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Measurement of the Forward-Backward Asymmetry in $Z \rightarrow b\bar{b}$ and $Z \rightarrow c\bar{c}$

The ALEPH Collaboration

Abstract

From a sample of 150 000 hadronic Z decays collected with the ALEPH detector at LEP, events containing prompt leptons are used to measure the forward-backward asymmetries for the channels $Z \rightarrow b\bar{b}$ and $Z \rightarrow c\bar{c}$, giving the results $A_{\text{FB}}^b = 0.126 \pm 0.028 \pm 0.012$ and $A_{\text{FB}}^c = 0.064 \pm 0.039 \pm 0.030$. These asymmetries correspond to the value of effective electroweak mixing angle at the Z mass $\sin^2 \theta_W(m_Z^2) = 0.2262 \pm 0.0053$.

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*See the following pages for the list of authors.

The ALEPH Collaboration

D. Decamp, B. Deschizeaux, C. Goy, J.-P. Lees, M.-N. Minard

Laboratoire de Physique des Particules (LAPP), IN²P³-CNRS, 74019 Annecy-le-Vieux Cedex, France

R. Alemany, J.M. Crespo, M. Delfino, E. Fernandez, V. Gaitan, Ll. Garrido, Ll.M. Mir, A. Pacheco

Laboratorio de Fisica de Altas Energias, Universidad Autonoma de Barcelona, 08193 Bellaterra (Barcelona), Spain⁸

M.G. Catanesi, D. Creanza, M. de Palma, A. Farilla, G. Iaselli,¹ G. Maggi, M. Maggi, S. Natali, S. Nuzzo, M. Quattromini, A. Ranieri, G. Raso, F. Romano, F. Ruggieri, G. Selvaggi, L. Silvestris, P. Tempesta, G. Zito

INFN Sezione di Bari e Dipartimento di Fisica dell' Università, 70126 Bari, Italy

Y. Gao, H. Hu,²¹ D. Huang, X. Huang, J. Lin, J. Lou, C. Qiao,²¹ T. Ruan,²¹ Wang, Y. Xie, D. Xu, R. Xu, J. Zhang, W. Zhao

Institute of High-Energy Physics, Academia Sinica, Beijing, The People's Republic of China⁹

W.B. Atwood,² L.A.T. Bauerdick, F. Bird, E. Blucher, G. Bonvicini, F. Bossi, D. Brown, T.H. Burnett,³ H. Drevermann, R.W. Forty, C. Grab, R. Hagelberg, S. Haywood, J. Hilgart, B. Jost, M. Kasemann, J. Knobloch, A. Lacourt, E. Lançon, I. Lehraus, T. Lohse, A. Marchioro, M. Martinez, P. Mato, S. Menary, A. Minten, A. Miotto, R. Miquel, H.-G. Moser, J. Nash, P. Palazzi, F. Ranjard, G. Redlinger, A. Roth, J. Rothberg,³ H. Rotscheidt, R. St.Denis, D. Schlatter, M. Takashima, M. Talby,⁴ W. Tejessy, H. Wachsmuth, S. Wasserbaech, S. Wheeler, W. Wiedenmann, W. Witzeling, J. Wotschack, W. Wu^{26,27}

European Laboratory for Particle Physics (CERN), 1211 Geneva 23, Switzerland

Z. Ajaltouni, M. Bardadin-Otwinowska, A. Falvard, R. El Fellous, P. Gay, P. Henrard, J. Jousset, B. Michel, J.-C. Montret, D. Pallin, P. Perret, J. Proriot, F. Prulhière, G. Stimpfl

Laboratoire de Physique Corpusculaire, Université Blaise Pascal, IN²P³-CNRS, Clermont-Ferrand, 63177 Aubière, France

J.D. Hansen, J.R. Hansen, P.H. Hansen, R. Møllerud, E.R. Nielsen, B.S. Nilsson

Niels Bohr Institute, 2100 Copenhagen, Denmark¹⁰

I. Efthymiopoulos, E. Simopoulou, A. Vayaki

Nuclear Research Center Demokritos (NRCD), Athens, Greece

J. Badier, A. Blondel, G. Bonneaud, J. Bourotte, F. Braems, J.C. Brient, G. Fouque, A. Gamess, R. Guirlet, S. Orteu, A. Rosowsky, A. Rougé, M. Rumpf, R. Tanaka, H. Videau

Laboratoire de Physique Nucléaire et des Hautes Energies, Ecole Polytechnique, IN²P³-CNRS, 91128 Palaiseau Cedex, France

D.J. Candlin, E. Veitch

Department of Physics, University of Edinburgh, Edinburgh EH9 3JZ, United Kingdom¹¹

G. Parrini

Dipartimento di Fisica, Università di Firenze, INFN Sezione di Firenze, 50125 Firenze, Italy

M. Corden, C. Georgiopoulos, M. Ikeda, J. Lannutti, D. Levinthal,¹⁶ M. Mermikides, L. Sawyer
Supercomputer Computations Research Institute and Dept. of Physics, Florida State Uni-
versity,

Tallahassee, FL 32306, USA ^{13,14,15}

A. Antonelli, R. Baldini, G. Bencivenni, G. Bologna,⁵ P. Campana, G. Capon, F. Cerutti,
V. Chiarella, B. D'Ettorre-Piazzoli,⁶ G. Felici, P. Laurelli, G. Mannocchi,⁶ F. Murtas, G.P. Mur-
tas, G. Nicoletti, L. Passalacqua, M. Pepe-Altarelli, P. Picchi,⁵ P. Zografou

