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COMMENT ON Z-Z' MIXING IN EXTENDED GAUGE THEORIES

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ABSTRACT

In gauge theories with gauge groups containing more than one $U(1)$ gauge group, it is possible to have kinetic mixing terms for the different $U(1)$ gauge fields in the Lagrangian. The physical consequences of these mixings are studied. It is shown that electromagnetic interactions are not affected by these mixing terms. The properties of interactions mediated by massive gauge bosons are modified however. Ways of eliminating the kinetic mixing terms are also discussed.

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The properties of the Z-boson are being carefully studied at the LEP and SLC facilities [1]. The experimental study of the Z-boson is one way to test the standard model (SM). It is clearly important to identify and study possible new physics which can modify the properties of the Z-boson from the SM prediction. The purpose of this note is to examine the occurrence and consequences of mixing in the kinetic energy terms of the gauge fields. We will see that in a large class of extensions to the SM, the effects of mixing in the kinetic terms are observable, and lead to mixing of the SM Z-boson with a Z' -gauge boson.

The standard model is a gauge theory with gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. The kinetic terms of the gauge fields are uniquely determined from the gauge principle

$$\mathcal{L}_{kinetic} = -\frac{1}{4} \sum_{a=1}^8 G_{\mu\nu}^a G^{a\mu\nu} - \frac{1}{4} \sum_{a=1}^3 W_{\mu\nu}^a W^{a\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad (1)$$

where $G_{\mu\nu}^a$, $W_{\mu\nu}^a$ and $F_{\mu\nu}$ are the field strength tensors associated with the gauge fields of $SU(3)_c$ (G_{μ}^a), $SU(2)_L$ (W_{μ}^a), and $U(1)_Y$ (B_{μ}) respectively. In particular note that the gauge fields B^{μ} and W_3^{μ} are necessarily orthogonal since no term of the form

$$W_{\mu\nu}^a F^{\mu\nu} \quad (2)$$

respects the gauge principle. Thus the SM does not allow for any mixing of the gauge fields in the kinetic terms. However, the SM is not the only gauge model consistent with present experiments. Other types of models can have mixing in the kinetic terms leading to non-trivial consequences as we will see.

We note first that a mixing term can only occur provided there are two or more field strength tensors $F_{\mu\nu}^1, F_{\mu\nu}^2, \dots$ which are each neutral under the gauge symmetry. This only arises for the abelian $U(1)$ group. Thus the simplest gauge model which has mixing in the kinetic terms is a two $U(1)$ model, with gauge group

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{Y'} \quad (3)$$

Denote the field strength tensors of the two $U(1)$ fields by $F_{\mu\nu}$ and $F'_{\mu\nu}$, then the kinetic terms take the form

$$\mathcal{L}_{kinetic} = a F_{\mu\nu} F^{\mu\nu} + b F'_{\mu\nu} F'^{\mu\nu} + c F_{\mu\nu} F'^{\mu\nu}. \quad (4)$$

By rescaling the gauge fields, the kinetic energy can be expressed in the form

$$\mathcal{L}_{kinetic} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} - \frac{2\alpha}{4} F_{\mu\nu} F'^{\mu\nu}. \quad (5)$$

The constant α is a physical parameter and can not be scaled away. The kinetic term can be diagonalised by making the non-unitary transformation

$$\begin{aligned} \tilde{B} &= B + \alpha B', \\ \tilde{B}' &= \sqrt{1 - \alpha^2} B'. \end{aligned} \quad (6)$$

The requirement of positive kinetic energy implies that $|\alpha| < 1$. To define our notation let us first set $\alpha = 0$. Denote the covariant derivative by

$$\mathcal{D}_\mu = i\partial_\mu + gW_{3\mu}I_3 + g_1B_\mu Y/2 + g'B'_\mu Y'/2 + \dots, \quad (7)$$

where we have only written down the diagonal component of the $SU(2)_L$ gauge fields $W_{3\mu}$ which can mix with the other $U(1)$ gauge fields (B_μ, B'_μ) . In general, for the symmetry breaking pattern:

$$\begin{aligned} & SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_{Y'} \\ & \quad \downarrow M_{Z'} \\ & SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \\ & \quad \downarrow M_W \\ & SU(3)_c \otimes U(1)_Q \end{aligned}$$

The gauge boson mass matrix takes the form:

$$\mathcal{L}_{mass} = V_0^T M V_0, \quad (8)$$

where

$$V_0^T = (A_0, Z_0, Z'_0), \quad M = \begin{pmatrix} 0 & 0 & 0 \\ 0 & M_{Z_0}^2 & \delta \\ 0 & \delta & M_{Z'}^2 \end{pmatrix}, \quad (9)$$

and

$$\begin{aligned} A_0 &= \cos\theta B - \sin\theta W_3, \\ Z_0 &= \sin\theta B + \cos\theta W_3, \\ Z'_0 &= B', \end{aligned} \quad (10)$$

