AC

TRI-PP-90-86
Nov 1990THE ROLE OF THE ETA MESON IN THE CALLAN-KLEBANOV
APPROACH TO THE SKYRME MODEL

CERN LIBRARIES, GENEVA

CM-P00068410

G. Pari

TRIUMF, 4004 Westbrook Mall, Vancouver B.C., Canada V6T 2A3

Abstract

We study, along lines similar to those developed by Callan and Klebanov for the treatment of the strange degrees of freedom within the context of the SU(3) Skyrme model, the question of including the eta meson alongside the kaons. To this end, we introduce an alternate Skyrme Lagrangian which provides a coupling between the Skyrminion and the eta field. We find an s-wave eta-Skyrminion bound state and quantize its rotational excitations. We identify the lowest member of the rotational band as the N(1535) negative parity nucleon resonance.

(submitted to Physics Letters B)

TRI-PP 90.86
(9117)

Following the initial phenomenological failure of the SU(3) Skyrme model based on unbroken SU(3) flavor symmetry [1], the bound state approach to strangeness of Callan and Klebanov [2,3] has emerged as a viable approach for the extension of the Skyrme model beyond the two flavor sector [4]. In this model, the large SU(3) flavor symmetry breaking suggested by the pion-kaon mass difference is taken into account by constructing hyperons as bound states of SU(2) hedgehog solitons and kaons. More precisely, the strangeness degrees of freedom are incorporated as vibrational modes about the zero modes corresponding to isospin rotations of the classical soliton. For the quantum numbers of the baryon states, it is found that the spin arises as a coupling between the kaon angular momentum and the spin of the Skyrminion, and, remarkably, that the bound kaon carries no isospin, therefore imposing that the isospin of the system be solely associated with the collective rotations of the hedgehog. The strangeness quantum number of the state, on the other hand, is carried by the kaons alone, with a contribution of $S = -1$ for each bound kaon. This scheme obtains good agreement with the experimental values of the hyperon mass splittings [3,5] and static observables such as the hyperon magnetic moments [6,7].

An interesting question which has been considered only rather briefly in the context of the bound state model of Callan and Klebanov is the role of the pseudoscalar eta meson ($m_\eta = 548.8$ MeV). Its comparable mass to that of the kaons ($m_{K^\pm} = 495.7$ MeV) suggests that it also be treated as a heavy degree of freedom relative to the pions ($m_\pi = 138$ MeV), and correspondingly as a vibrational perturbation about the SU(2) soliton background, in a way completely analogous to that followed for kaons. To implement this, we extend the previous kaon-Skyrminion ansätze to include an additional eta field. A natural choice [5,8], which introduces the kaons ($\phi_1, \dots, \phi_7$) and eta ($\phi_8 \equiv \eta$) symmetrically about the Skyrminion U_π is

$$U = \sqrt{U_{K+\eta}} U_\pi \sqrt{U_{K+\eta}} \quad (1)$$

where

$$\sqrt{U_{K+\eta}} = \exp i \sum_{a=1}^8 \frac{\lambda_a}{f_\pi} \hat{\tau}_a, \quad U_\pi = \exp i \sum_{a=1}^8 \lambda_a A_{1/2}^a F(r), \quad (1a)$$

Here, $F_\pi = 186$ MeV is the pion decay constant, λ_i are the Gell-Mann matrices for SU(3), $\hat{\tau}_i$ are the three components of a radial unit vector and $F(r)$ is the chiral angle. Collective coordinates allowing the hedgehog to acquire definite spin and isospin are introduced via the time-dependent unitary matrix $A \in \text{SU}(2)$. With this ansatz, we can consider different Lagrangians to investigate the issue of the existence of eta-Skyrminion bound states.