Laboratori Nazionali dell'INFN (LNF-INFN), 00044 Frascati, Italy

B. Altoon, O. Boyle, A.W. Halley, I. ten Have, J.L. Hearn, J.G. Lynch, W.T. Morton, C. Raine,
J.M. Scarr, K. Smith, A.S. Thompson, R.M. Turnbull

*Department of Physics and Astronomy, University of Glasgow, Glasgow G12 8QQ, United
Kingdom¹¹*

B. Brandl, O. Braun, R. Geiges, C. Geweniger, P. Hanke, V. Hepp, E.E. Kluge, Y. Maumary,
A. Putzer, B. Rensch, A. Stahl, K. Tittel, M. Wunsch

*Institut für Hochenergiephysik, Universität Heidelberg, 6900 Heidelberg, Fed. Rep. of
Germany¹⁷*

A.T. Belk, R. Beuselinck, D.M. Binnie, W. Cameron, M. Cattaneo, P.J. Dornan,¹ S. Dugeay,
A.M. Greene, J.F. Hassard, N.M. Lieske, S.J. Patton, D.G. Payne, M.J. Phillips, J.K. Sedgbeer,
G. Taylor, I.R. Tomalin, A.G. Wright

Department of Physics, Imperial College, London SW7 2BZ, United Kingdom¹¹

P. Girtler, D. Kuhn, G. Rudolph

Institut für Experimentalphysik, Universität Innsbruck, 6020 Innsbruck, Austria¹⁹

C.K. Bowdery,¹ T.J. Brodbeck, A.J. Finch, F. Foster, G. Hughes, N.R. Keemer, M. Nuttall,
A. Patel, B.S. Rowlingson, T. Sloan, S.W. Snow, E.P. Whelan

Department of Physics, University of Lancaster, Lancaster LA1 4YB, United Kingdom¹¹

T. Barczewski, K. Kleinknecht, J. Raab, B. Renk, S. Roehn, H.-G. Sander, M. Schmelling,
H. Schmidt, F. Steeg, S.M. Walther, B. Wolf

Institut für Physik, Universität Mainz, 6500 Mainz, Fed. Rep. of Germany¹⁷

J-P. Albanese, J-J. Aubert, C. Benchouk, V. Bernard, A. Bonissent, D. Courvoisier, F. Etienne,
S. Papalexiou, P. Payre, B. Pietrzyk, Z. Qian

*Centre de Physique des Particules, Faculté des Sciences de Luminy, IN²P³-CNRS, 13288
Marseille, France*

H. Becker, W. Blum, P. Cattaneo, G. Cowan, B. Dehning, H. Dietl, F. Dydak²⁸, M. Fernandez-Bosman, T. Hansl-Kozanecka,²² A. Jahn, W. Kozanecki,^{2,23} E. Lange, J. Lauber, G. Lütjens, G. Lutz, W. Männer, Y. Pan, R. Richter, J. Schröder, A.S. Schwarz, R. Settles, U. Stierlin, J. Thomas, G. Wolf

Max-Planck-Institut für Physik und Astrophysik, Werner-Heisenberg-Institut für Physik, 8000 München, Fed. Rep. of Germany¹⁷

V. Bertin, J. Boucrot, O. Callot,¹ X. Chen, A. Cordier, M. Davier, G. Ganis, J.-F. Grivaz, J. Harvey, Ph. Heusse, P. Janot, D.W. Kim,²⁰ F. Le Diberder, J. Lefrançois,¹ A.-M. Lutz, J.-J. Veillet, I. Videau, Z. Zhang, F. Zomer

Laboratoire de l'Accélérateur Linéaire, Université de Paris-Sud, IN²P³-CNRS, 91405 Orsay Cedex, France

D. Abbaneo, S.R. Amendolia, G. Bagliesi, G. Batignani, L. Bosisio, U. Bottigli, C. Bradaschia, M. Carpinelli, M.A. Ciocci, R. Dell'Orso, I. Ferrante, F. Fidecaro, L. Foà, E. Focardi, F. Forti, A. Giassi, M.A. Giorgi, F. Ligabue, A. Lusiani, E.B. Mannelli, P.S. Marrocchesi, A. Messineo, L. Moneta, F. Palla, G. Sanguinetti, J. Steinberger, R. Tenchini, G. Tonelli, G. Triggiani, C. Vannini, A. Venturi, P.G. Verdini, J. Walsh

Dipartimento di Fisica dell'Università, INFN Sezione di Pisa, e Scuola Normale Superiore, 56010 Pisa, Italy

J.M. Carter, M.G. Green,¹ P.V. March, T. Medcalf, I.S. Quazi, M.R. Saich, J.A. Strong, R.M. Thomas, L.R. West, T. Wildish

Department of Physics, Royal Holloway & Bedford New College, University of London, Surrey TW20 OEX, United Kingdom¹¹

D.R. Botterill, R.W. Clift, T.R. Edgecock, M. Edwards, S.M. Fisher, T.J. Jones, P.R. Norton, D.P. Salmon, J.C. Thompson

Particle Physics Dept., Rutherford Appleton Laboratory, Chilton, Didcot, OXON OX11 0QX, United Kingdom¹¹

B. Bloch-Devaux, P. Colas, C. Klopfenstein, E. Locci, S. Loucatos, E. Monnier, P. Perez, J.A. Perlas, F. Perrier, J. Rander, J.-F. Renardy, A. Roussarie, J.-P. Schuller, J. Schwindling, B. Vallage

Département de Physique des Particules Élémentaires, CEN-Saclay, 91191 Gif-sur-Yvette Cedex, France¹⁸

J.G. Ashman, C.N. Booth, C. Buttar, R. Carney, S. Cartwright, F. Combley, M. Dinsdale, M. Dogru, F. Hatfield, J. Martin, D. Parker, P. Reeves, L.F. Thompson

