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**Constructing Unified Models based on E_8
in Ten Dimensions**

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Abstract

We examine the virtues and difficulties of the attempts to construct realistic four-dimensional models from a gauge theory based on E_8 and defined in ten dimensions. The four-dimensional theories are obtained using the Coset Space Dimensional Reduction scheme. Some of our points and in particular the proposed mechanism for supersymmetry breaking might be useful in other dimensional reduction schemes.

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1. *Introduction* A phenomenologically viable unification of all the known interactions is still challenging the community of elementary particle physicists. The most ambitious schemes such as those based on the Kaluza-Klein [1] proposal cannot reproduce even the observed chiral world, while theoretically they cannot justify the assumed space-time configurations. On the other hand more moderate ideas like GUTs represented by the minimal $SU(5)$ GUT of Georgi and Glashow (G.G.) [2] had remarkable qualitative and to a certain extent quantitative success. However when the G.G. $SU(5)$ confronted with more accurate measurements on proton lifetime and $\sin^2 \theta$, it was ruled out [3]. Finally attractive ideas such as superstring [4] theories have very little impact to our understanding of physics below the Planck scale.

Taking a more optimistic view to the unification struggles of various kinds it is important to make some steps forward in overcoming the internal difficulties of the various schemes. In the present paper, using as a tool a specific scheme we make some suggestions that might be useful in other unification frameworks. More specifically we shall use the Coset Space Dimensional Reduction (CSDR) procedure to reduce a unified theory defined in higher dimensions in order to illustrate our points. The advantage of CSDR scheme is that it permits a detailed study of the four-dimensional theory resulting after dimensional reduction. The CSDR scheme is by itself a serious step forward within the Kaluza-Klein framework. In order to appreciate that, we recall that the introduction of gauge fields in higher dimensions was proposed as a solution to some of the problems of the Kaluza-Klein theories [5]. The virtue of the CSDR scheme as it has been developed [6-13,17] is that the introduction of gauge fields in higher dimensions can be considered now not as the price to be paid in order to make the Kaluza-Klein program viable but rather as a neat way to describe the low energy interactions. Despite some success of the CSDR scheme from this respect, the unified gauge theory used in higher dimensions was only free of gauge anomalies. It would be nice to start from a gauge theory free of gauge and gravitational anomalies such as $E_8 \times E_8$ or $SO(32)$ [14]. Thus our first point is to present models based on E_8 gauge group in ten dimensions, taking the usual point of view in string theories that the second E_8 is hidden and irrelevant for our world. Another point has to do with the recent speculations on dynamical symmetry breaking, which suggest that top quark condensates can substitute the role played in the Weinberg-Salam model by the fundamental scalars [15]. In our models here we show how scalars can be eliminated naturally from the spectrum of the four-dimensional theory. Recall that in four-dimensional theories obtained from dimensional reduction of gauge theories defined in higher dimensions, scalar fields appear naturally being just the components of the gauge field which corre-

spond to the extra dimensions. Without fundamental scalars the usual hierarchy problem that arises when the standard model is embedded in a GUT, disappears too. In turn one does not need supersymmetry either to cure the problem. However if the four-dimensional theory for some other reason has to be supersymmetric one should provide a mechanism for supersymmetry breaking at low energies. The lack of such a mechanism is one of the fundamental theoretical problems in four-dimensional supersymmetric theories obtained from superstrings. Our third point is to construct a supersymmetric four-dimensional theory obtained from a higher dimensional one, which is supplied with a mechanism that breaks supersymmetry. Such a mechanism could be used in superstrings theories too.

2. *CSDR scheme* Let us briefly recall the CSDR [6-13] procedure to reduce dimensionally a gauge theory defined in higher dimensions. Consider in D dimensions a Yang-Mills plus Dirac theory with gauge group G defined on a manifold M^D which will be compactified to $M^4 \times S/R$, $D=4+d$, $d = \dim S - \dim R$.

