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QUARKS, GLUONS, PIONS AND THE GOTTFRIED SUM RULE

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Abstract

It has been suggested that the violation of the Gottfried sum rule reported by the NMC group is due to the pionic contribution to the sea-quark distribution. After discussing estimates of this contribution in terms of the mean number of pions in the nucleon we show that, in effective chiral models with explicit quarks, it is simply proportional to the fraction of the nucleon's moment of inertia carried by the pions. We discuss the computation of this and related baryonic observables in the case of several models, taking into account the effect of one-gluon exchange, which is found to be crucial for their evaluation.

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Recent results [1] from the NMC experiment [2] seem to imply that the proton sea-quark distribution is not, as usually assumed, SU(2)-symmetric. The excess of $\bar{d}$ over $\bar{u}$ quarks in the proton may be related to the integrated difference of the proton and neutron electromagnetic structure functions by

$$S_G = \int_0^1 \frac{dx}{x} [F_2^{\pi^0}(x) - F_2^{\pi^+}(x)] = \frac{1}{3} + \frac{2}{3} \int_0^1 dx [\bar{u}(x) - \bar{d}(x)] . \quad (1)$$

The assumption of an SU(2)-symmetric sea, $\bar{u}(x) = \bar{d}(x)$, leads to the Gottfried sum rule [3] $S_G = 1/3$. The experimental result [1], derived by extrapolation from measurements in the region $0.004 < x < 0.8$,

$$S_G = 0.240 \pm 0.016 , \quad (2)$$

shows that this assumption is apparently invalid (the possibility that the discrepancy is mainly due to the domain $x < 0.004$ is discussed in Refs. [4-6]).

It has been suggested [6-8] that such an SU(2) breaking in the sea is due to the pionic contribution to the sea quark distribution, which has for some time been known to play a part in lepton-nucleon deep inelastic scattering [9,10]. In the following, we first discuss an estimate [8] for the pionic contribution to the Gottfried sum rule in terms of the mean number of pions in the nucleon. We then show that, in effective chiral models with explicit quarks, this contribution can be more directly related to the fraction J_π/J of the nucleon's moment of inertia carried by the pions. Other observables which are sensitive to this ratio are the neutron-proton mass difference, and the quark helicity contribution to the proton spin. We discuss the computation of J_π/J in the case of several models, and find that it depends strongly on the effect of residual gluon exchange. In the three-flavour quark-gluon-meson model, choosing the model parameters so as to optimize the values of the fully quantum-corrected nucleon and delta masses, we obtain $J_\pi/J \approx 0.3$, in accordance with the experimental values of S_G and of the above-mentioned observables.

As discussed by Henley and Miller [8], the pionic contribution to the difference between the number of $\bar{d}$ and $\bar{u}$ quarks in the proton is equal to the difference between the numbers of π^+ and π^- in the proton,

$$D = \int_0^1 dx [\bar{d}(x) - \bar{u}(x)] = N_{\pi^+} - N_{\pi^-} . \quad (3)$$

This difference may be estimated by assuming [8,10] the pions to be generated by the process $N \rightarrow N + \pi$, leading to the relative abundances 2, 1 and 0 for the π^+ , π^0 and π^- in the proton, hence

$$D \approx \frac{2}{3} N_\pi , \quad (4)$$

where N_π is the total number of pions in the proton. It then follows from the data (2) that

$$N_\pi \approx 0.21 \pm 0.04 , \quad (5)$$

which may be related in the cloudy bag model [11] to a bag radius $R \approx 0.9$ fm.

However, as pointed out in Refs. [6,7], one should also take into account the process $N \rightarrow \Delta + \pi$, which is expected to partly cancel the effect of the process $N \rightarrow N + \pi$ just considered; the contribution of other baryon resonances should also be taken into account [6]. To arrive at a rough estimate of the contribution of all these processes, let us make the (oversimplifying) assumption that the pions in the nucleon have been generated independently via the transition $q \rightarrow q + \pi$; the resulting relative abundances of π^+ , π^0 and π^- would now be, in a first approximation, 4, 3 and 2, so that eq.(4) would be replaced by

$$D \approx \frac{2}{9} N_\pi . \quad (6)$$

The data (2) would then lead to the larger mean pion number

$$N_\pi \approx 0.63 \pm 0.11 \quad (7)$$

(in accordance with an earlier cloudy bag model estimate [12] $N_\pi \approx 0.5$), corresponding [8] to a cloudy bag radius $R \approx 0.5$ fm.

