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Low Mass Dielectrons at LHC Energy

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ABSTRACT

We study the thermal production of low mass dielectrons in central nucleus-nucleus collisions at LHC energy. We assume initial quark-gluon plasma production, followed by a first order quark-hadron phase transition and subsequent hadronization; the expansion of the system is described by Bjorken's hydrodynamical model. In the quark-gluon plasma, we include both the basic $q\bar{q}$ annihilation process and lowest order QED and QCD corrections. In the hadronic phase, we consider pion annihilation as well as pion scattering with virtual bremsstrahlung leading to dielectron production. Our results are then compared to dielectron rates from π^0 and η Dalitz decays. We conclude that it will be rather difficult to disentangle the quark-gluon plasma contributions from competing dielectron production processes.

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1. Introduction

Probes of the early state of matter are crucial in the attempt to produce a quark-gluon plasma by high energy nuclear collisions. Thermal dileptons and photons were the first tools proposed for this.¹⁻¹¹ Since they can only interact electromagnetically, they will escape from the strongly interacting system essentially unaffected, carrying with them information about the temperature of the primordial state. Dileptons and photons are, however, produced in nuclear collisions by a variety of mechanisms, and we will learn something about the initial state only if it is possible to single out those which are thermally produced.

We shall consider here dilepton production, concentrating in particular on electron-positron pairs (dielectrons). It is useful to divide the range of the invariant mass $M(e^+e^-) \equiv M$ of the produced dielectrons into four regions. At high masses, for $M \gtrsim 3 - 4 \text{ GeV}$, we have Drell-Yan production: very early, hard (i.e. non-thermal) parton-antiparton annihilation and higher order interactions, producing a massive photon which decays into a lepton pair. This process appears rather well understood in terms of perturbative QCD and serves a reference for the study of the suppression of the J/ψ and higher charmonium/bottomonium states in a quark-gluon plasma.¹²⁻¹⁴ In the region $0.5 \text{ GeV} \lesssim M \lesssim 1.5 \text{ GeV}$, we have predominantly dilepton production by ρ, ω and ϕ decay. Unless it becomes somehow possible to measure a resonance mass shift or width broadening from decay inside dense matter, the resulting dileptons contain only information about the final hadronic state around freeze-out. Between this and the high mass region, we should observe thermal dileptons, if their production rate is sufficient, i.e., if the initial temperature was high enough.^{3,4,7,8} Otherwise, hadron decay and Drell-Yan production will “meet” here, as seems to be the case for present O-U and S-U collisions.¹⁵ Finally, below the hadron decay region, we have once more a chance for thermal production by $q\bar{q}$ and/or $\pi\pi$ annihilation, provided these can be disentangled from the background of other processes, such as the Dalitz decay of π^0 and η mesons.¹⁶⁻¹⁷

The accessibility of thermal dileptons above the hadron decay regime was recently studied for LHC energy.¹⁸ It was found that a conservative multiplicity estimate based on the p-p data does not leave much of a window for thermal dileptons; Drell-Yan production dominates down to $M \simeq 1.5 \text{ GeV}$, where hadron decay contributions are

expected to be sizeable. For much rescattering in A-A interactions with resulting higher multiplicities, the situation improves, however, and thermal dilepton production should become observable.

The aim of the present paper is to study the region below hadron decay production. We shall calculate the thermal production rate for a system which is initially a quark-gluon plasma, then expands, undergoes a first order hadronization transition, and eventually freezes out. The expansion will be described by the 1+1 dimensional Bjorken hydrodynamical model. For the plasma phase, we shall calculate the basic $q\bar{q}$ annihilation process (fig. 1a) and in addition, the lowest order QED (fig. 1b) and QCD (fig. 1c) corrections from photon and gluon emission, respectively. In the hadronic phase, which we take to be a pion gas, we consider $\pi\pi$ annihilation (fig. 2a) and $\pi\pi$ scattering with virtual bremsstrahlung leading to dilepton production (fig. 2b). Finally we compare the results thus obtained to the background of dileptons from the Dalitz decay of π^0 and η mesons.

The paper is organized as follows. In section 2, we calculate the different cross sections for e^+e^- pair production, both in a quark-gluon plasma and in a hadron gas. In section 3, the dielectron production rates are determined by kinetic theory. In section 4, the initial conditions and the expansion dynamics are specified and the formulas for the experimentally measured production rates are derived. In section 5, the background due to Dalitz decay of π^0 and η mesons is calculated. In section 6, the different low mass dielectron production rates are compared; in section 7, we make some final critical remarks.

2. Dilepton production in QCD matter

In a quark-gluon plasma, the lowest order process to produce lepton pairs is due to quark annihilation,

$$q\bar{q} \rightarrow \gamma^* \rightarrow e^+e^- \quad (2.1)$$

The cross section for the above basic reaction, σ_q , can be obtained from the QED cross section for the reaction $e^+e^- \rightarrow l^+l^-$,

$$\sigma(M) = \frac{4\pi}{3} \frac{\alpha^2}{M^2} \left(1 + \frac{2m_l^2}{M^2} \right) \left(1 - \frac{4m_l^2}{M^2} \right)^{\frac{1}{2}}, \quad (2.2)$$

multiplying it by the appropriate spin, colour and flavour factors,

$$\sigma_q(M) = (2S+1)N_c \sum_f e_q^2 \sigma(M), \quad (2.3)$$

where e_q are the electric charges for different quark flavours. We shall restrict our considerations to two light quarks, $e_u = 2/3$ and $e_d = -1/3$.