Department of Physics, University of Sheffield, Sheffield S3 7RH, United Kingdom¹¹

S. Brandt, H. Burkhardt, C. Grupen, H. Meinhard, L. Mirabito, E. Neugebauer, U. Schäfer, H. Seywerd

Fachbereich Physik, Universität Siegen, 5900 Siegen, Fed. Rep. of Germany¹⁷

G. Apollinari, G. Giannini, B. Gobbo, F. Liello, F. Ragusa,²⁵ L. Rolandi, U. Stiegler

Dipartimento di Fisica, Università di Trieste e INFN Sezione di Trieste, 34127 Trieste, Italy

L. Bellantoni, J.F. Boudreau, D. Cinabro, X. Chen, J.S. Conway, D.F. Cowen,²⁴ Z. Feng, D.P.S. Ferguson, Y.S. Gao, J. Grahl, J.L. Harton, J.E. Jacobsen, R.C. Jared,⁷ R.P. Johnson, B.W. LeClaire, Y.B. Pan, J.R. Pater, Y. Saadi, V. Sharma, Z.H. Shi, Y.H. Tang, A.M. Walsh, J.A. Wear, F.V. Weber, M.H. Whitney, Sau Lan Wu, G. Zobernig

Department of Physics, University of Wisconsin, Madison, WI 53706, USA¹²

¹Now at CERN.

²Permanent address: SLAC, Stanford, CA 94309, USA.

³Permanent address: University of Washington, Seattle, WA 98195, USA.

⁴Also Centre de Physique des Particules, Faculté des Sciences, Marseille, France

⁵Also Istituto di Fisica Generale, Università di Torino, Torino, Italy.

⁶Also Istituto di Cosmo-Geofisica del C.N.R., Torino, Italy.

⁷Permanent address: LBL, Berkeley, CA 94720, USA.

⁸Supported by CAICYT, Spain.

⁹Supported by the National Science Foundation of China.

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¹⁹Supported by Fonds zur Förderung der wissenschaftlichen Forschung, Austria.

²⁰Supported by the Korean Science and Engineering Foundation and Ministry of Education.

²¹Supported by the World Laboratory.

²²On leave of absence from MIT, Cambridge, MA 02139, USA.

²³Supported by Alexander von Humboldt Fellowship, Germany.

²⁴Now at California Institute of Technology, Pasadena, CA 91125, USA.

²⁵Now at Dipartimento di Fisica, Università di Milano, Milano, Italy.

²⁶On leave from Institute of High Energy Physics, Academia Sinica, Beijing, The People's Republic of China.

²⁷Now at Argonne National Laboratory, Argonne, IL 60439, USA.

²⁸Also at CERN, PPE Division, 1211 Geneva 23, Switzerland.

Introduction

In the theory of electroweak interactions, interference of the vector and axial-vector currents can result in a cross section for $e^+e^- \rightarrow q\bar{q}$ which is asymmetric with respect to the cosine of the angle between the e^- and q directions. Near the Z pole, in $e^+e^- \rightarrow Z \rightarrow q\bar{q}$, the forward-backward asymmetry, defined as

$$A_{\text{FB}} \equiv \frac{\sigma_F^q - \sigma_B^q}{\sigma_F^q + \sigma_B^q}, \quad (1)$$

arises from the difference between the left-handed and right-handed couplings, g_L and g_R , of the Z both to the initial-state electrons and the final-state quarks. Neglecting small QED and mass effects, at the Z pole it can be written as

$$A_{\text{FB}}^q = \frac{3}{4} \mathcal{A}_e \mathcal{A}_q; \quad \mathcal{A}_f \equiv \frac{g_{L_f}^2 - g_{R_f}^2}{g_{L_f}^2 + g_{R_f}^2} = 2 \frac{g_{V_f} g_{A_f}}{g_{V_f}^2 + g_{A_f}^2} \quad (2)$$

and is a sensitive measure of the ratio of the vector and axial-vector coupling constants, or in the Standard Model, of the effective electroweak mixing angle at the Z mass, $\sin^2 \theta_W(m_Z^2)$.

The forward-backward asymmetry can be measured in the case of heavy quarks by using leptons from semileptonic decays of c and b hadrons to tag the quark charge, although a correction must be made to account for B - $\bar{B}$ mixing, which causes the charge assignment to be incorrect in a fraction of the events. The event thrust axis may be used to approximate the quark direction.

In this letter we report on measurements of the $b\bar{b}$ and $c\bar{c}$ forward-backward asymmetries from a sample of 150 000 hadronic Z decays collected by the ALEPH detector at center-of-mass energies within 1 GeV of the Z peak. Both electrons and muons are identified in the hadronic jets, and the transverse momentum $p_\perp$ of the lepton with respect to the jet direction is used to distinguish the contributions of the $c\bar{c}$ and $b\bar{b}$ events to the observed asymmetry. Our previously published measurement of B - $\bar{B}$ mixing[1] is used to provide the necessary correction to the observed $b\bar{b}$ asymmetry.