$$A = \int d^4 x d^d y \sqrt{-g} \left[-\frac{1}{4} \text{Tr}(F_{MN} F_{KL}) g^{MK} g^{NL} + \frac{i}{2} \bar{\psi} \Gamma^M D_M \psi \right] \quad (1)$$

where $D_M = \partial_M - \theta_M - A_M$ with $\theta_M = \frac{1}{2} \theta_{MNA} \Sigma^{NA}$ the spin-connection of M^D , $F_{MN} = \partial_M A_N - \partial_N A_M - [A_M, A_N]$ and $M, N, \dots$ run over the D -dimensional space. We use Greek indices to denote the curved ones while Latin to denote the flat ones. The fields A_M and ψ are assumed to be symmetric in the sense that any transformation under symmetries of S/R (which form the isometry group S of S/R) is compensated by gauge transformations. The fermion field can be in any representation F of G unless a further symmetry such as supersymmetry is required. So if $g(s)$, $f(s)$ are gauge transformations in the adjoint and F irreps of G corresponding to the transformation s acting on S/R we require [6]

$$A_\mu(x, y) = g(s) A_\mu(x, s^{-1}y) g^{-1}(s) \quad (2)$$

$$A_\alpha(x, y) = g(s) \Sigma_\alpha^\beta A_\beta(x, s^{-1}y) g^{-1}(s) + g(s) \partial_\alpha g^{-1}(s) \quad (3)$$

$$\psi(x, y) = f(s) \Omega \psi(x, s^{-1}y) f^{-1}(s) \quad (4)$$

where Σ_α^β is the Jacobian matrix for the transformation s and Ω is the Jacobian matrix plus a local Lorentz rotation in the tangent space which is needed for the fermions when they transform in a curved space. These conditions imply certain constraints that the D -dimensional fields have to obey. The solution of these constraints will provide us with the four-dimensional unconstrained fields as well as with the gauge invariance that remains in the theory after dimensional reduction.

From eq.(2) it follows that the components $A_\mu(x, y)$ of the initial gauge field $A_M(x, y)$ become, after dimensional reduction, the four-dimensional gauge fields and furthermore they are independent of y . In addition, one can find that they have to commute with the elements of the R_G subgroup of G . Thus the four-dimensional gauge group H is the centralizer of R in G , $H = C_G(R_G)$.

Similarly, from eq.(3) the $A_\mu(x, y)$ components of $A_M(x, y)$ denoted by $\phi_a(x, y)$ from hereon, become scalars at four dimensions. These fields transform under R as a vector $\mathbf{v}$ specified by the embedding of $R \subset S$

$$\begin{aligned} S &\supset R \\ \text{adj } S &= \text{adj } R + \mathbf{v} \end{aligned} \quad (5)$$

Moreover $\phi_a(x, y)$ act as intertwining operators connecting induced representations of R acting on G and S/R . This implies, exploiting Schur's lemma that the transformation properties of the fields $\phi_a(x, y)$ under H can be found if we express the adjoint irrep of G in terms of $R_G \times H$

$$G \supset R_G \times H \quad \text{adj } G = (\text{adj } R_G, 1) + (1, \text{adj } H) + \sum (\mathbf{r}_i, \mathbf{h}_i) \quad (6)$$

Then if $\mathbf{v} = \sum s_i$, where each s_i is an irrep of R there survives an h_i multiplet for every pair $(\mathbf{r}_i, \mathbf{s}_i)$ where $\mathbf{r}_i$ and $\mathbf{s}_i$ are identical irreps of R .

Turning now to the fermion fields [7] we see from eq.(4) that they act as intertwining operators between induced representations of R acting on G and the tangent space of S/R , $SO(d)$. Proceeding along similar lines as in the case of scalars to obtain the representation of H under which the four-dimensional fermions transform, we have to decompose the representation F of the initial gauge group in which the fermions are assigned under $R_G \times H$

$$F = \sum (\mathbf{t}_i, \mathbf{h}_i) \quad (7)$$

and the spinor of $SO(d)$ under R

$$\sigma_d = \sum \sigma_i \quad (8)$$

Then for each pair $\mathbf{t}_i$ and σ_i where $\mathbf{t}_i$ and σ_i are identical irreducible representations there is an h_i multiplet of spinor fields in the four-dimensional theory. In order, however to obtain chiral fermions in the effective theory we have to impose further requirements. We first impose the Weyl condition in D dimensions. In $D=4n+2$ dimensions, which is the case

