CERN-TH.6063/91
UMSL-TH-2/91

Elastic Supersymmetric Relic-Nucleus Scattering Revisited

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Abstract

We survey the elastic scattering rates of neutralinos on many nuclei, incorporating LEP constraints and choosing model parameters that give a critical density of the lightest neutralino, using the odd-group model to estimate nuclear matrix elements and incorporating a simple model for the momentum dependence of the elastic spin and mass form factors. We find that the rates for elastic scattering on ^{19}F can be estimated reliably and are reasonably large for a relic mass, M_χ , in the range $M_\chi \approx 20 - 60 \text{ GeV}$. We present calculations of the elastic neutralino - ^{19}F scattering rates in regions of parameter space consistent with LEP and other laboratory experiments.

CERN-TH.6063/91
UMSTL-TH-2/91
April 1991

The issues are sharpening in the quest to identify the particle believed to constitute the dark matter. Several candidate particles have recently been excluded or constrained. New upper limits on the electron neutrino mass [1] seem to preclude its closing the Universe, and solar neutrino experiments [2] suggest that at least one other neutrino is not much heavier. A 17 keV neutrino [3] could not be stable enough to constitute the dark matter. LEP has excluded massive Dirac or Majorana neutrinos weighing up to almost $M_Z/2$ [4], and also sneutrinos. Massive charged particles have been considered [5] and discarded as likely dark matter candidates. The most prominent remaining candidates are the axion and the lightest spin- $\frac{1}{2}$ neutral supersymmetric particle (neutralino). The latter is significantly constrained by LEP and other experiments [6], but not excluded.

In view of the narrowing of the list of suspects, it is opportune to investigate the remaining candidates more carefully, and here we reexamine the prospects for detecting the lightest neutralino. It has in the past been proposed to look for products from neutralino annihilation in the galactic halo [7], the Sun [8] or the Earth [9], but we focus here on the most direct search, that for elastic supersymmetric relic - nucleus scattering in the laboratory [10]. This process has been considered by many previous authors in various approximations [11-14]. In this paper we survey many elements combining the following 7 features that previously have not been considered together. (1) We incorporate the LEP constraints on supersymmetric model parameters. (2) We use the full gaugino/Higgsino composition of the lightest neutralino in those regions of parameter space not already nibbled away by LEP. (3) We use the latest EMC [15] estimates of the spin-dependent nucleon matrix elements and also include the

strange-sea spin-independent contribution [16] to the scattering amplitude. (4) We use the odd group model [12] systematically to improve on naive shell model estimates of the elastic scattering matrix elements. (5) We model the momentum dependence of the elastic scattering spin and mass form factors. (6) We restrict the choice of supersymmetric model parameters by requiring the annihilation of neutralinos into all kinematically accessible fermion, Higgs and gauge boson channels to leave a critical density of neutralinos at present. (7) We use grand unification relations among squark and slepton masses [17], as well as the commonly used relations among gaugino masses.

We find that unless the lightest Higgs scalar be relatively light, $m_h \lesssim 60\text{ GeV}$, spin-independent scattering amplitudes are non-negligible only for heavy elements whose scattering rates are in any case small and uncertain. We find that the ^{19}F nucleus is to be preferred for several reasons. All nuclear physics techniques agree quite well on the relic scattering matrix element. The form factor does not affect the scattering amplitude significantly. Event rates that are accessible to proposed cryogenic detectors are possible for relic masses $M_X \approx 20 - 60\text{ GeV}$ over a sizable region of the parameter space. Chemical compounds of ^{19}F are suitable for detecting nuclear recoil via ionization and/or calorimetry [18]. We present calculations of the elastic relic - ^{19}F scattering rates in regions of the neutralino parameter space which have not been excluded by LEP or other laboratory experiments, and critical density is possible.