The ALEPH Detector and Lepton Identification

The ALEPH detector has been described in detail elsewhere [2], so only a brief description is presented here. Charged tracks are measured over the range $|\cos \theta| < 0.95$, with θ the polar angle, by an inner cylindrical drift chamber (ITC) and a large cylindrical time projection chamber (TPC). These are immersed in a magnetic field of 1.5 Tesla

and together measure the momentum of charged particles with a resolution of $\delta p/p = 0.0008p(\text{GeV}/c)^{-1} \oplus 0.003$ [2,3]. The TPC also provides up to 330 measurements of the specific ionization (dE/dx) of each charged track. For electrons in hadronic events, the dE/dx resolution is 4.6% for 330 ionization samples. The electromagnetic calorimeter (ECAL), which surrounds the TPC but is inside the coil of the superconducting solenoid, is used to measure electromagnetic energy and, together with the TPC, to identify electrons. It is a lead-proportional tube calorimeter with cathode-pad readout which has a resolution for electromagnetic showers of $\delta E/E = 0.025 \oplus 0.20/\sqrt{E}$, with E in GeV. It covers the angular region $|\cos \theta| < 0.98$ and is finely segmented into projective towers, each subtending an angle of approximately 0.8° by 0.8° . They are read out in three longitudinal segments corresponding to thicknesses of 4, 9, and 9 radiation lengths. Muons are identified by the hadron calorimeter (HCAL), composed of the iron of the magnet return yoke interleaved with 23 layers of streamer tubes, and the muon chambers, an additional two layers of streamer tubes surrounding the calorimeter. The tubes of the HCAL have a pitch of 1 cm and measure, in two dimensions, tracks from penetrating particles within the angular range $|\cos \theta| < 0.985$. The energy of hadronic showers is measured by means of a copper pad readout arranged in projective towers, each subtending an angle of approximately 3.8° by 3.8° , with an energy resolution of $\delta E/E = 0.85/\sqrt{E}$, with E in GeV. The muon chambers, covering the same angular range as the HCAL, are read out by cathode strips both parallel and perpendicular to the streamer tubes. Therefore each layer provides a three-dimensional coordinate for charged tracks which penetrate the 7.5 interaction lengths of material between the interaction point and the streamer tubes.

The selection of hadronic events is based on charged tracks. Each event is required to have at least five “good” charged tracks, where a “good” track is one that passes through a cylinder of 2 cm radius and 20 cm length around the interaction point, has $|\cos \theta| < 0.95$, and has at least four TPC coordinates. The sum of their energies must be greater than 20% of the center-of-mass energy. This selection has an efficiency of 95%, and the background from $\tau\bar{\tau}$ and two-photon events has been estimated to be less than 0.25%. Only data taken at center-of-mass energies within 1 GeV of the Z peak are used, giving a total of 148 243 events passing the selection, with 21 160 from the 1989 data set and the remainder from the 1990 data set. Table 1 shows how these events are distributed with respect to the center-of-mass energy.

Leptons are identified in the ALEPH detector by matching a charged track measured in the TPC and ITC with either an energy deposit consistent with being from an electron in the ECAL, or a pattern of hits in the HCAL and muon chambers consistent with being from a muon. The identification of leptons with the ALEPH detector has been discussed elsewhere [4], but some improvements to the methods presented there have been made. Electrons are identified by comparing the ECAL energy deposits in the four towers around the extrapolation of each charged track with that expected for an

Energy (GeV)	Electron Analysis	Muon Analysis
90.22	10102	7836
91.22	123112	106878
92.22	15029	12369
Total	148243	127083

Table 1: The number of hadronic events at each of three c.m.s. energy points for the electron and muon analyses. The muon analysis has fewer events because the muon chambers were not operational during 1989. The uncertainty in the energy scale is ± 0.02 GeV.

electron of the measured momentum. This energy must be greater than the expected value minus 1.6 standard deviations. The average depth of the energy deposition in the ECAL is also measured and required to be in the range -1.8σ to $+3.0\sigma$ of the value expected for an electron. At least 50 TPC ionization samples are required for an electron candidate, and a candidate is rejected if the dE/dx is more than 2.5 standard deviations below the expected value. Table 2 shows the efficiency for electron identification and the hadron misidentification probability. The same methods as presented in Ref. 4 are used to remove photon conversions and Dalitz pairs from the prompt electron signal and to estimate in the data the efficiency and background to electron identification.

Muons are identified using the pattern of fired planes in the HCAL and the three-dimensional coordinates of the muon chambers. The requirement of Ref. 4 of having at least one hit in the last three planes of the HCAL has been replaced by the requirement that the track extrapolates to within 4σ of a hit in the muon chambers. The rest of the cuts are the same. The technique of mapping the efficiency of the HCAL planes with $Z \rightarrow \mu^+\mu^-$ events has been extended to the full angular range, and the same method has been employed to measure the muon-chamber efficiency. Table 2 shows the efficiency for muon identification and the probability that a hadron shower penetrates through the HCAL, producing a fake muon signal. The efficiency and backgrounds to muon identification are estimated in the data by the techniques of Ref. 4.

Jet Reconstruction and $p_\perp$

Jets are formed using all good charged tracks and neutral electromagnetic and hadronic energy clusters in an event. The algorithm used to include both tracks and calorimeter clusters while avoiding double counting has been described in an earlier publication [5]. With the use of this algorithm, the average reconstructed energy in a well-contained

$p_{\perp}$	0.0–1.0 GeV/ c %	1.0–2.5 GeV/ c %
ϵ_e	61.1 $\pm$ 2.2	74.3 $\pm$ 2.2
ϵ_{π}	0.082 $\pm$ 0.016	0.034 $\pm$ 0.012
ϵ_{μ}	71.7 $\pm$ 2.2	73.8 $\pm$ 2.2
ϵ_h	0.094 $\pm$ 0.047	0.16 $\pm$ 0.08

Table 2: The combined efficiency (ϵ_e) for electron identification by ECAL and dE/dx including the inefficiency of the pair-rejection cuts, and the probability (ϵ_{π}) for a hadron to be misidentified as an electron when ECAL and dE/dx are combined. The efficiency (ϵ_{μ}) for muon identification by HCAL plus muon chambers, and the probability (ϵ_h) for a hadronic shower to penetrate through the calorimeters to the muon counters and be misidentified as a muon. All efficiencies and misidentification probabilities are given in percent, and the momentum range considered is between 3 and 23 GeV/ c .

hadronic event is 91 GeV with an r.m.s. of 8 GeV. The charged, electromagnetic and hadronic energy fractions observed are on average 59%, 26% and 15% respectively. From Monte-Carlo simulations we find that with this algorithm the angular resolution in the determination of the event thrust direction is better than 1.5°.

