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Finiteness of the Topological Models in the Landau gauge.

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Abstract

A general procedure for showing the perturbative finiteness of topological field theories is presented. As an explicit example the three-dimensional BF-system with a nonvanishing cosmological term is discussed in details.

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1 Introduction

Topological field theories [1], due to their peculiar properties, are responsible for a large number of interesting results and are objects of continuous investigations.

The main property of this class of models is represented by the general covariance, i.e. by the metric independence of the classical action.

This has many elegant consequences, as the characterization, within the standard perturbation theory [1,2], of topological quantities like the Gauss linking number and its generalization [3], the interpretation of the three-dimensional Einstein-Hilbert gravity [4], with or without a cosmological constant, in terms of a Chern-Simons gauge theory and the connection with the two-dimensional conformal models [1,2,5].

From a pure field theoretic point of view, the topological models are expected to be an example of finite field theories, the perturbative finiteness being a consistent and necessary step towards a correct topological interpretation of perturbative computations like a vacuum expectation value of Wilson lines [2].

The main purpose of this work is to give a general and systematic way to show the perturbative finiteness of a large class of topological models.

We checked the validity of our procedure on several situations, like, for instance, the three-dimensional Chern-Simons model [6,7], the four-dimensional antisymmetric tensor field model [8] and the three-dimensional BF-system. Actually, we believe that it is possible to show the finiteness of any topological model by means of a simple application of our arguments.

This is supported also by a recent extension of such a procedure to topological models built on curved space-time [9].

Our main tools are the existence of a supersymmetric algebra and the choice of a Landau gauge for the gauge-fixing part of the action.

The use of a Landau gauge should not be surprising, as it is known since a long time that this gauge exhibits remarkable finiteness properties [10], which, in the case of the topological models, lead to the perturbative finiteness thanks to the existence of a supersymmetric structure.

Such a supersymmetry, as it will be shown in details, turns out to be a general feature of the topological models, and its explicit realization is extremely simple in the Landau gauge.

Its existence is essentially due to the fact that the energy-momentum tensor of a topological model can be written as a pure BRS variation : $T_{\mu\nu} = s\Lambda_{\mu\nu}$. In the Landau gauge, both $T_{\mu\nu}$ and $\Lambda_{\mu\nu}$ are conserved, the supersymmetry being nothing but the Ward identity associated with the conservation of $\Lambda_{\mu\nu}$. It is interesting to note that the supersymmetry is realized in a non-linear way [11], and that the set of external fields needed to close the algebra off-shell coincides with the one which must be introduced to write the Slavnov identity associated with the BRS invariance.

In what follows, we shall take the three-dimensional BF-system as an explicit example to show in detail all the mentioned properties.

This choice is motivated by the simplicity of the quantization procedure and also by the fact that this model is related to the three-dimensional Einstein-Hilbert gravity with a nonvanishing cosmological constant.

The work is organized in the following way : in section 2 we introduce the on-shell supersymmetry; section 3 is devoted to the study of the Slavnov identity and to the off-shell supersymmetric algebra; section 4 concerns the quantum extension and the perturbative finiteness.

2 The model : BRS symmetry and supersymmetry

The model we consider is the three-dimensional BF-system whose classical action, including a cosmological constant α , is

$$S_{inv} = \frac{1}{2} \int_M d^3x \, \varepsilon^{\mu\nu\rho} \left\{ F_{\mu\nu}^a B_\rho^a + \frac{\alpha}{3} f^{abc} B_\mu^a B_\nu^b B_\rho^c \right\}, \quad (2.1)$$

where M is the flat space-time, $B_\rho^a(x)$ is a one-form and

$$F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + f^{abc} A_\mu^b A_\nu^c,$$

where f^{abc} are the structure constants of a compact simple gauge group G . Notice that the expression (2.1), being metric independent, has a topological character.

The action S_{inv} possesses two kinds of symmetries :

$$\begin{cases} \delta^{(1)} A_\mu^a &= -(D_\mu \theta)^a = -(\partial_\mu \theta^a + f^{abc} A_\mu^b \theta^c) \\ \delta^{(1)} B_\mu^a &= -f^{abc} B_\mu^b \theta^c \end{cases} \quad (2.2)$$

and

$$\begin{cases} \delta^{(2)} A_\mu^a &= -\alpha f^{abc} B_\mu^b \lambda^c \\ \delta^{(2)} B_\mu^a &= -(D_\mu \lambda)^a. \end{cases} \quad (2.3)$$

To quantize the model, one introduces two ghosts $(c^a(x), \phi^a(x))$, two antighosts $(\bar{c}^a(x), \bar{\phi}^a(x))$, two Lagrange multipliers $(b^a(x), d^a(x))$ and a nilpotent s -operation defined as :

$$\begin{aligned} s A_\mu^a &= -(D_\mu c)^a - \alpha f^{abc} B_\mu^b \phi^c \\ s c^a &= \frac{1}{2} f^{abc} (c^b c^c + \alpha \phi^b \phi^c) \\ s \bar{c}^a &= b^a \\ s b^a &= 0 \end{aligned} \quad (2.4)$$

$$\begin{aligned} s B_\mu^a &= -(D_\mu \phi)^a - f^{abc} B_\mu^b c^c \\ s \phi^a &= f^{abc} \phi^b c^c \\ s \bar{\phi}^a &= d^a \\ s d^a &= 0, \end{aligned}$$

$$s^2 = 0. \quad (2.5)$$

In a Landau-gauge, the gauge-fixing term S_{gf} reads :

$$\begin{aligned}
S_{gf} &= \int d^3x \, s(\bar{c}^a \partial^\mu A_\mu^a + \bar{\phi}^a \partial^\mu B_\mu^a) \\
&= \int d^3x \, \left\{ b^a \partial^\mu A_\mu^a + \bar{c}^a \partial^\mu \left[(D_\mu c)^a + \alpha f^{abc} B_\mu^b \phi^c \right] + d^a \partial^\mu B_\mu^a \right. \\
&\quad \left. + \bar{\phi}^a \partial^\mu \left[(D_\mu \phi)^a + f^{abc} B_\mu^b c^c \right] \right\} ,
\end{aligned} \tag{2.6}$$

and

$$sS = 0 , \tag{2.7}$$

where $S = S_{inv} + S_{gf}$.

