mainly for two reasons.

1. The Monte Carlo data evolve their smallest errors at the smallest distance.
2. If there is a non-empty perturbative small distance region at the moderate β values available, it encloses the smallest distance in lattice units. Starting here we determine the perturbative horizon.

Let us estimate the deviations from the asymptotic scaling behaviour. Changing β , but simultaneously fixing the lattice size $L_0 \times L_s \times L_s$, forces the renormalization group equations for the correlation functions to be governed by the beta-function W^1 ,

$$\begin{aligned} W(\beta, \frac{T}{\Lambda}, \frac{\mu}{L_s \Lambda}, \frac{\mu}{\Lambda}) &= \Lambda \frac{d}{d\Lambda} g_0(g, \mu, L_0, L_s, \Lambda) \Big|_{g, \mu, L_0, L_s, \text{fixed}} \\ &= -w_0 \left(\frac{2N_c}{\beta} \right)^{3/2} + O(\beta^{-5/2}), \end{aligned} \quad (40)$$

where $g_0^2 = 2N_c/\beta$, with the corresponding (1 loop) renormalization group flow

$$\left(\frac{\Lambda'}{\Lambda} \right)^2 = \exp \left(\frac{1}{\gamma_0} \frac{\beta' - \beta}{2N_c} \right). \quad (41)$$

[†] We prefer to write W instead of the commonly used β in order to avoid confusion with the bare coupling β .

Asymptotic scaling implies that $w_0 = \gamma_0 = \frac{11}{3} \frac{N_c}{16\pi^2}$. A similar flow follows for the renormalized coupling constant g under the change of the renormalization scale $\mu \rightarrow \mu'$. Now the scale μ is chosen in such a way that

$$\frac{\mu}{\mu'} = \frac{\Lambda}{\Lambda'},$$

so that

$$\frac{1}{(g')^2} - \frac{1}{g^2} = \frac{1}{\tau} \frac{\beta' - \beta}{2N_c}, \quad (42)$$

where $\tau = \tau(\beta, \frac{T}{\Lambda}, \frac{1}{2\Lambda}, \frac{\mu}{\Lambda})$ is equal to one in the case of asymptotic scaling. From Table 4 we infer that for the lattices

$$\begin{aligned} 4 \times 16^3 : \tau &= 0.88(3) \\ 4 \times 24^3 : \tau &= 0.86(3), \end{aligned} \quad (43)$$

which implies 12-14 % scaling violations.

Let us mention here that the violation of rotational invariance in the connected correlation function $P(\mathbf{x})$ for small separations $\mathbf{x}$ is at most of the size of the scaling violation at that distance, and it decreases with increasing separation. This is illustrated by Fig. 2, where $P(\mathbf{x})$ as a function of $R = |\mathbf{x}|$ for $R \leq 4$, on the 4×16^3 lattice at $\beta = 3.00$ is shown. The solid symbols show the on-axis correlations.

The MC and weak coupling results for the various lattice sizes and β values are shown in Figs. 3-6. In Figs. 5 and 6 we show the combined results for the two coupling values considered here. The MC results for the 4×32^3 lattice are compared to the weak coupling results computed on the 4×24^3 lattice, for the following reason. As already explained in Sect. 3, the linear divergence of the one loop correction as $L_s \rightarrow \infty$ starts to dominate the other corrections for $L_s = 32$. Already at distance 3 (in lattice units) it provides a 50 % contribution compared to the leading order, making at least third order corrections indispensable. It appears that we have reached the boundary of the weak coupling window. On the other hand, the Monte Carlo data show only weak finite-size dependence at short distances. That justifies the use of the 4×24^3 lattice instead, as long as we stay below the perturbative horizon.

