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Three-dimensional QED at finite temperature

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Abstract

We investigate the finite temperature behaviour of $(2+1)$ -dimensional QED with N flavours of fermions. The Landau gauge photon propagator is calculated to leading order in the large- N expansion at zero frequency. As in four-dimensional gauge theories the temporal component of the photon acquires a mass at non-zero temperature and static external electric fields are screened. We study the effect of thermal screening on dynamical fermion mass generation using an approximate treatment of the Schwinger-Dyson equation for the fermion self-energy. Numerical results for the temperature dependence of the dynamically generated fermion mass are presented for a range of values of the gauge coupling and of N . We find a non-zero critical temperature above which the fermion mass vanishes identically. In particular, we find that the ratio of the zero-temperature fermion mass to the critical temperature is approximately independent of N and is large compared to four-fermion theories.

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1 Introduction

Quantum Electrodynamics in (2+1)-dimensional spacetime (QED₃) has been the subject of several recent studies, both analytic [1, 2, 3] and numerical [4]. QED₃ provides a simple model of chiral symmetry breaking in QCD, and related theories also arise in the long-wavelength description of certain 2D condensed matter systems, including models of high- T_c superconductivity [5]. From the point of view of either of these applications, the finite temperature behaviour of the theory is of considerable interest. In this paper we study QED₃ with N flavours of light fermions at finite temperature. In Section 1 we calculate the Landau gauge photon propagator at zero frequency to leading order in $1/N$. In Section 2 we present an approximate analysis of dynamical mass generation at finite temperature based on an instantaneous-exchange approximation to the kernel of the Schwinger-Dyson equation for the fermion self-energy. At zero-temperature, dynamical mass generation can be understood qualitatively in terms of the static potential between charges [4, 6] and the aim of this investigation is to examine the effect of thermal corrections to this potential on the dynamical mass in a straightforward way. Our results indicate a non-zero critical temperature above which the dynamically generated fermion mass vanishes identically. In the final section we briefly discuss the possible significance of our results for models of high- T_c superconductivity based on (2+1)-dimensional gauge theory.

2 The photon propagator

The Lagrangian for massless QED₃ with N flavours of fermions is given by,

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}_i(i\not{D} - e\not{A})\psi_i \quad (1)$$

where $i = 1, 2, \dots, N$ and a reducible four-dimensional representation of the Dirac algebra has been chosen so that (1) has a continuous chiral symmetry. The conventions of [2] will be adopted throughout. It will also be convenient to consider a modification of QED₃ in which the gauge coupling in (1) is replaced by $e\tau_3$ where τ_3 is the 4×4 matrix $\text{diag}(1, 1, -1, -1)$ acting on spinor indices. We will refer to this variant as ‘ τ_3 -QED₃’. Because τ_3 anticommutes with γ_5 , τ_3 -QED₃ has no chiral symmetry in the sense of [2].

Although the ordinary perturbation expansion in the dimensionful coupling e is plagued with infrared divergences, the situation is dramatically improved by considering instead an expansion in powers of $1/N$ with $\alpha = e^2 N$ kept fixed. The leading order contribution to the photon propagator, $\Delta_{\mu\nu}$, in this expansion is given by the infinite sum of diagrams shown in Figure 1. Evaluating this sum in the Landau gauge (at zero-temperature) gives [1],

$$\Delta_{\mu\nu}(p) = \frac{\delta_{\mu\nu} - p_\mu p_\nu / p^2}{p^2(1 + \Pi(p))} \quad (2)$$

where $\Pi(p) = \alpha/8p$. The effective static potential between charges is given by the two-dimensional Fourier transform of $\Delta_{00}(p_0 = 0, \mathbf{p})$ [1]:

$$\begin{aligned} V(R) &\sim (\alpha/N) \log(R\alpha) & R \ll (\alpha)^{-1} \\ &\sim 1/NR & R \gg (\alpha)^{-1} \end{aligned} \quad (3)$$

The crossover from logarithmic to Coulombic attraction at large distance arises as a result of charge screening effects. A simple quantum mechanical treatment suggests that, for N sufficiently small, this attraction leads to fermion pair-condensation [6].