at hand, the decomposition of the left-handed, say spinor under $SO(4) \times SO(d)$ is $\sigma_D = (2, 1, \sigma_d) + (1, 2, \bar{\sigma}_d)$. So we have in this case the decompositions $\sigma_d = \sum \sigma_k$, $\bar{\sigma}_d = \sum \bar{\sigma}_k$. Let us start from a vectorlike representation F for the fermions. In this case each term $(\mathbf{t}_i, \mathbf{h}_i)$ in eq.(7) will be either selfconjugate or it will have a partner $(\bar{\mathbf{t}}_i, \bar{\mathbf{h}}_i)$. According to the rule described in eqs.(7) and (8) if we consider σ_d we will have at four dimensions left-handed fermions transforming according to $f_L = \sum h_k^L$. It is important to notice that since σ_d is non-selfconjugate f_L is non-selfconjugate too. Similarly from $\bar{\sigma}_d$ we will obtain the right-handed representation $f_R = \sum \bar{h}_k^R$ but as we have assumed that F is vectorlike $\bar{h}_k^R \sim h_k^L$. Therefore there will appear two sets of Weyl fermions with the same quantum numbers under H . This is already a chiral theory, but still one can go further and try to impose the Majorana condition in order to eliminate the doubling of the fermionic spectrum. We should remark now that if we had started with F complex we would have again a chiral theory since in this case $\bar{h}_k^R$ is different from h_k^L (σ_d non-selfconjugate). Nevertheless since starting with F vectorlike is a more appealing situation we proceed by considering the Majorana condition. The Majorana condition can be imposed in $D=2, 3, 4+8n$ dimensions and is given by $\psi = C(\bar{\psi})^T$ where C is the D -dimensional charge conjugation matrix. Majorana and Weyl conditions are compatible in $D=4n+2$ dimensions. Then in our case if we start with Weyl-Majorana spinors in $D=4n+2$ dimensions we force f_R to be the charge conjugate of f_L , thus arriving in a theory with fermions only in f_L . Furthermore if F is to be real then we have to have $D=2+8n$ while for F pseudoreal $D=6+8n$.

Next let us obtain the four-dimensional effective action. Assuming that the metric is block diagonal, taking into account all the constraints and integrating out the extra coordinates we obtain in four dimensions the following Lagrangian

$$A = C \int d^4x \left(-\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu a} + \frac{1}{2} (D_\mu \phi_a)^\dagger (D^\mu \phi^a) + V(\phi) + \frac{i}{2} \bar{\psi} \Gamma^\mu D_\mu \psi - \frac{i}{2} \bar{\psi} \Gamma^a D_a \psi \right) \quad (9)$$

where $D_\mu = \partial_\mu - A_\mu$ and $D_a = \partial_a - \theta_a - \phi_a$ with $\theta_a = \frac{1}{2} \theta_{abc} \Sigma^{bc}$ the connection of the coset space. C is the volume of the coset space and $V(\phi)$ is given by

$$V(\phi) = -\frac{1}{4} g^{ac} g^{bd} \text{Tr}(f_{ab}^c \phi_c - [\phi_a, \phi_b])(f_{cd}^d \phi_d - [\phi_c, \phi_d]) \quad (10)$$

where $(A=1, \dots, \dim S)$ and f 's are the structure constants of the Lie algebra of S ; $(i=1, \dots, \dim R)$ which form R , and the remaining ones Q_a ($a=\dim R+1, \dots, \dim S$) which form S/R

$$\begin{aligned} [Q_i, Q_j] &= f_{ij}^k Q_k \\ [Q_i, Q_a] &= f_{ia}^b Q_b \end{aligned}$$

$$[Q_a, Q_b] = f_{ab}{}^c Q_c + f_{ab}{}^i Q_i$$

The expression (10) for $V(\phi)$ is only formal because ϕ_a must satisfy the constraints coming from eq.(3) $f_{ai}{}^D \phi_D - [\phi_a, \phi_i] = 0$ where the ϕ_i generate R_G . These constraints imply that some components ϕ_a 's are zero, some are constants and the rest can be identified with the genuine Higgs fields. When $V(\phi)$ is expressed in terms of the unconstrained independent Higgs fields, it remains a quartic polynomial which is invariant under gauge transformations of the final gauge group H , and its minimum determines the vacuum expectation value of the Higgs fields [8].

In the fermionic Lagrangian the first term is just the kinetic term for the fermions while the second is the Yukawa term [10]. We note that since ψ is a Majorana-Weyl spinor in ten dimensions the representation under the gauge group in which the fermions are assigned must be real. The second term can be written as

$$\begin{aligned} L_Y &= -\frac{i}{2} \bar{\psi} \Gamma^a (\partial_a + \frac{1}{2} f_{bc} e_i^a \gamma_i \Sigma^{bc} + \frac{1}{2} G_{abc} \Sigma^{bc} - \phi_a) \psi \\ &= \frac{i}{2} \bar{\psi} \Gamma^a \nabla_a \psi + \bar{\psi} V \psi \end{aligned} \quad (11)$$

where

$$\nabla_a = -\partial_a - \frac{1}{2} f_{bc} e_i^a \gamma_i \Sigma^{bc} + \phi_a \quad (12)$$

