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Semi-Inclusive Deeply Inelastic Scattering at Electron-Proton Colliders

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Abstract

Measurement of the distribution of hadronic energy in the final state in deeply inelastic electron scattering at HERA can provide a good test of our understanding of perturbative QCD. For this purpose, we define an energy distribution function, which can be computed in terms of parton distribution functions for the incoming hadron without involving parton decay functions for the outgoing partons. We expand the energy distribution in terms of functions of certain angles times scalar functions that describe the hadronic structure. Finally, we compute these hadronic structure functions (away from the direction corresponding to zero transverse momentum) at order α_s . The gluon initiated contribution can yield as much as 50% of the total; therefore, this process can play a significant role in determining the gluon distribution.

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1 Introduction

The Drell-Yan process^[1] $hadron + hadron \rightarrow lepton + lepton + X$ has been an excellent testing ground for Quantum Chromodynamics. The crossed reaction $electron + positron \rightarrow hadron + hadron + X$ has also provided a good tool to test QCD. Here, the energy-energy correlation function^[2] in which one sums with an energy weighting over all flavors and energies of each of the two outgoing hadrons, has proved especially useful. The other crossed version of this reaction is semi-inclusive deeply inelastic scattering, $electron + hadron \rightarrow electron + hadron + X$. It is the oldest, but has been overshadowed by inclusive deeply inelastic scattering, $electron + hadron \rightarrow electron + X$ as a testing ground for QCD. With the advent of higher momentum transfers and 4π particle detection at HERA, the semi-inclusive process may be expected to play a greater role.

In this paper we discuss the theory of semi-inclusive deeply inelastic scattering. We pay particular attention to the analogues of the transverse momentum distribution and angular distribution of the Drell-Yan reaction. For the transverse momentum distribution, we treat the case of large transverse momentum, for which perturbation theory at order α_s is applicable. We leave for a future paper the case of small transverse momentum, for which contributions from graphs at arbitrary order must be summed. We do discuss in some detail the analogue of the dilepton angular distribution in the Drell-Yan process. The angular distribution carries information on the spin content of the theory, and allows one to see separate contributions from photon exchange and Z_0 exchange.

The reaction that we study is

$$e + A \rightarrow e + B + X. \quad (1)$$

Let us describe the particles by their energies and angles in the HERA laboratory frame, with the positive z -axis chosen in the direction of the proton beam and the negative z -axis in the direction of the electron beam. In completely inclusive deeply inelastic scattering, one measures only E' and θ' , the energy and angle of the scattered electron. In the semi-inclusive case studied in this paper, one also measures some basic features of the hadronic final state. In principle, one can measure the energy E_B and the angles θ_B, ϕ_B of the outgoing hadron B. However, it is much simpler to perform a purely calorimetric measurement, in which only the total energy coming into a calorimeter cell at angles θ_B, ϕ_B is measured. This calorimetric measurement gives the energy distribution

$$\frac{d\Sigma}{dE' d\cos\theta' d\cos\theta_B d\phi_B} = \sum_B \int dE_B (1 - \cos\theta_B) E_B \frac{d\sigma(e + p \rightarrow B + X)}{dE' d\cos\theta' dE_B d\cos\theta_B d\phi_B}. \quad (2)$$

Figure 1: The Born processes.

The sum runs over all species of produced hadrons B. We have included a factor $(1 - \cos\theta_B)$ in the definition because this factor is part of the Lorentz invariant dot product $P_{A,\mu} P_B^\mu = E_A E_B (1 - \cos\theta_B)$.

Notice that Σ measures the distribution of energy in the final state as a function of angle, without asking how that energy is split into individual hadrons moving in the same direction. For this reason, the theoretical expression for Σ will not involve parton decay functions that describe how partons decay into hadrons. We discuss this issue in the Conclusions.

At the Born level, the hard scattering process for the reaction (1) is $electron + quark \rightarrow electron + quark$ by means of virtual photon or Z_0 exchange, as illustrated in Fig. 1. At order α_s , one can have virtual corrections to the Born graph. In addition, one can have processes in which there are two scattered partons in the final state. Then the initial parton can be either a quark (or antiquark) or a gluon, while the observed hadron can come from the decay of either of the final state partons, as illustrated in Figures 2, 3, and 4.

Let us consider the effect of the emission of the additional, unobserved, "bremsstrahlung" parton. We can define a part of the photon momentum q^μ that is transverse to the momentum of the incoming hadron P_A^μ and to the momentum of the outgoing hadron P_B^μ . One merely subtracts from q^μ its projections along P_A^μ and P_B^μ :

$$q_t^\mu = q^\mu - \frac{P_B \cdot q}{P_A \cdot P_B} P_A^\mu - \frac{P_A \cdot q}{P_A \cdot P_B} P_B^\mu. \quad (3)$$

It is this transverse momentum that is analogous to the transverse momentum of produced W 's and Z 's or lepton pairs in the Drell-Yan process^[3,4,5,6,7] In the ("naive") parton model, there are no bremsstrahlung partons and all parton momenta are exactly collinear with the corresponding hadron momenta, so one has $q_t^\mu = 0$. At the order α_s , parton emission makes q_t^μ nonzero, in general. We will give formulas for the distribution of events as a function of $q_t^2 = -q_t^\mu \cdot q_{t\mu}$.

Figure 2: Feynman Diagrams for class I. Quark initiated process with a quark jet observed.

Figure 3: Feynman Diagrams for class II. Quark initiated process with a gluon jet observed.

We use order α_s perturbation theory, so we must consider q_T values that are not small. For small q_T , one would need to sum the leading contributions from soft gluon emission to all orders of perturbation theory. Such a summation is beyond the scope of the present paper, but we hope to return to it in a later paper.

This discussion of q_T can be translated to the observables of the HERA lab frame. In the naive parton model, the outgoing hadron B (along with all the other hadrons arising from the decay of the struck quark) emerges in the plane defined by the incoming and outgoing electrons at a precisely defined angle $\theta_B = \theta$, $\phi_B = 0$, which can be computed from the incoming particle momenta and the momentum of the scattered electron. As we shall see, q_T is a measure of the deviation of the angles (θ_B, ϕ_B) of hadron B from the parton model prediction.

The main object of study in this paper is the distribution of events at nonzero q_T as

Figure 4: Feynman Diagrams for class III. Gluon initiated process with a quark jet observed.

a function of the angles between the leptons and the hadrons, analogous to the angular distribution of the leptons in the Drell-Yan process. We will give predictions for the angular distribution at first order in α_s .

The method is straightforward. We write the energy distribution as the product of two tensors, $L_{\mu\nu} W^{\mu\nu}$. Here $L_{\mu\nu}$ describes the leptons and has a simple form, while $W^{\mu\nu}$ describes the hadrons. We expand $W^{\mu\nu}$ in the form

$$W^{\mu\nu} \propto \sum_{k=1}^9 \mathcal{V}_k^{\mu\nu} S_k, \quad (4)$$

where the $\mathcal{V}_k^{\mu\nu}$ are certain tensors made from the photon and hadron momenta. The nine structure functions S_k are Lorentz invariant functions of the photon and hadron momenta. With this decomposition, the energy distribution is proportional to

$$\sum_{k=1}^9 \mathcal{A}_k S_k, \quad (5)$$

where the $\mathcal{A}_k$ are functions of two angles ψ, ϕ that describe the direction of the leptons relative to the hadrons. The structure functions S_k can (at least in principle) be extracted from data and compared to the QCD predictions for them that we present in this paper.

There is an extensive literature on the theory of semi-inclusive deeply inelastic lepton scattering. Here we mention a few of the papers. The decomposition of the cross section in a form $\sum \mathcal{A}_k S_k$ was utilized for the case of electron scattering (with photon exchange) in the early 1970's. [6] With the advent of QCD as a serious theory in the late 1970's, there

was renewed interest in the distribution of final state hadrons as a function of transverse momentum and angles as a test of QCD.^[9] The complete angular decomposition was worked out, and QCD predictions were provided for the hadronic structure functions S_k .^[10] Here, one considered W exchange as well as photon exchange, so that all nine terms appear. Some authors also proposed measuring an energy distribution function similar to that proposed in this paper.^[11] A calculation at the one loop level of certain of the hadronic structure functions that vanish at the tree level was provided in the early 1980's.^[12] Calculations for deeply inelastic scattering at HERA, with some overlap with the present paper, were presented in reference [13].

In the present work, we take a fresh look at the physics that was investigated in the earlier work, given the much improved experimental situation at HERA compared to deeply inelastic scattering off a fixed target. Some changes are straightforward: the energy is high enough that Z exchange is relevant, and one has the kinematics of colliding beams instead of a fixed target. We also make the definition such that the measurement is not sensitive to how partons eventually decay into hadrons. First, we use a transverse momentum variable q_T that represents the transverse momentum of the vector boson relative to the hadron direction, instead of the transverse momentum of the final state hadron relative to the vector boson direction. Only the direction, but not the energy, of the observed final state hadron enters into the definition of q_T . Second, we form the energy distribution function (2), in which one measures the distribution of hadronic energy as a function of angles. As mentioned above, this observable is independent of the details of the hadron fragmentation.

Figure 5: The hadron frame.

2 Basic kinematics and the parton model

In this section, we choose variables that represent parton momentum fractions, a transverse momentum q_T , and angles ψ and ϕ of the leptons relative to the hadrons. We work by analogy to the Drell-Yan case, in which one defines the transverse momentum q_T of the virtual photon and angles θ and ϕ of the leptons relative to the hadrons. The choices that we make are a matter of convention, but the convention must be chosen so as to make the underlying parton physics as simple as possible. We first chose a reference frame, the “hadron frame,” in which the parton physics and the coupling of the hadron physics to the lepton physics is reasonably transparent. The hadron frame is essentially the Breit frame, with an additional specification of the direction of the x axis. It is not at all close to the HERA laboratory frame. We then describe how the chosen physics variables appear in the HERA laboratory frame.