At short distances R , the weak coupling results for $P(R)$ reproduce the numerical data very well. There is, of course, a perturbative horizon beyond which the results become worse. As the first criterion of the maximal distance where

second order perturbation theory is trustworthy, the weak coupling radius R_w , we demand that the second order contributions should not provide more than 30 % correction to the leading order (see Table 4). Furthermore, comparing the results for different lattice sizes, we see finite-size effects. As pointed out in Sect. 2, already the leading term P_{lead} has considerable volume dependence. This finite-size correction evolves the same distance dependence as the zero mode term P_{zm} , but it has opposite sign. On moderate sized lattices both contributions are of comparable magnitude. Hence, the total finite-size dependence appears to be smaller than one would expect from looking at each term separately. Nevertheless, in order to be sure that the finite volume does not influence the short distance range drastically, we restrict the latter to $R < L_s/3$. Let us mention again that the zero mode part P_{zm} of $P(R)$ contains a distance independent term of the order L_s^{-6} . Thus, it becomes leading for large R . Whereas on the 4×24^3 lattice it is always very small, it is non-negligible on the 4×16^3 lattice at distances $R \geq 5$. This, however, is already beyond the weak coupling radius according to the above criterion.

Let us now apply the program described in the introduction. First of all we need the perturbative horizon. From the data on the 4×16^3 lattice it is difficult to argue that we really observe screening. At distances where the weak coupling results are reliable to our second order calculation ($R \leq 4$), they agree very well with the data. Beyond, higher orders could give improved results. On the other hand, the larger lattices show that we can trust the perturbative results up to about twice that distance. But now there are deviations between the Monte Carlo and the analytical data for distances $R \geq 4$. We take that as the signal that the intermediate distance range is approached. Of course, higher orders may increase the perturbative horizon a little bit. Nevertheless, the correlations start to decrease faster, which signals the onset of screening. This is the starting point for the determination of the Debye mass.

Next we perform a detailed analysis of the functional form of $P(R)$ at larger separations. In Figs. 7 and 8 we show MC data for the effective screening mass $\mu(R)$, defined by

$$\frac{P(R-1)}{P(R)} = \frac{\cosh(\mu(R)(L_s/2 - R + 1))}{\cosh(\mu(R)(L_s/2 - R))}, \quad (44)$$

as a function of $t = \ln(\frac{R}{R-1})$. At large R , we have $t \approx 1/R$. A generic Debye

screened form of the connected correlation,

$$P(R) = c \frac{\exp(-\mu_D R)}{R^\alpha}, \quad (45)$$

corresponds to a linear dependence of $\mu(R)$ on t , $\mu(R) = \mu_D + \alpha t$, provided that periodicity effects can be neglected. This is at least the case for the larger lattices since, because of the rapid increase in the statistical uncertainty, all data are for $R \leq 6$. The figures show that finite-size effects are very small and support a screened form of the potential. The straight lines shown have a slope $\alpha = 2$ and are fits of the data from the larger lattices, which are discussed below. At distances $R \geq 4$ ($t \leq 0.3$) we see that the data are very well described by Eq. (45) with a power $\alpha = 2$. At smaller distances, on the other hand, we see significant deviations from this simple form. These deviations together with the fact that statistical uncertainties increase rapidly with R makes a determination of α including error bars difficult. The curvature seen in the data at small R suggests however a lower bound on α . While the slope at distance $R = 2 - 3$ corresponds to an effective power $\alpha \approx 1.2$, we estimate in this way that the value appropriate for the large distance behaviour satisfies $\alpha > 1.7$. It is not surprising to find that typical values for α obtained in earlier investigations on smaller lattices are below this bound. While these values may yield an effective description of the potential at intermediate distances, they do probably not reflect the large distance behaviour. This conclusion refers, of course, to the temperature range studied here. It is, for example, possible that α assumes a value near one closer to the phase transition, as indicated by the results in Refs. [12,13].

To further study the functional form of the Polyakov loop correlations, and to obtain estimates for the screening mass, we perform fits to the unsubtracted correlations. The functional form considered is

$$f(R) = c + b \left(\frac{\exp(-\mu_D R)}{R^\alpha} + \frac{\exp(-\mu_D(L_s - R))}{(L_s - R)^\alpha} \right), \quad (46)$$

where the second term takes periodicity into account, and where R lies in the interval $R_{min} \leq R \leq R_{max}$. In all the fits we choose $R_{max} = 8$; for the two smaller lattices this is the maximal distance accessible, while for the largest lattice the statistical errors in the correlations start to dominate. It is important to note that this value of $R_{max} = 2T^{-1}$ is large enough to allow for measurements beyond the estimated perturbative horizon. To get a reliable estimate for the

screening mass it is furthermore clear that we should require $R_{\min} > 3$, as we otherwise include points belonging to the perturbative regime (cf. Figs. 3-6). This important requirement has, unfortunately, not been fulfilled in most studies carried out so far.