The dimensions and the Faddeev-Popov ($\Phi\Pi$) charges of the quantum fields are displayed in Table 1.

	A_μ^a	c^a	$\bar{c}^a$	b^a	B_μ^a	ϕ^a	$\bar{\phi}^a$	d^a
dim	1	0	1	1	1	0	1	1
$\Phi\Pi$	0	1	-1	0	0	1	-1	0

Table 1. Dimensions and $\Phi\Pi$ charges of the quantum fields.

As observed for other topological models [6,8,12], the gauge fixed action ($S_{inv} + S_{gf}$), besides the BRS invariance (2.4), possesses another global symmetry whose generators δ_μ carry a Lorentz index and whose action on the fields is given by :

$$\begin{aligned}
\delta_\mu A_\nu^a &= -\varepsilon_{\nu\mu\rho} \partial^\rho \bar{\phi}^a \\
\delta_\mu c^a &= -A_\mu^a \\
\delta_\mu \bar{c}^a &= 0 \\
\delta_\mu b^a &= \partial_\mu \bar{c}^a \\
\delta_\mu B_\nu^a &= -\varepsilon_{\nu\mu\rho} \partial^\rho \bar{c}^a \\
\delta_\mu \phi^a &= -B_\mu^a \\
\delta_\mu \bar{\phi}^a &= 0 \\
\delta_\mu d^a &= \partial_\mu \bar{\phi}^a ,
\end{aligned} \tag{2.8}$$

and

$$\begin{aligned}
\{\delta_\mu, \delta_\nu\} &= 0 \\
\{s, \delta_\mu\} &= \partial_\mu + \text{equations of motion} ,
\end{aligned} \tag{2.9}$$

with

$$\delta_\mu S = 0 . \tag{2.10}$$

The equations (2.9) show that the algebra between δ_μ and s closes on-shell on the translations, thus allowing for a supersymmetric interpretation of the model.

We remark that the presence of such a supersymmetry is not a chance, but it is a general property of the topological models, since its existence is a consequence of the

metric independence of the starting action S_{inv} ; indeed, due to the general covariance, the physical part of the energy-momentum tensor vanishes, thus forbidding the usual particle interpretation of a standard field theory.

Moreover, it is interesting to stress that the supersymmetry appears only after the introduction of the Faddeev–Popov ghosts. This means that there is a pairing between the ghosts and the fields of the starting action S_{inv} ; in other words, the ghost degrees of freedom cancel those of S_{inv} in such a way that the gauge-fixed action $S_{inv} + S_{gf}$ has no local degrees of freedom.

For a better understanding of this point, let us relate the supersymmetry transformations (2.8) to the energy momentum tensor $T_{\mu\nu}$ of the model.

A straightforward computation shows that $T_{\mu\nu}$ is an exact s -variation

$$T_{\mu\nu} = \frac{1}{2} s \Lambda_{\mu\nu} , \quad (2.11)$$

where

$$\begin{aligned} \Lambda_{\mu\nu} = & \eta_{\mu\nu} [(\partial^\lambda \bar{c}^a) A_\lambda^a + (\partial^\lambda \bar{\phi}^a) B_\lambda^a] \\ & - (\partial_\mu \bar{c}^a) A_\nu^a - (\partial_\nu \bar{c}^a) A_\mu^a - (\partial_\mu \bar{\phi}^a) B_\nu^a - (\partial_\nu \bar{\phi}^a) B_\mu^a , \end{aligned} \quad (2.12)$$

and $\eta_{\mu\nu}$ is the constant diagonal flat metric.

Let us now compute the divergence of $\Lambda_{\mu\nu}$. After a little algebra one gets

$$\begin{aligned} \partial^\mu \Lambda_{\mu\nu} = & -\varepsilon_{\mu\nu\rho} (\partial^\mu \bar{\phi}^a) \frac{\delta S}{\delta A_\rho^a} + A_\nu^a \frac{\delta S}{\delta c^a} - (\partial_\nu \bar{c}^a) \frac{\delta S}{\delta b^a} \\ & -\varepsilon_{\mu\nu\rho} (\partial^\mu \bar{c}^a) \frac{\delta S}{\delta B_\rho^a} + B_\nu^a \frac{\delta S}{\delta \phi^a} - (\partial_\nu \bar{\phi}^a) \frac{\delta S}{\delta d^a} \\ & -\varepsilon_{\mu\nu\rho} \partial^\mu (\bar{c}^a \partial^\rho d^a + \bar{\phi}^a \partial^\rho b^a) , \end{aligned} \quad (2.13)$$

from which it follows that

$$\tilde{\Lambda}_{\mu\nu} = \Lambda_{\mu\nu} + \varepsilon_{\mu\nu\rho} (\bar{c}^a \partial^\rho d^a + \bar{\phi}^a \partial^\rho b^a) \quad (2.14)$$

is a conserved quantity.