The basic QED reaction for dilepton production in a hadron gas is pion annihilation,

$$\pi^+ \pi^- \rightarrow \gamma \rightarrow e^+ e^-. \quad (2.4)$$

The cross section for $\pi^+ \pi^-$ annihilation, σ_π , is calculated along the same lines as in a quark gas; the main difference is that now vector meson resonance formation plays a crucial role. The cross section σ_π is given by the well known expression,

$$\sigma_\pi(M) = \sigma(M) |F_\pi|^2 \left(1 - \frac{4m_\pi^2}{M^2}\right)^{\frac{1}{2}}. \quad (2.5)$$

In the Breit-Wigner approximation, assuming a ρ resonance contribution only, the pion form factor F_π becomes

$$|F_\pi|^2 = \frac{m_\rho^4}{(M^2 - m_\rho^2)^2 + m_\rho^2 \Gamma_\rho^2} \quad (2.6)$$

where $m_\rho = 0.775 \text{ GeV}$ is the ρ mass, and $\Gamma_\rho = 0.155 \text{ GeV}$ the ρ decay width.

When comparing the cross sections for quark and pion annihilation, eq. (2.3) and (2.5), one can immediately see that there are different kinematical boundaries for these processes. The pion annihilation cross section vanishes for the dielectron mass $M \leq 2m_\pi$, whereas for quark annihilation process this is the case for $M \leq 2m_q$. Thus, for $2m_q \leq M \leq 2m_\pi$, the only contribution to dielectron production appears to come from quark annihilation process. This is why the measurement of the dielectron distribution in this low invariant mass window could be of importance in looking for quark-gluon plasma formation in nuclear collisions.

For small mass pairs, however, there are obviously other sources than those indicated by eq. (2.1) and (2.4). In particular, in a pion gas we have bremsstrahlung emission of soft virtual photons, and in a quark gas we have the radiation corrections to the quark annihilation process, (2.1). In the following, we shall estimate the cross sections for both these processes.

The radiation corrections to the quark annihilation process have the following general structure,

$$q\bar{q} \rightarrow e^+ e^- + X, \quad (2.7)$$

where X refers to a soft photon or gluon.

The incoming energy of the $q\bar{q}$ system is shared in this case by three particles. Consequently, the dileptons thus produced can have a lower invariant mass than those coming from $q\bar{q} \rightarrow e^+ e^-$.

The differential cross section for dilepton production from $q\bar{q} \rightarrow e^+ e^- + \gamma$ can be calculated in QED, with the result:

$$\frac{d\sigma^{e^+e^-\gamma}}{ddM^2} = \frac{\alpha}{3\pi} \frac{1}{M^2} \left(1 + \frac{2m_l^2}{M^2}\right) \left(1 - \frac{4m_l^2}{M^2}\right)^{\frac{1}{2}} \frac{d\sigma^{\gamma\gamma^*}}{dt} \quad (2.8)$$

In eq.(2.8) the cross section for the annihilation process into a real (γ) and a virtual (γ^*) photon, $q(p_1)\bar{q}(p_2) \rightarrow \gamma(p)\gamma(p_3)$, where the symbols in brackets are the momenta carried by fields, is given by

$$\frac{d\sigma^{\gamma\gamma^*}}{dt} = 2\pi\alpha^2 (2S+1)^2 N_c \sum_{q=1}^{N_f} \frac{e_q^4}{s(s-4m_q^2)} A, \quad (2.9)$$

in terms of the invariant matrix element¹⁹

$$A \equiv \frac{1}{4} \left(\frac{t_1}{u_1} + \frac{u_1}{t_1} \right) + \frac{1}{2} \frac{M^4 - M^2 m_q^2}{t_1^2 u_1} \left(\frac{1}{t_1^2} + \frac{1}{u_1^2} \right) - \frac{M^2}{2} \left(\frac{1}{u_1} + \frac{1}{t_1} \right) - \left(\frac{m_q^2}{u_1} + \frac{m_q^2}{t_1} \right)^2 - \left(\frac{m_q^2}{u_1} + \frac{m_q^2}{t_1} \right) \quad (2.10)$$

and the Mandelstam variables $s_1 = (p_1 + p_2)^2$, $t_1 = (p - p_1)^2$, $s_1 + t_1 + u_1 = M^2$.

For the further analysis we need the average cross sections $d\sigma/dM^2$. This can be obtained from (2.8-2.10) by integrating over the allowed range of t_1 :

$$\frac{d\sigma^{e^+e^-\gamma}}{dM^2} = \int_{t_1^-}^{t_1^+} dt_1 \frac{d\sigma^{e^+e^-\gamma}}{dt_1 dM^2}, \quad (2.11)$$

with the bounds

$$t_1^\pm = -\frac{1}{2}(s - M^2)(1 \mp W(s)), \quad W(s) = \left(1 - \frac{4m_q^2}{s}\right)^{\frac{1}{2}} \quad (2.12)$$

For $N_c = 3, N_f = 2, S = 1/2, e_u = 2/3, e_d = -1/3$, we thus find:

$$\sigma^{\gamma\gamma^*} = \frac{68}{27} \frac{\pi\alpha^2}{s - 4m_q^2} \left\{ \left(1 - \frac{M^2}{s}\right) \left(1 + 2 \frac{M^4 - 4m_q^4}{(s - M^2)^2} + 2 \frac{M^2 + 2m_q^2}{(s - M^2)}\right) \right. \\ \left. \ln \frac{s}{4m_q^2} \left[1 + \left(1 - \frac{4m_q^2}{s}\right)^{1/2}\right]^2 - \left(1 - \frac{4m_q^2}{s}\right) \frac{s + 4m_q^2 + M^2}{s - M^2} \right\}. \quad (2.13)$$

Eq.(2.8), together with eq.(2.13), specify the QED cross section for the process $q\bar{q} \rightarrow e^+e^- + \gamma$.