$$V = -\frac{i}{2} \Gamma^a G_{abc} \Sigma^{bc} \quad (13)$$

and we have used the full connection with torsion [11] given by

$$\theta_c{}^a{}_b = f_{ib}{}^a e_i^b e_c^a + (D_{cb}{}^a + \frac{1}{2} T_c{}^a{}_b) = f_{ib}{}^a e_i^b e_c^a + G_{cb}{}^a \quad (14)$$

with

$$D_{cb}{}^a = g^{ad} \frac{1}{2} [f_{db}{}^c g_{ec} + f_{cb}{}^e g_{de} - f_{cd}{}^e g_{be}] \quad (15)$$

A useful choice for the torsion tensor is $T_{abc} = 2\tau(D_{abc} + D_{bca} - D_{cba})$. Also in this case T_{abc} is fully antisymmetric and when the radii are equal it is proportional to f_{abc} . As we already noticed, the CSDR constraints tell us that $\partial_a \psi = 0$. Furthermore since we can consider the Lagrangian at the point $y = 0$, due to its invariance under S -transformations one can show that $e_i^a = 0$ at that point. So eq.(13) becomes just $\nabla_a = \phi_a$ and it is exactly the Yukawa term.

Let us examine now the second term appearing in eq.(11). One can show easily that V anticommutes with the six-dimensional helicity operator. Furthermore one can show that the operator V commutes with the $T_i = -\frac{1}{2} f_{abc} \Sigma^{bc}$ (T_i close the R -subalgebra of $SO(6)$). In turn we can draw the conclusion - exploiting Schur's lemma - that the non-vanishing elements of V are only those which appear in the decomposition of both $SO(6)$ irreps 4 and $\bar{4}$ e.g. the singlets. Since this term is a pure geometric term we arrive at the conclusion that the singlets in 4 and $\bar{4}$ will acquire large geometrical masses, a fact that has interesting phenomenological implications. For example in supersymmetric theories defined in higher dimensions, it means that the gauginos obtained in four dimensions after dimensional reduction obtain superheavy masses i.e. supersymmetry is broken at the compactification scale. We note that for symmetric cosets the V operator is absent because $f_{ab}{}^c$ are zero.

3. Models In the following we construct models demonstrating the points advertised in the introduction.

1. $G=E_8$, $B = E_6/Z_2$, $B_0 = S/R = Sp(4) \times SU(2)/SU(2) \times SU(2) \times U(1)$ This is a ten-dimensional model based on the E_8 gauge group with fermions in the adjoint. The $R=SU(2) \times SU(2) \times U(1)$ is chosen to be identified with the $SU(2) \times SU(2) \times U(1)$ subgroup of E_8 appearing in the decomposition

$$\begin{aligned} E_8 \supset SU(2) \times E_7 & & E_7 \supset SU(2) \times SO(12) \\ 248 = (3, 1) + (1, 133) + (2, 56) & & 56 = (2, 12) + (1, 32) \\ & & 133 = (3, 1) + (2, 32') + (1, 66) \end{aligned}$$

$$SO(12) \supset SO(10) \times U(1)$$

$$12 = 1(2) + 1(-2) + 10(0) \quad 32' = 16(-1) + \bar{16}(1)$$

$$32 = 16(1) + \bar{16}(-1) \quad 66 = 1(0) + 45(0) + 10(2) + \bar{10}(-2)$$

One can easily check that the four-dimensional gauge group that survives after dimensional reduction over the specific coset space is

$$H = C_{E_8}(SU(2) \times SU(2) \times U(1)) = SO(10) \times U(1)^*$$

The $R=SU(2) \times SU(2) \times U(1)$ content of $Sp(4) \times SU(2)/SU(2) \times SU(2) \times U(1)$ vector and spinor are

$$6 = (2, 2)(0) + (1, 1)(2) + (1, 1)(-2); \quad 4 = (2, 1)(1) + (1, 2)(-1)$$

* The $U(1)$ in parenthesis can be omitted by putting equal to zero the corresponding coupling constant since the theory is renormalizable [12]

According to the CSDR rules the surviving matter fields are a) scalars transforming as $10(2) + 10(-2)$ under H and b) left-handed fermions transforming as $16(1)$ and $16(-1)$ if we apply Majorana and Weyl conditions in ten dimensions. Therefore the four-dimensional theory is an interesting GUT with the appropriate fermion multiplets. However the Higgs content of the theory is disastrous. Since the specific coset space is symmetric necessarily the theory breaks down spontaneously [8,13] leading to final unbroken gauge group $SO(9)$. The scalar field content could be appropriate for the electroweak symmetry breaking in the presence of another mechanism to provide the first superstrong breaking. Such a mechanism can be provided as will be discussed soon. will be discussed soon. However even in that case the electroweak breaking due to the Higgs fields will be totally wrong since it will take place at the compactification scale which is the only order parameter entering in the discussion. Therefore if we take seriously the extra dimensions which have to be unobservably small we better get rid of the scalars altogether.