Notice that $s (\varepsilon_{\mu\nu\rho} (\bar{c}^a \partial^\rho d^a + \bar{\phi}^a \partial^\rho b^a))$ is identically conserved, i.e.

$$\partial^\mu (s \varepsilon_{\mu\nu\rho} (\bar{c}^a \partial^\rho d^a + \bar{\phi}^a \partial^\rho b^a)) = 0 , \quad (2.15)$$

so that we can redefine the energy momentum tensor as

$$T_{\mu\nu} \longrightarrow \tilde{T}_{\mu\nu} = T_{\mu\nu} + \varepsilon_{\mu\nu\rho} \partial^\rho \left(\frac{b^a d^a}{2} \right) , \quad (2.16)$$

with

$$\tilde{T}_{\mu\nu} = \frac{1}{2} s \tilde{\Lambda}_{\mu\nu} . \quad (2.17)$$

Evidently

$$s \tilde{T}_{\mu\nu} = 0 , \quad (2.18)$$

and

$$\partial^\mu \tilde{T}_{\mu\nu} = (\text{contact terms}) , \quad (2.19)$$

as well as

$$\begin{aligned} \partial^\mu \tilde{\Lambda}_{\mu\nu} = & -\varepsilon_{\mu\nu\rho}(\partial^\mu \bar{\phi}^a) \frac{\delta S}{\delta A_\rho^a} + A_\nu^a \frac{\delta S}{\delta c^a} - (\partial_\nu \bar{c}^a) \frac{\delta S}{\delta b^a} \\ & -\varepsilon_{\mu\nu\rho}(\partial^\mu \bar{c}^a) \frac{\delta S}{\delta B_\rho^a} + B_\nu^a \frac{\delta S}{\delta \phi^a} - (\partial_\nu \bar{\phi}^a) \frac{\delta S}{\delta d^a} . \end{aligned} \quad (2.20)$$

Equations (2.17) and (2.18) show that $T_{\mu\nu}$ and $\tilde{T}_{\mu\nu}$ are equivalent. Integrating the equation (2.20), one gets

$$\begin{aligned} \int d^3x \left\{ -\varepsilon_{\mu\nu\rho}(\partial^\mu \bar{\phi}^a) \frac{\delta}{\delta A_\rho^a} + A_\nu^a \frac{\delta}{\delta c^a} - (\partial_\nu \bar{c}^a) \frac{\delta}{\delta b^a} \right. \\ \left. - \varepsilon_{\mu\nu\rho}(\partial^\mu \bar{c}^a) \frac{\delta}{\delta B_\rho^a} + B_\nu^a \frac{\delta}{\delta \phi^a} - (\partial_\nu \bar{\phi}^a) \frac{\delta}{\delta d^a} \right\} S = 0 , \end{aligned} \quad (2.21)$$

which describes the invariance of the gauge-fixed action S under the supersymmetric transformations (2.8).

It is now apparent that the existence of a supersymmetry is simply due to the fact that, in a topological model, both $\tilde{T}_{\mu\nu}$ and $\tilde{\Lambda}_{\mu\nu}$ are conserved.¹

It is easy to repeat the argument for other topological models as, for instance, the three-dimensional Chern-Simons model [6] and the four-dimensional BF-system [8], to convince oneself that the existence of a supersymmetry is indeed a general feature of the topological models.

3 The off-shell algebra

To write down a Slavnov identity corresponding to the transformations (2.4), we introduce a set of external sources coupled to the non-linear variations of the quantum fields :

$$\begin{aligned} S_s = \int d^3x \left\{ -\Omega^{a\mu} [(D_\mu c^a) + \alpha f^{abc} B_\mu^b \phi^c] + L^a \frac{1}{2} f^{abc} (c^b c^c + \alpha \phi^b \phi^c) \right. \\ \left. - \rho^{a\mu} [(D_\mu \phi)^a + f^{abc} B_\mu^b c^c] + D^a f^{abc} \phi^b c^c \right\} , \end{aligned} \quad (3.1)$$

where the dimensions and the Faddeev-Popov charges of $(\Omega^{a\mu}(x), L^a(x), \rho^{a\mu}(x), D^a(x))$ are listed in Table 2:

	$\Omega^{a\mu}$	L^a	$\rho^{a\mu}$	D^a
dim	2	3	2	3
$\Phi\Pi$	-1	-2	-1	-2

Table 2. Dimensions and $\Phi\Pi$ -charges of the external fields.

¹ We are indebted to F.Delduc for clarifying discussions on this point.

It is not difficult to see that the complete action

$$\Sigma = S_{inv} + S_{gf} + S_s, \quad (3.2)$$

satisfies the Slavnov identity

$$\mathcal{S}(\Sigma) = 0, \quad (3.3)$$

and the Ward identity

$$W_\mu \Sigma = \Delta_\mu^{cl}, \quad (3.4)$$

where

$$\mathcal{S}(\Sigma) = \int d^3x \left(\frac{\delta \Sigma}{\delta \Omega^{a\mu}} \frac{\delta \Sigma}{\delta A_\mu^a} + \frac{\delta \Sigma}{\delta L^a} \frac{\delta \Sigma}{\delta c^a} + \frac{\delta \Sigma}{\delta \rho^{a\mu}} \frac{\delta \Sigma}{\delta B_\mu^a} + \frac{\delta \Sigma}{\delta D^a} \frac{\delta \Sigma}{\delta \phi^a} + b^a \frac{\delta \Sigma}{\delta \bar{c}^a} + d^a \frac{\delta \Sigma}{\delta \bar{\phi}^a} \right), \quad (3.5)$$