In the spirit of the large N_c approximation, where to lowest order it is sufficient to treat U_π as a static background, we first set $A = 1$ (intrinsic frame) and substitute the meson field matrix U in the Skyrme model Lagrangian [9]

$$L_{\text{Skyrme}} = L_\sigma + L_{\text{st}} + L_{\text{mass}}. \quad (2)$$

L_σ is the nonlinear sigma model Lagrangian with two field derivatives, L_{st} is the four derivative term introduced by Skyrme to stabilize the classical soliton U_π and L_{mass} is a mass term which explicitly breaks the SU(3) $\times$ SU(3) $_R$ down to diagonal SU(3) (the explicit forms of these terms can be found, for example, in reference [5]). When we expand to second order in the meson fluctuations, we find the following results

which possibly explain why the role of the eta field has hitherto not been considered in the Callan-Klebanov model: (i) there is no eta-kaon coupling term. (ii) More importantly, the Skyrme stabilization term contributes no eta-hedgehog interaction whereas the nonlinear sigma model term only contributes the Klein-Gordon term of a free massless meson field [10]. (iii) Consideration of the mass term L_{mass} leads to the following eta meson Lagrangian

$$L_{\eta}^{\text{mass}} = \frac{1}{2} \int d^3x \{ \partial_{\mu}\eta \partial^{\mu}\eta + \gamma \eta^2 \}, \quad (3a)$$

with γ defined by

$$\gamma = \frac{1}{3}(m_{\pi}^2(2 - \cos F(r)) - 4m_K^2). \quad (3b)$$

Asymptotically, L_{η} represents the Lagrangian of a free massive eta meson due to the fact that

$$F(r) \rightarrow 0 \Rightarrow \gamma \rightarrow -m_{\eta}^2, \quad (4)$$

which follows after using the Gell-Mann-Okubo relation for meson masses

$$3m_{\eta}^2 = 4m_K^2 - m_{\pi}^2. \quad (5)$$

We find however that L_{η} does not support bound state solutions. (iv) Also, the Wess-Zumino action S_{WZ} must be included in the SU(3) Skyrme model [11]; to second order in the fields, this does not yield $\eta - \eta$ or $\eta - K$ interactions.

It is clear that to provide a non-trivial potential for the eta meson, which might bind the eta to the skyrmion, we must go beyond the basic Skyrme model [12]. To this end, we propose the replacement of the Skyrme stabilization term by the alternate term [13,14]

$$L_{st} = \frac{1}{16e^2}((Tr(l_{\mu}l_{\mu}))^2 - (Tr(l_{\mu}l^{\mu}))^2), \quad (6)$$

where $l_{\mu} = U^{\dagger} \partial_{\mu} U$. When l_{μ} is valued in the SU(2) Lie algebra,

$$L_{st} = L_{st}. \quad (7)$$

In the SU(3) sector, however, the two forms of stabilization terms are different, leading to contributions differing by a factor of 4 to the kaonic moment of inertia of the rotating hedgehog. A calculation of octet and decuplet mass splittings using L_{st} and carefully chosen chiral symmetry breaking terms (terms which lift the $F_{\pi} = F_K = F_{\eta}$ degeneracy) has provided results in excellent agreement with experiment [13]. For other static observables [13] such as charge radii and magnetic moments, comparable results as those found for L_{st} are obtained. Here, we note that L_{st} also differs from the Skyrme stabilization term by providing a non-trivial potential between the eta and the skyrmion. L_{st} does not, however, provide an eta-kaon coupling, allowing us to study the issue of eta-Skyrmion bound states without making further reference to the strangeness degrees of freedom.

Replacing L_{st} by L_{st} in the Lagrangian of eq. (2), substituting the meson field U (with $A = 1$) of eq. (1) and expanding to second order in the fields, we find

$$L_{\text{Skyrme}}(U) = L_{\text{Skyrme}}(U_{\pi}) + L_{\eta} + \dots \quad (8)$$

where ... stands for second order terms involving kaon fields and higher order terms in the meson fluctuations. The Lagrangian for the eta field now takes the form

$$L_{\eta} = \frac{1}{2} \int d^3x \{ \alpha \dot{\eta}^2 - \partial_i \eta \beta_{ij} \partial_j \eta + \gamma \eta^2 \}, \quad (9)$$

where

$$\alpha = 1 + \frac{4}{e^2 F_{\pi}^2} \left(\frac{dF(r)}{dr} + \frac{2 \sin^2 F(r)}{r^2} \right), \quad (9a)$$

$$\beta_{ij} = \delta_{ij} + \frac{4}{e^2 F_{\pi}^2} \left(\frac{dF(r)}{dr} (\delta_{ij} - \hat{x}_i \hat{x}_j) + \frac{\sin^2 F(r)}{r^2} (\delta_{ij} + \hat{x}_i \hat{x}_j) \right), \quad (9b)$$