2.1 The hadron frame

Define the hadron frame as follows, as pictured in Fig. 5. In the hadron frame, the time component of the virtual photon momentum is zero and its space components lie along the negative z -axis:

$$q^\mu = (0, 0, 0, -Q), \quad (6)$$

where

$$Q = \sqrt{-q^\mu q_\mu}. \quad (7)$$

The space-components of the momenta of the incoming hadron, A , are exactly along the positive z -axis in the hadron frame. We define a momentum fraction variable x_{bj} conventionally:

$$x_{bj} = Q^2 / (2q \cdot P_A). \quad (8)$$

Then the components of P_A in the hadron frame are

$$P_A^\mu = \left(\frac{Q}{2x_{bj}}, 0, 0, \frac{Q}{2x_{bj}} \right). \quad (9)$$

We align the x-axis of the hadron frame so that the space-components of P_B^μ lie in the x-z plane, with $P_B^y > 0$. (This is possible as long as q_T , defined in the Introduction and below, is nonzero. For $q_T = 0$, $\vec{P}_B$ lies along the negative z-axis of the hadron frame and the direction of the x-axis becomes undefined.)

As discussed in the Introduction, we define $q_T^2 = -q_\mu^2 \cdot q_\mu$, where

$$q_\mu^2 = q^\mu - \frac{P_B \cdot q}{P_A \cdot P_B} P_A^\mu - \frac{P_A \cdot q}{P_A \cdot P_B} P_B^\mu. \quad (10)$$

Note that in the naive parton model, in which parton momenta are exactly parallel to hadron momenta and the hard scattering is treated at the Born level, one has $q_T = 0$. In general, q_T is not zero. Non-zero q_T arises from "intrinsic" parton transverse momentum and, more significantly, from parton emission in the hard scattering process.

The definition (10) allows one to express q_T in terms of dot products:

$$\frac{q_T^2}{Q^2} = 1 + \frac{q \cdot P_B}{x_{bj} P_A \cdot P_B}. \quad (11)$$

The direction of P_B^μ as seen from the hadron frame is fixed once q_T/Q is given. One finds easily that

$$P_B^\mu = z \frac{Q}{2} \left(1 + \frac{q_T^2}{Q^2}, 2 \frac{q_T}{Q}, \frac{q_T^2}{Q^2} - 1 \right), \quad (12)$$

where z is a momentum fraction variable that specifies the normalization of P_B^μ . (14)

$$z = \frac{2x_{bj} P_A \cdot P_B}{Q^2}. \quad (13)$$

and thus

$$q \cdot P_B = \frac{-z Q^2}{2} \left(1 - \frac{q_T^2}{Q^2} \right). \quad (14)$$

In this paper, we discuss experiments that integrate over the energy carried by hadron B (as seen in the HERA lab frame). Therefore, we are more concerned with the direction of P_B^μ than with its normalization. It is thus convenient to work with a vector $\vec{P}_B^\mu$ that lies in the same direction as P_B^μ , but has a conventional normalization. We choose

$$\vec{P}_B^\mu = P_B^\mu / z = \frac{Q}{2} \left(1 + \frac{q_T^2}{Q^2}, 2 \frac{q_T}{Q}, \frac{q_T^2}{Q^2} - 1 \right), \quad (15)$$

It will prove useful to express the unit vectors along the axes of the hadron frame in terms of the vectors P_A^μ , $\vec{P}_B^\mu$, and q^μ :

$$\begin{aligned} T^\mu &= \frac{1}{Q} \{ q^\mu + 2x_{bj} P_A^\mu \}, \\ X^\mu &= \frac{1}{q_T} \left\{ \vec{P}_B^\mu - q^\mu - \left[1 + \frac{q_T^2}{Q^2} \right] x_{bj} P_A^\mu \right\}, \\ Y^\mu &= \epsilon^{\mu\nu\rho\sigma} Z_\nu T_\rho X_\sigma, \\ Z^\mu &= -\frac{q^\mu}{Q}. \end{aligned} \quad (16)$$

These vectors are orthogonal to one another and are normalized to $T_\mu T^\mu = 1$, $X_\mu X^\mu = -1$, $Y_\mu Y^\mu = -1$, $Z_\mu Z^\mu = -1$.

We are now ready to consider the leptons. We let l^μ denote the momentum of the incoming electron and $l'^\mu = l^\mu - q^\mu$ denote the momentum of the outgoing electron. We wish to define angles that describe the directions of l^μ and l'^μ in the hadron frame. Here, we face a fundamental difference with the Drell-Yan case. In the Drell-Yan case, q^μ is time-like, and one chooses a certain reference frame in which it points along the time axis. Then we define polar angles θ, ϕ in this reference frame. In the present case of deeply inelastic scattering, q^μ is spacelike, and we have chosen a certain reference frame in which it points along the negative z-axis. The angles that we want are thus angles that describe (l^μ, l'^μ) , instead of (l^μ, l'^μ, l'^μ) . We first note that the mass shell conditions $l \cdot l = l' \cdot l' = 0$ imply $2l \cdot q = -Q^2$, so that

$$l^\mu = -l \cdot Z = -Q/2. \quad (17)$$

We thus choose angles ψ, ϕ defined by

$$\begin{aligned} l^\mu &= l \cdot T = \frac{Q}{2} \cosh \psi, \\ l'^\mu &= -l \cdot X = \frac{Q}{2} \sinh \psi \cos \phi, \\ l'^\nu &= -l \cdot Y = \frac{Q}{2} \sinh \psi \sin \phi. \end{aligned} \quad (18)$$

For $l'^\mu = l^\mu - q^\mu$ we then have

$$l'^\mu = (l^\mu, l'^\nu, l'^\nu, +Q/2). \quad (19)$$

We choose ψ to be positive by definition. The angles ψ, ϕ can be written in terms of Lorentz invariants by using Eq. (18) with T^μ, X^μ , and Y^μ as given by Eq. (16).

Figure 6: The HERA lab frame.

There is an important difference between the lepton-hadron angles that describe electron-positron annihilation or the Drell-Yan process and the angles ψ, ϕ used here for deeply inelastic scattering. In deeply inelastic scattering at a given c.m. energy $\sqrt{s}$, once one has measured Q and x_B for an event, the 'hyperbolic' angle ψ is determined. Using $s = 2P_A^\mu l_\mu$ with P_A^μ given by Eq. (9) and l_μ given by Eq. (17) and Eq. (18), one finds

$$\cosh \psi = \frac{2x_B s}{Q^2} - 1. \quad (20)$$

Thus to see the dependence of the energy distribution on ψ while holding Q^2 and x_B constant, one must do the experiment at a range of values of s .

2.2 The HERA lab frame

We need to relate the physics variables defined in the hadron frame to the variables observed directly in the HERA laboratory frame, as illustrated in Fig. 6. The momentum components in the lab frame are given in terms of energies and angles by:

$$\begin{aligned} P_A^\mu &= (E_A, 0, 0, E_A), \\ P_B^\mu &= (E_B, E_B \sin \theta_B \cos \phi_B, E_B \sin \theta_B \sin \phi_B, E_B \cos \theta_B), \\ l^\mu &= (E, 0, 0, -E), \\ l'^\mu &= (E', -E' \sin \theta, 0, -E' \cos \theta), \\ q^\mu &= (E - E', E' \sin \theta, 0, E' \cos \theta - E). \end{aligned} \quad (21)$$

The conventionally normalized vector $\hat{P}_B^\mu$, Eq. (15), that gives the direction of the final state hadron is related to P_B^μ by $\hat{P}_B^\mu = P_B^\mu/z$ where

$$z = \frac{2x_B E_A E_B [1 - \cos \theta_B]}{Q^2}. \quad (22)$$

Let us discuss the physics variables $\sqrt{s}, Q, x_B, q_T$, and ϕ as seen in the lab frame. The variables $\sqrt{s}, x_B$, and Q are the familiar variables measured using only the initial beam energies and the energy and angle of the outgoing electron.

$$\begin{aligned} s &= 4EE_A \\ Q^2 &= 2EE'[1 - \cos \theta] \\ x_B &= \frac{EE'[1 - \cos \theta]}{E_A(2E - E'[1 + \cos \theta])} \end{aligned} \quad (23)$$

To measure the variables q_T and ϕ one must look at the final state hadron, B. In a typical event, there will be a jet of produced particles that recoils against the outgoing electron. One thinks of this jet as being made of the remnants of the struck quark. In the parton model, this recoil jet would be infinitely narrow and would be produced at angles $(\theta, \phi) = (\theta, 0)$, where θ is easily computed using the kinematics of massless partons,

$$\cot(\theta/2) = \frac{2x_B E_A}{Q} \left[1 - \frac{Q^2}{x_B s} \right]^{1/2}. \quad (24)$$

In real events, one will typically see a broad jet roughly centered at these angles, or else multiple jets. When the observed hadron B is produced in exactly the parton model direction $(\theta_B, \phi_B) = (\theta, 0)$, then q_T is zero. In general, one finds that q_T^2 may be expressed in terms of the lab frame variables by

$$\begin{aligned} q_T^2 &= \frac{8E^2 - 4E'(2E - E')(1 + \cos \theta)}{1 - \cos \theta_B} \\ &\times \{ \sin^2[(\theta_B - \theta)/2] + \sin \theta_B \sin \theta \sin^2(\phi_B/2) \}. \end{aligned} \quad (25)$$

This formula illustrates that q_T^2 is a measure of how far the observed hadron direction is from the parton model jet-direction. This is indicated in Figures 7 and 8, in which contour lines of q_T^2 as a function of θ_B and ϕ_B are shown for typical choices of E_A, E, E' , and θ .