$$W_\mu = \int d^3x \left\{ -\varepsilon_{\nu\mu\rho} (\partial^\rho \bar{\phi}^a + \rho^{a\rho}) \frac{\delta}{\delta A_\nu^a} - A_\mu^a \frac{\delta}{\delta c^a} + (\partial_\mu \bar{c}^a) \frac{\delta}{\delta b^a} - \varepsilon_{\nu\mu\rho} (\partial^\rho \bar{c}^a + \Omega^{a\rho}) \frac{\delta}{\delta B_\nu^a} \right. \\ \left. - B_\mu^a \frac{\delta}{\delta \phi^a} + (\partial_\mu \bar{\phi}^a) \frac{\delta}{\delta d^a} - L^a \frac{\delta}{\delta \Omega^{a\mu}} - D^a \frac{\delta}{\delta \rho^{a\mu}} \right\}, \quad (3.6)$$

and

$$\Delta_\mu^{cl} = \int d^3x \{ \varepsilon_{\mu\nu\rho} \rho^{a\nu} \partial^\rho b^a - \Omega^{a\nu} \partial_\mu A_\nu^a + \varepsilon_{\mu\nu\rho} \Omega^{a\nu} \partial^\rho d^a + L^a \partial_\mu c^a + D^a \partial_\mu \phi^a - \rho^{a\nu} \partial_\mu B_\nu^a \}. \quad (3.7)$$

Introducing the linearized Slavnov operator B_Σ

$$B_\Sigma = \int d^3x \left(\frac{\delta \Sigma}{\delta \Omega^{a\mu}} \frac{\delta}{\delta A_\mu^a} + \frac{\delta \Sigma}{\delta A_\mu^a} \frac{\delta}{\delta \Omega^{a\mu}} + \frac{\delta \Sigma}{\delta L^a} \frac{\delta}{\delta c^a} + \frac{\delta \Sigma}{\delta c^a} \frac{\delta}{\delta L^a} \right. \\ \left. + \frac{\delta \Sigma}{\delta \rho^{a\mu}} \frac{\delta}{\delta B_\mu^a} + \frac{\delta \Sigma}{\delta B_\mu^a} \frac{\delta}{\delta \rho^{a\mu}} + \frac{\delta \Sigma}{\delta D^a} \frac{\delta}{\delta \phi^a} + \frac{\delta \Sigma}{\delta \phi^a} \frac{\delta}{\delta D^a} \right. \\ \left. + b^a \frac{\delta}{\delta \bar{c}^a} + d^a \frac{\delta}{\delta \bar{\phi}^a} \right), \quad (3.8)$$

one verifies the following algebra :

$$\begin{aligned} \{B_\Sigma, B_\Sigma\} &= 0 \\ \{W_\mu, W_\nu\} &= 0 \\ \{W_\mu, B_\Sigma\} &= \mathcal{P}_\mu, \end{aligned} \quad (3.9)$$

where

$$\mathcal{P}_\mu = \sum_{\text{all fields } \phi} \int d^3x (\partial_\mu \phi) \frac{\delta}{\delta \phi}. \quad (3.10)$$

The introduction of the external sources $(\Omega^{a\mu}(x), L^a(x), \rho^{a\mu}(x), D^a(x))$ allows to put off-shell the original algebra (2.9), i.e. the external sources play the role of the auxiliary fields of the standard linearly realized supersymmetry. Notice that Δ_μ^{cl} in eq. (3.4), (3.7),

being linear in the quantum fields, is a classical breaking; such a feature seems to be a general property of a non-linearly realized supersymmetric algebra [9,10].

The Slavnov identity (3.3) and the Ward identity (3.4) are not the unique constraints which characterize the classical action, indeed, as in the usual gauge theories [13], we can impose the two gauge conditions

$$\begin{aligned}\frac{\delta \Sigma}{\delta b^a(x)} &= \partial^\mu A_\mu^a(x) \\ \frac{\delta \Sigma}{\delta d^a(x)} &= \partial^\mu B_\mu^a(x),\end{aligned}\quad (3.11)$$

and, thanks to the algebraic properties of the Landau gauge [7], the two integrated ghost equations of motion :

$$\begin{aligned}\mathcal{F}^a \Sigma &= \Delta_{(f)}^a \\ \mathcal{G}^a \Sigma &= \Delta_{(g)}^a,\end{aligned}\quad (3.12)$$

where

$$\begin{aligned}\mathcal{F}^a &= \int d^3x \left(\frac{\delta}{\delta c^a} + f^{abc} \bar{c}^b \frac{\delta}{\delta b^c} + f^{abc} \bar{\phi}^b \frac{\delta}{\delta d^c} \right) \\ \mathcal{G}^a &= \int d^3x \left(\frac{\delta}{\delta \phi^a} + f^{abc} \bar{\phi}^b \frac{\delta}{\delta b^c} + \alpha f^{abc} \bar{c}^b \frac{\delta}{\delta d^c} \right)\end{aligned}\quad (3.13)$$

and

$$\begin{aligned}\Delta_{(f)}^a &= \int d^3x f^{abc} (\Omega^{b\mu} A_\mu^c - L^b c^c + \rho^{b\mu} B_\mu^c - D^b \phi^c) \\ \Delta_{(g)}^a &= \int d^3x f^{abc} (\rho^{b\mu} A_\mu^c - D^b c^c + \alpha \Omega_\mu^b B_\mu^c - \alpha L^b \phi^c).\end{aligned}\quad (3.14)$$

Notice that, once again, $\Delta_{(f)}^a$ and $\Delta_{(g)}^a$, being linear in the quantum fields, are classical breakings. The equations (3.12) represent a general feature of the Landau-gauge which, in the absence of anomalies, give strong constraints on the quantum corrections. Actually, as we shall see in the next section, eqs. (3.12) are crucial for the proof of the perturbative finiteness of the model.

