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NEW THEORETICAL LOWER BOUNDS ON THE HIGGS SECTOR OF MINIMAL SUSY

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ABSTRACT

We show the existence of a new theoretical one-loop lower bound on the lightest Higgs boson h of the minimal supersymmetric standard model. This bound restricts considerably the parameter space of the model for $\tan \beta < 1$ and allows for the pseudoscalar Higgs field A to be actually lighter than h in some regions of parameter space. The lower bound on the heavy Higgs field H is correspondingly increased. Our numerical results are obtained for the case of degenerate squark masses but can be readily generalized. We conclude that the one-loop analysis favors the embedding of the model in the SUSY GUTs scenario with radiative electroweak symmetry breaking.

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1. Introduction

Low-energy supersymmetry is hoped to exist to alleviate the gauge hierarchy problem of models with very high mass scales. This implies that the masses of the superpartners of the usual Standard Model particles should be at most $\mathcal{O}(1\text{ TeV})$. The Higgs bosons in minimal supersymmetric models offer an ideal testing ground for these ideas since their masses are highly constrained by supersymmetry. This class of models possesses two Higgs doublets which after electroweak symmetry breaking give rise to two scalar (h, H), one pseudoscalar (A), and two charged ($H^\pm$) Higgs bosons. At *tree-level* their masses can be given in terms of just two parameters, which we take to be m_h and the ratio of Higgs vacuum expectation values $\tan \beta = v_2/v_1$. The following mass relations then follow (see *e.g.*, [1])

$$m_A^2 = m_h^2(M_Z^2 - m_h^2)/(M_Z^2 \cos^2 2\beta - m_h^2), \quad (1)$$

$$m_H^2 = M_Z^2 \cos^2 2\beta (M_Z^2 - m_h^2)/(M_Z^2 \cos^2 2\beta - m_h^2), \quad (2)$$

$$m_{H^\pm}^2 = m_A^2 + M_W^2. \quad (3)$$

Examination of these relations immediately yields the following tree-level bounds on the Higgs boson masses

$$m_h \leq |\cos 2\beta| M_Z \leq M_Z \leq m_H, \quad (4)$$

$$m_h \leq m_A |\cos 2\beta| \leq m_A \leq m_H, \quad (5)$$

$$m_{H^\pm} \geq M_W. \quad (6)$$

The above mass relations and bounds have been used extensively to constrain the parameters of the model since the LEP data has become available. In particular, experimental bounds on m_h and m_A currently stand at $m_{h,A} \gtrsim 40\text{ GeV}$ for $\tan \beta > 1$ [2]. Also, since for $\tan \beta$ close to one $m_h^2 \approx 0$, bounds have been placed on $\tan \beta$ as well, excluding the interval $0.5 \lesssim \tan \beta \lesssim 2.0$ [1,3]. Another relevant point is that at tree-level the model possesses a $\tan \beta \leftrightarrow 1/\tan \beta$ symmetry, at least as far as the mass relations are concerned. However, experimentally the region $\tan \beta < 1$ is much more difficult to explore [4]. On the other hand, only $\tan \beta > 1$ is compatible with the usual radiative breaking scenario in SUSY GUTs models (see *e.g.*, [5]).

Furthermore, perturbative unification arguments indicate that $\sin \beta \gtrsim m_t/200$ [1], favoring $\tan \beta > 1$ also.

All the above results apply at tree-level only. Radiative corrections to the Higgs boson masses have been recently computed [6,7,8,9,10,11] and have been found to be very significant (depending strongly on the top quark and squark masses), and basically invalidate the tree-level upper bound on the lightest Higgs boson mass m_h . Earlier calculation of one-loop corrections to the Higgs boson masses had either concentrated on large values of $\tan \beta$ [12] or considered only corrections to the tree-level Higgs mass sum rules [13,14,15], although it was recognized that effects due to large left-right stop mixing [14] or a heavy top quark [15] could be significant.