The relationship of ϕ to the lab frame variables is

$$\cos \phi = \frac{Q}{2q_T} \left[1 - \frac{Q^2}{x_B s} \right]^{-1/2} \left\{ 1 - \frac{Q^2}{x_B s} + \frac{q_T^2}{Q^2} - \left(\frac{Q}{2x_B E_A} \right)^2 \cot^2(\theta_B/2) \right\}. \quad (26)$$

This formula appears quite complicated, but is qualitatively simple. Consider any of the curves $q_T = \text{const}$ in Figures 7 and 8. As one moves around this curve, ϕ increases from 0 to 2π . The lines of constant ϕ are illustrated in the figure.

Q= 30 GeV x=0.02

Figure 7: Contours in ϕ_B and $\cos(\theta_B)$ for $Q = 30 \text{ GeV}$ and $x_{ij} = 0.02$. The circular rings are contours of constant q_T , and the radial arcs are contours of constant ϕ .

3 The lepton and hadron tensors

In this section, we begin the analysis of the energy distribution by writing it in terms of a lepton tensor $L_{\mu\nu}$ and a hadron tensor $W^{\mu\nu}$, which we define. This will enable us to proceed to the decomposition of the hadron tensor in the following section.

We begin with the calorimetric energy distribution

$$\frac{d\Sigma}{dE' d\cos\theta' d\cos\theta_B d\phi_B} = \sum_B \int dE_B (1 - \cos\theta_B) E_B \frac{d\sigma(e+p \rightarrow B+X)}{dE' d\cos\theta' dE_B d\cos\theta_B d\phi_B}. \quad (27)$$

It will be useful to express the energy distribution in terms of the physics variables Q^2, x_{ij}, q_T^2, ϕ instead of the lab frame variables $E', \theta', \theta_B, \phi_B$. At the same time, we change the integration variable from E_B to z . Thus the energy distribution of interest is

$$\frac{d\Sigma}{dQ^2 dx_{ij} dq_T^2 d\phi} = \sum_B \int dz (1 - \cos\theta_B) E_B \frac{d\sigma(e+p \rightarrow B+X)}{dQ^2 dx_{ij} dq_T^2 d\phi dz}, \quad (28)$$

where now we express $(1 - \cos\theta_B)E_B$ as

$$(1 - \cos\theta_B)E_B = z \frac{Q^2}{2x_{ij}E_A}, \quad (29)$$

The Jacobian for this change of variables is given by

$$E' dE' d\cos\theta = \frac{Q^2}{2x_{ij}^2 s} dQ^2 dx_{ij} \quad (30)$$

and

$$E_B dE_B d\cos\theta_B d\phi_B = \frac{1}{2} z dz dq_T^2 d\phi. \quad (31)$$

Q= 30 GeV x=0.04

Figure 8: Contours in ϕ_B and $\cos(\theta_B)$ for $Q = 30 \text{ GeV}$ and $x_{ij} = 0.04$. The circular rings are contours of constant q_T , and the radial arcs are contours of constant ϕ .

In this way, the energy distribution can be written in the form

$$\frac{d\Sigma}{dQ^2 dx_{ij} dq_T^2 d\phi} = \frac{1}{32(2\pi)^5} \frac{Q^4}{x_{ij}^3 s^2 E_A} \sum_{V_1 V_2} \frac{L_{\mu\nu} W^{\mu\nu}}{(Q^2 + M_{V_1}^2)(Q^2 + M_{V_2}^2)}. \quad (32)$$

Here we are summing over vector mesons V_1 and V_2 with $V_j = \gamma, Z$. The tensors $L_{\mu\nu}$ and $W^{\mu\nu}$ depend on V_1, V_2 in addition to the momenta in the problem.

The lepton tensor has the form

$$L_{\mu\nu} = \frac{1}{4} \text{Tr} \left\{ \not{V} \cdot \gamma \Gamma_\mu(V_1) \not{V} \cdot \gamma \bar{\Gamma}_\nu(V_2) \right\} \quad (33)$$

where

$$\Gamma_\mu(V) = i\gamma_\mu (g_v^c(V) + g_a^c(V)\gamma_5) \quad (34)$$

and the couplings $g_v^c(V)$ and $g_a^c(V)$ are listed in Table 1. Thus

$$L_{\mu\nu} = L_{\mu\nu}^S + L_{\mu\nu}^A \quad (35)$$

where the symmetric (S) and antisymmetric (A) tensors are

$$L_{\mu\nu}^S = [g_v^c(V_1)g_v^c(V_2) + g_a^c(V_1)g_a^c(V_2)] \left[l_\mu l_\nu + l'_\mu l'_\nu - \frac{1}{2} g_{\mu\nu} Q^2 \right], \quad (36)$$

$$L_{\mu\nu}^A = [g_v^c(V_1)g_a^c(V_2) + g_a^c(V_1)g_v^c(V_2)] i\epsilon_{\mu\nu\rho\sigma} l^\rho l'^\sigma. \quad (37)$$

Fermions	$g_u(\gamma)$	$g_s(\gamma)$	$g_v(Z)$	$g_a(Z)$
e^-	$-e$	0	$-e \frac{1-4\sin^2\theta_W}{4\cos\theta_W\sin\theta_W}$	$+e \frac{1}{4\cos\theta_W\sin\theta_W}$
u, c, t	$\frac{2}{3}e$	0	$+e \frac{1-\frac{2}{3}\sin^2\theta_W}{4\cos\theta_W\sin\theta_W}$	$-e \frac{1}{4\cos\theta_W\sin\theta_W}$
d, s, b	$-\frac{1}{3}e$	0	$-e \frac{1-\frac{4}{3}\sin^2\theta_W}{4\cos\theta_W\sin\theta_W}$	$+e \frac{1}{4\cos\theta_W\sin\theta_W}$

Table 1. Boson-fermion couplings.

The hadronic tensor is defined by

$$W_{\mu\nu}(P_A, \tilde{P}_B, q; V_1, V_2) = \frac{1}{2} \sum_{s, s', B, N} \int d^4x e^{-iq \cdot x} \int_0^1 dz z^2 \times \langle P_A, s | J_\nu^{V_2}(0) | z \tilde{P}_B, s', B, N \rangle \langle z \tilde{P}_B, s', B, N | J_\mu^{V_1}(x) | P_A, s \rangle. \quad (38)$$

Here $J_\mu^V(x)$ is the electroweak current operator that couples to vector boson V . The notation implies an average over the initial proton spin and a sum over the final hadron spins. There are also sums over the flavor B of the observed final state hadron and over the unobserved final state hadrons N . The observed final state hadron carries momentum $z \tilde{P}_B$. We integrate over the magnitude of this momentum, so that $W_{\mu\nu}$ depends only on its direction, which is specified by $\tilde{P}_B$. The hadronic tensor depends on the vector boson labels V_1, V_2 . Since the lepton tensor is symmetric in these labels, only the part of $W_{\mu\nu}$ that is symmetric in V_1, V_2 will contribute to the energy distribution.

Because of the hermiticity of the current operator, the hadronic tensor satisfies

$$W_{\mu\nu}(P_A, \tilde{P}_B, q; V_1, V_2) = [W_{\mu\nu}(P_A, \tilde{P}_B, q; V_2, V_1)]^*. \quad (39)$$

Current conservation implies

$$\begin{aligned} q^\mu W_{\mu\nu}(P_A, \tilde{P}_B, q; V_1, V_2) &= 0 \\ q^\nu W_{\mu\nu}(P_A, \tilde{P}_B, q; V_1, V_2) &= 0. \end{aligned} \quad (40)$$

In the case of axial currents, the current is only partially conserved. However, the extra terms in $W_{\mu\nu}$ are proportional to q_μ or q_ν and do not contribute to the energy distribution because they give zero when multiplied by $L^{\mu\nu}$.

Using Lorentz invariance, current conservation, and the hermiticity of the current operator, we express $W_{\mu\nu}$ in a basis of nine independent structure functions S

$$W^{\mu\nu} = \sum_{k=1}^9 \mathcal{V}_k^{\mu\nu} S_k. \quad (41)$$

Here $\{\mathcal{V}_1^{\mu\nu}, \dots, \mathcal{V}_9^{\mu\nu}\}$ are nine tensors constructed from $P_A^\mu, \tilde{P}_B^\mu$ and q^μ . The structure functions S_k contain the information about hadrons. They are Lorentz invariant functions of $P_A^\mu, \tilde{P}_B^\mu$ and q^μ . Thus they depend only on the Lorentz invariant combinations x_b, Q , and q_T .