The above results are easily generalised to non-zero temperatures using the formalism of Ref[7]. Each integral over the temporal component of a loop momentum is replaced by an infinite sum over Matsubara frequencies according to the prescription

$$\int \frac{dk_0}{(2\pi)} \rightarrow \frac{1}{\beta} \sum_{n=-\infty}^{+\infty} \quad k_0 \rightarrow (2n+1)\frac{\pi}{\beta} \quad (4)$$

where $\beta = (k_B T)^{-1}$. At zero frequency, $p_0 = 0$, the loop integrals and summations may be performed analytically using Feynman parameters and a linear shift in spatial momenta. After some calculation we obtain,

$$\Delta_{\mu\nu}(p_0 = 0, |\mathbf{p}| = P, \beta) = \frac{\delta_{\mu 0} \delta_{0\nu}}{P^2(1 + \Pi_{00}(P, \beta))} + \frac{\delta_{\mu i}(\delta_{ij} - P_i P_j / P^2) \delta_{j\nu}}{P^2(1 + \Pi_T(P, \beta))} \quad (5)$$

where the temporal and transverse components of vacuum polarisation are given by

$$\Pi_{00}(P, \beta) = \frac{2\alpha}{\pi\beta P^2} \int_0^1 dx \log \left(2 \cosh \left[\beta P \sqrt{x(1-x)}/2 \right] \right) \quad (6)$$

$$\Pi_T(P, \beta) = \frac{\alpha}{\pi P} \int_0^1 dx \sqrt{x(1-x)} \tanh \left[\beta P \sqrt{x(1-x)}/2 \right] \quad (7)$$

In either of the limits $P \rightarrow \infty$ or $\beta \rightarrow \infty$ the above expression clearly coincides with (2). However, as $P \rightarrow 0$, the term $P^2 \Pi_{00}(P, \beta)$ tends to a constant limit, $M_{00}^2(\beta)$ giving

$$\Delta_{\mu\nu}(p_0 = 0, P \rightarrow 0, \beta) = \frac{\delta_{\mu 0} \delta_{0\nu}}{P^2 + M_{00}^2} + \frac{\delta_{\mu i}(\delta_{ij} - P_i P_j / P^2) \delta_{j\nu}}{P^2(1 + \alpha\beta/12\pi)} \quad (8)$$

where the temporal component of the propagator has acquired a mass given by,

$$M_{00}(\beta) = \sqrt{\frac{2\alpha \log 2}{\pi\beta}} \quad (9)$$

These results are entirely analogous to the corresponding situation in four-dimensional gauge theory (see, for example [8] and references therein). The mass of the temporal photon implies that external electric fields are screened by thermal excitations. Transverse photons, however, remain massless and so external magnetic fields are unscreened. The massless pole in the spatial part of the photon propagator (8) potentially leads to IR divergences at higher order, although these are not apparant in the approximate analysis of the next section. Using the propagator (5) to calculate the static potential, we find that

$$V(R, \beta) \sim \exp(-M_{00}R) \quad R \gg (M_{00})^{-1} \quad (10)$$

while $V(R)$ retains its zero temperature behaviour (3) for $R \ll (M_{00})^{-1}$.

3 Dynamical mass generation

Both analytic [1, 2, 3] and numerical [4] investigations of massless QED₃ at zero temperature indicate that, at least for N sufficiently small, a fermion mass is generated dynamically and chiral symmetry is spontaneously broken. Frequently in field theory a symmetry that is broken at zero temperature is restored above some critical temperature. However, in three-dimensional spacetime, conventional spontaneous symmetry breaking cannot occur at non-zero temperature due to the Coleman-Mermin-Wagner theorem [9]. In fact this theorem does not preclude dynamical mass generation as is illustrated by the chirally-symmetric Gross-Neveu model [10, 11] (see also [12] for the (2+1)-dimensional case at finite temperature). This subtlety can be ignored if we consider instead τ_3 -QED₃ which, as stated above, has no chiral symmetry. The following analysis applies equally to both variants.