with $e = g\sin\theta$ and Lorentz indices are implicit. A non-zero δ causes the standard $Z - Z'$ mixing which arises from the symmetry breaking. When α is unequal to zero, the fields A_0, Z_0 and Z'_0 are not orthogonal. They can be related to orthogonal fields A, Z and Z' by the non-unitary transformation:

$$V_0 = S V, \quad (11)$$

where

$$V^T = (A, Z, Z'), \quad (12)$$

are the orthogonal fields given by

$$\begin{aligned} A &= \cos\theta \tilde{B} - \sin\theta W_3, \\ Z &= \sin\theta \tilde{B} + \cos\theta W_3, \\ Z' &= \tilde{B}', \end{aligned} \quad (13)$$

and S is the transformation matrix

$$S = \begin{pmatrix} 1 & 0 & \frac{-\cos\theta\alpha}{\sqrt{1-\alpha^2}} \\ 0 & 1 & \frac{-\sin\theta\alpha}{\sqrt{1-\alpha^2}} \\ 0 & 0 & \frac{1}{\sqrt{1-\alpha^2}} \end{pmatrix}. \quad (14)$$

The covariant derivative expressed in terms of the orthogonal fields $\tilde{B}$, $\tilde{B}'$ and W_3 has the form

$$\mathcal{D}_\mu = i\partial_\mu + gW_{3\mu}I_3 + g_1\tilde{B}_\mu Y/2 + \tilde{B}'_\mu \left(\frac{g'}{\sqrt{1-\alpha^2}}Y'/2 - \frac{g_1\alpha}{\sqrt{1-\alpha^2}}Y/2 \right). \quad (15)$$

Note in particular that $\tilde{B}$ couples to the same generator as B , that is $Y/2$.

In terms of the orthogonal fields A, Z, Z' , the gauge boson mass matrix is

$$\mathcal{L}_{mass} = V^T S^T M S V. \quad (16)$$

Note that the true mass matrix - $\mathcal{M} = S^T M S$ has only the lower block 2×2 submatrix non-zero. This means that the field A is massless and is the physical mass-eigenstate field, the photon. The photon couples to the same generator independent of the value of α since the photon is composed of a fixed (i.e. independent of α) combination of $\tilde{B}$ and W_3 , and the fields $\tilde{B}$ and W_3 couple to the generators $Y/2$ and I_3 independently of α (see eq.(15)). Thus if the theory has electric charge quantization to begin with (i.e. when $\alpha = 0$) then it will retain electric charge quantization when α is non-zero.

The photon decouples from the mass matrix of eq.(16) leaving a 2×2 mass matrix describing the masses of the Z and Z' gauge bosons. This mass matrix has the form:

$$\mathcal{L} = W^T X^T M X W, \quad (17)$$

where

$$W^T = (Z, Z'), \quad X = \begin{pmatrix} 1 & \frac{-\sin\theta\alpha}{\sqrt{1-\alpha^2}} \\ 0 & \frac{1}{\sqrt{1-\alpha^2}} \end{pmatrix}, \quad M = \begin{pmatrix} M_{Z_0}^2 & \delta \\ \delta & M_{Z'}^2 \end{pmatrix}. \quad (18)$$

Thus, the true mass matrix has the form

$$\mathcal{M} = X^T M X. \quad (19)$$

Evaluating this mass matrix we find that

$$\mathcal{M} = \begin{pmatrix} a & b \\ b & c \end{pmatrix}, \quad (20)$$

with

$$\begin{aligned} a &= M_{Z_0}^2, \\ b &= \frac{-\sin\theta\alpha}{\sqrt{1-\alpha^2}}M_{Z_0}^2 + \frac{\delta}{\sqrt{1-\alpha^2}}, \\ c &= \frac{M_{Z'}^2}{1-\alpha^2} + \frac{-2\sin\theta\alpha}{1-\alpha^2}\delta + \frac{\sin^2\theta\alpha^2}{1-\alpha^2}M_{Z_0}^2. \end{aligned} \quad (21)$$

Thus the Z and Z' fields mix with mixing angle

$$\tan 2\phi = \frac{2b}{a-c}. \quad (22)$$

Note that the mixing effects on the Z -boson will cancel provided $b = 0$. This occurs when

$$\alpha = \frac{\delta}{M_{Z_0}^2 \sin \theta}. \quad (23)$$

Of course the mixing of the kinetic terms can only cancel with the mixing due to symmetry breaking providing that $|\alpha| < 1$ which follows from positivity of the kinetic energy. It is clear that $Z - Z'$ mixing is α dependent. In general the analysis of $Z - Z'$ gauge bosons in extended gauge models should take into account the α dependence.