To take into account the effect of correlations between measurements of different $c(R)$'s we use so-called correlated χ^2 fits [2,38]. We thus calculate χ^2 using the formula

$$\chi^2 = \sum_{R_1, R_2} (c(R_1) - f(R_1)) V_{R_1, R_2}^{-1} (c(R_2) - f(R_2)), \quad (47)$$

where $c(R)$ denotes the (unsubtracted) Polyakov loop correlation (Eq. (35)). V_{R_1, R_2} is the covariance matrix for the correlation of measurements at distances R_1 and R_2 , estimated as

$$V_{R_1, R_2} = \frac{1}{N-1} \frac{1}{N} \sum_{k=1}^N (c_k(R_1) c_k(R_2) - c(R_1) c(R_2)), \quad (48)$$

where the data are divided into N blocks, with $c_k(R)$ the average over the k -th block.

Ideally one would like to fit all the four parameters in Eq. (46). With limited statistics at hand, we decide instead to keep the power α fixed, using the value $\alpha = 2$ suggested by our findings above. In Tables 5 and 6 we list the values obtained from the resulting three-parameter fits for lattice sizes 4×24^3 at $\beta = 2.80$ and 4×32^3 at $\beta = 3.00$ respectively. The same fits on the 4×16^3 lattice produced similar results, as expected from the small finite-size effects observed above. In agreement with Figs. 7 and 8 we find that good fits require $R_{\min}$ to be not smaller than 3-4. For comparison, results for the same lattice size and β value as in Table 5, but with $\alpha = 1$ are shown in Table 7. We see that from the goodness of the fits alone one cannot rule out $\alpha = 1$, which underlines the difficulty in precisely determining this parameter. However, we note that the fitted parameters are much more stable with respect to variations of $R_{\min}$ for the choice $\alpha = 2$ than for $\alpha = 1$, which we take as additional evidence for a value of α near two. In estimating the screening mass, we therefore assume $\alpha = 2$ which, using $R_{\min} = 4$, gives

$$\begin{aligned} \beta = 2.80 : \quad \mu_D/T &= 2.05(10) \\ \beta = 3.00 : \quad \mu_D/T &= 2.02(7). \end{aligned} \quad (49)$$

Let us compare our data with continuum perturbation theory in infinite spatial volume, which, for the connected correlation at large R , predicts the screened form in Eq. (45) with $\alpha = 2$ [1,18]. As discussed above, such a behaviour is completely consistent with our results. The value we find for the screening mass is, furthermore, surprisingly close to the "classical" value μ_{cl} , derived from the vacuum polarization at zero momentum in the infinite volume scaling limit [1] by applying some (non-rigorous) resummation of the perturbation series [18]. Using g from the larger lattices, as given in Table 4, we have values

$$\mu_{cl}/T = 2 \left(\frac{2}{3} \right)^{1/2} g \approx \begin{cases} 1.84, & \beta = 2.80; \\ 1.77, & \beta = 3.00, \end{cases} \quad (50)$$

which are around 10% smaller than our results in Eq. (49). Similar behaviour for the gauge group SU(3) has been obtained by Gao [13].

Of course, as already mentioned in the introduction, the ideal situation would be to extract the Debye mass at very large distances where only the screening tail of the complicated gluon dynamics survives. However, it is difficult to obtain such data with a sufficient statistical accuracy. Therefore, a careful comparison with the weak coupling expansion, which has been missing in most previous studies, is important. From the comparison in Figs. 3-6, we find a clear indication that our calculations cover a regime where nonperturbative effects are important; we estimate the perturbative horizon to be $R \simeq T^{-1}$. The observation of screening, on the other hand, refers to the region $T^{-1} \leq R \leq 2T^{-1}$, which increases our confidence in the results obtained. Our results furthermore suggest that the potential approaches a screened form already slightly beyond the perturbative horizon. It thus appears to us that the intermediate range between the perturbative short distance domain and the long distance screening range is rather small, at least at temperatures 3.5-6 T_c .