We assume the minimal supersymmetric extension of the Standard Model (MSSM) with two Higgs doublets [19], and hence 4 neutralinos that are mixtures of the standard gauginos ($\tilde{W}_3, \tilde{B}$) and Higgsinos ($\tilde{H}_1, \tilde{H}_2$). Their well known

mixing matrix [19] is characterized by the ratio of the hypercharge +1 to the hypercharge -1 Higgs vacuum expectation values $v_1/v_2 = \tan\beta$, a superpotential $H_1 H_2$ mixing coefficient μ , and SU(2) and U(1) gaugino masses M_2 and M_1 respectively, which we assume as usual to be related by grand unification to a common gaugino mass, $m_{1/2} : M_i = (\alpha_i/\alpha_{GUT})m_{1/2}$ ($i=1,2$), so that $M_1 = \frac{5}{3}\tan^2\theta_W M_2$. The same parameters also characterize the 2×2 chargino mass matrix. LEP and other experiments provide $\tan\beta$ - dependent constraints [6] in the (μ, M_2) plane that we will implement when we discuss elastic neutralino-nucleus scattering rates later in the paper. The masses of other particles (squarks, sleptons and Higgses) enter into the elastic scattering amplitudes and the calculation of the relic abundance. For the squarks and sleptons we assume conventional grand unification relations:

$$m_i^2 = m_0^2(1 + C_i t^2) + m_{1/2}^2 \pm \frac{1}{2}(v_1^2 - v_2^2) \left\{ \frac{1}{2} Y_i g_i^2 - T_{3i} g_i^2 \right\}, \quad (1)$$

where the + (-) sign is for $\tilde{f}_L$ ($\tilde{f}_R$), the scalar partner of the left- (right-) handed component of the fermion f , $\tilde{\xi} = m_{1/2}/m_0$ and [17] $C_{q_L} \approx C_{q_R} \approx 10C_{l_L} \approx 30C_{l_R} \approx 5$. An extra parameter is required to specify the Higgs spectrum at the tree level, and radiative corrections to the Higgs masses are significant [20], introducing sensitivity to extra parameters including the top quark mass. These radiative corrections mean that the LEP lower bounds on Higgs masses cannot be used to exclude $\tan\beta = 1$, which is why we do not discard this extreme case in our subsequent analysis.

We assume, as usual, that most of the dark matter halo surrounding our galaxy is made of elementary particle dark matter, so that the typical dark matter particle velocities are $\approx 300 \text{ km s}^{-1}$. At these very small velocities the energy transfer involved is very small, $\Delta E < M_\chi v^2 = 10 \text{ keV} (M_\chi/10 \text{ GeV})$, which is much smaller than any elementary particle mass scale in the problem. We can therefore use an effective Lagrangian to describe low-energy neutralino-quark interactions.

Neglecting kinetic energy terms:

$$L_{eff} = \bar{\chi} \gamma_\mu \gamma_5 \bar{q} q (A P_L + B P_R) + \bar{\chi} \chi C m_q \bar{q} q, \quad (2)$$

where $P_{R,L} = \frac{1}{2}(1 \pm \gamma_5)$. The first term causes a nuclear-spin-dependent interaction^[1] and is due to sfermion and Z boson exchange. The second term contributes a spin-independent term to the cross section that is proportional to the square of the nuclear mass, and is due to Higgs [21] and sfermion [22] exchange as well as sfermion mixing [10,23]. We never find the spin-independent interaction due to sfermion exchange [22] to be important, except in cases where the rates are very small anyway. We present detailed expressions for the coefficients A, B and C elsewhere [24], where we also compare to the previous literature.

In order to obtain the interaction rate we seek we must first evaluate $\langle \chi, nucleon | L_{eff} | \chi, nucleon \rangle$, which involves baryonic matrix elements to which we now turn. For a general neutralino, the spin-dependent invariant amplitude for scattering off a proton (neutron) p (n) is [10,11]

[1] It also causes a spin-independent interaction if sfermion masses are not degenerate, but this is always small [11].