Commuting equations (3.11), (3.12) with the Slavnov identity (3.3), one finds another set of constraints :

$$\begin{aligned}\overline{\mathcal{F}}^a(x) \Sigma &= \left(\partial^\mu \frac{\delta}{\delta \Omega^{a\mu}(x)} + \frac{\delta}{\delta \bar{c}^a(x)} \right) \Sigma = 0 \\ \overline{\mathcal{G}}^a(x) \Sigma &= \left(\partial^\mu \frac{\delta}{\delta \rho^{a\mu}(x)} + \frac{\delta}{\delta \bar{\phi}^a(x)} \right) \Sigma = 0,\end{aligned}\quad (3.15)$$

$$\begin{aligned}\mathcal{H}_{rig}^a \Sigma &= \int d^3x f^{abc} \left(A_\mu^b \frac{\delta}{\delta A_\mu^c} + c^b \frac{\delta}{\delta c^c} + \bar{c}^b \frac{\delta}{\delta \bar{c}^c} + b^b \frac{\delta}{\delta b^c} + B_\mu^b \frac{\delta}{\delta B_\mu^c} + \phi^b \frac{\delta}{\delta \phi^c} + \bar{\phi}^b \frac{\delta}{\delta \bar{\phi}^c} \right. \\ &\quad \left. + d^b \frac{\delta}{\delta d^c} + \Omega^{b\mu} \frac{\delta}{\delta \Omega^{c\mu}} + L^b \frac{\delta}{\delta L^c} + \rho^{b\mu} \frac{\delta}{\delta \rho^{c\mu}} + D^b \frac{\delta}{\delta D^c} \right) \Sigma = 0\end{aligned}\quad (3.16)$$

and

$$\begin{aligned}
\mathcal{N}^a \Sigma = \int d^3x f^{abc} \Bigg\{ & A_\mu^b \frac{\delta}{\delta B_\mu^c} + c^b \frac{\delta}{\delta \phi^c} + \bar{\phi}^b \frac{\delta}{\delta \bar{c}^c} + d^b \frac{\delta}{\delta b^c} + \rho^{b\mu} \frac{\delta}{\delta \Omega^{c\mu}} + D^b \frac{\delta}{\delta L^c} \\
& + \alpha \left(B_\mu^b \frac{\delta}{\delta A_\mu^c} + \phi^b \frac{\delta}{\delta c^c} + \bar{c}^b \frac{\delta}{\delta \bar{\phi}^c} + b^b \frac{\delta}{\delta d^c} \right. \\
& \left. + \Omega^{b\mu} \frac{\delta}{\delta \rho^{c\mu}} + L^b \frac{\delta}{\delta D^c} \right) \Bigg\} \Sigma = 0, \tag{3.17}
\end{aligned}$$

Equation (3.16) represents the Ward identity for the invariance of Σ under the rigid gauge transformations. Notice that the global gauge symmetry turns out to be a consequence of the Landau gauge [7].

To summarize, the classical action Σ in (3.2) is characterized by the Slavnov identity (3.3), the Ward identity (3.4), the gauge conditions (3.11), the integrated ghost equations of motion (3.12), the equations of motion of the antighosts (3.15) and the Ward identities (3.16) and (3.17).

The relevant part of the algebra given by these constraints is :

$$\begin{aligned}
& \bar{\mathcal{F}}^a(x) \mathcal{S}(\gamma) + B_\gamma \bar{\mathcal{F}}^a(x) \gamma = 0 \\
& \bar{\mathcal{G}}^a(x) \mathcal{S}(\gamma) + B_\gamma \bar{\mathcal{G}}^a(x) \gamma = 0 \\
& \mathcal{F}^a \mathcal{S}(\gamma) + B_\gamma (\mathcal{F}^a \gamma - \Delta_{(f)}^a) = \mathcal{H}_{rig}^a \gamma \\
& \mathcal{G}^a \mathcal{S}(\gamma) + B_\gamma (\mathcal{G}^a \gamma - \Delta_{(g)}^a) = \mathcal{N}^a \gamma \\
& B_\gamma \mathcal{S}(\gamma) = 0 \\
& \{W_\mu, W_\nu\} = 0 \\
& \{\bar{\mathcal{F}}^a(x), W_\mu\} = -\partial_\mu \frac{\delta}{\delta b^a(x)} ; \{\bar{\mathcal{G}}^a(x), W_\mu\} = -\partial_\mu \frac{\delta}{\delta d^a(x)} \tag{3.18} \\
& W_\mu \mathcal{S}(\gamma) + B_\gamma (W_\mu \gamma - \Delta_\mu^d) = \mathcal{P}_\mu \\
& [\mathcal{N}^a, \mathcal{N}^b] = -\alpha f^{abc} \mathcal{H}_{rig}^c \\
& [\mathcal{F}^a, \mathcal{N}^b] = -f^{abc} \mathcal{G}^c ; [\mathcal{G}^a, \mathcal{N}^b] = -\alpha f^{abc} \mathcal{F}^c \\
& \mathcal{N}^a \mathcal{S}(\gamma) - B_\gamma \mathcal{N}^a \gamma = 0 \\
& [\mathcal{N}^a, W_\mu] = 0,
\end{aligned}$$

where γ is a generic functional with even Faddeev–Popov charge.