5. Concluding Remarks.

We investigated the heavy quark potential in the deconfined phase of the SU(2) gauge theory. The temperature range considered is 3.5-6 T_c . At small distances the potential is well described by finite volume perturbation theory. The short distance horizon, beyond which nonperturbative effects become measurable, is of the order of the inverse temperature. Slightly beyond the horizon the potential approaches the screening form

$$V(R) \approx c \frac{\exp(-\mu_D R)}{R^2}.$$

The values we obtain for the Debye mass μ_D are around 10% higher than the classical value Eq. (50). In the temperature range considered, the ratio $\mu_D(T)/T$ depends only weakly on T .

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Appendix.

A. Weak coupling results.

We give the perturbative result for the Polyakov loop correlation function $P(R)$, Eq. (21). We again set the lattice spacing a equal to 1. The gauge group is assumed to be $SU(N_c)$ with arbitrary N_c . Feynman sums on the $L_0 \times L_s^3$ lattice range over a discrete set of the Brillouin zone, i.e.

$$\frac{T}{L_s^3} \sum_{\mathbf{k}} = \frac{1}{L_0 L_s^3} \sum_{\mathbf{k}} \quad (\text{A.1})$$

ranges over

$$\begin{aligned} k_0 &= \frac{2\pi}{L_0} \nu_0, & -\frac{L_0}{2} < \nu_0 \leq \frac{L_0}{2}, & \nu_0 \in \mathbb{Z}, \\ k_i &= \frac{2\pi}{L_s} \nu_i, & -\frac{L_s}{2} < \nu_i \leq \frac{L_s}{2}, & \nu_i \in \mathbb{Z}, \quad i = 1, 2, 3. \end{aligned} \quad (\text{A.2})$$

Some common notations are as follows. Latin indices range from 1 to 3, greek indices from 0 to 3.

$$\begin{aligned} \widehat{k}_\mu &= 2 \sin \frac{k_\mu}{2} \\ c_{k_\mu} &= \cos \frac{k_\mu}{2} \\ \widehat{\mathbf{k}}^2 &= \sum_{i=1}^3 \widehat{k}_i \widehat{k}_i \\ \widehat{k}^2 &= \widehat{\mathbf{k}}^2 + \widehat{k}_0^2 = \sum_{\mu=0}^3 \widehat{k}_\mu \widehat{k}_\mu. \end{aligned} \quad (\text{A.3})$$

The Polyakov loop correlation function $P(R)$ is given as follows.

$$P(R) = P_{\text{lead}}(R) + P_1 \text{ loop}(R) + P_{\text{zm}}(R) \quad (\text{A.4})$$

with:

1. Lowest order

$$P_{\text{lead}}(R) = \frac{N_c^2 - 1}{8N_c^2} T^{-2} g^4 (\Delta(R))^2, \quad (\text{A.5})$$

with the propagator

$$\Delta(R) = \frac{1}{L_s^3} \sum_{\mathbf{k} \neq 0} \frac{\cos k_1 R}{\widehat{\mathbf{k}}^2}. \quad (\text{A.6})$$

2. Zero mode contribution

$$P_{\text{zm}}(R) = \frac{N_c^2 - 1}{8N_c^2} T^{-2} \left\{ 2g^2 \Delta(R) L_s^{-3} (-\Pi_{00}(0))^{-1} + [L_s^{-3} (-\Pi_{00}(0))^{-1}]^2 \right\}, \quad (\text{A.7})$$

where Π_{00} is the one loop vacuum polarization as given below.