To construct the tensors $\mathcal{V}_k$, we make use of the vectors $T^\mu, X^\mu, Y^\mu, Z^\mu$ defined in Eq. (16). We define

$$\begin{aligned} \mathcal{V}_1^{\mu\nu} &= X^\mu X^\nu + Y^\mu Y^\nu \\ \mathcal{V}_2^{\mu\nu} &= g^{\mu\nu} + Z^\mu Z^\nu \\ \mathcal{V}_3^{\mu\nu} &= T^\mu X^\nu + X^\mu T^\nu \\ \mathcal{V}_4^{\mu\nu} &= X^\mu X^\nu - Y^\mu Y^\nu \\ \mathcal{V}_5^{\mu\nu} &= i(T^\mu X^\nu - X^\mu T^\nu) \\ \mathcal{V}_6^{\mu\nu} &= i\epsilon^{\mu\nu\alpha\beta} Z_\alpha T_\beta = i(X^\mu Y^\nu - Y^\mu X^\nu) \\ \mathcal{V}_7^{\mu\nu} &= i\epsilon^{\mu\nu\alpha\beta} Z_\alpha X_\beta = i(T^\mu Y^\nu - Y^\mu T^\nu) \\ \mathcal{V}_8^{\mu\nu} &= T^\mu Y^\nu + Y^\mu T^\nu \\ \mathcal{V}_9^{\mu\nu} &= X^\mu Y^\nu + Y^\mu X^\nu. \end{aligned} \quad (42)$$

The inverse tensors will be important to project out the structure functions from $W_{\mu\nu}$.

$$\begin{aligned}
\bar{\mathcal{V}}_1^{\mu\nu} &= \left(\frac{1}{2}\right)(2T^\mu T^\nu + X^\mu X^\nu + Y^\mu Y^\nu) \\
\bar{\mathcal{V}}_2^{\mu\nu} &= (T^\mu T^\nu) \\
\bar{\mathcal{V}}_3^{\mu\nu} &= \left(\frac{-1}{2}\right)(T^\mu X^\nu + X^\mu T^\nu) \\
\bar{\mathcal{V}}_4^{\mu\nu} &= \left(\frac{1}{2}\right)(X^\mu X^\nu - Y^\mu Y^\nu) \\
\bar{\mathcal{V}}_5^{\mu\nu} &= \left(\frac{1}{2}\right)(T^\mu X^\nu - X^\mu T^\nu) \\
\bar{\mathcal{V}}_6^{\mu\nu} &= \left(\frac{-1}{2}\right)(X^\mu Y^\nu - Y^\mu X^\nu) \\
\bar{\mathcal{V}}_7^{\mu\nu} &= \left(\frac{1}{2}\right)(T^\mu Y^\nu - Y^\mu T^\nu) \\
\bar{\mathcal{V}}_8^{\mu\nu} &= \left(\frac{-1}{2}\right)(T^\mu Y^\nu + Y^\mu T^\nu) \\
\bar{\mathcal{V}}_9^{\mu\nu} &= \left(\frac{1}{2}\right)(X^\mu Y^\nu + Y^\mu X^\nu).
\end{aligned} \tag{43}$$

The first five of these tensors are even under a parity transformation while tensors 6 through 9 are odd under parity. Tensors 1,2,3,4,8, and 9 are symmetric under interchange of μ, ν while tensors 5,6, and 7 are antisymmetric. We have defined the antisymmetric tensors with a factor i so that the hermiticity condition Eq. (39) implies that the parts of all of the structure functions S that are symmetric in the vector boson indices V_1, V_2 are real.

One can sort the tensors according to their properties under rotations about the Z -axis. The tensors $\mathcal{V}_1^{\mu\nu}, \mathcal{V}_2^{\mu\nu}$, and $\mathcal{V}_6^{\mu\nu}$ are invariant under such rotations and thus carry (Z -component of) angular momentum zero. The tensors $\mathcal{V}_3^{\mu\nu}, \mathcal{V}_5^{\mu\nu}$, and $\mathcal{V}_8^{\mu\nu}$ carry angular momentum ± 1 , while $\mathcal{V}_4^{\mu\nu}$ and $\mathcal{V}_9^{\mu\nu}$ carry angular momentum ± 2 .^[15]

The tensors $\mathcal{V}_1^{\mu\nu}$ and $\mathcal{V}_6^{\mu\nu}$ are especially important because the energy distribution at $q_T^2 = 0$ at order α_s^0 consists of terms that are proportional to these tensors. To see this, recall that the energy distribution at order α_s^0 is proportional to $\text{Tr}\{(P_A \cdot \gamma)\gamma^\mu(\tilde{P}_B \cdot \gamma)\gamma^\nu\}$ for the parity conserving part and to $\text{Tr}\{(P_A \cdot \gamma)\gamma^\mu(\tilde{P}_B \cdot \gamma)\gamma^\nu\gamma_5\}$ for the parity violating part. When $q_T^2 = 0$, a simple calculation shows that these two tensors are proportional to the two tensors $\mathcal{V}_1^{\mu\nu}$ and $\mathcal{V}_6^{\mu\nu}$.

In order to express $L_{\mu\nu}W^{\mu\nu}$, and thus the energy distribution, in terms of the structure functions S_k , we write the products $L_{\mu\nu}\mathcal{V}_k^{\mu\nu}$ in the form

$$L_{\mu\nu}\mathcal{V}_k^{\mu\nu} = \frac{Q^2}{2} G_k^i(V_1, V_2) \mathcal{A}_k. \tag{44}$$

Here the G_k^i are the combinations of couplings that appear in $L_{\mu\nu}$:

$$\begin{aligned}
G_k^i(V_1, V_2) &= [g_s^i(V_1)g_s^i(V_2) + g_s^i(V_1)g_s^i(V_2)] & k = 1, 2, 3, 4, 8, 9 \\
G_k^i(V_1, V_2) &= [g_s^i(V_1)g_s^i(V_2) + g_s^i(V_1)g_s^i(V_2)] & k = 5, 6, 7.
\end{aligned} \tag{45}$$

The $\mathcal{A}_k$ are then functions of the angles ψ, ϕ :

$$\begin{aligned}
\mathcal{A}_1 &= (+1) & (1 + \cosh^2(\psi)) \\
\mathcal{A}_2 &= (-2) \\
\mathcal{A}_3 &= (-1) \cos(\phi) & \sinh(2\psi) \\
\mathcal{A}_4 &= (+1) \cos(2\phi) & \sinh^2(\psi) \\
\mathcal{A}_5 &= (+2) \sin(\phi) & \sinh(\psi) \\
\mathcal{A}_6 &= (+2) & \cosh(\psi) \\
\mathcal{A}_7 &= (-2) \cos(\phi) & \sinh(\psi) \\
\mathcal{A}_8 &= (-1) \sin(\phi) & \sinh(2\psi) \\
\mathcal{A}_9 &= (+1) \sin(2\phi) & \sinh^2(\psi)
\end{aligned}$$

Note, for instance, the analogy between the angular coefficient $\mathcal{A}_1, 1 + \cosh^2(\psi)$, which appears in the order α_s^0 energy distribution, and the corresponding coefficient in the case of the Drell-Yan energy distribution, $1 + \cos^2(\theta)$.^[15]

We can now combine eqs. (32), (41), and (44) to obtain an expression for the energy distribution in terms of the structure functions S_k :

$$\begin{aligned}
\frac{d\Sigma}{dQ^2 dx_{bj} dq_T^2 d\phi} &= \frac{1}{64(2\pi)^5 x_{bj} s^2 E_A} \sum_k \sum_{V_1 V_2} \frac{G_k^i(V_1, V_2)}{(Q^2 + M_{V_k}^2)(Q^2 + M_{V_2}^2)} \\
&\quad \times \mathcal{A}_k(\psi, \phi) S_k(x_{bj}, Q^2, q_T^2; V_1, V_2).
\end{aligned} \tag{46}$$

The energy distribution may be written in the compact form

$$\frac{d\Sigma}{dx_{bj} dQ^2 dq_T^2 d\phi} = C(Q^2, x_{bj}, s, E_A) \sum_k \mathcal{A}_k(\psi, \phi) S_k(x_{bj}, Q^2, q_T^2). \tag{47}$$

Here the factor C is the factor in front of the summations in eq. (46),

$$C(Q^2, x_{bj}, s, E_A) = \frac{1}{64(2\pi)^5 x_{bj} s^2 E_A} Q^8. \tag{48}$$

The modified structure functions S_k are defined by

$$S_k(x_{bj}, Q^2, q_T^2) = \sum_{V_1 V_2} \frac{G_k^i(V_1, V_2)}{(Q^2 + M_{V_k}^2)(Q^2 + M_{V_2}^2)} S_k(x_{bj}, Q^2, q_T^2; V_1, V_2). \tag{49}$$

Figure 9: Scaled structure functions $S'_k(x_{ij}, Q^2, q_T^2)$ vs. q_T . The Log_{10} of the absolute value is plotted. Here, $Q = 15 \text{ GeV}$, and $x_{ij} = 0.1$. The scaled structure functions are in units of $\text{GeV}^{-3}/200 \text{ pb}^{-1}$ where 200 pb^{-1} is a nominal value for the annual integrated luminosity.

4 Results

In examining eq. (47), we see that the structure functions $S_k(x_{ij}, Q^2, q_T^2)$ are related to the energy distribution $d\Sigma$ by a constant $C(Q^2, x_{ij}, s, E_A)$ and an angular function $\mathcal{A}_k(\psi, \phi)$. The relative magnitudes of the various structure functions does not directly correspond to the relative contribution to the physical energy distribution; this is because the angular function $\mathcal{A}_k(\psi, \phi)$ can contain large factors arising from the hyperbolic functions of ψ .