The scaled-invariant-mass clustering algorithm [6] is used to form jets. The jet-resolution parameter has been chosen such that sets of tracks and calorimeter objects are merged together only as long as their squared invariant mass, $M_{12}^2 = 2E_1E_2(1 - \cos \theta_{12})$, is less than $(6 \text{ GeV})^2$. This corresponds to a $y_{\text{cut}} \equiv M_{12}^2/E_{\text{vis}}^2$ value of 0.0044. The $p_{\perp}$ of the lepton is then defined as the transverse momentum carried by the lepton with respect to the associated jet. In a departure from our previous definition [1,4], the lepton momentum is not excluded from the jet momentum vector.

Measurement of A_{FB}

In the following sections are described two approaches taken to measure the forward-backward asymmetry. Both methods define the production angle θ_a as follows. The flight direction of the heavy quark is estimated by the direction of the event thrust axis, which is given a sign according to the lepton charge: if $\hat{t}$ is chosen to be the thrust direction which is (of the two possibilities) closer in angle to the lepton direction, then $\cos \theta_a = -Q \hat{t} \cdot \hat{e}$, where $\hat{e}$ is the direction of the e^- beam and Q is the lepton charge.

In the first analysis, high- $p_{\perp}$ leptons are used to isolate an enriched sample of $b \rightarrow \ell$ decays. In order to extract the $b\bar{b}$ asymmetry, the observed asymmetry in the dis-

Reaction	Electron	Muon	Average
$b \rightarrow \ell$	83.1	75.0	79.1
$b \rightarrow c \rightarrow \ell$	7.0	7.5	7.2
$b \rightarrow W \rightarrow \bar{c}s \rightarrow \ell$	1.0	1.1	1.1
$c \rightarrow \ell$	4.6	5.3	5.0
Background	4.3	11.1	7.9

Table 3: The relative composition (in %) of the high- $p_{\perp}$ lepton sample, as calculated using our Monte-Carlo simulation. The contribution from $b \rightarrow \tau \rightarrow \ell$ is included in the $b \rightarrow \ell$ fraction. The systematic uncertainties estimated for the various contributions are given in Table 5.

tribution of $\cos \theta_a$ for these events is corrected for the presence of various sources of contamination, such as leptons from charm decays or cascade b decays ($b \rightarrow c \rightarrow \ell$), as well as for B - $\bar{B}$ mixing.

In the second approach, the whole $p, p_{\perp}$ spectrum of prompt lepton candidates, for $p > 3 \text{ GeV}/c$, is included in a fit in which both the $b\bar{b}$ and $c\bar{c}$ asymmetries are allowed to vary freely. The $p, p_{\perp}$ spectra for muon and electron candidates are divided into bins, and a fitting function is used which predicts, as a function of the asymmetries, the populations of the forward and backward $\cos \theta_a$ regions in each bin. Thus both asymmetries may be fitted simultaneously, with p and $p_{\perp}$ providing the necessary separation between the $c\bar{c}$ and $b\bar{b}$ contributions.

A_{FB}^b from High- $p_{\perp}$ Leptons

Selecting lepton candidates with $p \geq 3.0 \text{ GeV}/c$ and $p_{\perp} \geq 1.0 \text{ GeV}/c$ gives a high- $p_{\perp}$ sample which is dominated by leptons from semileptonic decays of b hadrons. Its fractional composition for electrons, muons and the combined sample is listed in Table 3.

The forward-backward asymmetry is measured in the following way. The distribution of the signed polar angle of the thrust axis in events with high- $p_{\perp}$ leptons is plotted in 20 bins of $\cos \theta_a$ between -1 and 1 . Polar-angle dependent corrections are then applied to account for (a) hadronic event selection efficiency, (b) track reconstruction efficiency and (c) variation in the lepton identification efficiency between the barrel region and the endcaps. These corrections are small at all polar angles except in the region $|\cos \theta_a| \geq 0.9$, which is excluded from the subsequent fit. This efficiency corrected $\cos \theta_a$ distribution is fitted by a binned maximum-likelihood procedure to the functional

form

$$\frac{1}{N} \frac{dN}{d \cos \theta_a} \propto 1 + \cos^2 \theta_a + \frac{8}{3} A_{\text{FB}}^{\text{obs}} \cos \theta_a \quad (3)$$

to extract the observed forward-backward asymmetry $A_{\text{FB}}^{\text{obs}}$.

