TOP MASS PREDICTION FROM
SUPERSYMMETRIC GUTS

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Abstract

We consider a supersymmetric GUT framework motivated by $SO(10)$ or $E_6$ unification in which the parameter $\tan \beta (\equiv v_2 / v_1)$ of the minimal supersymmetric standard model is constrained by the condition that the Yukawa couplings $h_u$, $h_d$ and $h_{\tau}$ are all equal at the GUT scale. With $\alpha_S(M_Z) = 0.106 \pm 0.006$, the estimate for the $b$ quark mass, which depends on $\tan \beta$, lies in the 'observed' range $m_b(m_b) = 4.25 \pm 0.10 \text{ GeV}$, provided that the top quark mass is $142^{+28}_{-48} \text{ GeV}$.

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The minimal supersymmetric extension of the standard model (MSSM) introduces an important new parameter \( \tan \beta \equiv \frac{v_1}{v_2} \), the ratio of the vacuum expectation values that provide masses for u type and d type quarks (plus the charged leptons)[1]. Phenomenological considerations require that \( 1 < \tan \beta < \frac{m_t}{m_b} \)[2]. Embedding the MSSM in SUSY \( SU(5) \)[3,4,5] leaves \( \tan \beta \) undetermined, which means that the \( SU(5) \) prediction for \( m_b \) depends on an additional free parameter[6].

In this note we consider a supersymmetric grand unified framework, based on groups such as \( SO(10) \) and \( E_6 \), in which \( \tan \beta \) is constrained by the condition that the Yukawa couplings \( h_t \), \( h_b \) and \( h_\tau \) are all equal at the GUT breaking scale \( M_X \). For \( \mu < M_X \), \( \tan \beta \) differs from \( \frac{m_t}{m_b} \) by a (small) calculable amount. With \( \alpha_S(M_Z) = 0.106 \pm 0.006 \), the estimated b quark mass lies within the 'measured' range \( (m_b(m_b) = 4.25 \pm 0.10 \text{ GeV})[7] \) provided that the top quark mass is \( 142^{+38}_{-28} \text{ GeV} \).

Our starting point is the assumption that the third generation fermions acquire mass from the coupling \( 16.16.10 \), where the 10-plet contains the two higgs doublets that develop vevs \( v_1 \) and \( v_2 \) in an \( SO(10) \) theory, or from the coupling \( 27^3 \) in an \( E_6 \) theory. This implies that the Yukawa couplings \( h_t \), \( h_b \) and \( h_\tau \) are all equal at \( M_X \) (See Table 1 for an estimate of \( M_X \) to one loop). For \( M_S < \mu < M_X (M_S = 1 \text{ TeV} \) denotes the SUSY breaking scale and \( t = \frac{\ln \mu}{16 \pi^2} \) the evolution equations for the gauge and Yukawa couplings to one loop are[6,8]:

\[
\begin{align*}
\frac{d g_1}{dt} &= (2 n_\psi + \frac{3}{6}) g_1^3 \\
\frac{d g_2}{dt} &= (-6 + 2 n_\psi + 1) g_2^3 \\
\frac{d g_3}{dt} &= (-9 + 2 n_\psi) g_3^3 \\
\frac{d h_t}{dt} &= h_t (6 h_t^2 + h_b^2 - \frac{16}{3} g_3^2 - 3 g_2^2 - \frac{13}{15} g_1^2) \\
\frac{d h_b}{dt} &= h_b (h_t^2 + 6 h_b^2 - \frac{16}{3} g_3^2 - 3 g_2^2 - \frac{7}{15} g_1^2) \\
\frac{d h_\tau}{dt} &= h_\tau (3 h_t^2 - 3 g_2^2 - \frac{9}{6} g_1^2)
\end{align*}
\]

(1)

For \( M_Z < \mu < M_S \), the equations are
\[ \frac{dg_1}{dt} = \left( \frac{3}{4} n_\tau + \frac{3}{16} \right) g_1^3 \]
\[ \frac{dg_2}{dt} = \left( -\frac{22}{3} + \frac{4}{3} n_\tau + \frac{1}{3} \right) g_2^3 \]
\[ \frac{dg_3}{dt} \simeq \left( -11 + \frac{4}{3} n_\tau \right) g_3^3 \]
\[ \frac{dh_1}{dt} = h_t(9h_t^2 + h_b^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{20}g_1^2) \]
\[ \frac{dh_2}{dt} = h_b(h_t^2 + 9h_t^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{20}g_1^2) \]
\[ \frac{dh_r}{dt} = h_r(6h_t^2 - \frac{9}{4}g_2^2 - \frac{9}{4}g_1^2) \]

At tree level the Yukawa couplings are given by

\[ h_t = \frac{m_t\sqrt{1 + \tan^2 \beta}}{174 \tan \beta} \]
\[ h_b = \frac{m_b\sqrt{1 + \tan^2 \beta}}{174} \]
\[ h_r = \frac{m_r\sqrt{1 + \tan^2 \beta}}{174} \]

where \( \sqrt{v_1^2 + v_2^2} = 174 \text{ GeV} \).