The estimates (5) or (7) may be compared with theoretical estimates for N_π obtained in the chiral quark-meson model [13] (which is simply the modern version of the two-flavour linear sigma model [14]). In the variational treatment of this model, states associated with non-strange baryons are constructed as products of valence quarks and a coherent superposition of mesons [15,16]. Extremizing the energy of these states leads to the coupled mean field equations for the quark and meson wave functions. These

equations are usually solved with the aid of the hedgehog ansatz [17], leading to baryonic states which we shall term "coherent hedgehogs". The mean number of pions in a coherent hedgehog was found to be $N_\pi \approx 1.2$ for a choice of the quark-pion coupling constant $g \approx 5.4$ [16]; we have shown [18,19] that realistic values for the quantum-corrected baryonic masses can only be obtained for a lower value $g \approx 4$ of the quark-pion coupling constant, leading to the smaller mean pion number $N_\pi \approx 0.5$, in qualitative agreement with (7). However, this is not yet the full story. Coherent hedgehogs are neither eigenstates of spin $\vec{J}$ nor of isospin $\vec{I}$, but only of the sum $\vec{J} + \vec{I}$. Physical hadrons may be obtained from these either by Peierls-Yoccoz projection [20] or cranking [21]. Both methods lead [16,19] to a reduction in the mean pion number in a nucleon. Residual gluon exchange between quarks, which (as discussed below) is necessary to account for the observed nucleon-delta mass difference, seems [19] to further lower N_π to a value more in agreement with (5).

As we shall now show, the estimates (4) or (6) can be replaced by a more transparent evaluation of the difference D , in the case where the nucleon is generated from a mean-field hedgehog solution of a generic chiral soliton or chiral bag model by the semi-classical cranking method [22-24]. This consists in transforming the mean quark and meson fields by a time-dependent isorotation, and solving the isorotated mean field equations. The resulting cranked mean spin and isospin are, for slow rotations, linear in the angular velocity ω ,

$$\langle \vec{J} \rangle = - \langle \vec{I} \rangle = \vec{J} \omega + O(\omega^2), \quad (8)$$

thus defining the hadronic moment of inertia $\vec{J}$. In a field theory where isospin is carried by quark (ψ) and pion (ϕ) fields, the isospin operator $\vec{I}$ may be decomposed as the sum

$$\vec{I} = \vec{I}_q + \vec{I}_\pi = \int d^3x \left(\psi^\dagger \frac{\vec{\tau}}{2} \psi \right) + \int d^3x (\phi \times \vec{\pi}), \quad (9)$$

leading to a similar decomposition

$$\vec{J} = \vec{J}_q + \vec{J}_\pi \quad (10)$$

for the moment of inertia (the mean field π canonically conjugate to ϕ is generated by the time-dependent rotation, $\vec{\pi} = -\vec{\omega} \times \vec{\phi}$).

The relevance of this to the computation of the difference D between the numbers of π^+ and π^- in the proton is obvious, if one realizes that this difference measures the fraction Q_π of the proton electric charge carried by the pions, also equal to the expectation value of the third component $(I_\pi)_3$ of the pion isospin operator, so that

$$D = Q_\pi = \langle (I_\pi)_3 \rangle_p = \frac{J_\pi}{J} \langle I_3 \rangle_p = \frac{1}{2} \frac{J_\pi}{J}. \quad (11)$$

There is a priori no simple relation between Q_π and N_π , the ratio of which depends on the model parameters. Returning to eq.(1), we find that the quantity S_G measured in deep-inelastic scattering is

$$S_G = \frac{1}{3} \left(1 - \frac{J_\pi}{J} \right), \quad (12)$$

i.e. the Gottfried sum rule is broken because a (clearly non-vanishing) fraction of the nucleon isospin is carried by the pions. Of course eq.(12) also means that S_G is proportional to the fraction of the nucleon moment of inertia carried by the (valence) quarks,

$$S_G = \frac{1}{3} \frac{J_q}{J}, \quad (13)$$

which is our final result. The above derivation may easily be extended, with the same result (13), to the more general case where other isovector meson fields (e.g. vector mesons) are present. Interestingly enough, the ratio J_q/J thus measured by the NMC experiment also comes into play in the computation of the strong contribution to the neutron-proton mass difference [25,26]. It is also related to (but different from) the fraction J_{qs}/J of the nucleon moment of inertia carried by the quark helicities, which we have shown [27] to be equal in the two-flavour case to the quark helicity contribution Δq to the proton spin, much investigated theoretically after the EMC measurements [28] of the polarized proton structure function.