The observed forward-backward asymmetry is related to the actual forward-backward asymmetry A_{FB}^b by the equation:

$$\begin{aligned} A_{\text{FB}}^{\text{obs}} = & A_{\text{FB}}^b (f_{b \rightarrow \ell} - f_{b \rightarrow c \rightarrow \ell} + f_{b \rightarrow W \rightarrow \bar{c} s \rightarrow \ell}) \cdot (1 - 2\chi) \\ & - A_{\text{FB}}^c f_{c \rightarrow \ell} - A_{\text{FB}}^{\text{bkgnd}} f_{\text{bkgnd}}. \end{aligned} \quad (4)$$

Here χ is the average B - $\bar{B}$ mixing probability at LEP, and $f_{b \rightarrow \ell}$, $f_{c \rightarrow \ell}$, *etc.* are the fractional populations of various sources in the sample of high- $p_{\perp}$ leptons. It is assumed, in order to obtain $f_{b \rightarrow c \rightarrow \ell}$ and $f_{b \rightarrow W \rightarrow \bar{c} s \rightarrow \ell}$, that $(14 \pm 7)\%$ of b decays result in a $\bar{c}$, based on phase-space calculations [11], and that 98% of b decays produce a c quark. Contaminations such as $b \rightarrow c \rightarrow \ell$ and $c \rightarrow \ell$ have leptons of the opposite charge with respect to $b \rightarrow \ell$ decays and thus cancel some of the asymmetry of the signal.

The observed asymmetry is corrected for the presence of B - $\bar{B}$ mixing using the ALEPH measurement of $\chi = 0.132 \pm 0.027$ [1]. $A_{\text{FB}}^{\text{bkgnd}}$, the residual forward-backward asymmetry in the background, is estimated from Monte-Carlo simulation of hadronic events to be $2 \pm 4\%$. In order to account for the asymmetry in the small ($\approx 5\%$) $c \rightarrow \ell$ contribution to the high- $p_{\perp}$ sample, Standard-Model couplings are used to relate A_{FB}^c to A_{FB}^b . At the Z peak, for example, this implies that $A_{\text{FB}}^c = 0.73 A_{\text{FB}}^b$.¹

The forward-backward asymmetry varies as a function of center-of-mass energy. Therefore, A_{FB}^b has been analysed separately at each of the three energy points listed in Table 1. The results of the fits are displayed in Table 4 for the electron and muon channels combined. A final value for the asymmetry at the Z peak is obtained by correcting each value for the expected energy dependence and then taking a weighted average. The result is

$$A_{\text{FB}}^b = 0.126 \pm 0.032 \pm 0.012, \quad (5)$$

where the second error is systematic. In Fig. 1 the acceptance-corrected polar angle distribution of the b -quark direction is shown for the data taken at the Z peak. Table 5 lists the major contributions to the systematic error in the measurement of A_{FB}^b . They are dominated by the uncertainties in the measured B - $\bar{B}$ mixing rate and the semileptonic branching ratios of beauty and charm hadrons.

¹The program EXPOSTAR [7] was used to calculate these asymmetries. The ratio of $c\bar{c}$ to $b\bar{b}$ asymmetry depends only very weakly on the top-quark mass.

Figure 1: The polar-angle distribution of the b -quark direction. Only data collected at the peak energy are included, and the electron and muon channels have been combined. The smooth curve is the fit of Eqn. 3 to the data.

Channel	Peak $- 1$ GeV	Peak	Peak $+ 1$ GeV
Electron + Muon	0.153 ± 0.117	0.123 ± 0.036	0.134 ± 0.095

Table 4: The extracted asymmetry A_{FB}^b for the three energy points. The errors are statistical only.

Source	Variation	ΔA_{FB}^b
$\chi = 0.132$ [1]	± 0.027	0.0095
$B(b \rightarrow \ell)$	10%	0.0045
$B(b \rightarrow c \rightarrow \ell)$	10%	0.003
$B(b \rightarrow W \rightarrow \bar{c}s)$	50%	0.0015
$B(c \rightarrow \ell)$	20%	0.0023
$\varepsilon_b = 0.006^{+0.004}_{-0.003}$ [4]	1σ	0.0027
$\varepsilon_c = 0.024^{+0.047}_{-0.022}$ [4]	1σ	0.0008
$A_{\text{FB}}^{\text{bkgnd}} = 2 \pm 4\%$	1σ	0.004
Monte Carlo statistics	–	0.0018
Total		0.012

Table 5: Contributions to the systematic error on A_{FB}^b in the fit of the high- $p_{\perp}$ lepton sample. The central values used for the semileptonic branching ratios are the same as in our previous publication, Ref. 4.

Fitting over the Full Range in $p_\perp$

Dividing the $p, p_\perp$ spectrum of the leptons into bins and fitting over the full range of $p_\perp$ gives a result for the $b\bar{b}$ asymmetry with no assumption made on the relation between it and the $c\bar{c}$ asymmetry, and with better statistical precision than can be obtained with a single bin at high $p_\perp$. In addition, since both asymmetries are allowed to vary freely in the fit, a result is obtained for A_{FB}^c as well as for A_{FB}^b .

The fitting method is similar to that used in Ref. 4 to measure the partial widths of the Z decaying to $b\bar{b}$ and $c\bar{c}$, with the exception that here the angular dependence is added to the fitting function. Taking the fit of the muon spectrum as an example, the prediction for the number of muons above background in each bin labelled $p, p_\perp, \theta_a$ is given by a sum over the four sources of prompt leptons, primary- c decay, primary- b decay, cascade decays and $b \rightarrow \tau \rightarrow \ell$:

$$\begin{aligned}
N_\mu(p, p_\perp, \theta_a) - N_{\text{bkgrnd}}(p, p_\perp, \theta_a) \\
= \epsilon(p, p_\perp, \theta_a) \times \left[2N_{c\bar{c}} B(c \rightarrow \mu) \sum_k W^k(\langle x_E^c \rangle) P_{\text{cpri}}^k(p, p_\perp) \mathcal{A}(\theta_a, -A_{\text{FB}}^{c, \text{eff}}) \right. \\
+ 2N_{b\bar{b}} B(b \rightarrow \mu) \sum_k W^k(\langle x_E^b \rangle) P_{\text{bpri}}^k(p, p_\perp) \mathcal{A}(\theta_a, A_{\text{FB}}^{b, \text{eff}}) \quad (6) \\
+ 2N_{b\bar{b}} B(c \rightarrow \mu) \sum_k W^k(\langle x_E^b \rangle) P_{\text{csec}}^k(p, p_\perp) \\
\left. \times [B(b \rightarrow \bar{c}) \mathcal{A}(\theta_a, A_{\text{FB}}^{b, \text{eff}}) + B(b \rightarrow c) \mathcal{A}(\theta_a, -A_{\text{FB}}^{b, \text{eff}})] \right. \\
\left. + 2N_{b\bar{b}} B(b \rightarrow \tau) B(\tau \rightarrow \mu) \sum_k W^k(\langle x_E^b \rangle) P_\tau^k(p, p_\perp) \mathcal{A}(\theta_a, A_{\text{FB}}^{b, \text{eff}}) \right].
\end{aligned}$$

The $\sum_k$ represent sums over six bins in x_E , the b or c hadron energy divided by the LEP beam energy, while the $W^k(\langle x_E \rangle)$ are fragmentation functions derived from the LUND-6.3 parton-shower model [8]. Within the LUND program, the fragmentation function of Peterson *et al.* [9] is used at the quark level, as a function of $z = (E + p_\parallel)_{\text{hadron}} / (E + p)_{\text{quark}}$.² $P_i^k(p, p_\perp)$ represents the probability that a heavy hadron undergoing semileptonic decay with x_E in bin k results in a lepton in the bin $p, p_\perp$ and is derived from the Monte-Carlo simulation, including detector effects. The function $\mathcal{A}(\theta_a, A_{\text{FB}}^{\text{eff}})$ is the expected angular distribution as a function of the asymmetry, integrated over the $\cos \theta_a$ bin:

$$\mathcal{A}(\theta_a, A_{\text{FB}}^{\text{eff}}) = \int_{\text{bin } \theta_a} \left[\frac{3}{8}(1 + \cos^2 \theta) + A_{\text{FB}}^{\text{eff}} \cos \theta \right] d \cos \theta, \quad (7)$$

where $A_{\text{FB}}^{\text{eff}}$ is the effective asymmetry after dilution by mixing, according to $A_{\text{FB}}^{\text{eff}} = A_{\text{FB}} \cdot (1 - 2\chi)$. The ALEPH mixing measurement; $\chi = 0.132 \pm 0.027$ [1], is used for $b\bar{b}$,

²There is no significant change in the asymmetry results if the Peterson *et al.* form is used directly as a function of x_E , instead of relying of the LUND model to generate the x_E distribution.

Channel	A_{FB}^b	A_{FB}^c	correlation coef.
muons	0.110 ± 0.039	0.085 ± 0.057	0.21
electrons	0.141 ± 0.039	0.045 ± 0.052	0.18
combined	0.126 ± 0.028	0.064 ± 0.039	0.20

Table 6: The forward-backward asymmetries fitted over the full $p_{\perp}$ range separately for muons and electrons. Only statistical errors are given, which do not include the effects of allowing the branching ratios and fragmentation parameters to vary in the fit.

while for $c\bar{c}$ it is assumed that there is no significant mixing [10]. The Standard-Model values 0.217 and 0.171 are used for $N_{b\bar{b}}/N_{\text{had}}$ and $N_{c\bar{c}}/N_{\text{had}}$ respectively, $B(b \rightarrow \tau)$ is set to $0.31 \cdot B(b \rightarrow \mu)$, according to phase-space calculations [11], and $B(\tau \rightarrow \ell)$ is taken to be 0.178 ± 0.004 [10].

The fitted asymmetries correspond to the values at the Z peak. For $\mathcal{A}(\theta_a, A_{\text{FB}}^{\text{eff}})$ in Eqn. 6, the inclusion of data taken at ± 1 GeV from the peak is accounted for by using Standard-Model predictions, including corrections for initial-state radiation, for the difference in asymmetry with respect to that at the peak [7,12]. This correction changes the fitted peak asymmetry by an amount which is very small with respect to the measurement errors. That is because, as can be seen from Table 1, the events off the peak are relatively few and are distributed to both sides.

Six variables are allowed to float freely in the fit: A_{FB}^b , A_{FB}^c , $B(b \rightarrow \ell)$, $B(c \rightarrow \ell)$, $\langle x_E^b \rangle$ and $\langle x_E^c \rangle$. The muons and electrons first are fit separately in order to check for consistency. The fits are made with six bins in p , and six bins in $p_{\perp}$. Only two bins in $\cos \theta_a$ (forward and backward) are fitted, although efficiency corrections in $\cos \theta_a$ are made with a finer binning.³ The asymmetry results, with statistical errors only, are given in Table 6. The values preferred by the fit for the branching ratios and fragmentation parameters are consistent with our published results from the first LEP run [4]. They will be presented in a forthcoming publication after further study of systematic effects which, while being important to improved understanding of the branching ratios, only give small contributions to the error in the asymmetry. To give a visual impression of the fit, projections onto the p axis of the data and the best fit are shown in Fig. 2 for electrons and muons, and projections onto the $p_{\perp}$ axis are shown in Fig. 3. The fit of the asymmetries is restricted to the range $p > 3$ GeV/c, for a total of 7822 electron candidates and 10350 muon candidates. The model fits the data well, and the muon and electron asymmetries agree well with each other, both for $b\bar{b}$ and $c\bar{c}$. The result for

³Dividing the $\cos \theta_a$ distribution into more bins in the fit neither changes the result nor significantly reduces the statistical error of the asymmetry. Also, the dependence of the efficiency correction on $|\cos \theta_a|$ need not be known to a high accuracy, as it has only a second-order effect on the asymmetry.