$$M_{S,D}(\chi p(n) \rightarrow \chi p(n)) = 4 \bar{S}_\chi \cdot \bar{S}_{p(n)} \sum_{q \in p(n)} (B - A) \Delta q, \quad (3)$$

where Δq is the fraction of the nucleon's spin, $\bar{S}_{p(n)}$, due to quarks of flavor q . We use the most recent analysis of the EMC [15] combined with the most recent analyses of hyperon data [25] to determine the Δq 's: $\Delta u = .77 \pm .08$, $\Delta d = -.49 \pm .08$ and $\Delta s = -.15 \pm .08$. Second, the spin-independent part in (2) involves the baryonic matrix elements $\langle p(n) | m_q \bar{q} q | p(n) \rangle$. Because of the m_q factor, heavy quarks do not decouple [26], and $\langle p(n) | \sum_{\text{heavy}} m_q \bar{q} q | p(n) \rangle = \frac{2}{9} M_{p(n)} \bar{\psi} \psi$. This, however, assumes no strangeness in the nucleons; as emphasized in ref. [26], the strange quarks are neither light nor heavy in the heavy-quark expansion. Recent analyses suggest a substantial strange sea in the nucleons (see e.g. ref. [27]), and perhaps a significant fraction of the nucleon mass comes from the strange-quark contribution $m_s \bar{s} s$ [27]. Including this contribution, the spin-independent invariant amplitude is [16]:

$$M_{S,L}(\chi p(n) \rightarrow \chi p(n)) = \left(\hat{f} \frac{m_u C_u + m_d C_d}{m_u + m_d} + \hat{f} C_s \right. \\ \left. + \frac{2}{27} (1 - \hat{f} - \bar{f}) (C_c + C_b + C_t) \right) \bar{\chi} \chi \cdot M_{p(n)} \bar{\psi} \psi, \quad (4)$$

where $\langle p(n) | (m_u + m_d) (\bar{u} u + \bar{d} d) | p(n) \rangle = 2 \hat{f} \cdot M_{p(n)} \bar{\psi} \psi$ and $\langle p(n) | m_s \bar{s} s | p(n) \rangle = \hat{f} M_{p(n)} \bar{\psi} \psi$; $\hat{f} \approx .05$ and $\bar{f} \approx .2$ [27].

We must now turn to the nuclear matrix element of the scalar ($\bar{\psi} \psi$) and axial-vector ($\bar{\psi} \gamma_\mu \gamma_5 \psi$) nucleon operators at vanishing momentum transfer. The scalar operator simply gives the mass of the nucleus $|N\rangle$, $\langle N | M_{p(n)} \bar{\psi} \psi | N \rangle = M_N \bar{N} N$. Secondly, $\langle N | \bar{\psi} \gamma_2 \gamma_3 \psi | N \rangle = \langle N | \bar{S}_p N \rangle = \lambda \langle N | \bar{J} N \rangle$, where $\bar{J}$ is the total

nuclear angular momentum. In the single particle shell model, $\lambda = \pm(2L + 1)^{-1}$ for a nucleon in an orbital L and $J = L \pm \frac{1}{2}$. This model is known to be a fairly good description near closed shells, but it is poor away from closed shells. This can easily be seen in the discrepancies between the predicted and the experimental values of the magnetic moments [11]. Engel and Vogel [12] have noted, however, that a better approximation is to relax the assumption that a single nucleon of the odd group of nucleons contributes all the nuclear angular momentum, while still assuming that the even group of nucleons has vanishingly small angular momentum, as indicated by nuclear transfer data. Then λ is directly related to the measured magnetic moment, μ_{ex} , by [12]

$$J\lambda = \frac{\mu_{ex} - g_{odd}^L J}{g_{odd}^S - g_{odd}^L}, \quad (5)$$

where g_{odd}^L (g_{odd}^S) is the orbital (spin) nucleon g -factor ($= 1(5.586)$, $0(-3.826)$ for protons, neutrons). We compare the single-particle and odd-group values of $\lambda^2(J+1)$ in Table 1, and see that the corrections can be substantial [2].