4 Renormalization and perturbative finiteness

To discuss the renormalization of the model, let us begin showing that the symmetries and the constraints considered in the previous sections are not anomalous.

The great number of constraints which form the algebra (3.18) renders the analysis of the possible anomalies quite long. For that reason, we shall limit ourselves to show in detail, as an explicit example, the analysis of the supersymmetric Ward identity (3.4), omitting the straightforward but tedious study of the other equations.

Following [7], it is not difficult to show the absence of anomalies for the gauge conditions (3.11), for the ghost equations (3.12), for the antighost equations (3.15) and for the rigid Ward identity (3.16).

Therefore, we start assuming that the quantum vertex functional Γ is such that :

$$\begin{aligned} \frac{\delta \Gamma}{\delta b^a(x)} &= \partial^\mu A_\mu^a(x) & \frac{\delta \Gamma}{\delta d^a(x)} &= \partial^\mu B_\mu^a(x) \\ \overline{\mathcal{F}}^a(x)\Gamma &= 0 & \overline{\mathcal{G}}^a(x)\Gamma &= 0 \\ \mathcal{F}^a\Gamma &= \Delta_{(f)}^a & \mathcal{G}^a\Gamma &= \Delta_{(g)}^a . \end{aligned} \quad (4.1)$$

and

$$\mathcal{H}_{rig}^a \Gamma = 0 . \quad (4.2)$$

Let us now write a broken supersymmetric Ward identity

$$W_\mu \Gamma = \Delta_\mu^{cl} + \mathcal{A}_\mu \cdot \Gamma , \quad (4.3)$$

where $\mathcal{A}_\mu \cdot \Gamma$ represents the possible breaking induced by radiative corrections. According to the Quantum Action Principle [14], the lowest order nonvanishing contribution to the breaking is a local field functional of dimension 4 and Faddeev–Popov charge -1 . According to the algebra (3.18), the local insertion $\mathcal{A}_\mu \cdot \Gamma$ has to satisfy the following constraints :

$$\begin{aligned} \frac{\delta}{\delta b^a(x)}(\mathcal{A}_\mu \cdot \Gamma) &= \frac{\delta}{\delta d^a(x)}(\mathcal{A}_\mu \cdot \Gamma) = 0 \\ \overline{\mathcal{F}}^a(x)(\mathcal{A}_\mu \cdot \Gamma) &= \overline{\mathcal{G}}^a(x)(\mathcal{A}_\mu \cdot \Gamma) = 0 \end{aligned} \quad (4.4)$$

$$\begin{aligned} \mathcal{F}^a(\mathcal{A}_\mu \cdot \Gamma) &= 0 \\ \mathcal{G}^a(\mathcal{A}_\mu \cdot \Gamma) &= 0 \end{aligned} \quad (4.5)$$

and

$$\begin{aligned} \mathcal{H}_{rig}^a(\mathcal{A}_\mu \cdot \Gamma) &= 0 \\ W_\mu(\mathcal{A}_\nu \cdot \Gamma) + W_\nu(\mathcal{A}_\mu \cdot \Gamma) &= 0 . \end{aligned} \quad (4.6)$$

To the lowest nonvanishing order

$$\mathcal{A}_\mu \cdot \Gamma = \mathcal{A}_\mu + O(\hbar \mathcal{A}_\mu) . \quad (4.7)$$

The conditions (4.4) imply that $\mathcal{A}_\mu$ is independent from $(b^a(x), d^a(x))$, and that the fields $(\bar{c}^a(x), \Omega^{a\mu}(x))$, $(\bar{\phi}^a(x), \rho^{a\mu}(x))$ enter only through the combinations

$$\begin{aligned} \hat{\Omega}^{a\mu}(x) &= \Omega^{a\mu}(x) + \partial^\mu \bar{c}^a(x) \\ \hat{\rho}^{a\mu}(x) &= \rho^{a\mu}(x) + \partial^\mu \bar{\phi}^a(x) , \end{aligned} \quad (4.8)$$

i.e.

$$\mathcal{A}_\mu = \mathcal{A}_\mu(A, c, B, \hat{\Omega}, L, \hat{\rho}, D) . \quad (4.9)$$

From (4.5) and (4.6) it follows that $\mathcal{A}_\mu$ can be parametrized as :

$$\begin{aligned} \mathcal{A}_\mu = \int d^3x \quad & \left\{ a_1 \widehat{\Omega}_\lambda^a \partial_\mu A^{a\lambda} + a_2 \widehat{\Omega}_\lambda^a \partial^\lambda A_\mu^a + a_3 \widehat{\Omega}^{a\lambda} \partial_\mu B_\lambda^a + a_4 \widehat{\Omega}_\lambda^a \partial^\lambda B_\mu^a + a_5 \widehat{\rho}_\lambda^a \partial_\mu A^{a\lambda} \right. \\ & + a_6 \widehat{\rho}_\lambda^a \partial^\lambda A_\mu^a + a_1 \widehat{\rho}^{a\lambda} \partial_\mu B_\lambda^a + a_7 \widehat{\rho}_\lambda^a \partial^\lambda B_\mu^a + (a_1 + a_2) c^a \partial_\mu L^a \\ & + (a_3 + a_4) \phi^a \partial_\mu L^a + (a_5 + a_6) c^a \partial_\mu D^a + (a_1 + a_7) \phi^a \partial_\mu D^a \\ & + \alpha_1^{a[bc]} \widehat{\Omega}_\lambda^a A_\mu^b A^{c\lambda} + \alpha_2^{a[bc]} A_\mu^a \widehat{\Omega}_\lambda^b B^{c\lambda} - \alpha_2^{a[bc]} A^{a\lambda} \widehat{\Omega}_\lambda^b B_\mu^c \\ & + a_8 f^{abc} \widehat{\Omega}_\lambda^a B_\mu^b B^{c\lambda} - \alpha_1^{a[bc]} B^{a\lambda} A_\mu^b \widehat{\rho}_\lambda^c + \alpha_1^{a[bc]} B_\mu^a A^{b\lambda} \widehat{\rho}_\lambda^c \\ & \left. + a_9 f^{abc} \widehat{\rho}_\lambda^a A_\mu^b A^{c\lambda} + \alpha_2^{a[bc]} \widehat{\rho}_\lambda^a B_\mu^b B^{c\lambda} \right\} , \end{aligned} \quad (4.10)$$