In order to estimate the low energy QCD cross section $\sigma(q\bar{q} \rightarrow e^+e^- + g)$, we assume that it can be approximated by the QED result (2.8-2.13) multiplied by the factor $(4\alpha_s/9\alpha)$. A more detailed discussion of QCD perturbative calculations can be found in the literature.²⁰

Finally, let us consider the cross section for e^+e^- production due to pion scattering with bremsstrahlung. In the reaction $\pi\pi \rightarrow \pi\pi$ one expects the radiation of real or virtual photons, simply because the charged particles are accelerated. In the soft photon approximation, to which we restrict our discussion, photons can be radiated only from the initial or from the final charged lines. The differential cross section for e^+e^- pairs obtained from the decay of virtual photons emitted in pion-pion scattering can be calculated from the expression²¹

$$\frac{d\sigma_{\pi\pi}^{e^+e^-}}{dM^2} = \frac{\alpha^2}{8\pi^4} \frac{1}{M^2} \int |\epsilon \cdot J|^2 \frac{d\sigma_{\pi\pi}}{dt} \delta(M^2 - (p_+ + p_-)^2) \frac{d^3p_+ d^3p_-}{E_+ E_-} dt. \quad (2.14)$$

Here t is four-momentum transfer in $\pi\pi$ collision, M is the invariant mass of the virtual photon, p_+ and p_- are the momenta of e^+ and e^- s, and

$$|\epsilon \cdot J|^2 = \sum_{\delta} J \cdot \epsilon_{\delta} J \cdot \epsilon_{\delta}. \quad (2.15)$$

The sum is taken over all photon polarizations, and the current J in the reaction $\pi_a \pi_b \rightarrow \pi_c \pi_d$ is defined as:

$$J^{\mu} = -Q_{\pi_a} \frac{p_a^{\mu}}{p_a \cdot q} - Q_{\pi_b} \frac{p_b^{\mu}}{p_b \cdot q} + Q_{\pi_c} \frac{p_c^{\mu}}{p_c \cdot q} + Q_{\pi_d} \frac{p_d^{\mu}}{p_d \cdot q}. \quad (2.16)$$

The above equation is particularly simple, if we assume that in the initial state one of the incoming pions is electrically neutral. Then, for $|t| < 4m_{\pi}^2$ we can make the approximation²²

$$|\epsilon \cdot J|^2 \simeq \frac{2}{3} \frac{1}{q_0^2} \left[\frac{-t}{m_{\pi}^2} \right], \quad (2.17)$$

where for the virtual photon q_0^2 is replaced by the symmetrized combination $E(E^2 - M^2)$, with $E = E_+ + E_- = |\mathbf{p}_+| + |\mathbf{p}_-|$. With these approximations, the momentum integrations in (2.12) give²²

$$\frac{d\sigma_{\pi\pi}^{e^+e^-}}{dM^2} = \frac{\alpha^2}{3\pi^2} \frac{\bar{\sigma}(s)}{M^2} \ln \left[\frac{s^{1/2} - 2m_{\pi}}{M} \right], \quad (2.18)$$

where $\bar{\sigma}$ is the momentum transfer weighted cross section

$$\bar{\sigma}(s) \sim \int_{(4m_{\pi}^2 - s)}^0 \frac{-t}{m_{\pi}^2} \sum_{\pi=\pi^{\pm}} \frac{d\sigma_{\pi\pi}^{e^+e^-}}{dt}. \quad (2.19)$$

In an isospin symmetric pion gas, the differential cross section under the integral in (2.19) is a symmetric function of t and u . Then the integral in (2.19) can be performed exactly, with the result:

$$\bar{\sigma}(s) \sim 2\sigma_{el} \left[\frac{s}{4m_{\pi}^2} - 1 \right], \quad (2.20)$$

where $\sigma_{el} \sim \sigma_{\pi^0\pi^+} + \sigma_{\pi^0\pi^-}$.

The elastic $\pi\pi$ cross section for low energy collisions is known to be well described by the chiral model.²³ In terms of this model, the elastic cross section in (2.20) is given by

$$\sigma_{el}(s) = \frac{2}{3} \frac{1}{F_{\pi}^4} \frac{1}{16\pi} s \left[1 - \frac{5m_{\pi}^2}{s} + \frac{7m_{\pi}^4}{s^2} \right], \quad (2.21)$$

with the pion decay constant $F_{\pi} = 0.098\text{GeV}$.

In the case when the collision energy is of the order of the ρ meson mass, the largest contribution to $\pi\pi$ scattering amplitude will be due to resonance formation. Then the elastic pion cross section in (2.20) in the limit of small $|t|$ takes the form

$$\sigma_{el}(s) = \frac{g_{\rho\pi\pi}^4}{48\pi s} \frac{(s - 4m_{\pi})^2}{(s - m_{\rho})^2 + \Gamma_{\rho}^2 m_{\rho}^2}, \quad (2.22)$$

where the coupling constant $g_{\rho\pi\pi} \sim 6$ and m_{ρ} , Γ_{ρ} are defined in the same way as in eq.(2.6).

Finally, for large collision energy one expects $\sigma_{\pi\pi}$ to become energy independent, and the value of σ_{el} in (2.20) can be approximated by $\sigma_{el}(s) \sim (4/3) \sigma_{\pi^+\pi^+} \sim 5\text{mb}$.

For the pion-pion elastic cross section, we shall therefore take the chiral model form for $\sqrt{s} \leq 0.6\text{GeV}$, the ρ -resonance form for $0.6 \leq \sqrt{s} \leq 1.5\text{GeV}$, and above $\sqrt{s} = 1.5\text{GeV}$ the constant value just quoted.

assumption is to take an ideal gas equation of state. Here the quark-gluon plasma is considered to be a non-interacting gas of quarks and gluons; similarly, the hadron gas is assumed to be an ideal pion system. In order to obtain a phase transition between these two states of QCD matter, one still needs an additional parameter in the quark-gluon plasma, the bag pressure. The resulting QCD phase transition is by construction always strongly first order. This input leads to the following scenario of the expansion dynamics of QCD matter. At an initial time τ_0 , the system is created as a pure quark-gluon plasma of initial temperature T_0 . Then the system cools by hydrodynamic expansion from the initial down to the critical temperature T_c ; this is reached at the proper time τ_1 . The system remains in the mixed phase until a time τ_2 , at which the quark-gluon plasma is totally converted into a hadron gas. Then the system continues to expand up to a time τ_f , when the temperature drops to its final value T_f . Afterwards, hadrons expand freely until detection; their kinetic energy determined by the final temperature.