For light nuclei one can get a far better estimate of λ . As pointed out in [12,28], β -decay transitions between mirror-pair nuclei combined with the measured magnetic moments allow one to obtain both the odd and even contributions to $\bar{J}$. The results indeed show that the even system contributes a small amount to $\bar{J}$ [12,28]. A comparison of the mirror-pair estimates with full,

[2] Note also that for odd neutron systems the correction is adequately described by $(\mu_{ex}/\mu_n)^2$ as expected [11].

numerical shell model calculations [13] shows very good agreement. Since the odd-group estimate (5) is a fairly good approximation to the more exact, mirror-pair estimate for light nuclei (except for ${}^7\text{Li}$), and a comparison with numerical results for heavy nuclei [14] also shows fairly good agreement (to within 10% or better, except for ${}^{127}\text{I}$ and ${}^{135}\text{Ba}$), we shall use (5) throughout our calculations.

For large $M_X \gtrsim 100 \text{ GeV}$, the typical momentum transfer $\vec{q} = M_X \Delta \vec{v} \approx 10^{-3} M_X \gtrsim 100 \text{ MeV} \approx (2 \text{ fm})^{-1}$, so that $|\vec{q}|^{-1}$ gets comparable to the size of a large nucleus $\approx A^{1/3} \text{ fm}$, and we must correct the nuclear matrix elements by the appropriate form factors. The spin-independent amplitude is corrected by the mass form factor

$$F_{\text{mass}}(\vec{q}) = \frac{1}{M_N} \int d^3x \rho_N(\vec{x}) e^{-i\vec{q} \cdot \vec{x}} \quad (6)$$

This correction has been considered in the literature [9] assuming a gaussian form factor, $|F_{\text{mass}}(\vec{q})|^2 = \exp(-\vec{q}^2/2q_0^2)$, with $q_0^2 = 3/c_{\text{charge}}^2$ and the rms charge radius taken from a fit to the data [29]: $c_{\text{charge}} = (0.30 + 0.39A^{1/3}) \text{ fm}$.

We have calculated $F_{\text{mass}}(\vec{q})$ in the Fermi model, $\rho_N(r) \propto (1 + e^{(r-R)/a})^{-1}$, with the half-radius R chosen to fit c_{charge} [3], and $a = .55 \text{ fm}$ in order to have the skin thickness $(= 4.4a) = 2.4 \text{ fm}$, which is a reasonable fit to the data [29]. We find that $F_{\text{mass}}(\vec{q})$ deviates drastically from the gaussian fit for $\vec{q}^2/q_0^2 \gtrsim 2$. *This should be borne in mind when dealing with the differential cross section.* In

[3] The mass and charge rms radii should not differ significantly since $Z \approx A \cdot Z$. We find explicitly that in the shell model described below these radii differ by $\lesssim 1\%$ for all the nuclei considered here, and we shall not distinguish them in our discussion.

general, $|F_{\text{mass}}(\vec{q})|^2$ falls off faster (slower) than $\exp(-\vec{q}^2/q_0^2)$ for $A \gtrsim (\lesssim) 20$. The integral of $|F_{\text{mass}}(\vec{q})|^2$, however, differs from that of the gaussian fit by $\lesssim 10\%$ for $M_X \lesssim 100 \text{ GeV}$, because most of the integral comes from the low $\vec{q}^2$ region where the behavior is universal. Since we shall be interested here only in the total elastic scattering rate, we have merely corrected the spin-independent amplitude throughout with the gaussian fit. Integrating over a Maxwellian neutralino velocity distribution one gets the following correction factor ζ for the cross section (assuming no energy threshold) [9]:

$$\zeta = \frac{0.573}{b} \left[1 - \frac{\exp\left(-\frac{b}{1+b}\right) \text{erf}\left(\sqrt{\frac{1}{1+b}}\right)}{\sqrt{1+b} \text{erf}(1)} \right] \quad (7)$$

where $b = (M_X^2 M_N^2 / (M_X + M_N)^2) \frac{g}{g_{\text{charge}}}^2$

The spin-dependent amplitude is corrected by a corresponding spin form factor. This is readily calculable in the single particle shell model, provided we can make the approximation that the spin polarization of the odd group of nucleons is due primarily to the nucleons in the same shell. This seems reasonable in light of the Pauli exclusion principle, which provides the basis for the individual nucleon treatment of the nucleus. We have calculated the spin form factor in a shell model with an infinite square well potential (with $\vec{L} \cdot \vec{s}$ and $\vec{L}^2$ couplings to give the appropriate shell structure to the nuclei considered here). For an unpaired nucleon in an orbital nL_J ,