We shall show that using the Wilson flux [16,17] mechanism we can achieve the spontaneous symmetry breaking of the GUT at hand and the natural disappearance of the Higgs fields from the four-dimensional theory. In order to do so we have to substitute the simply connected coset space $B_0 = Sp(4) \times SU(2)/SU(2) \times SU(2) \times U(1)$ with $B = B_0/Z_2$, where Z_2 is a freely acting discrete group on B_0 . The B_0 has [17] a $(Z_2)^2$ that belongs to the center of S and a $(Z_2)^2$ that belongs to $W = W_S/W_R$, where W_S, W_R are the Weyl groups of S, R respectively. We choose the Z_2 used in B to be the one that corresponds to the $W_{SU(2)}/W_{U(1)}$ in W . The Z_2 is freely acting and the resulting space B is non-simply connected since $\pi_1(B_0) = Z_2$. Then there will be contours not contractible to a point due to holes in the manifold B . It turns out that the final unbroken gauge group is the centralizer of $R \times T^G$ in G , where T^G is the subgroup of G generated by the Wilson loops [16]. As far as the surviving fields is concerned they have to be invariant under the diagonal sum $Z_2 \oplus T^G$. We choose T^G to be the Z_2 subgroup of the hypercharge in the decomposition $SU(5) \supset SU(3)_c \times SU(2)_L \times U(1)_Y$ where $SU(5)$ appears in the decomposition $SO(10) \supset SU(5) \times U(1)$. Furthermore the embedding of $Z_2 \in W$ in the hypercharge is chosen in such a way that all the components of the fundamental representation transform evenly. One the other hand the R decompositions of the vector and the spinor are transforming as follows [17]:

$$\begin{array}{l} \text{vector} \left\{ \begin{array}{l} (1,1)(2) \longleftrightarrow (1,1)(-2) \\ (2,2)(0) \longleftrightarrow (2,2)(0) \end{array} \right\}; \quad \text{spinor} \left\{ \begin{array}{l} (2,1)(1) \longleftrightarrow (2,1)(-1) \\ (1,2)(1) \longleftrightarrow (1,2)(-1) \end{array} \right\} \end{array}$$

As a result of the dimensional reduction of the gauge theory over B we obtain
a) Final unbroken gauge group $SU(3) \times SU(2) \times U(1)$.

- b) Two families of quarks and leptons resulting from the two left-multiplets $16(1)$ and $16(-1)$ under $SO(10) \times U(1)$. The Z_2 can substitute the Majorana condition in ten dimensions which is therefore relaxed.
- c) The scalars do not survive since the invariants under the diagonal sum of the discrete groups does not have definite transformation properties under the gauge group.

Therefore we obtain the standard model with even number of families since we can add more fermion multiplets in the adjoint of E_8 in ten dimensions. The spontaneous symmetry breaking of the electroweak group has then to rely on the condensates of the heavy flavours [15]. Finally the state of proton decay problem remains as in the G.G. $SU(5)$ model, given that the final spontaneous symmetry breaking step of the theory coincides with the breaking of $SU(5)$.

2. $G = E_8, B = B_0/Z_2, B_0 = S/R = (SU(2)/U(1))^3$ This is the same ten-dimensional gauge theory as in the previous example which however now is dimensionally reduced over a different manifold $B_0 = S/R = (SU(2)/U(1))^3$ is a symmetric coset space as a result of the direct product of three symmetric homogeneous spaces $SU(2)/U(1)$. The $R=U(1)^3$ is chosen to be identified with the three $U(1)$ subgroups of E_8 appearing in the decomposition

$$\begin{aligned} E_8 &\supset E_7 \times SU(2) \supset E_7 \times U(1) \\ 248 &= (133, 1) + (1, 3) + (56, 2) \\ &= 133(0) + 1(0) + 1(2) + 1(-2) + 56(1) + 56(-1) \end{aligned}$$

$$\begin{array}{ll} E_7 \supset E_6 \times U(1) & E_6 \supset SO(10) \times U(1) \\ 56 = 1(3) + 27(1) + \overline{27}(-1) + 1(-3) & 27 = 1(4) + 16(1) + 10(-2) \\ 133 = 78(0) + 1(0) + 27(-2) + \overline{27}(2) & 78 = 1(0) + 45(0) + 16(-3) + \overline{16}(3) \end{array}$$