In the above model, the dependence of the temperature on the proper time in the quark-gluon plasma as well as in the hadron gas is simply given by

$$T = T_i \left[\frac{\tau_i}{\tau} \right]^{\frac{1}{3}}, \quad (4.2)$$

where τ_i and T_i indicate for the quark-gluon plasma the initial time τ_0 and initial temperature T_0 of the system. For the pure hadron gas phase, we have $T_i = T_c$ and $\tau_i = \tau_2$.

At the time τ_1 , when the coexistence phase begins, the temperature is fixed and equal to its critical value; however, the system is still expanding, with the entropy determined at each value of time by the equation

$$s(\tau) = f(\tau) s_{QG}(T_c) + [1 - f(\tau)] s_{HG}(T_c). \quad (4.3)$$

The function $f(\tau)$ describes the fraction of the entropy which is in the pure quark-gluon plasma phase.

In the Bjorken model, the volume element is given by

$$d^4x = d^2x_t dy \tau d\tau, \quad (4.4)$$

where τ as before is the proper time, y the rapidity of a fluid element and x_t the transverse coordinate. In the central region of an A-A collision, the integration over x_t leads to a constant factor πR_A^2 , where R_A is the nuclear radius.

With the measure (4.4) and the production rates (3.1), one can calculate the experimentally measured dilepton distributions $dN/dy dM$. Converting the time to a temperature integration according to eqs.(4.1) and (4.2) the production rates in the pure quark and hadron phases can be calculated from

$$\frac{dN}{dy dM} = 3\pi R_A^2 \left(\frac{T_0}{T_c} \right)^6 \tau_0^2 \times \left\{ \begin{array}{l} R^2 \int_{T_f/T_c}^{T_0/T_c} dz z^{-7} \frac{dN^h}{d^4x dM} \\ \int_{T_0/T_c}^{T_0/T_c} dz z^{-7} \frac{dN^q}{d^4x dM} \end{array} \right\}, \quad (4.5)$$

Similarly, in the mixed phase the separate contributions of quark-gluon and pion gas terms give

$$\frac{dN^M}{dy dM} \Big|_{T=T_c} = \frac{3}{6} \pi R_A^2 \left(\frac{T_0}{T_c} \right)^6 \tau_0^2 \times \left\{ \begin{array}{l} R(R-1) \frac{dN^h}{d^4x dM} \\ (R-1) \frac{dN^p}{d^4x dM} \end{array} \right\}. \quad (4.6)$$

In the above equations R denotes the ratio between the number of degrees of freedom of the quark-gluon plasma and that of a pion gas. For the ideal gas equation of state and two quark flavours and with three degrees of freedom for an ideal pion gas, $R \simeq 12$. The rates $dN^q/d^4x dM$ and $dN^h/d^4x dM$ in eqs.(4.5)/(4.6) are given by eq.(3.4).

The results for the dilepton spectra, eq.(4.5)/(4.6), evidently depend on a number of parameters. These are the initial temperature T_0 , the initial thermalization time τ_0 , the critical temperature T_c , and the final temperature, T_f .

For isentropic longitudinal expansion, the initial temperature and the initial time can be related to the hadron multiplicity in a central A-A collision by²⁶

$$\frac{dN}{dy} = \frac{\pi R_A^2 \tau_0 s(T_0)}{3.6}; \quad (4.7)$$

here $R_A \sim 1.2A^{1/3}$ is the nuclear radius. The entropy density $s(T_0)$ of a plasma droplet becomes in the ideal gas approximation

$$s(T_0) = \frac{\pi^2}{45} (32 + 2N_f) T_0^3. \quad (4.8)$$

In the central region of A-A collisions, the multiplicity of the secondary hadrons can be parameterized by

$$\frac{dN}{dy} = 0.8A^\alpha \ln \sqrt{s}, \quad (4.9)$$

with $\alpha \geq 1$. The value of $\alpha = 1$ corresponds to independent p-p collisions, whereas $\alpha > 1$ is obtained by rescattering between protons and secondary hadrons. The present experimental results from heavy nuclei collision suggests $\alpha \sim 1.1$.²⁷

At the LHC, the CMS energy for central collisions between heavy nuclei will be $s = 6.3 \text{ TeV}/A$.¹³ At this energy, the initial temperature in U-U interactions for $\tau_0 = 1 \text{ fm}$ varies between 0.24 and 0.30 GeV, depending on α and N_f . In the following, we take $N_f = 2$ and $T_0 = 0.25 \text{ GeV}$. The physical value of the critical deconfinement temperature is also not yet free of uncertainties. The most recent lattice gauge theory results²⁸⁻²⁹ indicate $T_c \simeq 0.15 \text{ GeV}$, which we shall use here. Finally, in our numerical computations we shall take $T_f = 0.1 \text{ GeV}$ as the value of the freeze-out temperature.

5. Dalitz pairs

Until now, we have considered the dilepton spectrum due to thermal emission. There is, however, a large non-thermal background to dilepton production due to different channels of meson decays into e^+e^- pairs. For the low invariant mass dielectrons, this background comes mainly from the π^0 and the η Dalitz decay,

$$\pi^0 \rightarrow e^+e^- + \gamma, \quad (5.1)$$

$$\eta \rightarrow e^+e^- + \gamma \quad (5.2)$$

In addition, there are other decay channels of π^0 and η mesons into e^+e^- , such as $(\pi^0, \eta) \rightarrow e^+e^-$. However, the reaction $\pi^0 \rightarrow e^+e^-$ has a decay rate which is around 10^{-5} lower than Dalitz decay. Similarly, $\eta \rightarrow e^+e^-$ is around 10^{-1} down from Dalitz decay. Thus, both π^0 and $\eta \rightarrow e^+e^-$ effects are small. Furthermore, the η meson can decay into dielectrons as $\eta \rightarrow e^+e^-\pi^+\pi^-$; however, the branching ratio $\Gamma_{\eta \rightarrow e^+e^-\gamma} / \Gamma_{\eta \rightarrow e^+e^-\pi^+\pi^-} \sim 0.25$ is small as well, so that the above processes can be neglected as compared to Dalitz decay. The differential branching ratio for the reactions (5.1)/(5.2) for a dilepton mass M is³⁰