γ is defined in eq. (3b), and $\dot{\eta}$ denotes the time derivative of the field. Using the canonical momentum

$$\pi^0 = \frac{\delta L_{\eta}}{\delta \dot{\eta}} = \alpha \dot{\eta}, \quad (10)$$

we obtain, after the usual Legendre transformation, the Hamiltonian

$$H_{\eta} = \frac{1}{2} \int d^3x \{ \alpha \dot{\eta}^2 + \partial_i \eta \beta_{ij} \partial_j \eta - \gamma \eta^2 \}. \quad (11)$$

It is useful to introduce at this stage a partial wave decomposition of the eta field

$$\eta(\vec{r}, t) \rightarrow Y_m^l(\Omega) \eta_l(r, t). \quad (12)$$

This allows us to write the Hamiltonian for η as

$$H_{\eta} = \frac{1}{2} \int_0^{\infty} dr r^2 \alpha (\dot{\eta}^2 - \ddot{\eta} \eta), \quad (13)$$

where in the last equation, we have used the Euler-Lagrange equation for η obtained from the usual variation of the Lagrangian:

$$\alpha \ddot{\eta} - \frac{1}{r^2} \frac{d}{dr} (r^2 \beta \frac{d\eta}{dr}) + \left(\frac{l(l+1)}{r^2} \delta - \gamma \right) \eta = 0 \quad (14a)$$

$$\delta = 1 + \frac{4}{e^2 F_{\pi}^2} \left(\frac{dF(r)}{dr} + \frac{\sin^2 F(r)}{r^2} \right), \quad (14b)$$

$$\beta = \delta_{ij} \hat{x}_i \hat{x}_j. \quad (14c)$$

These equations are the same, except for the terms coming from L_{mass} , as the equations obtained when the eta field is introduced as a phase about the SU(2) hedgehog.

The time-translational invariance of the Lagrangian allows us to extract the time dependence of η_l as

$$\eta_l(r, t) = e^{-i\omega t} \eta_l(r). \quad (15)$$

With this form substituted in eq. (14), we have a classical eigenvalue equation which can be investigated numerically, given values of the parameters e^2 and F_{π} , for bound states $\omega^2 < m_{\eta}^2$. Using the experimental values for the masses of the pion and kaon, we find an $l = 0$ bound state of energy $\omega = 377.8$ MeV for F_{π} fixed at its experimental

value of 186 MeV and $e = 4.25$ (Set I) chosen to reproduce the nucleon- Δ split. Another case of interest is the one which reproduces both the nucleon and Δ masses. For this purpose we use $e = 4.84$ and $F_\pi = 108$ MeV (Set II), and find an energy of $\omega = 313.7$ MeV for the s-wave state. We find no other s-wave bound state. In the other partial waves, $l > 0$, the centrifugal term is repulsive enough to impede the formation of eta-hedgehog bound states.

The s-wave bound state we have described does not have well defined spin or isospin quantum numbers. These can be generated in the standard way by quantizing collective coordinates associated with rotations of the hedgehog. Since the eta is an isoscalar field, the introduction of collective coordinates transforming the soliton from the intrinsic to the laboratory frame leaves it unaffected. Letting the SU(2) matrix $A \equiv A(t)$ in eq. (1a) be time-dependent, we obtain (omitting the kaons as before) an additional contribution to the Lagrangian

$$\delta L^{ime} = 2 \mathfrak{S} \dot{K}_i \dot{K}_i + \frac{8}{e^2 F_\pi^2} \int d^3x \sin^2 F(r) \left(\epsilon_{ijk} \frac{\hat{x}_k}{r} \dot{K}_j \eta \partial_i \eta + (\delta_{ij} - \hat{x}_i \hat{x}_j) \dot{K}_i \dot{K}_j \partial_k \eta \partial_k \eta \right), \quad (16)$$