For fixed $\{Q^2, x_{ij}, s, E_A\}$, we note that ψ is also fixed; therefore, in order to facilitate a meaningful comparison among the various structure functions, we define a scaled structure function, $S'_k(x_{ij}, Q^2, q_T^2)$,

$$S'_k(x_{ij}, Q^2, q_T^2) = C(Q^2, x_{ij}, s, E_A) \mathcal{P}_k(\psi) S_k(x_{ij}, Q^2, q_T^2), \quad (50)$$

where we introduce the function $\mathcal{P}_k(\psi)$ which is simply $\mathcal{A}_k(\psi, \phi)$ with the ϕ -dependence removed. Specifically, we shall find it convenient to define:

Figure 10: Scaled structure functions $S'_k(x_{ij}, Q^2, q_T^2)$ vs. q_T . The Log_{10} of the absolute value is plotted. Here, $Q = 30 \text{ GeV}$, and $x_{ij} = 0.1$. The scaled structure functions are in units of $\text{GeV}^{-3}/200 \text{ pb}^{-1}$ where 200 pb^{-1} is a nominal value for the annual integrated luminosity.

$$\begin{aligned} \mathcal{N}_1(\phi) &= 1 & \mathcal{P}_1(\psi) &= (+1) (1 + \cosh^2(\psi)) \\ \mathcal{N}_2(\phi) &= 1 & \mathcal{P}_2(\psi) &= (-2) \\ \mathcal{N}_3(\phi) &= \cos(\phi) & \mathcal{P}_3(\psi) &= (-1) \sinh(2\psi) \\ \mathcal{N}_4(\phi) &= \cos(2\phi) & \mathcal{P}_4(\psi) &= (+1) \sinh^2(\psi) \\ \mathcal{N}_5(\phi) &= \sin(\phi) & \mathcal{P}_5(\psi) &= (+2) \sinh(\psi) \\ \mathcal{N}_6(\phi) &= 1 & \mathcal{P}_6(\psi) &= (+2) \cosh(\psi) \\ \mathcal{N}_7(\phi) &= \cos(\phi) & \mathcal{P}_7(\psi) &= (-2) \sinh(\psi) \\ \mathcal{N}_8(\phi) &= \sin(\phi) & \mathcal{P}_8(\psi) &= (-1) \sinh(2\psi) \\ \mathcal{N}_9(\phi) &= \sin(2\phi) & \mathcal{P}_9(\psi) &= (+1) \sinh^2(\psi) \end{aligned}$$

If we now express the physical energy distribution in terms of these scaled structure functions, we have:

$$\begin{aligned} \frac{d\Sigma}{dx_{ij} dQ^2 dq_T^2 d\phi} &= \sum_k \mathcal{N}_k(\phi) S'_k(x_{ij}, Q^2, q_T^2) = \sum_k \Sigma_k \\ &= S'_1 + S'_2 + S'_3 \cos(\phi) + S'_4 \cos(2\phi) + S'_5 \sin(\phi) + \\ &\quad + S'_6 + S'_7 \cos(\phi) + S'_8 \sin(\phi) + S'_9 \sin(2\phi), \end{aligned} \quad (51)$$

where we introduce the shorthand Σ_k to represent the partial contribution to the energy distribution function.

Figure 11: Scaled structure functions $S'_k(x_b, Q^2, q_T^2)$ vs. q_T . The Log_{10} of the absolute value is plotted. Here, $Q = 120 \text{ GeV}$, and $x_b = 0.3$. The scaled structure functions are in units of $\text{GeV}^{-3}/200 \text{ pb}^{-1}$ where 200 pb^{-1} is a nominal value for the annual integrated luminosity.

4.1 Structure Functions

In Figs. 9, 10, and 11, we show how the scaled structure functions behave as functions of q_T for three choices of $\{x_b, Q^2\}$.^[16] We have plotted the logarithm of the absolute values of the structure functions. Notice that S'_3 and S'_7 change sign for $q_T/Q \sim 0.6$; both are negative for small q_T , and become positive for large q_T . S'_2 is negative, and $\{S'_1, S'_4, S'_6\}$ are positive.

We first take note that three of the structure functions, namely S'_5 , S'_8 , and S'_9 , are zero at this order of perturbation theory. In fact, by applying time reversal invariance to the Feynman diagrams used in the calculation of $W^{\mu\nu}$, one finds that

$$W^{\mu\nu}(P_A, \bar{P}_B, q; V_1, V_2) = \Lambda_\mu^\alpha \Lambda_\nu^\beta \left[W^{\nu'\mu'}(\Lambda P_A, \Lambda \bar{P}_B, \Lambda q; V_2, V_1) \right]_{i \leftrightarrow -i}, \quad (52)$$

where Λ_μ^α is the Lorentz transformation matrix that reverses the space components of vectors. At the order α , level of calculation, the i terms in the Feynman denominators do not have any effect. One therefore concludes that the terms in $W^{\mu\nu}(P_A, \bar{P}_B, q; V_1, V_2) + W^{\mu\nu}(P_A, \bar{P}_B, q; V_2, V_1)$ that are odd under parity must be odd under interchange of the indices μ, ν , while the terms that are even under parity must be even under interchange of μ, ν . Thus

$$S_8 = S_9 = S_9 = 0 \quad (53)$$

at order α . Because the T-odd parts of $W^{\mu\nu}$ vanish at leading order in α , they are of particular interest. They have been calculated by Hagiwara, Hikasa, and Kai.^[12]

Let us now examine the q_T dependence of the structure functions as $q_T \rightarrow 0$. The Born level hadron tensor $W^{\mu\nu}$ consists of two terms, which are proportional to a delta function of q_T times the tensors $\mathcal{V}_1^{\mu\nu}$ and $\mathcal{V}_6^{\mu\nu}$ that are associated with the structure functions S'_1 and S'_6 . At order α , there are contributions to $W^{\mu\nu}$ that are singular as $q_T \rightarrow 0$, behaving like $1/q_T^2$ times a logarithm of q_T . These terms arise from the emission of partons that are collinear to the incoming parton, collinear to the outgoing parton, or soft. The corresponding tensor structure is the same as for the Born term. Thus S'_1 and S'_6 have a $1/q_T^2$ behavior, which can be seen in the graphs. We can now imagine expanding $W^{\mu\nu}$ in powers of q_T . With one extra power of q_T , one can get a $\cos(\phi)$ angular function. This gives contributions to S'_3 and S'_7 that are of order $|q_T|/q_T^2 = 1/q_T$. With two extra powers of q_T , one can get a $\cos(2\phi)$ dependence on ϕ . This gives contributions to S'_4 that are of order $|q_T|^2/q_T^2 = q_T^0$. With two extra powers of q_T , one can also return to an angular function that is independent of ϕ but has a different dependence on the hyperbolic angle ψ than does the Born term. This gives contributions to S'_2 of order q_T^0 , up to logarithms. This dependence of the various structure functions on q_T can be seen in the graphs.

We should emphasize that one cannot trust fixed order perturbation theory when q_T/Q is small – precisely because of the singular behavior discussed above. When q_T/Q is small, only the S_1 and S_6 structure functions contribute significantly to $d\Sigma/dx_b dQ^2 dq_T^2 d\phi$, but these receive contributions from all orders of perturbation theory, leading to a Sudakov exponentiation similar to that found in the Drell-Yan process at small q_T/Q . The analysis of this exponentiation is beyond the scope of the present paper.^[17,18,19]

In examining the evolution of the structure functions as Q^2 increases, we note that at larger values of Q^2 , the parity violating structure functions (S'_6 and S'_7) increase in proportion to the parity conserving structure functions (S'_1 , S'_3 , S'_5 , and S'_4). This is, of course, because the parity violating structure functions are overcoming the $Q^2/(M_Z^2 + Q^2)$ suppression arising from the Z^0 propagator. Understandably, it is the $Z^0 - \gamma$ interference which gives the dominant contribution.

Also note that the ordering of the structure functions is preserved for the different choices of $\{x_b, Q^2\}$: $S'_6 > S'_7$ for the parity violating structure functions; and $S'_1 > S'_3 > S'_4 > S'_5$ (for small q_T/Q) for the parity conserving functions.

As Q^2 increases, the range in magnitude of the structure functions decreases. For $Q = 15 \text{ GeV}$, the structure functions span nearly 6 decades, while for $Q = 120 \text{ GeV}$ the structure functions span only 3 decades. Therefore, decreased sensitivity is required to measure the smaller structure functions (i.e., S'_2 and S'_4) at large Q^2 ; however, since the

Figure 12: ϕ -distribution, $d\Sigma/dx_{ij} dQ^2 dq_T^2 d\phi$ for $Q = 30 \text{ GeV}$, and $x_{ij} = 0.03$. These distributions are scaled by the ϕ -independent energy distribution functions, $\Sigma_1 + \Sigma_2 + \Sigma_6 = \Sigma_A$. (Note, Σ_1 , which is the dominant contribution, does not fall in the plot range.)

number of events falls steeply with Q^2 , the accuracy of this measurement will be limited by the statistics.

4.2 ϕ -Distributions

In Figs. 12 and 13 we show the ϕ distribution of the individual contributions to the energy distribution scaled by the ϕ -independent energy distribution functions $\Sigma_1 + \Sigma_2 + \Sigma_6 = \Sigma_A$, (where we introduce the shorthand Σ_A to represent the sum of Class A, cf., Table 2.) Let us note the general features of these curves. First, the overall magnitude is determined by the scaled structure functions, S'_k . Second, as mentioned above, three of the energy distribution functions, namely Σ_5 , Σ_8 , and Σ_9 , are zero at this order of perturbation theory. All three of these energy distribution functions are odd functions of ϕ : either $\sin(\phi)$, or $\sin(2\phi)$. Conversely, these are the only energy distribution functions which are odd functions of ϕ . Therefore, to order α_s , QCD predicts that the ϕ -distribution is symmetric. Certainly, this will be an easy prediction to test.