The Schwinger-Dyson equation for the (zero-temperature) fermion propagator, $S_F(p)$, is

$$S_F^{-1}(p) = S_F^{(0)-1}(p) - e \int \frac{d^3k}{(2\pi)^3} \gamma^\mu S_F(k) \Delta_{\mu\nu}(q) \Gamma^\nu(k, p) \quad (11)$$

where $\Gamma^\nu(k, p)$ is the fermion-photon vertex and $q = k - p$. This relation is shown graphically in Figure 2. The simplest possible truncation of this equation is to replace $\Delta_{\mu\nu}$ by the expression (2) and replace Γ^ν by the bare vertex, $e\gamma^\nu$. Both these substitutions are correct to $O(1/N)$. Writing the fermion propagator as $S_F^{-1}(p) = (1 + A(p))\not{p} + \Sigma(p)$ and separating eqn(11) into scalar and spinor parts yields coupled integral equations for $A(p)$ and $\Sigma(p)$. The wave-function renormalization $A(p)$ is suppressed by an explicit power of $1/N$ in the self-energy equation and, following [2], we assume that it may be safely neglected giving the following closed integral equation for $\Sigma(p)$.

$$\Sigma(p) = \frac{2\alpha}{N} \int \frac{d^3k}{(2\pi)^3} \frac{1}{q^2(1 + \Pi(q))} \frac{\Sigma(k)}{k^2 + \Sigma^2(k)} \quad (12)$$

Equation (12) was first analysed in Ref[1] where the further simplifying assumption $\Sigma(p) \simeq \Sigma(0) = m$ was made. In this case (12) becomes

$$1 = \frac{2\alpha}{N} \int_{k < \Lambda} \frac{d^3k}{(2\pi)^3} \frac{1}{k^2(1 + \Pi(k))} \frac{1}{k^2 + m^2} \quad (13)$$

For continuum QED₃, the UV momentum cutoff Λ is naturally identified with the coupling α . In order to make contact with QED₃ on the lattice and with the applications of QED₃ as a long-wavelength effective theory of 2D condensed matter systems we retain α/Λ as an adjustable parameter. In the strong-coupling limit, $\alpha/\Lambda \rightarrow \infty$, the non-trivial solution of (13) is given by,

$$m \simeq \Lambda \exp(-N\pi^2/8) \quad (14)$$

In arriving at the simple gap equation, (13), of Ref[1] we have made a series of drastic approximations to the exact Schwinger-Dyson equation (11). In the zero-temperature case, several more exact treatments of the fermion self-energy have been given in the literature.

In particular, equation (12), has been solved numerically [2] including the full momentum-dependence of $\Sigma(p)$. This more accurate approach indicates that $\Sigma(0)$ is non-zero only for $N < N_c = 32/\pi^2$ where it is given by,

$$\Sigma(0) \simeq \alpha \exp \left(\frac{-2\pi}{\sqrt{N_c/N - 1}} \right) \quad (15)$$

Other authors [3] have reinstated the wavefunction renormalization $A(p)$ and employed an ansatz for the vertex $\Gamma^\nu(k, p)$ which satisfies the Ward identity thereby going beyond a simple perturbative truncation of (11). This leads to a behaviour qualitatively similar to (14) in which the fermion mass decreases exponentially with N but remains non-zero even for large N . Unfortunately, lattice simulations have so far been unable to distinguish between the two possible behaviours. This is because the fermion mass decays exponentially with N and the corresponding length scales quickly become much larger than the size of the lattice.

