Top-quark-mass prediction from supersymmetric grand unified theories

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We consider a supersymmetric grand-unified-theory (GUT) framework motivated by SO(10) or E_6 unification in which the parameter $\tan\beta (\equiv v_2/v_1)$ of the minimal supersymmetric standard model is constrained by the condition that the Yukawa couplings h_t, h_b , and h_τ are all equal at the GUT scale. With $\alpha_s(M_Z) = 0.106 \pm 0.006$, the estimate for the b -quark mass, which depends on $\tan\beta$, lies in the “observed” range $m_b(m_b) = 4.25 \pm 0.10$ GeV, provided that the top-quark mass is 142^{+26}_{-49} GeV.

The minimal supersymmetric extension of the standard model (MSSM) introduces an important new parameter $\tan\beta \equiv v_2/v_1$, the ratio of the vacuum expectation values that provide masses for u -type and d -type quarks (plus the charged leptons) [1]. Phenomenological considerations require that $1 < \tan\beta < m_t/m_b$ [2]. Embedding the MSSM in supersymmetric (SUSY) SU(5) [3–5] leaves $\tan\beta$ undetermined, which means that the SU(5) prediction for m_b depends on an additional free parameter [6].

In this Brief Report we consider a supersymmetric grand unified framework, based on groups such as SO(10) and E_6 , in which $\tan\beta$ is constrained by the condition that the Yukawa couplings h_t, h_b , and h_τ are all equal at the grand-unified-theory (GUT) breaking scale M_X . For $\mu < M_X$, $\tan\beta$ differs from m_t/m_b by a (small) calculable amount. With $\alpha_s(M_Z) = 0.106 \pm 0.006$, the estimated b -quark mass lies within the “measured” range [$m_b(m_b) = 4.25 \pm 0.10$ GeV] [7] provided that the top-quark mass is 142^{+26}_{-49} GeV.

Our starting point is the assumption that the third-generation fermions acquire mass from the coupling $16 \times 16 \times 10$, where the 10-plet contains the two Higgs doublets that develop vacuum expectation values (VEV's) v_1 and v_2 in an SO(10) theory, or from the coupling 27^3 in an E_6 theory. This implies that the Yukawa couplings h_t, h_b , and h_τ are all equal at M_X (see Table I for an estimate of M_X to one loop). For $M_S < \mu < M_X$ [$M_S = 1$ TeV denotes the SUSY-breaking scale and

$t \equiv \ln(\mu(\text{GeV})/16\pi^2)$] the evolution equations for the gauge and Yukawa couplings to one loop are [6,8] (with $\alpha_i \equiv g_i^2/4\pi$, $i = 1, 2$ and $\alpha_s \equiv g_3^2/4\pi$)

$$\begin{aligned} dg_1/dt &= (2n_g + \frac{3}{5})g_1^3, \\ dg_2/dt &= (-6 + 2n_g + 1)g_2^3, \\ dg_3/dt &= (-9 + 2n_g)g_3^3, \\ dh_t/dt &= h_t(6h_t^2 + h_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2), \\ dh_b/dt &= h_b(h_t^2 + 6h_b^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2), \\ dh_\tau/dt &= h_\tau(3h_b^2 - 3g_2^2 - \frac{9}{5}g_1^2). \end{aligned} \quad (1)$$

For $M_Z < \mu < M_S$, the equations are

$$\begin{aligned} dg_1/dt &= (\frac{4}{3}n_g + \frac{2}{10})g_1^3, \\ dg_2/dt &= (-\frac{22}{3} + \frac{4}{3}n_g + \frac{1}{3})g_2^3, \\ dg_3/dt &= (-11 + \frac{4}{3}n_g)g_3^3, \\ dh_t/dt &= h_t(9h_t^2 + h_b^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{17}{20}g_1^2), \\ dh_b/dt &= h_b(h_t^2 + 9h_b^2 - 8g_3^2 - \frac{9}{4}g_2^2 - \frac{5}{20}g_1^2), \\ dh_\tau/dt &= h_\tau(6h_b^2 - \frac{9}{4}g_2^2 - \frac{9}{4}g_1^2). \end{aligned} \quad (2)$$