Dimensional reduction of the ten-dimensional theory over B_0 gives in four dimensions a gauge theory based on the group $H=C_{E_8}(U(1)^3) = SO(10) \times U(1)^3$. The $R=U(1)^3$ content of $(SU(2)/U(1))^3$ vector and spinor are

$$6 = (2a, 0, 0) + (0, 2b, 0) + (0, 0, 2c) + (-2a, 0, 0) + (0, -2b, 0) + (0, 0, -2c)$$

and

$$4 = (a, b, c) + (-a, -b, c) + (-a, b, -c) + (a, -b, -c)$$

With $a = b = c = 1$ we find that the four-dimensional fields transform under $SO(10)$ as singlets and fermions transforming as 16-plets if we apply Majorana and Weyl conditions in ten dimensions.

Thus again we are led in four dimensions to an interesting GUT with the right fermion multiplets to describe a family of quarks and leptons. The Higgs scalar being an $SO(10)$ singlet is irrelevant for the spontaneous symmetry breaking of the theory. In order to break further the $SO(10)$ we have to employ the flux breaking mechanism. This can be done by replacing B_0 by $B = B_0/Z_2$ where Z_2 belongs to any $W_{SU(2)}/W_{U(1)}$ of W of B_0 . The action of each Z_2 is to change the sign of the $U(1)$ charges. The Z_2 's act freely and B is non-simply connected. By embedding the Z_2 of B into the Z_2 subgroup of the hypercharge in the decomposition of $SU(5) \supset SU(3)_c \times SU(2)_L \times U(1)_Y$ as we did in the first model we manage to produce the spontaneous symmetry breaking of the theory down to the standard model. Again the breaking of the electroweak gauge group has to rely on the condensates of the heavy flavours.

3. $G=E_8$, $B = B_0/Z_2$, $B_0 = S/R = Sp(4)/(SU(2) \times U(1))_{non-max}$ Here we start again with the same gauge theory in ten dimensions as in the previous cases but we use a non-symmetric space B_0 . Using the decompositions

$$E_8 \supset SU(5) \times SU(5), \quad SU(5) \supset SU(3) \times SU(2) \times U(1)$$

we choose $SU(2) \times U(1)$ of the second one to be identified with R . Then the resulting four-dimensional gauge group is $H = C_{E_8}(SU(2) \times U(1)) = SU(5) \times U(1) \times U(1)$. The $R = SU(2) \times U(1)$ content of $Sp(4)/(SU(2) \times U(1))_{non-max}$ vector and spinor is given by

$$6 = 1(2) + 1(-2) + 2(1) + 2(-1); \quad 4 = 1(0) + 1(2) + 2(-1)$$

Then from the decomposition of E_8 under $SU(5) \times SU(3) \times (SU(2) \times U(1))_R$

$$\begin{aligned} 248 = & [24, 1; 1(0)] + [1, 1; 1(0)] + [1, 8; 1(0)] + [1, 1; 3(0)] \\ & + [1, 3; 2(5)] + [1, \bar{3}; 2(-5)] + [5, 1; 1(6)] + [\bar{5}, 1; 1(-6)] \\ & + [5, 3; 1(-4)] + [\bar{5}, \bar{3}; 1(4)] + [5, \bar{3}; 2(1)] + [\bar{5}, 3; 2(-1)] \\ & + [10, 1; 2(-3)] + [\bar{10}, 1; 2(3)] + [10, 3; 1(2)] + [\bar{10}, \bar{3}; 1(-2)] \end{aligned}$$

we find that the four-dimensional gauge theory contains scalars transforming under $H \times R$ as $[\bar{5}, 3; 2(-1)]$ and $[10, 3; 1(2)]$ and fermions transforming as $[\bar{5}, 3; 2(-1)]$, $[10, 3; 1(2)]$, $[24, 1; 1(0)]$, $[1, 1; 1(0)]$ and $[1, 8; 1(0)]$. The resulting four-dimensional theory is supersymmetric and among others one has to face the problem of supersymmetry breaking.