$$\frac{d\Gamma^{(\alpha)}}{dM} = \frac{4\alpha}{3\pi} \frac{\Gamma_{(\alpha) \rightarrow 2\gamma}}{M} \left(1 - \frac{M^2}{m_a^2}\right)^3 \left(1 + \frac{2m_e^2}{M^2}\right) \left(1 - \frac{4m_e^2}{M^2}\right)^{\frac{1}{2}}, \quad (5.3)$$

where the index a refers to η or π^0 meson. Integrating this over all M (with $2m_e \leq M \leq m_a$), we get

$$R_{(\alpha)} \equiv \frac{\Gamma_{(\alpha)}}{\Gamma_{(\alpha) \rightarrow 2\gamma}} = \frac{2\alpha}{3\pi} \left(2 \ln \left[\frac{m_a}{m_e} \right] - \frac{7}{2}\right). \quad (5.4)$$

This gives for the branching ratios $R_{\pi^0} \sim 1.2\%$ and $R_\eta \sim 1.6\%$, which agrees with experimental measurements.³¹

The number of dileptons obtained from Dalitz decay per unit rapidity y and dielectron mass M can be easily calculated from

$$\frac{dN}{dM dy} = \frac{1}{\Gamma_i^{(\alpha)}} \frac{d\Gamma^{(\alpha)}}{dM} \frac{dN^{(\alpha)}}{dy}, \quad (5.5)$$

where $dN^{(\alpha)}/dy$ indicates the multiplicity and $\Gamma_i^{(\alpha)}$ the overall decay width of π^0 or η mesons.

6. Results and discussion

To begin our analysis, we shall first discuss the general structure of the dielectron mass distributions $dN^{e^+e^-}/dM dy$, assuming quark and pion annihilation processes as the only source of e^+e^- pairs. In particular, we shall briefly consider the dependence of the above spectrum on the initial conditions. A more detailed presentation of this problem can be found in the literature.³

In figs. 3 and 4 we show the dilepton distributions calculated from eqs.(4.5)/(4.6). In both figures the contributions of hadrons and quarks to the overall dilepton spectrum are shown separately. The hadron gas part of the dielectron spectrum is obtained as a sum of the production rates calculated in the hadronic and the mixed phase, using the pion annihilation cross section. Similarly, the quark production rate is considered to be the sum of all dilepton pairs produced in the quark and the mixed phase due to the quark annihilation process. Resonance pion annihilation dominates quark annihilation in the mass range $2m_\pi < M < 1.2 \text{ GeV}$, due to the ρ peak in the pion form factor. In the invariant mass region $M \leq 2m_\pi$, the only contribution to the e^+e^- spectrum is due to quark annihilation, since pion annihilation is not possible in this invariant mass region.

In fig. 3, we see in particular the strong dependence of the dilepton production rates on the initial temperature. As expected from eqs.(4.5)/(4.6), the production rates are strongly increasing functions of the initial temperature. For production in the hadronic phase this increase is due to the relation between the production rates, the multiplicities of secondary hadrons, and the initial temperature T_0 : the distribution $dN^{e^+e^-}/dM dy$ is proportional to $(dN/dy)^2$, so that it will increase with the sixth power of the initial temperature. Normalizing $dN^h/dM dy$ by $(dN/dy)^2$, we get a result independent of T_0 . This is not the case for production in the quark phase.

The increase of $dN^q/dMdy$ with initial temperature comes from two different effects. It is also related to the quadratic dependence of the production rates on the secondary hadron multiplicity, but in addition, with increasing temperature, the system spends more time in the pure quark-gluon plasma phase. Consequently, more dilepton pairs can be produced in this phase. The increase in production rates appears especially in the large invariant mass region, $M > 2\text{GeV}$, because with increasing temperature, the production of large invariant mass electron pairs is kinematically favored.

In fig. 4 we study the dependence of dilepton spectra on the freeze-out temperature. A change of T_f will evidently not influence the thermal rates in the pure quark phase and in the mixed phase; thus $dN^q/dMdy$ is independent of the final temperature. In contrast, the hadron part of the spectrum shows an increase with decreasing T_f , particularly in the low mass region $M < 1\text{GeV}$. This is due to the fact that with decreasing final temperature the system will spend more time in the pure pion phase. Since the pions will also have lower energies, they will predominantly produce low mass e^+e^- pairs. In the mass range studied in fig. 4, we also see that the change in $dN^h/dMdy$ is greater when decreasing T_f from 0.15GeV to 0.1GeV than from 0.1GeV to 0.05GeV . This is because $T_f = 0.15\text{GeV}$ corresponds by assumption to the critical temperature, so that here $dN^h/dMdy$ gets a contribution only from the mixed phase. Decreasing T_f to 0.1GeV will lead to an additional contribution from the pure pion phase. When comparing $dN^h/dMdy$ at $T_f = 0.1\text{GeV}$ with the value at 0.05GeV , one notices a change mainly for $M < m_\rho$. This means that for $T < 0.1\text{GeV}$, the energy of the colliding pions is already too low to produce abundantly lepton pairs with mass larger than 0.8GeV .

As indicated in Section 4, the initial temperature can be fixed from the relation between the secondary hadron multiplicity and the initial entropy. The final temperature should be considered as a free parameter. The value of T_f could perhaps be extracted eventually from experimental low mass dilepton distributions, because, as we just discussed, a change of the initial temperature influences mainly the low mass dilepton spectrum. In the remainder, we shall assume the freeze-out temperature to be 0.1GeV .