Equation (13) is easily generalised to finite temperature employing the prescription (4) to give an equation for $m(\beta)$.

$$1 = \frac{2\alpha}{N\beta} \sum_{n=-\infty}^{+\infty} \int \frac{d^2 K}{(2\pi)^2} \Delta(\tilde{k}_n, K, \beta) \frac{1}{k_n^2 + K^2 + m^2(\beta)} \quad (16)$$

where $k_n = (2n + 1)\pi/\beta$, $\tilde{k}_n = 2n\pi/\beta$ and

$$\Delta(\tilde{k}_n, K, \beta) = \frac{1}{8} \text{Tr}(\gamma^\mu \Delta_{\mu\nu}(\tilde{k}_n, K, \beta) \gamma^\nu) \quad (17)$$

The photon exchange kernel Δ depends on the full finite temperature photon propagator at arbitrary frequency. However, for the present purpose of obtaining a qualitative picture of mass generation at non-zero temperature we make the further simplification of retaining only the instantaneous-exchange part of this kernel. This approximation corresponds to considering the quantum statistical mechanics of an ensemble of Dirac fermions interacting via the static two-body potential $V(R, \beta)$ in place of a full field-theoretic treatment. A similar approximation has also been adopted in studies of chiral symmetry restoration in QCD [13]. Unlike the case of QCD, dynamical mass generation at zero temperature in QED₃ can be understood qualitatively in terms of the static potential [4, 6]. By adopting the instantaneous-exchange approximation we are assuming that this is also true at finite temperature. The approximation is equivalent to setting $\Delta(k_n, K, \beta) = \Delta_{00}(k_0 = 0, K, \beta)$ or, using (5),

$$\Delta(K, \beta) = \frac{1}{K^2(1 + \Pi_{00}(K, \beta))} \quad (18)$$

Substituting (18) for Δ in (16) and performing the angular integral and the Matsubara frequency sum analytically gives

$$1 = \frac{\alpha}{2N\pi} \int_0^\Lambda \frac{dK}{K} \frac{\tanh[\beta\sqrt{K^2 + m^2(\beta)}/2]}{(1 + \Pi_{00}(K, \beta))\sqrt{K^2 + m^2(\beta)}} \quad (19)$$

Equation (19) was solved numerically for several values of N and α/Λ . The results show clearly that the fermion mass is a monotone decreasing function of temperature up to a critical temperature, T_c , above which m vanishes identically. The graph of m against $k_B T$ for $N = 1$ is shown in Figure 3. The values of $m(T = 0)$ and $k_B T_c$ for a range of N at intermediate and strong coupling ($\alpha/\Lambda = 1$ and ∞) are given in Tables 1 and 2 respectively. These quantities are also given at $N = 1$ for a range of α/Λ in Table 3 and, as expected, both grow as the coupling is increased. For $\alpha < \Lambda$ the fermion mass is proportional to α only and is thus insensitive to Λ . This confirms the expectation that the theory is naturally cut-off in the UV by α , unless of course a lower cutoff Λ is imposed. Even in the instantaneous-exchange approximation, the zero-temperature fermion mass retains its characteristic exponential dependence on N . In the strong-coupling limit, for $N > 1$, the mass is given by

$$m(T = 0) \simeq 2\Lambda \exp(-N\pi/4) \quad (20)$$

A quantity of interest, especially for condensed matter applications, is the ratio of twice the zero-temperature mass to the critical temperature, $r = 2m(T = 0)/k_B T_c$. We find that r varies only slowly with N and α/Λ . In particular, for $1 < N < 5$ and $1 \leq \alpha/\Lambda \leq \infty$, r is in the range 9.36-9.70. This ratio has also been calculated recently for the Gross-Neveu model in (2+1)-dimensions [14] where the result 2.77 was found. Another relevant comparison is with the value $r = 3.54$ which characterizes BCS superconductors. In both of these cases the effective two-body potential $V(R)$ is largely independent of temperature and for comparison we have repeated our calculation for QED₃ replacing $\Pi_{00}(K, \beta)$ by the zero-temperature kernel $\Pi_{00}(K) = \alpha/8K$ in (18). The resulting values of the ratio are shown as r^* in Table 3 and are comparable in magnitude with the BCS value. Hence the comparatively large values of r (ie low T_c in units of $m(T = 0)$) shown in Tables 1 and 2 are directly due to thermal screening of the two-body potential $V(R, \beta)$.