Figure 13: ϕ -distribution, $d\Sigma/dx_{ij} dQ^2 dq_T^2 d\phi$ for $Q = 120 \text{ GeV}$, and $x_{ij} = 0.3$. These distributions are scaled by the ϕ -independent energy distribution functions, $\Sigma_1 + \Sigma_2 + \Sigma_6 = \Sigma_A$. (Note, Σ_1 , which is the dominant contribution, and Σ_6 , which is $\sim 25\%$ of the dominant contribution, do not fall in the plot range.)

Fig. 12 shows four separate values of q_T for $Q = 30 \text{ GeV}$ and $x_{ij} = 0.03$ with $q_T/Q = 0.2, 0.4, 0.6$, and 0.8 . Note that for small values of Q^2 , the $\cos(\phi)$ contribution arising from Σ_3 are $\sim 10\%$ of Σ_A , and the $\cos(2\phi)$ contribution arising from Σ_4 are $\sim 3\%$ of Σ_A . As Q^2 increases, the $\cos(\phi)$ contributions decrease to $\sim 8\%$ of Σ_A , and the $\cos(2\phi)$ contributions increase to $\sim 12\%$ of Σ_A . The Σ_2 energy distribution functions is roughly -1% , the Σ_6 energy distribution functions is roughly $+1\%$, and the Σ_7 energy distribution functions is roughly $\pm 0.01\%$ for this choice of $\{x_{ij}, Q^2\}$.

Fig. 13 shows four separate values of q_T for $Q = 120 \text{ GeV}$ and $x_{ij} = 0.3$ with $q_T/Q = 0.2, 0.4, 0.6$, and 0.8 . The features of this curve are similar to Fig. 12. In particular, we note that the parity violating contributions, Σ_6 and Σ_7 , have increased relative to the parity conserving contributions. Specifically, Σ_6 (not shown) is $\sim 25\%$ of Σ_A , and Σ_7 is $\sim 2\%$ of Σ_A .

From these plots, it is clear that we can hope to extract four classes of energy distribution functions from the data, as shown in Table 2.

Figure 14: Fractional gluon contribution (Gluon/Total) for the total energy distribution $d\Sigma/dx_b, dQ^2 dq_T^2$ vs. q_T for a range of $\{x_b, Q^2\}$.

Class	Structure Functions	ϕ -Dependence	Angular Momentum	Magnitude
A	$\{S'_1, S'_2, S'_6\}$	1	0	$\sim 100\%$
B	$\{S'_3, S'_7\}$	$\cos(\phi)$	1	$\sim 10\%$
C	$\{S'_4\}$	$\cos(2\phi)$	2	$\sim 10\%$
D	$\{S'_5, S'_8, S'_9\}$	$\sin(\phi)$ or $\sin(2\phi)$	1 or 2	$\leq O(\alpha_s^2)$

Table 2. Structure Function Classes.

4.3 Gluon Contributions

Measurement of the structure functions at finite values of q_T/Q can provide important information on the distribution of gluons in the proton. We recall that the Born term in deeply inelastic scattering has contributions from quarks and antiquarks, but not from gluons. Gluon contributions occur at order α_s (if one uses the $\overline{MS}$ definition of parton distributions). However, in order to extract both the quark and the gluon distributions from the same data, one must use the Q^2 dependence of the data. Thus it is not easy

Figure 15: Fractional gluon contribution (Gluon/Total) for the scaled structure functions $S'_k(x_b, Q^2, q_T^2)$ vs. q_T . Here, $Q = 30 \text{ GeV}$, and $x_b = 0.1$.

to determine the gluon distribution accurately from inclusive deeply inelastic scattering. However, if one has measured the quark distributions via the inclusive cross section, then the gluon distribution can be determined from the data on the energy distribution at finite q_T/Q . One will be limited by statistics, since the main contributions to the energy distribution at finite q_T/Q come from a fraction α_s of the events – those events in which there are two current jets instead of one. Nevertheless, one is helped because both quarks and gluons contribute at the Born (i.e., $O(\alpha_s)$) level for $q_T/Q \neq 0$. [20]

To estimate the sensitivity of this process, we compute the contribution of the gluon distribution to the total energy distribution. This result is shown in figure 14 for a variety of $\{x_b, Q^2\}$ values. As anticipated, the gluon contribution decreases with increasing Q^2 , and increases with decreasing x_b . Thus, for small values of x_b and Q^2 , the gluon distribution can be responsible for over half of the total energy distribution; this should provide us with a powerful tool in probing the gluon distribution.

In Figs. 15 and 16 we show the fractional gluon contribution for the scaled structure functions as a function of q_T for each individual structure function. This fraction is defined to be the contribution of the scaled structure functions due to the gluon divided by the total quark plus gluon contribution. Note the large variation of the S'_3 and S'_7 ratios are due to the zeros of these structure functions. Whereas the total energy distribution is dominated by S'_1 , we see that the other structure functions can receive different contribution from the gluons. For example, S'_2 and S'_4 characteristically receive a larger contribution from the

Figure 16: Fractional gluon contribution (Gluon/Total) for the scaled structure functions $S'_k(x_B, Q^2, q_T^2)$ vs. q_T . Here, $Q = 15 \text{ GeV}$, and $x_{Bj} = 0.1$.

gluons than the S'_1 term. Conversely, the parity violating structure functions (arising from $Z - \gamma$ or $Z - \gamma$) receive essentially no contribution from the gluons.

5 Conclusions

Semi-inclusive deeply inelastic scattering at HERA will provide a good opportunity for testing quantum chromodynamics. One will be able to investigate the final state for transverse momenta q_T that are large enough that there should be little confusion from non-perturbative effects. Tests of QCD using nearly on-shell photons will also be important, but the investigation of the final state of deeply inelastic scattering should play a special role because the measurement of the structure functions such as $F_2(x, Q^2)$ in inclusive deeply inelastic scattering is so important. According to perturbative QCD, the dependence of these structure functions on Q^2 is controlled by final states containing large transverse momentum partons. These high P_T final states occur in a fraction α_s of the events. If this theory is right, then one must be able to measure directly the energy flow in these final states.

One kind of measurement that can be performed consists of measuring directly the distribution of single hadrons B in the final state as a function of θ_B, ϕ_B and E_B . In the corresponding QCD calculation, there appear collinear singularities, which are factored into parton decay functions. However, these parton decay functions must be determined

experimentally. One expects that more conclusive results can be obtained from measurements that are independent (in the high Q^2 and q_T^2 limit) of how the final state partons fragment into hadrons. That is, the calculation at the parton level should be "infrared safe" in the sense of producing a finite answer when computed perturbatively without the infrared cutoffs that are provided in nature by parton masses and hadron wavefunctions.

We have proposed that the energy distribution function, analogous to the energy-energy correlation function in electron annihilation,^[2] is suitable for such a measurement. Why is quantity infrared safe? The physics, if not the mathematics, is quite simple. The physical question is the following: is the energy distribution function sensitive to the details of parton decay? For a given x_{Bj} and Q^2 , the energy correlation function is defined as the average over events of the energy $(\sum E_B) d\theta_B d\phi_B$ in an infinitesimal calorimeter cell at angles θ_B, ϕ_B . However, as long as the energy distribution is a smooth function of the angles, one can obtain the same result by averaging over a finite range of angles θ_B, ϕ_B . Now, according to the theory, the energy E_B comes from partons, and the parton's energy will be distributed energy in a jet that covers a small angular range of the calorimeter. Insensitivity to the details of parton decay is maintained because when we average over a range of θ_B, ϕ_B that is large compared to the jet size, we pick up all the energy of the jet. By energy conservation, this energy equals the energy of the initial parton. Notice that the crucial physical requirement is that the angular distance over which the energy distribution varies appreciably should be small compared to the angular size of a typical jet.

We have proposed describing the direction θ_B, ϕ_B using the transverse momentum variable q_T , which gives the transverse momentum of the virtual vector boson relative to the initial and final state hadron directions. In the earlier literature,^[3] one finds a transverse momentum measured by the transverse momentum of the final state hadron relative to the direction defined by the initial state hadron and the virtual vector boson. Our choice is made by analogy to the Drell-Yan process, in which one measures the transverse momentum of the virtual vector boson relative to the direction defined by the two incoming hadrons. This choice is essential for us because q_T depends only on the direction of the final state hadron, but not on its energy, so that it can be utilized in a measurement in which one integrates over the energy.

We should mention that there is another kind of infrared safe measurement possible for investigating the final state of deeply inelastic scattering. One could measure a differential cross section $d\sigma/dE' d\cos\theta' dE_J d\cos\theta_J d\phi_J$ for producing one jet plus anything in the

hadronic final state. As long as one is careful about the precise definition of what one means by a jet, the jet cross section will be subject to the same angular decomposition that appears in this paper. This jet cross section has a nominal advantage for making precision QCD tests: one gains a variable, E_J , in addition to the two angular variables θ_J, ϕ_J . However, we suspect that the number of deeply inelastic scattering events, large though it may be, will not be sufficient for a detailed analysis in a total of two electron variables and three hadron variables.

Finally, we have noted that the hadronic structure functions at large q_T receive substantial contributions from incoming gluons. Thus the measurement of the energy distribution at large q_T can provide a rather direct measurement of the gluon distribution in a proton. To make such an analysis precise, one should supplement the order α_s calculation of the hadronic structure functions given in this paper to an order α_s^2 calculation. Given the order α_s^2 results presently available, [21] this would presumably not be an impossible task.