$$F_{spin}(\vec{q}) = \frac{2}{|j_L|(x_{nL})|^2} \int_0^1 x^2 j_0(|\vec{q}|R_{\infty}) j_L(x_{nL}) dx \quad (8)$$

where x_{nL} is the n th zero of the spherical Bessel function $j_L(x)$ and R_{∞} is the radius of the square well, chosen here to fit r_{charge} . We find that $F_{spin}(\vec{q})$ also deviates drastically from a gaussian fit for $\vec{q}^2/\bar{q}_0^2 \geq 2$, where $\bar{q}_0^2$ is defined in terms of the rms spin radius $r_{spin} : \bar{q}_0^2 = 3/r_{spin}^2$ and $|F_{spin}(\vec{q})|^2$ falls off relatively faster for lower n and higher L . The integrated $|F_{spin}(\vec{q})|^2$ differs from the gaussian fit by a small amount again, so we have just corrected the spin-dependent cross section by $\zeta(r_{spin}) = \zeta(r_{charge} \times (r_{spin}/r_{charge}))$. The calculated ratios r_{spin}/r_{charge} are given in Table 1. The correction ζ is larger (smaller) than the spin-independent correction for $r_{spin}/r_{charge} < 1$ (> 1), for example by as much as $\approx 30\%$ for $M_{\chi} = 100$ GeV and $A \approx 200$.

We now combine the above results to obtain the rate of neutralino-nucleus elastic scattering per unit mass of target material. From (3) and (4) we get (for 100% isotopic abundance)

$$R = (R_{S.D.} + R_{S.I.}) \left[\frac{4 M_{\chi} M_N}{(M_{\chi} + M_N)^2} \right] \left[\frac{\rho}{3 \text{ GeV} \cdot \text{cm}^{-3}} \right] \left[\frac{\langle \vec{v}_E \rangle}{320 \text{ km} \cdot \text{s}^{-1}} \right] \frac{\text{events}}{\text{kg} \cdot \text{day}}, \quad (9)$$

where ρ is the density of dark matter in the solar neighborhood and $\langle \vec{v}_E \rangle$ is the average velocity with which the particles hit the detector. Here

$$R_{S.D.} = 5.5 \left[\frac{10^4 \text{ GeV}^2}{m_0^2 - M_{\chi}^2} \right]^2 \lambda^2 J(J+1) \zeta(r_{spin}) \left[\frac{m_0^2 - M_{\chi}^2}{e^2} \sum_{q \in p(n)} (B - A) \Delta q \right]^2 \quad (9a)$$

with $e = g_2 \sin \theta_W$, and

$$R_{S.I.} = 210 M_N^2 M_Z^4 \zeta(r_{charge}) \left(\hat{r} \frac{m_u C_u + m_d C_d}{m_u + m_d} + \frac{2}{27} (1 - \hat{r}) (C_c + C_b + C_t) \right)^2. \quad (9b)$$

We have evaluated the rate (9) for the nuclei of Table 1 assuming a closure density of neutralinos for a present-day Hubble expansion rate of $50 \text{ kms}^{-1} \text{ Mpc}^{-1}$. To estimate this, we have calculated [24] the density of relics for a general neutralino composition, including all channels kinematically allowed for $M_{\chi} \leq M_W$ and fully expanding the annihilation cross sections to first order in $\vec{p}_{\chi}/M_{\chi}^2$. We present our results in Table 2 for various masses of gaugino-like and Higgsino-like neutralinos, and in the figure of the elastic scattering rate in the (μ, M_2) plane. In the region $M_2 < |\mu|$ of the parameter space, where the neutralino is primarily gaugino-like, we have chosen m_0 to obtain a closure density, whereas in the region $M_2 > |\mu|$, where the neutralino is primarily Higgsino-like, we have chosen $\tan \beta$. In the region $M_2 \approx |\mu|$, where the neutralino is fairly mixed and the spin-independent interactions most important, the relic density is typically much less than the critical value. The choice of other parameters is such that these rates are representative of what we find in our survey of the (μ, M_2) plane [24], as can be seen in the figure. Heavy nuclei such as Tl may give the best rates if the lightest Higgs scalar were