where $\dot{K}_i = -\frac{i}{2} \text{Tr}(\dot{A}^\dagger \dot{A})$, $i = 1, 2, 3$, are the collective velocities and $\mathfrak{S}$ is the Skyrmin moment of inertia [15]. We have expanded the interaction to second order in the collective velocities, rather than to first order only as is done in the strangeness case, because here the linear term vanishes identically for a purely radial s-wave eta. From eq. (16), applied to s-wave, the conjugate momentum to $\dot{K}_i$ is

$$\begin{aligned} \pi_i^K &= \frac{\delta L^{ime}}{\delta \dot{K}_i} \\ &= 4 \dot{K}_i \left(\mathfrak{S} + \frac{32\pi}{3e^2 F_\pi^2} \int_0^\infty dr r^2 \sin^2 F(r) \left(\frac{d\eta}{dr} \right)^2 \right), \end{aligned} \quad (17)$$

which implies the following form for the Hamiltonian:

$$\delta H^{ime} = \frac{1}{2} \frac{\vec{J} \cdot \vec{J}}{\mathfrak{S} + \epsilon}, \quad (18)$$

where $\vec{J}$ is the angular momentum of the Skyrmin and we use the shorthand ϵ for the integral in eq. (17). The effect of the s-wave bound state on the mass relation is therefore just an effective shift of the Skyrmin moment of inertia. The eta fields appearing in ϵ are to be interpreted as field operators. To this end, we introduce a partial wave decomposition

$$\hat{\eta}_l = \sum_n \left(\eta(r) \hat{a}_n^{\dagger} e^{i\omega_n t} + \eta(r) \hat{a}_n e^{-i\omega_n t} \right), \quad (19)$$

with n running over all bound states and $\eta(r) \hat{a}_n$ the given bound state profile obtained from solving eq. (14). The creation and annihilation operators $\hat{a}_n$, $\hat{a}_n^\dagger$ satisfy the usual

commutation rules for boson operators and we understand the integral ϵ as normal ordered.

We evaluate the rotational mass spectrum arising from δH^{ime} in first order perturbation theory by using a product state for the eta-Skyrmin system. Substituting (19) and expanding the denominator in eq. (18), we find the mass relation

$$M = M_d + \frac{1}{2\mathfrak{S}} I(I+1) + \omega + \left(\kappa - \frac{\epsilon}{\mathfrak{S}^2} \right) I(I+1). \quad (20)$$

Numerically, $\epsilon = 6.1 \times 10^{-5}$ MeV (Set I) or $\epsilon = 8.3 \times 10^{-5}$ MeV (Set II) for the bound state η_l profile normalized with respect to the nonlinear metric [3] $r^2 \alpha$

$$e^2 \int_0^\infty dr r^2 \alpha (\omega_n + \omega_m) \eta_l^\dagger \eta_l^\dagger = \delta_{mn}. \quad (21)$$

Using this normalization equation along with the decomposition (19) in H_m of eq. (13), provides the result that ω in the mass formula is just the classical frequency. A parameter κ has been introduced in the mass relation to represent both the omitted higher order terms in the $\epsilon/\mathfrak{S}$ expansion and the fact that this series suffers from operator ordering ambiguities [3].

The mass spectrum we have discussed is now compared to experiment [16]: we first make the obvious observation that since the eta meson is a pseudoscalar particle, the bound state we have is a negative parity baryon state. The lowest mass rotational band member has the quantum numbers $J^P = \frac{1}{2}^-$, $I = \frac{1}{2}$ (recall that for the Skyrmin, $I = S$). A clear candidate for this is the N(1535) negative parity S₁₁ 55% branching ratio to $\eta + N$ suggesting a large η content of the state. A previous description of N(1535) in the context of the Skyrme model required the introduction of an isoscalar scalar σ meson [17]; here, we obtain very naturally a state with the correct quantum numbers, albeit bound by a large amount. For Set II of parameters we find a mass 256 MeV below ηN threshold (we have used the calculated m_η arising from the Gell-Mann-Okubo relation), whereas the larger value of F_π of Set I leads to a smaller binding energy of 191 MeV. In both cases, the free parameter κ was set to zero. Using κ to bring the mass of the state in closer agreement with experiment is of dubious value since we expect κ to be at most of the order of a few MeV [5]. We also note that it would have been more pleasing to have the rotational contribution sufficient to push the bound state energy above threshold (this scheme has previously been advocated by Blom et al. [5] for the strange resonance N(1520)). This is not the case here; furthermore, the small size of ϵ would preclude this.