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6 Appendix A

6.1 Fundamental Formulas

We now give some explicit formulas for computation of the structure functions and energy distribution contributions. The fundamental formula is:

$$\frac{d\Sigma}{dx_{ij} dQ^2 d\hat{q}_T^2 d\phi} = C(Q^2, x_{ij}, s, E_A) \sum_k A_k(\psi, \phi) S_k(x_{ij}, Q^2, q_T^2). \quad (54)$$

where

$$C(Q^2, x_{ij}, s, E_A) = \frac{1}{64(2\pi)^5 x_{ij} s^2 E_A} Q^6 \quad (55)$$

The function $S_k(x_{ij}, Q^2, q_T^2)$ is written as a sum over the intermediate vector bosons;

$$S_k(x_{ij}, Q^2, q_T^2) = \sum_{V_1 V_2} \frac{G_k^2(V_1, V_2)}{(Q^2 + M_{V_1}^2)(Q^2 + M_{V_2}^2)} S_k(x_{ij}, Q^2, q_T^2; V_1, V_2), \quad (56)$$

where $G_k^2(V_1, V_2)$ are the combinations of couplings which appear in $L_{\mu\nu}$ as defined in eq. 45.

6.2 At the parton level

To relate the above hadron quantities to the perturbatively calculable ones of the parton level, we have:

$$S_k(x_{ij}, Q^2, q_T^2; V_1, V_2) = \sum_q \int_{x_{ij}}^1 \frac{d\xi_q}{\xi_q} f_{q/A}(\xi_q, \mu) \hat{S}_k^q(\hat{x}, Q^2, q_T^2; V_1, V_2). \quad (57)$$

Here, $\hat{x} = x_{ij}/\xi_q = q^2/(2k_a \cdot q)$. The parton level function $\hat{S}_k^q(\hat{x}, Q^2, q_T^2; V_1, V_2)$ is calculated using incoming and outgoing partons in place of hadrons. It is obtained from the parton level $\hat{W}_a^{\mu\nu}$ by inverting the relation: $\hat{W}_a^{\mu\nu}(k_a^\mu, \hat{k}_b^\mu, q^\mu, V_1, V_2) = \sum_k \mathcal{V}_k^{\mu\nu} \hat{S}_k^q(\hat{x}, Q^2, q_T^2; V_1, V_2)$ to find:

$$\hat{S}_k^q(\hat{x}, Q^2, q_T^2; V_1, V_2) = (\mathcal{V}_k^{-1})_{\mu\nu} \hat{W}_a^{\mu\nu}(k_a^\mu, \hat{k}_b^\mu, q^\mu, V_1, V_2) \quad (58)$$

where $(\mathcal{V}_k^{-1})_{\mu\nu}$ represents the inverse of $\mathcal{V}_k^{\mu\nu}$ as given in equation (44). We calculate $\hat{W}_a^{\mu\nu}$ using

$$\hat{W}_a^{\mu\nu}(k_a^\mu, \hat{k}_b^\mu, q^\mu; V_1, V_2) = \int_0^1 d\hat{z} \hat{z}^3 B_a^{\mu\nu}(k_a^\mu, \hat{z}\hat{k}_b^\mu, q^\mu; V_1, V_2) 2\pi \delta((q^\mu + k_a^\mu - \hat{z}\hat{k}_b^\mu)^2). \quad (59)$$

Here $B_a^{\mu\nu}(k_a^*, \hat{z}_0 k_a^*, q^*, V_1, V_2)$ is the Feynman graph with the on-shell delta function for the unobserved final state parton factored out. To be explicit, the generic form of $B_a^{\mu\nu}$ is:

$$B_a^{\mu\nu}(k_a^*, \hat{z}_0 k_a^*, q^*, V_1, V_2) = \frac{1}{C_{Q,G}} \frac{1}{2} \text{Tr}(t_a^{\mu\nu}) (4\pi\alpha_s) \frac{1}{\hat{p}_1^2 \hat{p}_2^2} T^{\mu\nu}, \quad (60)$$

where

$$T^{\mu\nu} = \text{Tr}[\gamma^\mu (g_v(V_1) + g_a(V_1)\gamma^5) \dots \gamma^\nu (g_v(V_2) + g_a(V_2)\gamma^5) \dots], \quad (61)$$

$\mathcal{P}_1^2$ and $\mathcal{P}_2^2$ are the appropriate parton propagators (*c.f.* diagrams 2, 3, and 4), $C_{Q,G}$ is the color average factor for quarks ($C_Q = 3$) and gluons ($C_G = 8$), and $g_{v,a}(V_{1,2})$ are the vector and axial-vector couplings of the intermediate vector boson ($V_{1,2}$) to the partons.

After a bit of algebra, one finds

$$\delta((q^\mu + k_a^\mu - \hat{z} \hat{k}_b^\mu)^2) = \frac{1}{Q^2} \frac{\hat{z}}{1 - \hat{z}} \hat{z}_0 \delta(\hat{z} - \hat{z}_0), \quad (62)$$

where

$$\hat{z}_0 = \frac{(1 - \hat{x})}{1 - \hat{x}[1 - q_T^2/Q^2]}. \quad (63)$$

Thus

$$\hat{W}_a^{\mu\nu}(k_a^*, \hat{k}_b^*, q^*, V_1, V_2) = \frac{2\pi}{Q^2} \frac{\hat{z}}{1 - \hat{z}} \hat{z}_0^3 B_a^{\mu\nu}(k_a^*, \hat{z}_0 \hat{k}_b^*, q^*, V_1, V_2). \quad (64)$$

Therefore, we have:

$$\begin{aligned} \hat{S}_k^2(\hat{z}, Q^2, q_T^2, V_1, V_2) &= (V_k^{-1})_{\mu\nu} \hat{W}_a^{\mu\nu}(k_a^*, \hat{k}_b^*, q^*, V_1, V_2) \\ &= \frac{2\pi}{Q^2} \frac{\hat{z}}{1 - \hat{z}} \hat{z}_0^3 (V_k^{-1})_{\mu\nu} B_a^{\mu\nu}(k_a^*, \hat{z}_0 \hat{k}_b^*, q^*, V_1, V_2). \end{aligned} \quad (65)$$

Finally, to relate these parton quantities to the hadron quantities,

$$\begin{aligned} S_k(x_b, Q^2, q_T^2, V_1, V_2) &= \sum_a \int_{x_{a,j}} \frac{d\xi_a}{\xi_a} f_{a/A}(\xi_a) \hat{S}_k^2(x_{b,j}/\xi_a, Q^2, q_T^2, V_1, V_2) \\ &= \sum_a \int_{x_{a,j}} \frac{d\xi_a}{\xi_a} f_{a/A}(\xi_a) \frac{2\pi}{Q^2} \frac{\hat{z}}{1 - \hat{z}} \hat{z}_0^3 (V_k^{-1})_{\mu\nu} B_a^{\mu\nu}(k_a^*, \hat{z}_0 \hat{k}_b^*, q^*, V_1, V_2). \end{aligned} \quad (66)$$

7 Appendix B

To be explicit, we list the traces which enter equation (60). First, the diagrams with an external quark and a quark measured:

$$T_1^{\mu\nu} = \frac{\text{Tr}[\not{\epsilon}_a \gamma^\rho (\not{k}_b - \not{q}) \gamma^\mu (g_1^2 + g_1^2 \gamma^5) \not{k}_b \gamma^\nu (g_2^2 + g_2^2 \gamma^5) (\not{k}_b - \not{q}) \gamma_\rho]}{((k_b - q)^2 (k_b - q)^2)} \quad (67)$$

$$T_2^{\mu\nu} = \frac{\text{Tr}[\not{\epsilon}_a \gamma^\mu (g_1^2 + g_1^2 \gamma^5) (\not{q} + \not{k}_a) \gamma^\rho \not{k}_b \gamma_\rho (\not{q} + \not{k}_a) \gamma^\nu (g_2^2 + g_2^2 \gamma^5)]}{((k_a + q)^2 (k_a + q)^2)} \quad (68)$$

$$T_3^{\mu\nu} = \frac{\text{Tr}[\not{\epsilon}_a \gamma^\rho (\not{k}_b - \not{q}) \gamma^\mu (g_1^2 + g_1^2 \gamma^5) \not{k}_b \gamma_\rho (\not{q} + \not{k}_a) \gamma^\nu (g_2^2 + g_2^2 \gamma^5)]}{((k_b - q)^2 (k_a + q)^2)} \quad (69)$$

$$T_4^{\mu\nu} = \frac{\text{Tr}[\not{\epsilon}_a \gamma^\mu (g_1^2 + g_1^2 \gamma^5) (\not{q} + \not{k}_a) \gamma^\rho \not{k}_b \gamma_\rho (g_2^2 + g_2^2 \gamma^5) (\not{k}_b - \not{q}) \gamma_\rho]}{((k_a + q)^2 (k_b - q)^2)} \quad (70)$$