Experimentally, there is no sign of any mixing in the neutral current sector, despite the experiments at LEP and SLC [1]. It might be worthwhile to look at ways in which the mixing of the kinetic terms can be eliminated naturally. This may happen if the theory is an effective theory of a gauge theory with no more than one $U(1)$ factor. This is because the introduction of a mixing term in the effective theory is inconsistent with the underlying symmetry of the Lagrangian. Alternatively, the existence of discrete symmetries can prevent the mixing of the Z and Z' . To see this, consider a gauge model with gauge group

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_{Y_1} \otimes U(1)_{Y_2} \quad (24)$$

with the discrete symmetry

$$B_1 \leftrightarrow B_2, \quad (25)$$

where B_1 and B_2 are the gauge bosons of the two $U(1)$'s. To obtain a realistic model with such a gauge and discrete symmetry structure, one could define the first generation to transform under one of the $U(1)$'s, and the second generation to transform under the other $U(1)$, with the third to transform under both but with half of the $U(1)$ charges, so that the diagonal subgroup generated by $Y = Y_1 + Y_2$ is the actual hypercharge. In this case the discrete symmetry obviously interchanges the first and second generation fermions with the third generation and Higgs doublet being neutral. Then, the kinetic terms of the $U(1)$ gauge bosons which respects this discrete symmetry have the form:

$$\mathcal{L}_{kinetic} = -\frac{1}{4}F_{1\mu\nu}F_1^{\mu\nu} - \frac{1}{4}F_{2\mu\nu}F_2^{\mu\nu} - \frac{2}{4}\alpha F_{1\mu\nu}F_2^{\mu\nu}. \quad (26)$$

One might think that there is mixing however this is not the case. The reason is that the linear combination of B_1 and B_2 which couples to hypercharge is $(B_1 + B_2)/\sqrt{2}$ (since $U(1)_Y$ is the diagonal subgroup). Rewriting the kinetic terms in terms of $B = (B_1 + B_2)/\sqrt{2}$ and $B' = (B_1 - B_2)/\sqrt{2}$, then we find

$$\mathcal{L}_{kinetic} = a F_{\mu\nu}F^{\mu\nu} + b F'_{\mu\nu}F'^{\mu\nu}. \quad (27)$$

Note that in this basis there is no mixing term, since under the discrete symmetry $F \rightarrow F, F' \rightarrow -F'$. Thus the mixing terms are forbidden by the discrete symmetry, and consequently, there is no mixing between the Z and Z' coming from the kinetic mixing. Note however that the rescaling of the gauge fields B and B' necessary to put $\mathcal{L}_{kinetic}$ into canonical form is not unphysical. This physical rescaling parameter

scales the coupling constants of the B and B' $U(1)$ symmetries differently so that they are unequal in general (despite the existence of the discrete symmetry). This simple example illustrates that a Lagrangian which possesses discrete symmetry can eliminate mixing of the Z and Z' gauge bosons, although the mixing of the kinetic terms can have other physical effects.

In the above we have discussed in some detail the physical consequences of kinetic mixing terms of the gauge bosons in theories with two $U(1)$ factor gauge groups. The discussions can be easily generalized to theories with any number of $U(1)$ factors. We have shown that the kinetic mixing term does not affect the generator of the photon field for theories with two $U(1)$ factors. It is interesting to note that this is also true even with more than two $U(1)$ groups. This can be shown as follows.

Consider a general gauge theory with many $U(1)$ factor groups, which has after symmetry breaking, only one $U(1)$ unbroken by the vacuum (the photon). Let us temporarily set the kinetic mixing terms of the gauge bosons to zero. Then, after symmetry breaking one can write down the gauge boson mass term as

$$\mathcal{L}_m = B^T M B, \quad (28)$$

where M is the diagonalised mass matrix which has one eigenvalue equal to zero, the corresponding field B_0 of the vector field $B^T = (B_0, \dots)$ is identified with the photon. After the kinetic mixing terms are introduced, the new mass eigenstates become $B'^T = (B'_0, \dots)$ which are related to B by $B = U B'$, where U is a non-unitary matrix whose inverse exists. The new mass matrix M' is given by

$$M' = U^T M U. \quad (29)$$

Such transformation does not change the rank of the matrix and therefore there is also a zero mass eigenstate B'_0 . This zero mass eigenstate is the photon field in the new theory. From eq.(29), we obtain

$$\sum_i m_i^2 U_{i0} U_{i0} = 0. \quad (30)$$

Since $m_i^2 > 0$ for $i > 0$, then $U_{i0} = 0$ for $i > 0$. This implies that the photon field B'_0 couples to the same generator of the photon B_0 of the theory without kinetic mixings up to a constant U_{00} . The constant U_{00} can be rescaled into the coupling constant.

The kinetic mixing terms will induce mixings for massive gauge bosons in the general case of many $U(1)$ factors, just like they have done in our two $U(1)$ analysis. With more than two $U(1)$ factors, the mixings are complicated due to more parameters in the kinetic mixing terms. We will not discuss them here.

In conclusion, we have discussed possible physical consequences of kinetic mixing terms in gauge theories with more than one $U(1)$ gauge group. We have shown that the generator of the photon field is not affected by the kinetic mixing terms. This implies that the new parameters introduced by the kinetic mixing terms do not cause new problems for electric charge quantization. However, the interactions of massive

gauge bosons are modified when kinetic mixing terms are present. In particular, the properties of the Z -boson will be modified from the SM predictions. We have also shown that a Lagrangian with discrete symmetry can prevent the Z -boson from mixing with the Z' , although in general the mixing in the kinetic terms may have other manifestations in this case.

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