At the tree level the Yukawa couplings are given by

$$\begin{aligned} h_t &= \frac{m_t \sqrt{1 + \tan^2\beta}}{174 \tan\beta}, \\ h_b &= \frac{m_b \sqrt{1 + \tan^2\beta}}{174}, \\ h_\tau &= \frac{m_\tau \sqrt{1 + \tan^2\beta}}{174}, \end{aligned} \quad (3)$$

where $\sqrt{v_1^2 + v_2^2} = 174$ GeV.

In Fig. 1 we plot $\tan\beta$ vs $m_t(m_t)$, where $\tan\beta$ is deter-

TABLE I. One-loop predictions for $\sin^2\theta_w(M_Z)$ and M_X with SUSY SO(10) or E_6 GUT broken directly to the minimal supersymmetric extension of the standard model.

$\alpha_s(M_Z)$	M_S (TeV)	M_X (GeV)	$\sin^2\theta_w(M_Z)$
0.100	1.0	0.42×10^{16}	0.235
0.106	1.0	0.71×10^{16}	0.233
0.112	1.0	1.02×10^{16}	0.231

FIG. 1. Plot of $\tan\beta$ vs $m_t(m_t)$ with $\alpha_s=0.106$ and $M_S=1$ TeV.

mined by the requirement that for a given $m_t(m_t)$, the three Yukawa couplings h_t , h_b , and h_τ meet at the GUT scale M_X . In Fig. 2 an example of the evolution of the Yukawa couplings as functions of the momentum scale is shown. It may be noticed that h_t/h_b is of order 1 in the entire range and asymptotically reaches 1 from above. In Fig. 3 we plot $m_t(\text{physical}) \approx m_t(m_t)[1 + 4\alpha_s(m_t)/3\pi]$ vs $m_b(m_b)$. Note that between M_Z and m_b the QCD corrections are included to two loops. For $\alpha_s(M_Z)$, following the first paper in Ref. [9], we take the range 0.106 ± 0.006 . Our conclusion from this is that the top-quark mass is 142^{+26}_{-49} GeV. A larger value for $\alpha_s(M_Z)$, say 0.12, leads to a top-quark mass in the range 171–182 GeV.

Independent of the constraint from $m_b(m_b)$, one can approximately bound h_t by setting the right-hand side of the evolution equation for its logarithm to zero. It turns out that, for $h_t \lesssim 1.05$, the system of equations lies in the

FIG. 2. Plot of Yukawa couplings vs $\log_{10}\mu(\text{GeV})$ for the case $\alpha_s=0.106$, $M_S=1$ TeV and $m_t(\text{physical})=142$ GeV.

perturbative domain [6,10]. In the first paper of Ref. [6], $\tan\beta$ was set to unity which gives an approximate bound on the top-quark mass of $(1.05)(1/\sqrt{2})(174 \text{ GeV}) \approx 130$ GeV. Our study involves large values of $\tan\beta$ and as a consequence, we end up with an approximate upper

FIG. 3. Plots of $m_b(m_b)$ vs $m_t(\text{physical})$ for typical choices of parameters.

bound on m_t of $(1.05)(174 \text{ GeV}) \simeq 183 \text{ GeV}$, which is similar to the second paper of Ref. [6].

In conclusion, some recent investigations [9] suggest that supersymmetric grand unified theories directly broken to the MSSM are in striking agreement with data. For instance, the predicted value for $\sin^2\theta_W$ is in excellent agreement with recent results. Moreover, the observed gauge couplings when extrapolated to high energies appear to meet at a common scale close to 10^{16} GeV (with $M_S \simeq 1 \text{ TeV}$). Our results on the top-quark mass

take us a step further in this direction. We have shown that certain supersymmetric GUT's also predict a heavy top quark.

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