3. Correction to the order g^6 . We write

$$P_1 \text{ loop} = g^6 (P_{\text{vac}} + P_{\text{vert}} + P_{3g}). \quad (\text{A.8})$$

3a. Propagator corrections

$$P_{\text{vac}}(R) = \left(\frac{N_c^2 - 1}{8N_c^2} T^{-2} \right) 2 \Delta(R) \cdot \frac{1}{L_s^3} \sum_{\mathbf{k} \neq 0} \frac{\cos k_1 R}{(\widehat{\mathbf{k}}^2)^2} \Pi_{\text{ren}}(\mathbf{k}), \quad (\text{A.9})$$

$$\Pi_{\text{ren}}(\mathbf{k}) = \Pi_{00}(\mathbf{k}) - \frac{1}{2} \widehat{\mathbf{k}}^2 \Sigma_{11}(\kappa_0), \quad (\text{A.10})$$

where

$$\kappa_0 = \pi T (1, 1, 1), \quad (\text{A.11})$$

$$\Pi_{00}(\mathbf{k}) = N_c T.$$

$$\begin{aligned} & \left[\frac{1}{L_s^3} \sum_{\mathbf{k}' \neq 0, \mathbf{k}} \frac{1}{(\mathbf{k} - \mathbf{k}')^2 \widehat{\mathbf{k}}^2} \left\{ (\widehat{\mathbf{k}' - 2\mathbf{k}})^2 - \frac{((\mathbf{k}' - \mathbf{k})^2 - \widehat{\mathbf{k}}^2)^2}{\widehat{\mathbf{k}}^2} \right\} \right. \\ & + \frac{1}{2} \frac{1}{L_s^3} \sum_{\mathbf{k}_0' \neq 0} \sum_{\mathbf{k}'} \frac{1}{(\mathbf{k}' - \mathbf{k})^2 \widehat{\mathbf{k}}^2} \cdot \\ & \cdot \left\{ 2\widehat{\mathbf{k}}_0'^2 \sum_{i=1}^3 c_{k_i}^2 + 4c_{k_0}^2 \sum_{i=1}^3 (\widehat{\mathbf{k}}_i^2 + (\widehat{\mathbf{k}' - \mathbf{k}_i})^2) c_{k_i}^2 \right. \\ & + \left. \left(\widehat{\mathbf{k}}^2 + (\mathbf{k}' - \mathbf{k})^2 - \widehat{\mathbf{k}}^2 \right)^2 \frac{c_{k_0}^2}{\widehat{\mathbf{k}}_0'^2} \right\} \\ & + \frac{L_s^3 - 1}{12L_s^3} - 2v_1 - T^{-1}(2m_1 - m_2 + l_1 - \frac{1-T}{2}) \\ & + \widehat{\mathbf{k}}^2 \left\{ \frac{5}{12} v_1 + (1 - \frac{2}{N_c^2}) \frac{L_s^3 - 1}{24L_s^3} + \frac{1}{3T} (m_1 - \frac{1}{2N_c^2} m_2 + \frac{1}{2} l_1 - \frac{1-T}{4N_c^2}) \right\} \Big] \\ & - \frac{N_c}{12T}, \end{aligned} \quad (A.12)$$

with

$$\begin{aligned} m_1 &= \frac{T}{L_s^3} \sum_{\mathbf{k}_0 \neq 0} \sum_{\mathbf{k}} \frac{1}{\widehat{\mathbf{k}}_0^2 \widehat{\mathbf{k}}^2} \\ m_2 &= \frac{T}{L_s^3} \sum_{\mathbf{k}_0 \neq 0} \sum_{\mathbf{k}} \frac{\widehat{\mathbf{k}}_0^2}{\widehat{\mathbf{k}}^2} \\ l_1 &= T \sum_{\mathbf{k}_0 \neq 0} \frac{1}{\widehat{\mathbf{k}}_0^2} = \frac{1-T^2}{12T} \\ v_1 &= \Delta(0) = \frac{1}{L_s^3} \sum_{\mathbf{k} \neq 0} \frac{1}{\widehat{\mathbf{k}}^2}. \end{aligned} \quad (A.13)$$

$\Sigma_{11}(\mathbf{k})$ is the discretized version of the second derivative of $\Pi_{00}(\mathbf{k})$ with respect to the lattice momentum $\widehat{\mathbf{k}}_1$, cf. e.g. Appendix A in [10]. Furthermore, the mixed second derivatives are left out in the definition of $\Pi_{\text{ren}}(\mathbf{k})$. They provide only scaling violations and are of the order of 0.01%.