fairly light, $m_h \lesssim 40 \text{ GeV}$, a possibility that is all but excluded by LEP, or $\tan\beta \gg 1$. In the former case, radiative corrections [20] make the analysis of the Higgs-mediated interaction unreliable, which is why we refrain from presenting such results. However, a light nucleus such as ^{19}F gives the best rates everywhere in the (μ, M_2) plane if $m_h \gtrsim 60 \text{ GeV}$ and $\tan\beta \lesssim 6$. We find that current LEP constraints restrict the region in the (μ, M_2) plane where a critical density of neutralinos with $M_\chi \leq M_W$ is possible to regions where $R \lesssim 10(\text{kg}\cdot\text{day})^{-1}$, and typically $R \approx (1 - 0.1)(\text{kg}\cdot\text{day})^{-1}$. While these rates are certainly well below the current level of sensitivity of ionization detectors, $R \gtrsim 10(\text{kg}\cdot\text{day})^{-1}$, they seem large enough to be interesting for proposed cryogenic detectors [18].

Acknowledgements

We thank P. F. Smith for encouragement.

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Table Captions

Table 1 : Shell Model predictions for $\lambda^2 J(J+1)$ and $r_{\text{spin}}/r_{\text{charge}}$ for nuclei of interest in dark matter searches. Tables (1a) and (1b) contain the nuclei which are described by one unpaired proton or neutron respectively. The first column identifies the nucleus by chemical symbol and atomic number $A = Z + N$. The second column gives the nuclear spin J . The third column is the natural abundance of the species, except for those indicated by a superscript e for which a commercially available enriched sample is assumed. The fourth column gives the shell model description of the nucleus. Column 5 contains the single-particle shell model prediction, and column 6 the measured value, of the magnetic dipole moment. The

next two columns give the single-particle (7) and odd-group (8) model predictions for $\lambda^2 J(J+1)$. Finally, the last column gives the value of the ratio of charge and spin rms radii in the (infinite square well) shell model.

Table 2 : Representative elastic scattering rates, eq. (9), for various isotopes. The model parameter m_o ($\tan\beta$) was chosen to give a critical density of neutralinos of a given mass M_χ in the gaugino-like (Higgsino-like) region with $\mu = -(200 - 400)$ GeV ($M_2 = 1000$ GeV). For the gaugino-like neutralino, $\tan\beta = 4$ ($\mu = -200$ GeV) or $\tan\beta = 8$ ($\mu = -400$ GeV). Here $m_h = 80$ GeV. Isotopes of the same element have been combined.

Figure Caption

Elastic neutralino-nucleus scattering rate, eq. (9), in the (μ, M_2) plane. For the left (right) panels $\mu < 0$ ($\mu > 0$). In the upper panels the elastic scattering rate $R(\chi N \rightarrow \chi N)/(\text{kg day}) > 0.1$ (0.01) in the dark (light) shaded regions for $N = {}^{19}\text{F}$; here $\tan\beta = 2$ and $m_h = 50$ GeV. In the lower panels $N = {}^{203}\text{Tl} + {}^{205}\text{Tl}$, and $\tan\beta = 8$ and $m_h = 80$ GeV. In the upper (lower) panels the spin-dependent (spin-independent) interaction dominates. In the region marked XLEP the lightest chargino is excluded by experiments at LEP, whereas in the region marked CDF, the gluino is unacceptably light if $M_1 = (\alpha_i/\alpha_{\text{GUT}})m_{1/2}$ ($i=2,3$). Finally, in the dense-dotted region Z decays into neutralinos are excluded by experiments at LEP [4,6].