In the above discussion we have analyzed the dilepton spectra, assuming that the system is created initially at $\tau_0 \sim 1\text{fm}$ as quark-gluon plasma and then undergoes

a phase transition to a hadron gas. We cannot, however, exclude the case that there is no quark-gluon plasma formation, but that initially, at the time $\tau_0 \sim 1\text{fm}$, the system thermalizes as a pion gas. The number of degrees of freedom in the initial state will then be smaller by more than an order of magnitude. Thus, to get the same initial entropy of the system, the initial temperature has to increase $R^{1/3}$ times, where R is the ratio of the degrees of freedom in the two cases. With $T_0 = 0.25\text{GeV}$ for quark-gluon plasma, we thus need $T_0^h \sim 0.6\text{GeV}$ in the pion gas. In fig. 3 we have also included the dielectron spectrum for this case. The production rates now are evidently higher than the hadronic contribution $dN^h/dMdy$ calculated in the model with a phase transition, because of the higher temperature in the purely hadronic system. Also, since in the pion gas temperatures up to 0.6GeV are possible, there is abundant production of dielectrons with the invariant mass $M > 1\text{GeV}$. However, a crucial difference arises in the low mass region of the dilepton spectrum, since for the pion gas there is no quark-gluon plasma contribution to the overall dielectron spectrum. The crucial question is whether it is possible to pick out the quark-gluon plasma contribution in the experimentally measured low mass distributions. The most recent results of the HELIOS experiment³² indicate that the major contamination to low mass dileptons is due to three body Dalitz decay of π^0 and the η mesons. The lifetime of the π^0 and the η is so long that only very few will decay during the mixed or hadronic phase. Most of the background e^+e^- pairs are obtained after decoupling, when the particles have stopped interacting and their momenta remain constant. The final state pion multiplicities are determined by the initial conditions from eq.(4.7). Assuming 80% of charged particles to be pions, and taking the charged to neutral pion ratio equal to two, we can estimate the e^+e^- spectrum due to the π^0 Dalitz decay from eq.(5.5). It is not clear how the ratio of η to π^0 multiplicities depends on the collision energy. The highest energy data give $\eta/\pi^0 \sim 0.27 \pm 0.1$.³³ In our calculations we shall take the upper limit of this ratio, $\eta/\pi^0 \sim 0.37$.

The results for the mass distribution of e^+e^- pairs due to Dalitz decay of the π^0 and the η are shown in fig. 5 for the LHC initial conditions. The π^0 decay is evidently dominating in the kinematical region $M < m_\pi$. The three body η Dalitz decay is crucial in the mass window $m_\pi < M < 2m_\pi$. The main conclusion from

fig. 5 is that the contribution of $q\bar{q} \rightarrow e^+e^-$, in the region where there is no pion annihilation production, is on the average several orders of magnitude lower than the Dalitz decay.

We have already indicated in this section that $dN^q/dMdy$ increases at least quadratically with dN/dy . In contrast, the dielectron spectrum due to Dalitz decay increases linearly with dN/dy . Thus, if the initial temperature is sufficiently high, the quark annihilation process could dominate the Dalitz background. In fig. 6 we show the over-all e^+e^- production rates calculated with an initial temperature $T_0 = 1.0\text{GeV}$. Here, the quark annihilation process dominates Dalitz decay in the mass window $m_\pi < M < 2m_\pi$. We have examined in more detail the relative magnitude of $dN^q/dMdy$ compared to the Dalitz decay spectrum. For Dalitz pairs, the normalized spectrum gives a constant value, while for $q\bar{q} \rightarrow e^+e^-$ process it is an increasing function of T_0 . To observe $dN^q/dMdy$ above the Dalitz η background, one needs an initial temperature $T_0 > 0.6\text{GeV}$, which is not expected in present or future heavy ion experiments.¹³ It will therefore be necessary to subtract the Dalitz background from experimentally measured e^+e^- spectrum.

In addition to the production mechanisms considered so far, the production via bremsstrahlung is of crucial importance for low invariant mass dielectron. The total production rate for pion bremsstrahlung, just as for pion annihilation, is calculated as the sum of the mixed and the pure pion phase contribution. Using the elastic pion-pion cross section described in section 2, we obtain the e^+e^- spectrum shown in fig. 7. The interesting point in this figure is that the pion bremsstrahlung is very large as compared to quark annihilation. The difference is especially significant in the low mass region $M < 2m_\pi$.

The rates for the mass distribution of pion bremsstrahlung shown in fig. 7 were calculated assuming that one of the pions in the initial state is charge neutral. Obviously there are other pion-pion reactions which will contribute via bremsstrahlung to the overall dielectron spectrum and which have not been included here. Thus our results should be considered as a lower bound for the pion bremsstrahlung contribution to the e^+e^- production rate.

The quark contribution to the dielectron production rates shown in fig's. 3 to 7 includes only the lowest order quark annihilation process. However, since both quark

and antiquark in the initial state can radiate a photon or a gluon, the total cross section for e^+e^- pair productions will be modified by reactions such as $q\bar{q} \rightarrow e^+e^- + X$, as explained in the Section.2. The cross sections for these processes (see eqs.(2.8 - 2.13)), are infrared divergent. The divergences appear in the limit of real photons or gluons with vanishing four-momentum. However, in our model the volume of matter created in heavy ion collisions is always finite. Thus there is a natural infrared momentum cut-off determined by the size of the system. As the system expands, the value of this cut-off will in general be time dependant. Initially it is determined by the size of the system in transverse direction. For U - U collisions we have $\lambda \sim 15\text{ fm}$ which gives $p_{\min}^7 \sim 13\text{MeV}$. When the mixed phase is totally converted to a pion gas, λ is given by the length of the system in longitudinal direction. With the initial conditions appropriate for U - U collision at LHC energy we get $\lambda \sim 50\text{fm}$ and $p_{\min}^7 \sim 4\text{MeV}$. In our numerical calculations we have taken the infrared cut-off to be time independent and equal to $2m_\pi$; thus in eq.(3.3) we take $z_{\min} = M + 2m_\pi$. The e^+e^- mass distributions calculated with the $q\bar{q} \rightarrow e^+e^- + X$ cross sections are shown in fig. 8. The contribution coming from the higher order QED process is found to be very small for $M > 20\text{MeV}$; this is what one could expect since the cross section for $\sigma_{q\bar{q} \rightarrow e^+e^- + \gamma}$ is proportional to α^3 . However, for $M < 20\text{MeV}$ the production rates from the higher order QED process start to dominate the spectrum. To estimate the contribution to the dielectron spectrum coming from $q\bar{q} \rightarrow e^+e^- + g$, we have assumed that the cross section for this process is just given by the previous QED result, multiplied by the factor $4\alpha_s/9\alpha$. Renormalization leads in general to a temperature dependent strong coupling constant α_s , as well as temperature dependent quark masses. From lattice results, $\alpha_s(T)$ can be parameterized as³⁴

$$\alpha_s(T) = \frac{6\pi}{(33 - 2n_f) \ln[a(T/T_c)]} \quad (6.1)$$

with $a \sim 8$.