Figure 2: Projections onto p of the spectrum of observed (a.) electron candidates and (b.) muon candidates with no cut on $p_{\perp}$. Also shown are the fits to Eqn. 6, broken down into their various contributions.

Figure 3: Projections onto $p_{\perp}$ of the spectrum of observed (a.) electron candidates and (b.) muon candidates for $p > 3 \text{ GeV}/c$. Also shown are the fits to Eqn. 6, broken down into their various contributions.

Source	Error on A_{FB}^c	Error on A_{FB}^b
$B(c \rightarrow \ell), B(b \rightarrow \ell), \langle x_c \rangle, \langle x_b \rangle$	0.017	0.0043
Efficiency	0.0001	0.0001
$B(b \rightarrow c)$	0.0084	0.0006
χ (B-B mixing)	0	0.0093
Background level	0.013	0.0012
Background asymmetry	0.011	0.005
Instrumental asymmetry	0.015	0.0029
Total	0.030	0.012

Table 7: Systematic errors of the asymmetry measurements from the fit of the full $p_\perp$ spectrum.

A_{FB}^b agree well with that obtained from fitting the angular distribution at high $p_\perp$.

Systematic errors have been studied by varying the efficiencies, backgrounds, fragmentation parameters, branching ratios, and the mixing parameter within their estimated errors. These factors all play a role in the prediction, via Eqn. 6, of the expected observed asymmetry in each $p, p_\perp$ bin as a function of the $b\bar{b}$ and $c\bar{c}$ asymmetries. In addition, sources of electrons and muons in the data (for example, electrons from photon conversions and muons from the decay $Z \rightarrow \mu^+\mu^-$) have been used to establish upper limits on the errors in the asymmetries due to instrumental asymmetries in the efficiencies and background probabilities. The results are summarized in Table 7 for both $c\bar{c}$ and $b\bar{b}$. Although the central values for the asymmetries have been derived allowing the branching ratios and fragmentation parameters to float freely in the fit, for the purpose of deriving the systematic errors in Table 7 these parameters have been varied by the amounts listed in Table 5, which are larger than the statistical constraints given by the fit to the data. The resulting error on the $b\bar{b}$ asymmetry, which is sensitive mainly to the relatively pure high- $p_\perp$ region, is small. The $c\bar{c}$ asymmetry has a larger systematic error, because it is sensitive to the low $p_\perp$ region, where the background is relatively high. As in the analysis of the previous section, the error on the $b\bar{b}$ asymmetry is dominated by the uncertainty in the mixing parameter χ , which is itself largely statistical. From the combined fit to electrons and muons,

$$\begin{aligned}
A_{\text{FB}}^b &= 0.126 \pm 0.028 \pm 0.012, \\
A_{\text{FB}}^c &= 0.064 \pm 0.039 \pm 0.030.
\end{aligned}
\tag{8}$$

The second error is in each case systematic and has been obtained by adding in quadrature the individual contributions. This result for A_{FB}^b is in agreement with other measurements at LEP [13].

The Electroweak Mixing Angle from A_{FB}

In the framework of the improved Born approximation [14], the forward-backward asymmetry may be interpreted in terms of the ratio of the vector and axial-vector coupling constants as shown in Eqn. 2. A result is derived for $\sin^2 \theta_W(m_Z^2)$, the effective electroweak mixing angle at the Z mass, by inserting into Eqn. 2 the relation

$$g_{V_f}/g_{A_f} = 1 - \frac{2Q_f}{T_f^3} (\sin^2 \theta_W(m_Z^2) + C_f) , \quad (9)$$

where Q_f is the charge and T_f^3 the third component of weak isospin of the fermion, and C_f is a flavour-dependent weak vertex correction.

The vertex correction modifies $\sin^2 \theta_W(m_Z^2)$ by +0.0007. Since most of the sensitivity to $\sin^2 \theta_W$ comes from the factor A_e in Eqn. 2, this correction has very little dependence on the top-quark mass. In deriving $\sin^2 \theta_W(m_Z^2)$, we also take into account corrections for initial-state radiation [15], the photon-exchange diagram and the difference between the peak energy point ($\sqrt{s} = 91.22 \text{ GeV}$) and the Z mass ($m_Z = 91.193 \text{ GeV}/c^2$ [16]). They have a combined effect equivalent, for example, to a change in the $b\bar{b}$ asymmetry by +0.003. To account for effects of gluon bremsstrahlung, a first order QCD correction [12] is made, which changes the $b\bar{b}$ asymmetry by $+0.003 \pm 0.001$. Including all of these corrections, this measurement of the $b\bar{b}$ asymmetry give the value $\sin^2 \theta_W(m_Z^2) = 0.2256 \pm 0.0055$. Combining the $b\bar{b}$ and $c\bar{c}$ asymmetry measurements, taking into account the 20% correlation, the result is $\sin^2 \theta_W(m_Z^2) = 0.2262 \pm 0.0053$.

Conclusion

We have measured the forward-backward asymmetry at $\sqrt{s} = 91.22 \text{ GeV}$ in $e^+e^- \rightarrow b\bar{b}$ to be $A_{\text{FB}}^b = 0.126 \pm 0.030$ and in $e^+e^- \rightarrow c\bar{c}$ to be $A_{\text{FB}}^c = 0.064 \pm 0.049$. Combining the results on the two asymmetries, we obtain a value of

$$\sin^2 \theta_W(m_Z^2) = 0.2262 \pm 0.0053 \quad (10)$$

for the effective electroweak mixing angle at the Z mass.

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