Next, the diagrams with an external quark and a gluon measured:

$$T_5^{\mu\nu} = \frac{\text{Tr}[\not{\epsilon}_a \gamma^\rho (\not{k}_a - \not{k}_b) \gamma^\mu (g_1^2 + g_1^2 \gamma^5) (\not{k}_a + \not{q} - \not{k}_b) \gamma^\nu (g_2^2 + g_2^2 \gamma^5) (\not{k}_a - \not{k}_b) \gamma_\rho]}{((k_a - k_b)^2 (k_a - k_b)^2)} \quad (71)$$

$$T_6^{\mu\nu} = \frac{\text{Tr}[\not{\epsilon}_a \gamma^\mu (g_1^2 + g_1^2 \gamma^5) (\not{q} + \not{k}_a) \gamma^\rho (\not{k}_a + \not{q} - \not{k}_b) \gamma_\rho (\not{q} + \not{k}_a) \gamma^\nu (g_2^2 + g_2^2 \gamma^5)]}{((q + k_a)^2 (q + k_a)^2)} \quad (72)$$

$$T_7^{\mu\nu} = \frac{\text{Tr}[\not{\epsilon}_a \gamma^\rho (\not{k}_a - \not{k}_b) \gamma^\mu (g_1^2 + g_1^2 \gamma^5) (\not{k}_a + \not{q} - \not{k}_b) \gamma_\rho (\not{q} + \not{k}_a) \gamma^\nu (g_2^2 + g_2^2 \gamma^5)]}{((k_a - k_b)^2 (q + k_a)^2)} \quad (73)$$

$$T_8^{\mu\nu} = \frac{\text{Tr}[\not{\epsilon}_a \gamma^\mu (g_1^2 + g_1^2 \gamma^5) (\not{q} + \not{k}_a) \gamma^\rho (\not{k}_a + \not{q} - \not{k}_b) \gamma^\nu (g_2^2 + g_2^2 \gamma^5) (\not{k}_a - \not{k}_b) \gamma_\rho]}{((q + k_a)^2 (k_a - k_b)^2)} \quad (74)$$

Finally, the diagrams with an external gluon and a quark or anti-quark measured:

$$T_9^{\mu\nu} = \frac{\text{Tr}[(\not{\epsilon}_a + \not{q} - \not{k}_b) \gamma^\rho (\not{q} - \not{k}_b) \gamma^\mu (g_1^2 + g_1^2 \gamma^5) \not{k}_b \gamma^\nu (g_2^2 + g_2^2 \gamma^5) (\not{q} - \not{k}_b) \gamma_\rho]}{((k_b - q)^2 (k_b - q)^2)} \quad (75)$$

$$T_{10}^{\mu\nu} = \frac{\text{Tr}[(\not{\epsilon}_a + \not{q} - \not{k}_b) \gamma^\mu (g_1^2 + g_1^2 \gamma^5) (\not{k}_a - \not{k}_b) \gamma^\rho \not{k}_b \gamma_\rho (\not{k}_a - \not{k}_b) \gamma^\nu (g_2^2 + g_2^2 \gamma^5)]}{((k_a - k_b)^2 (k_a - k_b)^2)} \quad (76)$$

$$T_{11}^{\mu\nu} = \frac{\text{Tr}[(\not{\epsilon}_a + \not{q} - \not{k}_b) \gamma^\rho (\not{q} - \not{k}_b) \gamma^\mu (g_1^2 + g_1^2 \gamma^5) \not{k}_b \gamma_\rho (\not{k}_a - \not{k}_b) \gamma^\nu (g_2^2 + g_2^2 \gamma^5)]}{((k_b - q)^2 (k_a - k_b)^2)} \quad (77)$$

$$T_{12}^{\mu\nu} = \frac{\text{Tr}[(\not{\epsilon}_a + \not{q} - \not{k}_b) \gamma^\mu (g_1^2 + g_1^2 \gamma^5) (\not{k}_a - \not{k}_b) \gamma^\rho \not{k}_b \gamma_\rho (g_2^2 + g_2^2 \gamma^5) (\not{q} - \not{k}_b) \gamma_\rho]}{((k_a - k_b)^2 (k_b - q)^2)} \quad (78)$$

8 Appendix C

To facilitate the estimation of the statistical accuracy which can be achieved with the energy distribution measurement, we provide in Table 3a, the number of events per year ($\mathcal{L} = 200 \text{ pb}^{-1}$) in bins of $\{x_{ij}, Q^2\}$. We have imposed a cut of $y \geq 0.1$ to ensure that we are in a kinematic region where we can accurately measure both the outgoing lepton and jet.^[22] Table 3b is similar to table 3a, but with $y \geq 0.01$ to illustrate to effect of this cut. The x_{ij} and Q^2 values associated with the number of events represents the lower limits $\{x_{ij}(i), Q^2(i)\}$ of the integration region, and the upper limits for the table are $\{x_{ij}(i_{\max}), Q^2(i_{\max})\} = \{1.0, 230 \text{ GeV}^2\}$. Specifically,

$$\# \text{ events} = \mathcal{L}(200 \text{ pb}^{-1}) \times \int_{x_{ij}(i)}^{x_{ij}(i+1)} \int_{Q^2(i)}^{Q^2(i+1)} \frac{d\sigma_{\text{Born}}}{dx_{ij} dy}. \quad (79)$$

The Q^2 bins are 10 GeV^2 , and the x_{ij} bins are either 0.01 or 0.10. Note, these event rates are for the Born process only; we have not assumed any “ K -factors” in this calculation. The number of events integrated over x_{ij} and Q^2 appear in the rightmost column and bottom row, respectively.

BORN NEUTRAL CURRENT: $Y>0.10$ 200 pb ⁻¹ (t)																					203072	
	$x = 0.01$	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	TOTAL			
$Q = 2$	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
10	86794	16922	2896	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	106612	0		
20	22389	13342	9226	5777	2884	1434	643	168	0	0	0	0	0	0	0	0	0	0	55863	0		
30	5025	4062	2919	2237	1785	1468	1234	1055	844	1427	0	0	0	0	0	0	0	0	22056	0		
40	228	1460	1201	937	761	634	538	463	404	2156	143	0	0	0	0	0	0	0	8925	0		
50	0	91	514	470	383	322	275	239	210	1221	476	46	0	0	0	0	0	0	4247	0		
60	0	0	17	190	222	185	158	138	121	717	312	143	24	0	0	0	0	0	2227	0		
70	0	0	0	0	55	111	101	87	77	456	200	96	45	12	1	0	0	0	1241	0		
80	0	0	0	0	0	6	44	59	52	310	137	66	31	13	4	1	0	0	723	0		
90	0	0	0	0	0	0	0	7	27	222	98	47	22	9	3	1	0	0	436	0		
100	0	0	0	0	0	0	0	0	0	138	72	35	16	7	2	1	0	0	271	0		
110	0	0	0	0	0	0	0	0	0	70	55	26	12	5	2	0	0	0	170	0		
120	0	0	0	0	0	0	0	0	0	31	43	21	10	4	1	0	0	0	110	0		
130	0	0	0	0	0	0	0	0	0	8	34	16	8	3	1	0	0	0	70	0		
140	0	0	0	0	0	0	0	0	0	0	23	13	6	3	1	0	0	0	46	0		
150	0	0	0	0	0	0	0	0	0	0	11	11	5	2	1	0	0	0	30	0		
160	0	0	0	0	0	0	0	0	0	0	3	9	4	2	1	0	0	0	19	0		
170	0	0	0	0	0	0	0	0	0	0	0	7	3	1	0	0	0	0	11	0		
180	0	0	0	0	0	0	0	0	0	0	0	3	3	1	0	0	0	0	7	0		
190	0	0	0	0	0	0	0	0	0	0	0	1	2	1	0	0	0	0	4	0		
200	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	2	0		
210	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1	0		
220	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0		
TOT	114436	35877	16773	9611	6090	4160	2993	2216	1735	6756	1607	540	192	66	17	3	0	0	203072	0		

Table 3a. Number of events per year for the Born process. $\mathcal{L} = 200 \text{ pb}^{-1}$, and $y \geq 0.1$.

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BORN NEUTRAL CURRENT: $\gamma s \rightarrow 0.01$ 200 $pb^{-1}(\gamma)$																
	x = 0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.2	0.3	0.4	0.5	0.6	TOT
Q = 2	935442	284827	122723	61260	32804	17921	9510	4488	721	0	0	0	0	0	0	1469696
10	134188	75021	50783	37730	29634	24155	20219	17265	14973	56498	8427	1086	0	0	0	469969
20	22388	13342	9226	6905	5433	4425	3696	3148	2723	15442	6595	3225	1318	365	82	98327
30	5025	4062	2918	2237	1785	1468	1234	1055	915	5206	2211	1070	508	218	79	30017
40	228	1460	1201	937	761	634	538	463	404	2323	991	478	225	96	35	10784
50	0	91	514	470	383	322	275	239	210	1221	526	253	119	51	18	4698
60	0	0	17	190	222	185	158	138	121	717	312	150	70	30	11	2324
70	0	0	0	0	55	111	101	87	77	456	200	96	45	19	7	1256
80	0	0	0	0	0	6	44	59	52	310	137	66	31	13	5	724
90	0	0	0	0	0	0	0	7	27	222	98	47	22	9	3	436
100	0	0	0	0	0	0	0	0	0	138	72	35	16	7	2	271
110	0	0	0	0	0	0	0	0	0	70	55	26	12	5	2	170
120	0	0	0	0	0	0	0	0	0	31	43	21	10	4	1	110
130	0	0	0	0	0	0	0	0	0	8	34	16	8	3	1	70
140	0	0	0	0	0	0	0	0	0	0	23	13	6	3	1	46
150	0	0	0	0	0	0	0	0	0	0	11	11	5	2	1	30
160	0	0	0	0	0	0	0	0	0	0	3	9	4	2	1	19
170	0	0	0	0	0	0	0	0	0	0	0	7	3	1	0	11
180	0	0	0	0	0	0	0	0	0	0	0	3	3	1	0	7
190	0	0	0	0	0	0	0	0	0	0	0	1	2	1	0	4
200	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	2
210	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1
220	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0
TOT	1097272	378803	187383	109729	71077	49227	35775	26949	20223	82632	19738	6613	2408	834	249	2088973

Table 3b. Number of events per year for the Born process. $\mathcal{L} = 200 \text{ pb}^{-1}$, and $y \geq 0.01$.

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