What do model computations have to say about the value of (13)? In the two-flavour chiral bag model [29], the ratio $J_q/3J$ varies from 0 for $R = 0$ fm to 0.20 for R

= 1 fm [23], which is significantly lower than the experimental result (2). However for such large radii the nucleon-delta mass-splitting, given in the cranking approach by

$$\Delta M_{N\Delta} = M_{\Delta} - M_N = \frac{3}{2J} \quad (14)$$

is too small. This implies [23] the need to take into account residual gluon exchange between quarks [30], which would reduce the quark moment of inertia J_q and thus increase $\Delta M_{N\Delta}$, but also decrease the ratio $J_q/3J$ well below its experimental value. The inclusion of explicit sea-quark effects [31] does not significantly alter these conclusions.

In the two-flavour chiral quark-meson model [13], the ratio $J_q/3J$ varies [27] from 0.13 for $g = 5.4$ [24] to 0.27 for $g = 3.95$ (g being the quark-pion coupling constant), so that values of S_G consistent with experiment are obtained for low values of g , $3.95 \leq g \leq 4.15$. Here again, the effect of gluon exchange, necessary to obtain a satisfactory value for the nucleon-delta mass splitting [19], would be to significantly reduce S_G to $S_G \approx 0.20$.

We shall discuss this last point in fuller detail in the more realistic three-flavour case, using the SU(3) quark-gluon-meson (QGM) model introduced in Ref [32]. The philosophy behind this model, which generalizes the SU(3) $\times$ SU(3) linear σ -model [33,34], is the same as that behind the chiral bag model, i.e. to provide a smooth interpolation between QCD and a Skyrme-like non-linear σ -model. The QGM model is defined by the Lagrangian density

$$\mathcal{L} = -\frac{1}{4} \kappa F_{\mu\nu}^a F^{\mu\nu a} + \bar{\psi} [\gamma^\mu D_\mu - g(\Sigma + i\gamma^5 \Pi)] \psi + \frac{1}{4} \text{Tr} (\partial_\mu \Sigma \partial^\mu \Sigma + \partial_\mu \Pi \partial^\mu \Pi) - U \quad (15)$$

involving the quark field ψ coupled to the scalar and pseudoscalar meson nonet fields Σ and Π , as well as to a flavour-singlet gluon color field through the covariant derivative

$$D_\mu \psi = (\partial_\mu + ig_s \frac{\lambda_a}{2} A_\mu^a) \psi \quad (a = 1, \dots, 8). \quad \text{The meson interaction potential is the sum}$$

$$U = \frac{\lambda}{4} \left[\frac{1}{2} \text{Tr} (M^\dagger M) - v^2 \right]^2 + \frac{\mu}{4} \left[\frac{3}{4} \text{Tr} (M^\dagger M)^2 - \frac{1}{4} (\text{Tr} (M^\dagger M))^2 \right] + \gamma (\det M + \det M^\dagger) + \frac{1}{2} \text{Tr} (C\Sigma) \quad (16)$$

of three chirally symmetric terms (M is the matrix $\Sigma - i\Pi$) and of a dynamical symmetry breaking term. Six of the potential parameters (μ , v , γ and the diagonal 3×3 matrix C) may be fitted [34,26] to the physical pseudoscalar meson masses (after subtracting the effect of electromagnetic mass splittings) and axial decay constants, leaving one free parameter λ^2 in (16). The gluon field is also coupled to mesons through the colour-confining dielectric function $\kappa(M)$ [35]. We shall use

$$\kappa = \left(1 - \frac{\det M + \det M^\dagger}{2 \det \langle M \rangle_0} \right)^n \quad (17)$$

($n > 1$), which goes to zero as the meson fields go to their vacuum expectation values.

Choosing as in Refs.[32,26] $\lambda^2 = 28$, and $g = 4.04$, which optimizes the nucleon mass, we obtain from eq.(13) $S_G = 0.31$ [26] for a vanishing effective strong fine structure constant α_s . This is well above the experimental value (2). The values of the other observables which, as discussed above, are related to the ratio J_q/J , namely the strong contribution ΔM_N^S to the neutron-proton mass difference, and the first moment of the polarized proton structure function

$$\langle g_1^p \rangle = \int_0^1 dx g_1^p(x) = \frac{1}{2} \left[\frac{4}{9} \Delta u + \frac{1}{9} \Delta d + \frac{1}{9} \Delta s - \frac{\alpha_s(Q^2)}{3\pi} \Delta g(Q^2) \right] \left(1 - \frac{\alpha_s(Q^2)}{\pi} \right) \quad (18)$$