In the temperature range $0.15\text{GeV} < T < 0.25\text{ GeV}$, the coupling constant α_s varies according to (6.1) as $0.25 < \alpha_s < 0.31$ for $N_f = 2$. Since this variation of α_s is small in this temperature window, we have assumed that α_s is temperature independent and equal to 0.3. The quark masses are also assumed to be T -independent and are taken to have their bare values, $m_u = m_d = 5\text{MeV}$. The results for the quark

bremsstrahlung cross section, are included in fig. 8. Low mass pairs ($M < 0.1\text{GeV}$) are found to be dominantly produced by the process $q\bar{q} \rightarrow e^+e^- + g$. Thus the QCD corrections to $q\bar{q}$ annihilation are crucial to describe the properties of low mass electron pairs coming from the quark-gluon plasma. It is possible, however, that in the low invariant mass region the application of finite temperature perturbation theory may be of importance.³⁵ The quantitative results shown in fig. 8 will be obviously rescaled when α_s is changed. If the infrared cut-off would be a time dependent parameter, the results for the spectrum would also be modified; one would expect a still steeper increase of the spectrum with decreasing M . Finally, we have to point out, that for soft gluon emission the application of perturbative results might be questionable.

In fig. 9 we show all the different contributions to the low mass spectrum that we had considered here.

7. Closing Remarks

We have analyzed the origin, size and behaviour of low invariant mass dielectron rates in central collisions between lead nuclei at LHC energy. In this analysis, we have assumed the (1+1) dimensional Bjorken hydrodynamical model to describe the expansion of the system. For a (3+1) dimensional expansion, one could expect a modification of the results shown in fig. 9. The effects of transverse collective flow were found to be important in the late stages of heavy ion collisions.^{3,24,9} Thus, the transverse expansion will generally not influence the production rates calculated in the pure quark and in the mixed phase. The most important changes would be expected when the system reaches the hadronic phase. Because of the more rapid expansion and hence shorter life time contributions from pion bremsstrahlung and pion annihilation will decrease in this case.

We have further assumed an ideal gas equation of state and a first order hadronisation phase transition. An ideal gas assumption of QCD matter is known from lattice calculations to be very crude approximation near T_c .³⁶ Taking a more realistic equation of state with interactions in the system will modify our results in two points. First, it will change the time dependence of temperature derived from eq.(3.1), which

can change the relative importance of quark and hadron gas contributions to the overall dielectron spectrum.^{7,9,11} Secondly, interactions in the system could modify the quark and pion dispersion relations, which might increase the e^+e^- production in the hadron gas and decrease the contribution of the quark phase to the overall dielectron spectrum.³⁷

We thus have to note that our study of the low mass dilepton spectrum from nuclear collisions still contains a considerable number of uncertainties. Nevertheless, we believe that some conclusions can be drawn.

In the mass range $M \lesssim 2m_\pi$, the contributions from π^0 and η Dalitz decays are expected to be two and more orders of magnitude above the $q\bar{q}$ annihilation spectrum. This “background” would therefore have to be eliminated with high precision.

Once this is done, there still remain contributions from pion bremsstrahlung which are also about two orders of magnitude above the $q\bar{q}$ annihilation. Moreover, our calculation does not yet contain all possible bremsstrahlung contributions and thus only constitutes a lower bound.

In view of these factors, we believe that it will be very difficult to extract a meaningful $q\bar{q}$ annihilation contribution from any measured low mass dielectron spectrum.