Next we can use the Wilson flux mechanism in order to break the $SU(5) \subset H$ down to the standard model leaving the $SU(3) \subset H$ to play the role of gauge flavour symmetry. In order to apply the Wilson flux mechanism we consider instead of $B_0 = Sp(4)/(SU(2) \times U(1))_{non-max}$ the manifold $B = B_0/Z_2$ where Z_2 is the center of $Sp(4)$. Under the action of this Z_2 the vector and spinor transform to themselves. Then we embed this discrete symmetry into a discrete subgroup of $U(1)$ that appears in the decomposition $SU(5) \supset SU(3) \times SU(2) \times U(1)$ in a way that leaves invariant all the matter fields. In this way we have succeeded to leave invariant all the matter fields under the simultaneous action of the two discrete symmetries and therefore the only effect of the Wilson flux breaking mechanism is to break $SU(5)$ down to $SU(3) \times SU(2) \times U(1)$.

The main point we would like to illustrate using the present model is a way to break supersymmetry. In order to do so we have to examine the Yukawa term of the resulting four-dimensional theory in our model. The Yukawa term in general is given in eqs.(11)–(15). In order to determine its form in the present form we note the following [10]

a.) The most general $Sp(4)$ invariant metric g_{ab} of $Sp(4)/(SU(2) \times U(1))_{non-max}$ is

$$g_{ab} = \text{diag}(a, a, b, b, a, a)$$

and its fully antisymmetric structure constants are given by $f_{13}{}^6 = -f_{14}{}^5 = f_{23}{}^5 = f_{24}{}^6 = f_{71}{}^6 = -f_{72}{}^5 = f_{81}{}^5 = f_{82}{}^6 = f_{91}{}^2 = -f_{95}{}^6 = f_{10,1}{}^2 = f_{10,5}{}^6 = 1$, $f_{78}{}^9 = f_{10,3}{}^4 = 2$.

b.) Using the antisymmetric structure constants of $Sp(4)$ the D's in eq.(15) are found to be: $D_{136} = D_{613} = -D_{145} = -D_{514} = D_{235} = D_{523} = D_{246} = D_{624} = \frac{b}{2}$, $D_{361} = -D_{451} = D_{352} = D_{462} = a - \frac{b}{2}$ and in turn the $G_{abc} = D_{abc} + \frac{1}{2}T_{abc}$ in eq.(13) are given by

$$G_{136} = G_{613} = -G_{145} = -G_{514} = G_{235} = G_{523} = G_{246} = G_{624} = \frac{b - \tau(2a + b)}{2}$$

$$G_{361} = -G_{451} = G_{352} = G_{462} = \frac{2a - b - \tau(2a + b)}{2}$$

c.) Writing the ten-dimensional spinor ψ as

$$\psi = (\psi_1, \dots, \psi_4, \chi_1, \dots, \chi_4)^T$$

where ψ_i, χ_i $i=1, \dots, 4$ are four-dimensional spinors. In order to be specific let us select the Γ matrices, closing the algebra $[\Gamma^\mu, \Gamma^\nu] = 2\eta^{\mu\nu}I_{10}$ and $[\Gamma^a, \Gamma^b] = -2g^{ab}I_{10}$, to be

$$\Gamma^\mu = \gamma^\mu \otimes I_8$$

$$\Gamma^{p+3} = \frac{1}{\sqrt{a_p}} \gamma_5 \otimes \begin{pmatrix} 0 & -\tilde{\gamma}^p \\ \tilde{\gamma}^p & 0 \end{pmatrix} \quad p = 1, 2, 3, 5, 6$$

$$\Gamma^7 = \frac{1}{\sqrt{a_4}} \gamma_5 \otimes \begin{pmatrix} 0 & iI_4 \\ iI_4 & 0 \end{pmatrix}$$

where $a_1 = a_2 = a_5 = a_6 = a$ and $a_3 = a_4 = b$. γ^μ , γ_5 are the usual four-dimensional γ -matrices and $\tilde{\gamma}^p$ are given by

$$\begin{aligned} \tilde{\gamma}^1 &= \sigma^1 \otimes \sigma^2 & \tilde{\gamma}^2 &= \sigma^2 \otimes \sigma^2 & \tilde{\gamma}^3 &= -I_2 \otimes \sigma^3 \\ \tilde{\gamma}^5 &= \sigma^3 \otimes \sigma^2 & \tilde{\gamma}^6 &= -I_2 \otimes \sigma^1 \end{aligned}$$