3b. Vertex corrections [10].

$$\begin{aligned} P_{\text{vert}}(R) &= -\frac{N_c^2 - 1}{8N_c^2} \frac{N_c}{T} \frac{1}{L_s^3} \sum_{\mathbf{l}} \cos l_1 R \\ & \cdot \frac{1}{L_s^3} \sum_{\mathbf{k}_1 \neq 0, \mathbf{l}} \frac{1}{\widehat{\mathbf{k}}_1 (1 - \widehat{\mathbf{k}}_1)^2} \frac{1}{L_s^3} \sum_{\mathbf{k}_2 \neq 0, -\mathbf{l}} \frac{1}{\widehat{\mathbf{k}}_2 (1 + \widehat{\mathbf{k}}_2)^2} \cdot \\ & \cdot \left[\widehat{\mathbf{l}}^2 - (1 + \widehat{\mathbf{k}}_2 - \widehat{\mathbf{k}}_1)^2 \right] \left(\frac{1}{12} + \frac{1 - \delta_{\mathbf{k}_1 + \mathbf{k}_2, 0}^3}{\widehat{\mathbf{k}}_1 + \widehat{\mathbf{k}}_2} \right) \\ & + (1 - \delta_{\mathbf{k}_1 + \mathbf{k}_2, 0}^3) \frac{(\widehat{\mathbf{k}}_1^2 - \widehat{\mathbf{k}}_2^2)((\widehat{\mathbf{k}}_1 - 1)^2 - (\widehat{\mathbf{k}}_2 - 1)^2)}{((\widehat{\mathbf{k}}_1 + \widehat{\mathbf{k}}_2)^2)^2} \\ & + \frac{2N_c^2 - 3}{12N_c^2} \sum_{i=1}^3 \widehat{\mathbf{k}}_{1i} (1 - \widehat{\mathbf{k}}_1)_i \widehat{\mathbf{k}}_{2i} (1 + \widehat{\mathbf{k}}_2)_i \Big]. \end{aligned} \quad (A.14)$$

3c. Three gluon exchange.

$$P_{3g}(R) = \frac{N_c^2 - 1}{8N_c^2} T^{-3} \frac{N_c^2 - 4}{12N_c} (\Delta(R))^3. \quad (A.15)$$

In the case of quark-quark interaction this term changes its sign. For $\text{SU}(2)$, i.e. $N_c = 2$, it vanishes.

In contrast to [10] we prefer to renormalize the self energies of the Polyakov loops in a minimal way. That implies the additional UV-finite counterterm contributions as given in [10] drop out.

B. Critical volume.

Consider any $\text{SU}(N_c)$ Yang-Mills theory at finite temperature T . Suppose that all ultraviolet subtractions are done as usual for massless theories at zero temperature except that we do not necessarily apply a mass subtraction at the temperature T . Then the ordering of the weak coupling series is preserved if the spatial extension L_s of the system stays below some critical value related to the gauge coupling g according to

$$L_s T \leq \frac{\text{const}}{g^\epsilon}, \quad 0 < \epsilon < 1. \quad (B.1)$$

The proof is very similar to that of the corresponding inequality Eq. (13) for the case with mass subtractions as given in Sect.5 and Appendix C of [10]. Here we stress only those steps that are different.

Let I_Γ be a Feynman integral which is represented by a one-particle irreducible Feynman diagram Γ with $m(\Gamma)$ loops. All external momenta are assumed to be non-exceptional. First of all we can show that for all Zimmermann subspaces H , the (lattice-) IR divergence degrees [39] satisfy

$$\text{degr}_H I_\Gamma > -\delta \quad (B.2)$$

with

$$\delta = 2m(\Gamma) - 2 + 2|B_2|, \quad (B.3)$$

where $|B_2|$ denotes the number of 2-point vertices of Γ that result from the gauge field measure.