In Fig. 1 we plot \( \tan \beta \) vs. \( m_t(m_t) \), where \( \tan \beta \) is determined by the requirement that for a given \( m_t(m_t) \), the three Yukawa couplings \( h_t, h_b \) and \( h_r \) meet at the GUT scale \( M_X \). In Fig. 2 an example of the evolution of the Yukawa couplings as functions of the momentum scale is shown. It may be noticed that \( \frac{h_t}{h_b} \) is of \( O(1) \) in the entire range and asymptotically reaches 1 from above. In Fig. 3 we plot \( m_t(\text{physical}) \approx m_t(m_t)(1 + \frac{\alpha_s(m_t)}{3\pi}) \) vs. \( m_b(m_b) \). Note that between \( M_Z \) and \( m_t \) the QCD corrections are included to two loops. For \( \alpha_s(M_Z) \), following the first paper in ref[9], we take the range \( 0.106 \pm 0.006 \). Our conclusion from this is that the top quark mass is \( 142^{+120}_{-40} \text{ GeV} \). A larger value for \( \alpha_s(M_Z) \), say 0.12, leads to a top quark mass in the range 171 \( - \) 182 GeV.

Independent of the constraint from \( m_t(m_t) \), one can approximately bound \( h_t \) by setting the right hand side of the evolution equation for its logarithm to zero. It turns out that for \( h_t \simeq 1.05 \), the system of equations lies in the perturbative domain[6,10]. In the first paper of ref[6], \( \tan \beta \) was set to unity which gives an approximate bound on the top quark mass of \( (1.05)(\frac{1}{\sqrt{2}})(174 \text{ GeV}) \simeq 130 \text{ GeV} \). Our study involves large values of \( \tan \beta \) and as a consequence, we end up with an approximate upper bound on \( m_t \) of \( (1.05)(174 \text{ GeV}) \simeq 183 \text{ GeV} \), which is similar to the second paper of ref[6].
In conclusion, some recent investigations[9] suggest that supersymmetric grand unified theories directly broken to the MSSM are in striking agreement with data. For instance, the predicted value for $\sin^2 \theta_W$ is in excellent agreement with recent results. Moreover, the observed gauge couplings when extrapolated to high energies appear to meet at a common scale close to $10^{16}$ GeV (with $M_5 \simeq 1$ TeV). Our results on the top mass takes us a step further in this direction. We have shown that certain supersymmetric GUTS also predict a heavy top quark.

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REFERENCES

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<th>$\alpha_s(M_Z)$</th>
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Table 1.

One loop predictions for $\sin^2 \theta_W(M_Z)$ and $M_X$ with SUSY $SO(10)$ or $E_6$ GUT broken directly to the minimal supersymmetric extension of the standard model.

FIGURE CAPTIONS

1. Plot of $\tan \beta$ vs. $m_t(m_t)$ with $\alpha_s = 0.106$ and $M_S = 1$ TeV.
2. Plot of Yukawa couplings vs. $\log_{10} \mu$ (GeV) for the case $\alpha_s = 0.106$, $M_S = 1$ TeV and $m_t(physical) = 142$ GeV.
3. Plots of $m_b(m_t)$ vs. $m_t(physical)$ for typical choices of parameters.
\( \alpha_s = 0.106, \ M_S = 1 \text{ TeV} \)

FIG. 1
$\alpha_s = 0.106$, $M_s = 1 \text{ TeV}$, $m_t(\text{physical}) = 142 \text{ GeV}$

![Graph showing Yukawa couplings vs. $\log_{10} \mu$ (GeV). The graph includes solid, dotted, and dashed lines.](FIG. 2)
$\alpha_s = 0.106$, $M_S = 1$ TeV

FIG. 3b
\[ \alpha_s = 0.112, \ M_S = 1 \text{ TeV} \]

**FIG. 3c**