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References

1. E.L. Feinberg, Nuovo Cimento A34 (1976)391; E.V. Shuryak, Phys. Lett. B78 (1978)150; L.D. McLerran and T. Toimela, Phys. Rev. D31 (1985)545.
2. G. Domokos, Phys. Rev. D28 (1983)123.
3. For a recent review, see P. V. Ruuskanen, in Quark Gluon Plasma - Advanced Series on Directions in High Energy Physics Vol. 6, ed. R. Hwa, World Scientific, Singapore, 1990..
4. K. Kajantie, J. Kapusta, L. McLerran and A. Mekjian, Phys. Rev. D34 (1986)2746.
5. R. Yoshida, T. Miyazaki and M. Kadoya, Phys. Rev. D35 (1987)388; M. Neubert, Z.Phys. C42 (1990)231.
6. K. Redlich, Phys. Rev. D36 (1987)3387.
7. J. Cleymans, J. Fingberg and K. Redlich, Phys. Rev. D35 (1987)2153.
8. M. Kataja and P.V. Ruuskanen, Phys. Lett. B243 (1990)181; B. Kämpfer and O.P. Pavlenko, CERN preprint TH-5771/90.
9. J. Letessier and A. Tounsi, Phys. Rev. D40 (1989)2914.
10. S.A. Chin, Phys. Lett. B119 (1982)51.
11. J. Blaizot, and J. Ollitrault, Phys. Lett. B191 (1987)21; Phys. Rev. D36 (1987)916.
12. M. Matsui and H. Satz, Phys. Lett. B178 (1986)416.
13. For a recent review see also: H. Satz, in Proceedings of the ECFA Large Hadron Collider Workshop, Aachen, Germany, (1990); Volume I, CERN 1990.
14. F. Karsch, and H. Satz, CERN Preprint, TH-5900/90 (1991).
15. J.W. Harris (NA35 Collab.), Nucl. Phys. A498 (1989)133c, and refs. therein.
16. H.A. Weldon, Phys. Rev. Lett. 66 (1991)293.
17. H. Specht, in Proceedings of the ECFA Large Hadron Collider Workshop, Aachen, Germany, October (1990); Volume II, CERN 1991.
18. P.V. Ruuskanen, "Emission of Thermal Dileptons in Ultrarelativistic Heavy-Ion Collisions", VIIIth International Conference on Ultrarelativistic Nucleus/Nucleus Collisions, Menton, France, May 1990; Jyväskylä Preprint JYFL 12/90, 1990.
19. V. Cerny, P. Lichard, S. Olejnik and J. Pisut Acta Phys. Pol. B9 (1978)537.
20. G. Altarelli, Phys. Rep. 81 (1982)1; G. Altarelli, R.K. Ellis, M. Greco, and G. Martinelli, Nucl. Phys. B246 (1984)12.
21. R. Rückl, Phys. Lett. B64 (1976)39.
22. C. Gale and J. Kapusta, Phys. Rev. C38 (1988)2659; Phys. Rev. C35 (1987)2107.
23. S. Weinberg, Phys. Rev. Lett. B17 (1966)616; 18 (1967)188.
24. H. von Gersdorf, L. McLerran, M. Kataja, and P.V. Ruuskanen, Phys. Rev. D34 (1986)794; K. Kajantie, M. Kataja and L. McLerran, Phys. Rev. D34 (1986)811.
25. J.D. Bjorken, Phys. Rev. D27 (1983)140.
26. M. Gulassey, in Multiparticle Dynamics 1983, P. Yager and J.F. Guhion (Eds.), World Scientific, Singapore 1983; R. Hwa and K. Kajantie, Phys. Rev. D32 (1985)1109.
27. U. Heinz, in Proceedings of the ECFA Large Hadron Collider Workshop, Aachen, Germany, (1990); Volume II, CERN 1991.
28. For a recent survey, see B. Petersson, Lattice Results on Finite Temperature QCD a Status Report VIIIth International Conference on Ultrarelativistic Nucleus-Nucleus Collisions, Menton, France, May 1990; Bielefeld Preprint BI-TP 90/28.
29. S. Gottlieb et.al., Phys. Rev. D35 (1987)3972.
30. N.P. Samios, Phys. Rev. 121 (1961)275.
31. Review of Particle Properties, Phys. Lett. B204 (1988)1.
32. HELIOS Note 433 (1989).
33. F. Abe et.al., Phys. Rev. D41 (1990)2330.
34. F. Karsch, Z.Phys. C38 (1988)147.
35. R. Baier, B. Pire, and D. Schiff, Phys. Rev. D38 (1988)2814; T. Alther, P. Aurenche, and T. Becherawy, Nucl. Phys. B315 (1989)436; E. Braaten, R.D. Pisarski, and T.C. Yan, Brookhaven Nat. Lab. report TH-90-1, (1990).
36. J. Engels, J. Fingberg, K. Redlich, H. Satz, and M. Weber, Z.Phys. C42 (1989)341.
37. M.I. Gorenstein, and O.P. Pavlenko, Z.Phys. C37 (1988)611.

Figure Captions

Figure 1 : Dielectron production in a quark-gluon plasma. (a): quark anti-quark annihilation; (b): lowest order QED corrections from photon emission; (c): lowest order QCD corrections from gluon emission.

Figure 2 : Dielectron production in a hadron gas. (a): $\pi\pi$ annihilation; (b): $\pi\pi$ scattering with virtual bremsstrahlung.

Figure 3 : Dielectron mass distributions for two values of the initial temperature. Quark contribution: $T_0 = 1.0\text{GeV}$ (solid line); $T_0 = 0.25\text{GeV}$ (dotted line). Pion gas contributions: $T_0 = 1.0\text{GeV}$ (dot-dashed line); $T_0 = 0.25\text{GeV}$ (dashed line). Circle indicates the thermal production rates assuming that there is no phase transition.

Figure 4 : Dielectron mass distributions for fixed initial temperature $T_0 = 0.2\text{GeV}$ and for different values of the final temperature. Quark contribution (solid line). Pion gas contribution: $T_f = 0.15\text{GeV}$ (dot-dashed line); $T_f = 0.1\text{GeV}$ (dashed line); $T_f = 0.05\text{GeV}$ (dotted line).

Figure 5 : Thermal dielectron mass distributions and Dalitz background. Quark contribution (solid line); pion gas contribution (dot-dashed line); Dalitz π^0 decay (dotted line); Dalitz η decay (dashed line). The initial state parameters are taken for U-U collisions at LHC energy.

Figure 6 : As in fig. 5, but with the initial temperature $T_0 = 1.0\text{GeV}$.

Figure 7 : Dielectron invariant mass distributions calculated with pion-bremsstrahlung. Dot-dashed line pion gas contribution including only pion annihilation process; quark gas contributions (solid line); pion bremsstrahlung calculated with σ_{el} explained in section.2 (dashed line).

Figure 8 : Dielectron mass distributions obtained including QED and QCD corrections in the quark annihilation process. Pion gas contribution (dot-dashed line). Quark gas contributions: solid line - only $q\bar{q} \rightarrow e^+e^-$ process; dotted line - only $q\bar{q} \rightarrow e^+e^- + \gamma$ process; stars - only $q\bar{q} \rightarrow e^+e^- + g$ process. Parameters as in (fig. 7).

Figure 9 : Summary of contributions to the low invariant mass dielectron spectrum for U - U collisions at LHC energy. Pion gas contribution (dot-dashed line). Quark gas contributions: solid line - including only $q\bar{q} \rightarrow e^+e^-$ annihilation process,

stars - including only $q\bar{q} \rightarrow e^+e^- + g$ process. Pion bremsstrahlung (dashed line). Dalitz π^0 decay (dotted line). Dalitz η decay (long dashed line).

FIG. 1

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