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Abstract The standard derivations of the energy and phase of the synchronous particle in a proton accelerator assume, as if by definition, that said synchronous particle lies on the central orbit of the machine. This is manifestly unjustified in the particular case of the acceleration near transition of a mixture of ions, when a small difference in charge-to-mass ratio can produce a large radial separation of the different ion species. The development of a simple derivation of the parameters of the synchronous particle that involves no such *a priori* constraint has yielded some surprises; not least, a belated explanation for an apparent anomaly encountered in 1987, when a mixture of oxygen and sulphur ions was accelerated in the CERN Proton Synchrotron for the first time. These ideas are supported by measurements performed in 1990 during a second ion run.

1 Preamble

In the autumn of 1987, sulphur ions were accelerated at CERN to a world-record energy of 6.4 TeV. The $^{32}\text{S}^{16+}$ ions were supplied by an electron cyclotron resonance source which also produced $^{16}\text{O}^{8+}$ ions. Significantly, however, the oxygen ions constituted a ninety percent “contamination” of the total beam current. Since the relative charge-to-mass ratio difference of $^{16}\text{O}^{8+}$ with respect to $^{32}\text{S}^{16+}$ is only about -5.4×10^{-4} , the oxygen was essentially indistinguishable from sulphur throughout the acceleration process, with the exception of the region near transition. It was near transition in the PS that the oxygen was eliminated, albeit with some difficulty initially. It proved far more straightforward to remove the sulphur when the oxygen was required for setting-up.

The “asymmetry” of the selection process could not be satisfactorily explained. Despite the obvious fact of the beam control system being dominated by the oxygen, there seemed to be no convincing mechanism to cause an RF system which included a phase loop but no radial feedback to behave differently in the two cases.

2 The Synchronous Particle

According to the “standard creed”, the momentum of the synchronous particle is given by the fundamental guide field relation

$$p_{\text{nom}} = q_{\text{ion}} B_z \rho_{\text{nom}} \quad (1)$$

Here, q_{ion} is the particle’s charge and ρ_{nom} is its radius of curvature in the dipole magnetic field, B_z , of the machine. For the suffix nom to denote parameters which pertain to a central orbit of mean radius R_{nom} presupposes the ideal

RF (angular) frequency programme

$$\omega_{\text{RF}}(B_z) = \frac{hc}{R_{\text{nom}}} \left[1 + \left(\frac{E_{\text{rest,ion}}}{q_{\text{ion}} B_z \rho_{\text{nom}} c} \right)^2 \right]^{-1/2} \quad (2)$$

where $E_{\text{rest,ion}}$ is the rest-mass energy of the particle, c is the speed of light *in vacuo* and where the harmonic number, h , may be any positive integer. The corresponding synchronous phase (with respect to the zero-crossing of the RF), φ_{nom} , is given by

$$\sin \varphi_{\text{nom}} = \frac{2\pi R_{\text{nom}} \rho_{\text{nom}}}{\hat{V}_{\text{RF}}} \frac{dB_z}{dt} \quad (3)$$

where $\hat{V}_{\text{RF}}$ is the total (peak) RF voltage and t is time.

A full derivation of the parameters of the synchronous particle for arbitrary ω_{RF} is given elsewhere [1]. The starting point is a more general form of (1):

$$p_0 = q_{\text{ion}} B_z \rho_{\text{nom}} \left(\frac{R_0}{R_{\text{nom}}} \right)^{1/\alpha} \quad (4)$$

where the synchronous particle is now denoted by suffix zero and α is the momentum compaction factor. It follows that the synchronous energy, E_0 , is obtained by solving

$$E_0(E_0^2 - E_{\text{rest,ion}}^2)^{(\alpha-1)/2} = \frac{hc}{\omega_{\text{RF}} R_{\text{nom}}} (q_{\text{ion}} B_z \rho_{\text{nom}} c)^\alpha \quad (5)$$

While the synchronous phase is given by

$$\sin \varphi_0 = \frac{2\pi h}{q_{\text{ion}} \hat{V}_{\text{RF}} \omega_{\text{RF}}} \frac{dE_0}{dt} - \frac{2\pi R_0}{\hat{V}_{\text{RF}}} \mathcal{E}_\theta(R_0) \quad (6)$$

where $\mathcal{E}_\theta$ is the azimuthal component of the electric field which is produced in accordance with Faraday’s Law by the time-variation of the magnetic field. The second, “betatron acceleration” term is hereafter neglected as small.