Now let I_Γ be the Feynman sum that corresponds to Γ calculated in finite spatial volume L_s^3 . If $\delta > 0$, I_Γ will behave for large L_s according to

$$I(L_s) \sim L_s^{\delta-1} (\log L_s)^{m(\Gamma)}. \quad (B.4)$$

On the other hand, I_Γ contributes to the order $g^{\lambda(\Gamma)}$ with

$$\lambda(\Gamma) = 2m(\Gamma) - 2 + e(\Gamma) + 2|B_2|, \quad (B.5)$$

where $e(\Gamma)$ is the number of external lines joining the diagram Γ . Hence

$$I_\Gamma \sim g^{e(\Gamma)+1} (g L_s)^{2m(\Gamma)-3+2|B_2|} (\log L_s)^{m(\Gamma)}. \quad (B.6)$$

Now the constraint Eq. B.1 insures that for $L_s \leq L_s^c$

$$I_\Gamma(L_s) \sim g^{e(\Gamma)+1} (g^{1-\epsilon})^{2m(\Gamma)-3+2|B_2|} (\log g)^{m(\Gamma)}. \quad (B.7)$$

In order to prove Eq. B.2 we start with the theorem given in Appendix C of [10], which generalizes in the following way to our case of no mass subtraction. Let Γ be a one-particle irreducible diagram. Assume again that the ultraviolet renormalizations are done as usual, and that there are no IR mass subtractions of the 2-point functions at zero momentum. For all Zimmermann subspaces H , we then have

$$\text{degr}_H I_\Gamma > \min_{e, \lambda_1, \dots, \lambda_e} \eta_\Gamma(e; \lambda_1, \dots, \lambda_e), \quad (B.8)$$

with

$$\eta_\Gamma(e; \lambda_1, \dots, \lambda_e) = \sum_{\substack{B \in \mathcal{B}_\Gamma \\ r(B) \leq 3}} (r(B) - 3) + \sum_{i=1}^e \left[\sum_{\substack{B \in \mathcal{B}_\Gamma \\ r(B) \leq \lambda_i}} (r(B) - 3) - r(\lambda_i) \right]. \quad (B.9)$$

The minimum is over all collections of mutually disjoint, one-particle irreducible subdiagrams of Γ which do not contain external vertices of Γ , i.e. those with external lines. The prime on the first sum indicates summation only over internal vertices of Γ .

Next, we do the same graph theoretical rearrangements as in [10]. The only exception here is that Γ may have vertices B with two legs, namely those which arise from the gauge field measure. Let $|B_2|$ be their number. We end up with

$$\eta_\Gamma(e; \lambda_1, \dots, \lambda_e) \geq -m(\Gamma) + 1 - 2|B_2| - n_2,$$

where n_2 is the maximal number of disjoint one loop 2-point functions contained in Γ . Because there can be at most $m(\Gamma) - 1$ of them, we have $n_2 \leq m(\Gamma) - 1$, and hence Eq. B.2.

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Tables

Table 1.

Values for c and for $-\Pi_{00}(0)$ on lattices of various sizes.

$L_0 \times L_s^3$	c	$-\Pi_{00}(0)$
4×16^3	0.0874	0.0384
4×24^3	0.0758	0.0431
4×32^3	0.0689	0.0455
6×24^3	0.0465	0.0139

Table 2.

Values of $-3\Pi_{00}(0)/(T^2 N_c)$ for $SU(2)$ on various lattices.

L_s/L_0	L_0	$-3\Pi_{00}(0)/(2T^2)$
4	4	0.92
	∞	0.66
6	4	1.03
	∞	0.78
8	4	1.09
	20	0.83

Table 3.

The correlation function $c(R)$ for various lattice sizes and values of β . Also listed are the corresponding Polyakov loop values.