where $a_1, \dots, a_9$ are constant parameters and $\alpha_1^{a[bc]}, \alpha_2^{a[bc]}$ are invariant tensors. It can be verified that $\mathcal{A}_\mu$ can be written as

$$\mathcal{A}_\mu = W_\mu \mathcal{A} , \quad (4.11)$$

with

$$\begin{aligned} \mathcal{A} = \int d^3x \quad & \left\{ -\frac{1}{2} a_5 \varepsilon^{\lambda\sigma\tau} (\partial_\lambda A_\sigma^a) A_\tau^a - a_1 \varepsilon^{\lambda\sigma\tau} (\partial_\lambda A_\sigma^a) B_\tau^a - \frac{1}{2} a_3 \varepsilon^{\lambda\sigma\tau} (\partial_\lambda B_\sigma^a) B_\tau^a \right. \\ & - \frac{1}{6} a_9 f^{abc} \varepsilon^{\lambda\sigma\tau} A_\lambda^a A_\sigma^b A_\tau^c + (a_1 + a_2) \widehat{\Omega}_\lambda^a \partial^\lambda c^a + (a_3 + a_4) \widehat{\Omega}_\lambda^a \partial^\lambda \phi^a \\ & + (a_5 + a_6) \widehat{\rho}_\lambda^a \partial^\lambda c^a + (a_1 + a_7) \widehat{\rho}_\lambda^a \partial^\lambda \phi^a - \frac{1}{2} \alpha_1^{a[bc]} \varepsilon^{\lambda\sigma\tau} A_\lambda^a A_\sigma^b A_\tau^c \\ & \left. + \frac{1}{2} \alpha_2^{a[bc]} \varepsilon^{\lambda\sigma\tau} A_\lambda^a B_\sigma^b B_\tau^c - \frac{1}{6} a_8 f^{abc} \varepsilon^{\lambda\sigma\tau} B_\lambda^a B_\sigma^b B_\tau^c \right\} . \end{aligned} \quad (4.12)$$

Equation (4.12) shows that the breaking $\mathcal{A}_\mu$ can be reabsorbed as a local counterterm in a way which is compatible with the equations (4.1) and (4.2).

This proves that the supersymmetric Ward identity holds to the order considered, hence, by induction, to all perturbative orders.

From now on, we can therefore assume that it is possible to build a quantum functional Γ which satisfies, besides the conditions (4.1) and (4.2), also the supersymmetry identity

$$W_\mu \Gamma = \Delta_\mu^d . \quad (4.13)$$

We shall now discuss the quantum extension of (3.18) :

$$\mathcal{N}^a \Gamma = \Delta^a \cdot \Gamma , \quad (4.14)$$

$$\Delta^a \cdot \Gamma = \Delta^a + O(\hbar \Delta^a) , \quad (4.15)$$

where Δ^a is an integrated local functional with dimension 3 and zero Faddeev-Popov charge.

The equations (4.1), (4.2) and (4.13) imply the following conditions on Δ^a :

$$\begin{aligned} \frac{\delta \Delta^b}{\delta b^a(x)} &= \frac{\delta \Delta^b}{\delta d^a(x)} = 0 \\ \overline{\mathcal{F}}^a(x) \Delta^b &= \overline{\mathcal{G}}^a(x) \Delta^b = 0 \end{aligned} \quad (4.16)$$

and

$$\begin{aligned}\mathcal{F}^a \Delta^b &= \mathcal{G}^a \Delta^b = 0 \\ W_\mu \Delta^b &= 0 \\ \mathcal{N}^a \Delta^b - \mathcal{N}^b \Delta^a &= 0 ,\end{aligned}\tag{4.17}$$

from which it follows that $\Delta^a = 0$, i.e., there is no breaking of the symmetry $\mathcal{N}^a$:

$$\mathcal{N}^a \Gamma = 0.\tag{4.18}$$

It remains only the discussion of the quantum implementation of the Slavnov identity (3.3).

As in the previous cases, let us write a broken Slavnov identity :

$$\begin{aligned}\mathcal{S}(\Gamma) &= \Delta \cdot \Gamma \\ \Delta \cdot \Gamma &= \Delta + O(\hbar \Delta) ,\end{aligned}\tag{4.19}$$

where Δ is an integrated local functional with dimension 3 and Faddeev-Popov charge +1.