When the special case of an ideal frequency programme is considered by substituting (2) into (5), two roots are found. One solution corresponds to $R_0 = R_{\text{nom}}$ and is thus consistent with (1) and (3). The other lies on the opposite side of transition and, except very near to transition, would fall well outside the physical aperture of a real machine.

3 Mixtures of Ions in the PS

An Open-loop Analysis

The acceleration of particles other than the usual protons requires the modification of the RF frequency programme to cater for different charge-to-mass ratios. In the

PS, this is achieved by the introduction, on a cycle-to-cycle basis, of a rate multiplication factor, b , which modifies the so-called B-train measurement of the dipole magnetic field. Thus, in the absence of feedback, the RF frequency would be generated as a slightly modified form of (2):

$$\omega_{\text{RF}}(B_z; b) = \frac{hc}{R_{\text{nom}}} \left[1 + \left(\frac{E_{\text{rest,p}}}{q_p b B_z \rho_{\text{nom}} c} \right)^2 \right]^{-1/2} \quad (7)$$

with, ideally,

$$b = \frac{q_{\text{ion}}/E_{\text{rest,ion}}}{q_p/E_{\text{rest,p}}} \quad (8)$$

Consequently, (5) becomes

$$f_1(E_0; \text{ion}) = f_2(B_z; b) \quad (9)$$

where

$$f_1(E_0; \text{ion}) = q_{\text{ion}}^{-\alpha} E_0 (E_0^2 - E_{\text{rest,ion}}^2)^{(\alpha-1)/2} \quad (10)$$

is determined by ion species and

$$f_2(B_z; b) = (B_z \rho_{\text{nom}} c)^\alpha \left[1 + \left(\frac{E_{\text{rest,p}}}{q_p b B_z \rho_{\text{nom}} c} \right)^2 \right]^{1/2} \quad (11)$$

is a function only of the free variable B_z . Furthermore, assuming $\alpha = \gamma_{\text{tr}}^{-2}$ to be invariant,

$$\sin \varphi_0(B_z; \text{ion}, b) = \frac{2\pi R_{\text{nom}}}{q_{\text{ion}} \hat{V}_{\text{RFC}}} \left[1 + \left(\frac{E_{\text{rest,p}}}{q_p b B_z \rho_{\text{nom}} c} \right)^2 \right]^{1/2} \times \frac{dB_z}{dt} \frac{f_2'(B_z; b)}{f_1'(E_0; \text{ion})} \quad (12)$$

Equations (9) and (12) provide the synchronous energy and phase, respectively, for a given ion species under open-loop conditions. That is, the RF frequency of (7) is not considered to be influenced by feedback. However, it is a strength of the approach developed thus far that this frequency is not assumed to be ideal, so that b , although a fixed parameter, may be chosen freely.

Oxygen and Sulphur

The stage is now set for the (numerical) solution of (9) for $^{16}\text{O}^{8+}$ and $^{32}\text{S}^{16+}$ ions for the various permutations of ion species and b . The result is shown in fig.1, first in the case when b is calculated for oxygen and then in the case when the RF frequency programme for sulphur is chosen. Both cases exhibit a monotonically increasing solution for $\gamma_0 = E_0/E_{\text{rest,ion}}$ as a function of B_z for the ion species which corresponds to the RF programme; this is the "standard creed". There is also a decelerating solution for that species. However, for the other species, the solutions in the two cases are very different. For sulphur ions on the oxygen programme, the synchronous energy solutions above and below transition never meet. While for oxygen ions on the sulphur programme, there is a period during which there is no solution for the synchronous energy.

Figure 1: $(\gamma_0 - \gamma_{\text{tr}})$ vs. $B_z/[T]$ for $^{16}\text{O}^{8+}$ (thin lines) and $^{32}\text{S}^{16+}$ (thick lines) ions in the PS. (Dark/light lines correspond to synchronous particles which are below/above transition.)

Figure 2: $\sin \varphi_0$ vs. $B_z/[T]$ for $^{16}\text{O}^{8+}$ (thin lines) and $^{32}\text{S}^{16+}$ (thick lines) ions in the PS. (Dark/light lines correspond to synchronous particles which are below/above transition.)