$\langle L(0)L(R) \rangle$	4×16^3 $\beta = 2.80$	4×24^3 $\beta = 2.80$	4×16^3 $\beta = 3.00$	4×32^3 $\beta = 3.00$
R				
1	0.19809(5)	0.19818(8)	0.23736(9)	0.23752(5)
2	0.18496(5)	0.18496(8)	0.22593(9)	0.22610(5)
3	0.18272(5)	0.18271(8)	0.22399(9)	0.22417(5)
4	0.18216(6)	0.18216(8)	0.22351(9)	0.22370(5)
5	0.18199(6)	0.18191(8)	0.22337(9)	0.22355(5)
6	0.18193(6)	0.18194(8)	0.22331(9)	0.22350(5)
7	0.18192(6)	0.18191(8)	0.2233(1)	0.22348(5)
8	0.18191(6)	0.18190(8)	0.2233(1)	0.22347(5)
$\langle \bar{L} \rangle$	0.42649(7)	0.42649(9)	0.4725(1)	0.47272(6)

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Table 4.

Renormalized coupling constant g , weak coupling radius $R_w < L_s/3$ (in lattice units) and correction to the leading order at R_w . The errors quoted are due to the statistical uncertainties in the Monte Carlo data at unit distance.

β	g_0^2	lattice size	g^2	R_w	%
2.80	1.43	4×16^3	1.2598(12)	4	12
		4×24^3	1.2777(15)	7	30
3.00	1.33	4×16^3	1.1757(13)	4	10
		4×24^3	1.1889(15)	7	32

Table 5.

Fit to connected Polyakov loop correlations calculated on the 4×24^3 lattice at $\beta = 2.80$, using Eq. (46) with $\alpha = 2$.

b	μ	c	R_{min}	df	χ^2/df
0.0330(1)	0.493(3)	0.18189(6)	2	4	28.3
0.0374(10)	0.541(10)	0.18190(6)	3	3	1.6
0.0338(30)	0.513(25)	0.18188(6)	4	2	0.2

Table 6.

As in Table 5, but for a 4×32^3 lattice at $\beta = 3.00$

b	μ	c	R_{min}	df	χ^2/df
0.0284(1)	0.495(2)	0.22351(5)	2	4	22.3
0.0305(5)	0.520(6)	0.22346(5)	3	3	1.5
0.0288(17)	0.505(17)	0.22346(5)	4	2	0.7

Table 7.

As in Table 5, but now with $\alpha = 1$.

b	μ	c	R_{min}	df	χ^2/df
0.0397(2)	0.941(2)	0.18194(6)	2	4	100.9
0.0313(8)	0.851(9)	0.18192(6)	3	3	16.5
0.0222(19)	0.757(24)	0.18188(6)	4	2	0.8

Figure captions.

Fig.1 The vacuum polarization $\Pi_{00}(0)$ on the $L_0 \times L_s^3$ lattice in dependence on the lattice size, for $L_s/L_0 = 4$ ($\circ$), $L_s/L_0 = 6$ (∇), $L_s/L_0 = 10$ (Δ), and for $L_s/L_0 = 20$ ($\times$).

Fig.2 On-axis (solid Δ) and off-axis (Δ) connected correlations up to lattice distance 4 on the 4×16^3 lattice at $\beta = 3.00$. The line is a spline-connected fit to the on-axis data.

Fig.3 The connected correlation function $P(R)$ as a function of the lattice distance R , together with the weak coupling result for the 4×16^3 lattice at $\beta = 2.80$

Fig.4 As in Fig. 2, but for the 4×24^3 lattice. The weak coupling results are for the 4×24^3 lattice (see section 4).

Fig.5 Combined Monte Carlo and weak coupling results for $\beta = 2.80$, where (Δ) and ($\circ$) denote the 4×16^3 and 4×24^3 results respectively. The solid (broken) line denotes the weak coupling result for the 4×24^3 (4×16^3) lattice.

Fig.6 Combined Monte Carlo and weak coupling results for $\beta = 3.00$, where (Δ) and ($\circ$) denote the 4×16^3 and 4×32^3 results respectively. The weak coupling results are shown as in Fig. 5.

Fig. 7 Effective screening mass $\mu(R)$ obtained from Eq. (44) for the 4×16^3 ($\circ$) and 4×24^3 (Δ) lattices at $\beta = 2.80$. The straight line shown has slope equal to two.

Fig.8 As in Fig. 7, but for the 4×16^3 ($\circ$) and 4×32^3 (Δ) lattices at $\beta = 3.00$.

Fig. 3

Fig. 4

Fig. 1

Fig. 2