Our model also predicts a state with the same quantum numbers as the $\Delta(1232)$, except with negative parity. A possible candidate for this seems to be the π -N D-wave four star resonance $\Delta(1700)$. We poorly account, however, for the experimental splitting of 165 MeV between $\Delta(1700)$ and N(1535), finding splittings of 287 MeV and 283 MeV for Set I and Set II respectively (again $\kappa = 0$). The $\Delta(1700)$ has been well accounted for in Skyrme model adiabatic scattering calculations [18]. This fact, along with the discrepancy between experimental and theoretical splittings makes our identification uncertain.

The comparison we have made to experiment should be taken with caution until clear support for the alternate Skyrme term we have used is found in the underlying vector meson physics [13] and the physics it predicts is clarified. We have found that for the η -hedgehog system it leads to quite different results from those of the Skyrme term, providing a binding potential in the s-wave channel. This allows us to propose a simple model for the $N(1535)$ resonance. In view of these results, it becomes of interest to investigate how the strangeness sector of the Callan-Klebanov model is affected by L_{alt} , and how the physical discussions of hyperons based on this model are modified [19].

The author has benefited from many discussions with G. Gat, G. Holzwarth and B. K. Jennings during earlier work [15] on this problem. He also wishes to thank J. M. Eisenberg and B. K. Jennings for useful suggestions regarding the manuscript. References:

- [1] D. Masak, Phys. Rev. D **39** (1989) 305; and references therein.
- [2] C. G. Callan and I. Klebanov, Nucl. Phys. B **262** 1985 365.
- [3] C. G. Callan, K. Hornbostel and I. Klebanov, Phys. Lett. B **202** (1988) 269.
- [4] For an alternative point of view, see H. Yabu and K. Andō, Nucl. Phys. B **301** (1988) 601.
- [5] U. Blom, K. Dannbom and D. O. Riska, Nucl. Phys. A **493** (1989) 384.
- [6] J. Kunz and P. J. Mulders, Phys. Rev. D **41** (1990) 1578.
- [7] E. M. Nyman and D. O. Riska, Nucl. Phys. B **325** (1989) 593.
- [8] H. Schnitzer, Phys. Lett. B **139** (1984) 217.
- [9] T.H.R. Skyrme, Proc. R. Soc. London, Ser. A **260** (1961) 127.
- [10] For a similar result obtained with a different ansatz, see M. Karliner and M. P. Mattis, Phys. Rev. Lett. **56** (1986) 428.
- [11] E. Witten, Nucl. Phys. B **223** (1983) 422 and 433.
- [12] The other four-derivative chirally invariant term, the so-called symmetric quartic term of J. F. Donoghue, E. Golowich and B. R. Holstein, Phys. Rev. Lett. **53** (1984) 747, leads to an unacceptable singular eta-hedgehog potential. G. Pari and G. Gat, in TRIUMF Report Scientific Activities, (1989).
- [13] G. Pari, B. Schwesinger and H. Walliser, submitted for publication.
- [14] G. Pari, J. M. Eisenberg, A. Gal and B. K. Jennings, submitted for publication.

- [15] G. Adkins, C. Nappi and E. Witten, Nucl. Phys. B **228** (1983) 552.
- [16] Review of Particle Properties, Particle Data Group, Phys. Lett. B **204** (1988) 1.
- [17] B. Schwesinger and H. Walliser, Phys. Lett. B **231** (1989) 11.
- [18] M. P. Mattis and M. Karliner, Phys. Rev. D **31** (1985) 2833.
- [19] K. Dannbom, E. M. Nyman and D. O. Riska, Phys. Lett. B **227** (1989) 291.