4 Conclusion

One immediate application of this study is to the effective long-wavelength theory of high- T_c superconductivity proposed in [6, 15]. To leading-order in the large- N expansion, this theory coincides with strongly-coupled τ_3 -QED₃. The existence of a massless pole in the spatial part of the photon propagator (5) means that superconductivity persists in this model for all $T < T_c$ and the prediction for r in the physical case $N = 2$ is given in Table 2 as 9.70. While this numerical value is the result of a crude approximation to the Schwinger-Dyson equation, we expect the qualitative feature, that r is significantly greater for this theory than for theories with a temperature independent two-body potential, to survive in a more accurate treatment. Perhaps one of the most distinctive features of the quasi-planar high- T_c materials YBaCuO and La(Sr)CuO is that they exhibit values of r much larger than those successfully predicted by BCS theory for conventional superconductors (see for example [16]). Our results suggest that this feature is correctly reproduced in effective theories of high- T_c related to QED₃.

The analysis presented here could be refined in a number of ways. Firstly, it would be interesting to extend the calculation of the photon propagator presented in Section 1 to the case of non-zero frequency. It would then be possible to go beyond the approximation of instantaneous exchange and correctly incorporate retardation effects in the kernel of the Schwinger-Dyson equation. In particular, the role of the massless spatial modes and any resulting IR divergences at finite temperature should be examined carefully. Another more ambitious improvement would be to solve (12) directly at finite temperature for the momentum dependent self-energy $\Sigma(P, \beta)$. Finally, many of our considerations could be checked directly by Monte-Carlo simulations of non-compact QED₃ on asymmetric lattices.

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Numerical Results

N	1	2	3	4	5
$m(T=0)/\Lambda$	0.229	6.50×10^{-2}	2.46×10^{-2}	9.67×10^{-3}	4.23×10^{-3}
$k_B T_c/\Lambda$	4.90×10^{-2}	1.39×10^{-2}	5.09×10^{-3}	2.17×10^{-3}	6.86×10^{-3}
$r = 2m/k_B T_c$	9.35	9.38	9.65	9.36	12.34

Table 1: The zero-temperature fermion mass and the critical temperature at $\alpha/\Lambda = 1$ for $N = 1, 5$

N	1	2	3	4	5	6
$m(T=0)/\Lambda$	1.15	0.434	0.191	8.61×10^{-2}	3.87×10^{-2}	1.76×10^{-2}
$k_B T_c/\Lambda$	0.207	8.94×10^{-2}	4.02×10^{-2}	1.80×10^{-2}	8.03×10^{-3}	3.34×10^{-3}
$r = 2m/k_B T_c$	11.11	9.70	9.49	9.58	9.64	10.52

Table 2: The zero-temperature fermion mass and the critical temperature at $\alpha/\Lambda = \infty$ for $N = 1, 6$

α/Λ	0.01	0.1	1	10	100	1,000
$m(T=0)/\Lambda$	2.42×10^{-3}	2.84×10^{-3}	0.229	0.803	1.10	1.15
$k_B T_c/\Lambda$	6.22×10^{-4}	6.12×10^{-3}	4.90×10^{-2}	0.159	0.201	0.207
$r = 2m/k_B T_c$	7.69	9.29	9.35	10.10	10.94	11.11
r^*	3.24	3.76	3.74	3.83	3.87	3.89

Table 3: The zero-temperature fermion mass and the critical temperature at $N = 1$ for a range of α

Figures

$$\text{wavy line with central dot} = \text{wavy line} + \text{wavy line with fermion loop} + \text{wavy line with two fermion loops} + \dots$$

Figure 1: The photon propagator to leading order in $1/N$.

$$(\text{fermion line with central dot})^{-1} = (\text{bare fermion line})^{-1} - \text{fermion line with fermion loop}$$

Figure 2: The Schwinger-Dyson equation for the fermion propagator.

Figure 3: Graph of m/Λ against $k_B T/\Lambda$ at $N = 1$, $\alpha/\Lambda = 1$.