(taking into account the effect of the axial anomaly [36]), also come out too high. We obtain $\Delta M_N^S = (5.2 \pm 0.3)$ MeV [26], to be compared with the experimental value [37]

$$\Delta M_N^S = (2.05 \pm 0.30) \text{ MeV} \quad (19)$$

and $\langle g_1^p \rangle = 0.190$ for $\Delta g = 0$, $\alpha_s(Q^2) = 0.25$, vs. the experimental value [28]

$$\langle g_1^p \rangle = 0.126 \pm 0.018 \quad (20)$$

(our assumption of a vanishing gluonic helicity contribution Δg to the proton spin is in accordance with recent lattice gauge calculations [38], which show that the scale invariant anomalous contribution to (18) is small; $(\alpha_s/6\pi) |\Delta g| < 0.006$). At the same time, the nucleon-delta mass difference (14) is much too small; $\Delta M_{N\Delta} = 63$ MeV (vs. the experimental value 293 MeV).

Let us now turn on the attractive residual gluon interaction between quarks with coupling α_s . As discussed in Refs.[19,32], the mean gluon field vanishes before cranking, so that only cranking-dependent observables vary with α_s . We find (fig.1) that, as expected, the decrease of the quark moment of inertia $\mathcal{J}_q$ leads to a quasi-linear decrease of the three observables S_G , ΔM_N^S , and $\langle g_1^p \rangle$, and increase of $\Delta M_{N\Delta}$. Choosing for definiteness, as in Ref.[32], $n = 3/2$ (using a different value of n would essentially amount to a rescaling of α_s), we observe that these four observables converge towards their mean experimental values quasi-simultaneously, the values for $\alpha_s = 0.6$ being $S_G = 0.228$, $\Delta M_N^S = (2.23 \pm 0.13) \text{ MeV}$, $\langle g_1^p \rangle = 0.127$ and $\Delta M_{N\Delta} = 295 \text{ MeV}$ (other “spectator” observables, which depend little or not on α_s , are the quantum-corrected nucleon mass $M_N = 816 \text{ MeV}$, and the nucleon axial-vector decay constant $g_A(0) = 1.13$). In our opinion, this correlation between S_G and other cranking-dependent observables, which are simultaneously tuned to their experimental values by adjusting a single parameter α_s , is too sharp to be accidental. This can only mean that the two basic assumptions which led to our eq.(13) for S_G , namely that the reported violation of the Gottfried sum rule is caused by isospin breaking in the sea, itself essentially due to the pionic contribution, must be correct.

Finally we discuss briefly a model computation carried out in a model where other isovector mesons are present besides the pion (the isovector scalar mesons as well as the isospinor mesons do not contribute to the moment of inertia in the QGM model). Such a two-flavour chiral quark-meson model with vector and axial-vector mesons has been introduced in Ref.[39]. Cranking this model [40] leads to a moment of inertia to which contribute not only quarks and pions but also the ρ and A_1 :

$$\mathcal{J} = \mathcal{J}_q + \mathcal{J}_\pi + \mathcal{J}_\rho + \mathcal{J}_A. \quad (21)$$

Numerical evaluation for a given set of parameters shows [40] that most of the moment of inertia is due to pions, and yields $\mathcal{J}_q / 3\mathcal{J} = 0.076$, again (as in the case of the chiral bag model) well below the experimental value (2). However, as we have seen this ratio is generally rather sensitive to the choice of model parameters, and the parameters which in Refs.[39,40] lead to a sensible phenomenology would not necessarily do so if the nucleon mass was corrected for translational and rotational fluctuations, as we have done consistently in our computations. The same remark applies to the chiral bag model computations mentioned previously.

We have discussed the pionic contribution to the Gottfried sum rule. Estimates relating this to the mean pion number in the nucleon are at best qualitative. On the other hand we have shown that, in chiral bag or chiral soliton models with explicit quarks,

this contribution is proportional to the fraction of the nucleon’s moment of inertia carried by the pions. We have seen that corrections due to one-gluon exchange are crucial for the evaluation of the l.h.s. S_G of the Gottfried sum rule. When the effective strong fine structure constant α_s is varied in the three-flavour quark-gluon-meson model, S_G attains its experimental value quasi-simultaneously with three other α_s -dependent observables. This implies that the pionic contribution is indeed responsible for the reported violation of the Gottfried sum rule.

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Figure caption

Fig. 1. Variation with α_S of S_G , together with that of the nucleon-delta mass difference $\Delta M_{N\Delta}$, of the first moment $\langle g_1^p \rangle$ of the polarized proton structure function, and of the strong contribution ΔM_N^S to the neutron-proton mass difference, all normalized to their experimental values.

Fig. 1