Table I(a)

Isotope	J	Abundance	Shell Model	nL_J	μ_{th}	μ_{ex}	$\lambda^2(J+1)$	$\frac{f_{spin}}{f_{charge}}$
AElement		%					single particle	odd group
¹ H	1/2	100	1S _{1/2}	2.79	2.793	2.793	0.750	1
⁷ Li	3/2	100 ^e	1P _{3/2}	3.79	3.256	3.256	0.417	0.244
¹¹ B	3/2	100 ^e	1P _{3/2}	3.79	2.689	2.689	0.417	0.112
¹⁵ N	1/2	0.4	1P _{1/2}	-0.26	-0.283	-0.283	0.083	0.087
¹⁹ F	1/2	100	2S _{1/2}	2.79	2.629	2.629	0.750	0.647
²³ Na	3/2	100	1D _{5/2}	4.79	2.218	2.218	0.350	0.041
²⁷ Al	5/2	100	1D _{5/2}	4.79	3.642	3.642	0.350	0.087
³¹ P	1/2	100	2S _{1/2}	2.79	1.132	1.132	0.750	0.057
³⁵ Cl	3/2	100 ^e	1D _{3/2}	0.13	0.822	0.822	0.150	0.036
⁵¹ V	7/2	99.8	1F _{7/2}	5.79	5.151	5.151	0.321	0.167
⁵⁵ Mn	5/2	100	1F _{7/2}	5.79	3.450	3.450	0.321	0.060
⁵⁹ Co	7/2	100	1F _{7/2}	5.79	4.620	4.620	0.321	0.077
⁶⁹ Ga	3/2	60.1	2P _{3/2}	3.79	2.017	2.017	0.417	0.021
⁷¹ Ga	3/2	39.9	2P _{3/2}	3.79	2.562	2.562	0.417	0.089
⁷⁵ As	3/2	100	2P _{3/2}	3.79	1.439	1.439	0.417	0.000
⁷⁹ Br	3/2	50.7	2P _{3/2}	3.79	2.106	2.106	0.417	0.029
⁸¹ Br	3/2	49.3	2P _{3/2}	3.79	2.271	2.271	0.417	0.047
⁹³ Nb	9/2	100	1G _{9/2}	6.79	6.171	6.171	0.306	0.162
¹⁰⁷ Ag	1/2	57.8	2P _{1/2}	-0.26	-0.114	-0.114	0.083	0.054
¹⁰⁹ Ag	1/2	48.2	2P _{1/2}	-0.26	-0.131	-0.131	0.083	0.057
¹²¹ Sb	5/2	57.3	2D _{5/2}	1.71	3.363	3.363	0.194	0.050
¹²³ Sb	7/2	42.7	1G _{7/2}	1.71	2.550	2.550	0.194	0.055
¹²⁷ I	5/2	100	2D _{5/2}	4.79	2.813	2.813	0.250	0.007
¹³³ Cs	7/2	100	1G _{7/2}	1.71	2.582	2.582	0.194	0.052
¹³⁹ La	7/2	99.9	1G _{7/2}	1.71	2.783	2.783	0.194	0.031
¹⁹¹ Ir	3/2	37.3	2D _{3/2}	0.13	0.146	0.146	0.150	0.145
¹⁹³ Ir	3/2	62.7	2D _{3/2}	0.13	0.160	0.160	0.150	0.142

Table I(a) (cont'd)

²⁰³ Tl	1/2	29.5	3S _{1/2}	2.79	1.622	0.750	0.180	0.848
²⁰⁵ Tl	1/2	70.5	3S _{1/2}	2.79	1.638	0.750	0.185	0.848
²⁰⁹ Bi	9/2	100	1H _{9/2}	2.63	4.200	0.205	0.005	1.10

Table I(b)