From the algebra (3.18) and from the conditions (4.1), (4.2), (4.13) and (4.18), it follows that

$$\begin{aligned}\frac{\delta \Delta}{\delta b^a(x)} &= \frac{\delta \Delta}{\delta d^a(x)} = 0 \\ \overline{\mathcal{F}}^a(x) \Delta &= \overline{\mathcal{G}}^a(x) \Delta = 0 ,\end{aligned}\tag{4.20}$$

$$\begin{aligned}\mathcal{F}^a \Delta &= \mathcal{G}^a \Delta = 0 \\ W_\mu \Delta &= 0 \\ \mathcal{N}^a \Delta &= \mathcal{H}_{rig}^a \Delta = 0 ,\end{aligned}\tag{4.21}$$

and

$$B_\Sigma \Delta = 0 ,\tag{4.22}$$

where B_Σ is the linearized Slavnov operator defined in (3.8).

Again, one easily sees that the conditions (4.20), (4.21) and (4.22) imply that

$$\Delta = 0 ,\tag{4.23}$$

thus proving the quantum implementation of the Slavnov identity and the absence of anomalies for all the symmetries and constraints which define the model.

For what concerns the stability of the theory under radiative corrections, we limit ourselves to the perturbations of the classical action Σ which are integrated local functionals of dimension 3, zero Faddeev-Popov charge and are invariant under rigid transformations of the gauge group.

Using the algebra (3.18), the perturbation $\tilde{\Sigma}$ has to satisfy the stability conditions :

$$\begin{aligned}\frac{\delta}{\delta b^a(x)} \tilde{\Sigma} &= \frac{\delta}{\delta d^a(x)} \tilde{\Sigma} = 0 \\ \overline{\mathcal{F}}^a(x) \tilde{\Sigma} &= \overline{\mathcal{G}}^a(x) \tilde{\Sigma} = 0\end{aligned}\tag{4.24}$$

$$\mathcal{F}^a \tilde{\Sigma} = \mathcal{G}^a \tilde{\Sigma} = 0 \quad (4.25)$$

$$W_\mu \tilde{\Sigma} = 0 \quad (4.26)$$

$$\mathcal{N}^a \tilde{\Sigma} = 0 \quad (4.27)$$

and

$$B_\Sigma \tilde{\Sigma} = 0 . \quad (4.28)$$

Equations (4.24) imply that

$$\tilde{\Sigma} = \tilde{\Sigma}(A, c, B, \phi, \hat{\Omega}, L, \hat{\rho}, D) \quad (4.29)$$

and from (4.25) it follows that

$$\begin{aligned} \tilde{\Sigma}(A, c, B, \phi, \hat{\Omega}, L, \hat{\rho}, D) = & \int d^3x \quad \varepsilon^{\mu\nu\rho} \left\{ p_1 A_\mu^a \partial_\nu A_\rho^a + p_2 f^{abc} A_\mu^a A_\nu^b A_\rho^c + p_3 B_\mu^a \partial_\nu B_\rho^a + p_4 f^{abc} B_\mu^a B_\nu^b B_\rho^c \right. \\ & \left. + p_5 (\partial_\mu A_\nu^a) B_\rho^a + \gamma_1^{[ab]c} A_\mu^a A_\nu^b B_\rho^c + \gamma_2^{a[bc]} A_\mu^a B_\nu^b B_\rho^c \right\} \\ + \int d^3x \quad & \left(p_6 \hat{\Omega}^{a\mu} \partial_\mu c^a + p_7 \hat{\Omega}^{a\mu} \partial_\mu \phi^a + p_8 \hat{\rho}^{a\mu} \partial_\mu c^a + p_9 \hat{\rho}^{a\mu} \partial_\mu \phi^a \right) , \end{aligned} \quad (4.30)$$

where $p_1, \dots, p_9$ are constants and $\gamma_1^{[ab]c}$, $\gamma_2^{a[bc]}$ are invariant tensors. The Slavnov condition (4.28) implies that

$$\begin{aligned} \tilde{\Sigma}(A, c, B, \phi, \hat{\Omega}, L, \hat{\rho}, D) = & \int d^3x \quad \varepsilon^{\mu\nu\rho} \left\{ p_1 A_\mu^a \partial_\nu A_\rho^a + p_2 f^{abc} A_\mu^a A_\nu^b A_\rho^c + p_5 (\partial_\mu A_\nu^a) B_\rho^a + \frac{1}{2} f^{abc} (p_5 + p_6) A_\mu^a A_\nu^b B_\rho^c \right\} \\ + \int d^3x \quad & \left(p_6 \hat{\Omega}^{a\mu} \partial_\mu c^a + 2(3p_2 - p_1) \hat{\rho}^{a\mu} \partial_\mu c^a + p_6 \hat{\rho}^{a\mu} \partial_\mu \phi^a \right) , \end{aligned} \quad (4.31)$$

and, finally, from (4.26) one finds

$$p_1 = p_2 = p_5 = p_6 = 0, \quad (4.32)$$

so that

$$\tilde{\Sigma}(A, c, B, \phi, \hat{\Omega}, L, \hat{\rho}, D) = 0. \quad (4.33)$$

Equation (4.33) tells us that there are no counterterms compatible with the network of constraints (4.24) to (4.28), meaning that the model is finite to all orders of perturbation theory.

The same result can be obtained applying this procedure to other topological models, as the Chern-Simons model [7] and the more complex four-dimensional BF-system [15].

5 Conclusions

Topological models can be characterized by means of a supersymmetric structure whose origin relies on the peculiar properties of the energy-momentum tensor and whose explicit realization turns out to be extremely simple in a Landau-gauge.

Moreover, the properties of the Landau-gauge allow to prove in an algebraic way the perturbative finiteness of these models, i.e. the vanishing of the beta functions and of the anomalous dimensions.

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