In this representation Γ^{11} is found to be

$$\Gamma^{11} = \sqrt{\prod a_i} \Gamma^0 \dots \Gamma^9 = \gamma_5 \otimes \begin{pmatrix} -I_4 & 0 \\ 0 & I_4 \end{pmatrix}$$

while the charge conjugation operator is given by

$$C_{10} = C \otimes \begin{pmatrix} 0 & -\vartheta \\ \vartheta & 0 \end{pmatrix}$$

where $\vartheta = \sigma^2 \otimes iI_2$ and C is the four-dimensional charge conjugation matrix. When we apply the Weyl $\Gamma^{11} \psi = \psi$ and Majorana $\psi = C_{10}(\bar{\psi})^T$ the ten-dimensional spinor takes the form

$$\psi = (L\psi_1, L\psi_2, L\psi_3, L\psi_4, R\tilde{\psi}_1, R\tilde{\psi}_2, R\tilde{\psi}_3, R\tilde{\psi}_4)^T$$

where $\tilde{\psi}_i = (-1)^i C \gamma_0 \psi_i^*$. Calculating the embedding of $R=SU(2) \times U(1)$ into $SO(6)$ using the matrices $T_i = -\frac{1}{2} f_{iab} \Sigma^{ab}$, $\Sigma^{ab} = \frac{1}{4} [\Gamma^a, \Gamma^b]$ we find that the components of the spinor transform under R , ψ_1 as $1(0)$, ψ_2 as $1(2)$ and $(\psi_3, -\psi_4)$ as $2(-1)$.

We find that the Yukawa term becomes

$$\begin{aligned} & \left\{ \frac{1}{\sqrt{b}} \bar{\psi}_{2L} (\phi_4 - i\phi_3) \psi_{1L}^c + \frac{1}{\sqrt{a}} \bar{\psi}_{4L} (\phi_5 - i\phi_6) \psi_{1L}^c \right. \\ & + \frac{1}{\sqrt{a}} \bar{\psi}_{1L} (\phi_1 - i\phi_2) \psi_{3L}^c + \frac{1}{\sqrt{a}} \bar{\psi}_{4L} (\phi_1 + i\phi_2) \psi_{2L}^c \\ & \left. + \frac{1}{\sqrt{a}} \bar{\psi}_{3L} (\phi_5 + i\phi_6) \psi_{2L}^c + \frac{1}{\sqrt{b}} \bar{\psi}_{4L} (\phi_4 + i\phi_3) \psi_{3L}^c + \text{h.c.} \right\} + (1-3\tau) \frac{2a+b}{a\sqrt{b}} \bar{\psi}_{1L} \psi_{1L}^c \end{aligned} \quad (16)$$

where ϕ_i are the Higgs fields.

The fermion mass matrix that is obtained depends obviously on the v.e.v.'s of the Higgs fields involved. We note that the v.e.v.'s of the Higgs fields crucially depend on the relative values of the radii a, b appearing in the metric g_{ab} and does not concern us here. The details can be found in ref.[10]. For our purposes it is enough to consider the last term in eq.(16). This term does not involve any Higgs field. It is the purely geometrical V term appearing in eq.(13). The fermion fields involved are those that transform as $1(0)$ under R i.e. are the $[24, 1; 1(0)]$, $[1, 1; 1(0)]$ and $[1, 8; 1(0)]$ multiplets. In other words this geometrical mass term provides masses to the gauginos. Therefore it is a supersymmetry breaking term. In the lack of any torsion in the coset space under consideration the mass of the gauginos is of the order of the compactification scale. However, in the presence of torsion, by adjusting the value of τ , we obtain supersymmetry breaking at any scale. In particular for $\tau = \frac{1}{3}$ the theory remains supersymmetric. The torsion introduced here is a geometrical property of the manifold and can be fixed at any value. Of course in a more general theory that includes gravity a detailed choice of the value of τ would produce a naturality problem. Therefore in a more general study of the problem one should introduce an antisymmetric field instead of a rigid value of the torsion of the manifold and leave the dynamics to choose a particular value. Although the proposed mechanism for supersymmetry breaking has to be considered in this general framework we would like to stress that such a mechanism is not applicable in compactifications such as when torii are used, like in string theories, for the simple reason that they do not admit torsion. It seems that in order to apply this mechanism in string theories a necessary condition is to use compactification on manifolds that admit torsion [18].

4. Conclusions In the present paper we have presented some representative examples in order to demonstrate our points that (a) it is possible to construct four-dimensional gauge theories starting from a ten-dimensional theory free from gauge and gravitational anomalies (b) scalar fields can naturally disappear from the four-dimensional theory if the spontaneous symmetry breaking is chosen to be dynamical and (c) the supersymmetry of the four-dimensional theory can be broken at any scale by adjusting the torsion of the coset space.

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