Isotope	J	Abundance	Shell Model	μ_{th}	μ_{ex}	$\lambda^2(J+1)$	$\frac{f_{spin}}{f_{charge}}$
AElement		%	nL_J			single particle	odd group
³ He	1/2	100 ^e	1S _{1/2}	-1.91	-2.128	0.750	0.928
⁹ Be	3/2	100	1P _{3/2}	-1.91	-1.178	0.417	0.158
¹⁷ O	5/2	0.0	1D _{5/2}	-1.91	-1.890	0.350	0.342
²⁹ Si	1/2	4.7	2S _{1/2}	-1.91	-0.555	0.750	0.063
⁴⁷ Ti	5/2	7.3	1F _{7/2}	-1.91	-0.788	0.321	0.059
⁴⁹ Ti	7/2	5.5	1F _{7/2}	-1.91	-1.104	0.321	0.107
⁶⁷ Zn	5/2	4.1	1F _{5/2}	1.36	0.875	0.179	0.073
⁷³ Ge	9/2	7.8	1G _{9/2}	-1.72	-0.879	0.306	0.065
⁹¹ Zr	5/2	11.3	2D _{5/2}	-1.91	-1.304	0.350	0.163
⁹⁹ Ru	5/2	12.7	2D _{5/2}	-1.91	-0.640	0.350	0.039
¹⁰¹ Ru	5/2	17.0	2D _{5/2}	-1.91	-0.719	0.350	0.049
¹¹¹ Cd	1/2	12.8	3S _{1/2}	-1.91	-0.594	0.750	0.072
¹¹³ Cd	1/2	12.2	3S _{1/2}	-1.91	-0.622	0.750	0.079
¹¹⁵ Sn	1/2	0.4	3S _{1/2}	-1.91	-0.919	0.750	0.173
¹¹⁷ Sn	1/2	7.7	3S _{1/2}	-1.91	-1.001	0.750	0.205
¹²⁹ Xe	1/2	26.4	3S _{1/2}	-1.91	-0.778	0.750	0.124
¹³¹ Xe	3/2	21.2	2D _{3/2}	1.15	0.692	0.150	0.055
¹⁵⁵ Gd	3/2	14.8	3P _{3/2}	-1.91	-0.260	0.417	0.008
¹⁵⁷ Gd	3/2	15.7	3P _{3/2}	-1.91	-0.340	0.417	0.013
¹⁸³ W	1/2	14.3	3P _{1/2}	0.64	0.118	0.083	0.003
¹⁹⁹ Hg	1/2	16.8	3P _{1/2}	0.64	0.506	0.083	0.052
²⁰¹ Hg	3/2	13.2	3P _{3/2}	1.15	0.560	0.150	0.036
²⁰⁷ Pb	1/2	22.1	3P _{1/2}	0.64	0.593	0.083	0.072

Table 2

Elastic Scattering Rate $R(\chi N \rightarrow \chi N)/(kg \cdot day)$									
M_χ $\tan\beta$	25 GeV			75 GeV			60 GeV	75 GeV	
	4	8		4	8		≈ 4.5	≈ 8	
Isotope	Gaugino						Higgsino		
F	.041	.025		.0078	.0057		.020	.017	
V	.0090	.030		.0025	.041		.0057	.032	
Mn	.0032	.030		.0011	.047		.0020	.034	
Co	.0039	.034		.0014	.053		.0025	.038	
Ga	.0024	.042		.0011	.068		.0016	.048	
As	.000080	.045		.00050	.075		.000018	.052	
Br	.0018	.050		.0010	.082		.0012	.057	
Nb	.0061	.063		.0022	.099		.0039	.071	
Ag	.0022	.077		.0014	.12		.0015	.086	
Sb	.0017	.084		.0013	.13		.0011	.092	
I	.00033	.087		.00096	.14		.00015	.095	
Xe	.00059	.043		.00066	.066		.00062	.046	
Cs	.0015	.092		.0013	.14		.00086	.099	
La	.00096	.096		.0012	.15		.00051	.10	
Gd	.000081	.033		.00034	.050		.000042	.034	
Ir	.0033	.14		.0020	.19		.0020	.13	
Hg	.00017	.042		.00043	.059		.00012	.041	
Tl	.0040	.14		.0023	.20		.0026	.14	
Pb	.00017	.032		.00034	.044		.00014	.031	
Bi	.00030	.15		.0014	.20		.000073	.14