In this paper we show that one-loop radiative corrections, in addition to relaxing the upper bound on m_h , also give rise to a new *lower* bound which is mostly relevant for $\tan \beta \lesssim 1$, hence breaking the $\tan \beta \leftrightarrow 1/\tan \beta$ symmetry of the mass relations. This lower bound is such that the region $\tan \beta < 1$ becomes quite disfavored theoretically as the allowed range for m_h is rather restricted and even null in some cases. We also show that the “lightest” Higgs boson h can actually be heavier than the pseudoscalar A for non-negligible regions of parameter space. Finally, the tree-level lower bound $m_H \geq M_Z$ is also correspondingly relaxed.

2. Radiative corrections

There have been several calculations of the radiative corrections to the Higgs boson masses of the minimal supersymmetric Standard Model. To this end three methods have been used: minimization of the effective scalar one-loop potential [12,6,7,11], use of renormalization group evolution methods [9], and direct computation of one-loop Higgs self-energy diagrams [13,14,15,8]. Even though these methods are equivalent, not all these calculations have been carried out to the same level of detail. The most complete calculation available to date is given in [11]; we will use their results in what follows. These authors compute the radiative corrections to $m_{h,H,A}$ keeping the top-stop and bottom-sbottom contributions in all generality (*i.e.*, allowing for non-negligible left-right squark mixing). However, they neglect the contribution from the gauge, neutralino, and chargino sectors; these are expected to be relatively small for the large values of the top-quark mass used [13,14,8].

The squared mass matrix for the h and H mass eigenstates is given by [11]

$$\begin{pmatrix} m_A^2 \sin^2 \beta + M_Z^2 \cos^2 \beta + \gamma \hat{\Delta}_{11} M_Z^2 & -(m_A^2 + M_Z^2) \sin \beta \cos \beta + \gamma \hat{\Delta}_{12} M_Z^2 \\ -(m_A^2 + M_Z^2) \sin \beta \cos \beta + \gamma \hat{\Delta}_{12} M_Z^2 & m_A^2 \cos^2 \beta + M_Z^2 \sin^2 \beta + \gamma \hat{\Delta}_{22} M_Z^2 \end{pmatrix}, \quad (7)$$

where m_A is the physical one-loop corrected pseudoscalar mass and

$\gamma \equiv 3\alpha/(4\pi \sin^2 \theta_w \cos^2 \theta_w) = 0.0104$ (using $\sin^2 \theta_w = 0.233$ and $\alpha = 1/127.9$). The quantities $\hat{\Delta}_{11,12,22} \equiv \Delta_{11,12,22}/M_Z^4$ are complicated functions of the $\tilde{t}_{1,2}$ and $\tilde{b}_{1,2}$ squark masses and of the supersymmetry breaking parameters A_t, A_b , and μ [11]. Before dwelling into their specific forms, let us first analyze the squared Higgs mass matrix in search for one-loop bounds on the Higgs masses. In analogy with the tree-level case, we first calculate the lightest eigenvalue of the matrix (identified with h), and then solve the resulting expression for m_A^2 . We obtain

$$m_A^2 = [m_h^2(aM_Z^2 - m_h^2) - kM_Z^4]/[(m_h^{max})^2 - m_h^2], \quad (8)$$

where

$$a = 1 + \gamma(\hat{\Delta}_{11} + \hat{\Delta}_{22}), \quad (9)$$

$$k = \gamma(\hat{\Delta}_{11} \sin^2 \beta + \hat{\Delta}_{22} \cos^2 \beta + 2\hat{\Delta}_{12} \sin \beta \cos \beta) + \gamma^2(\hat{\Delta}_{11}\hat{\Delta}_{22} - \hat{\Delta}_{12}^2), \quad (10)$$

$$\frac{1}{M_Z^2}(m_h^{max})^2 = \cos^2 2\beta + \gamma(\hat{\Delta}_{11} \cos^2 \beta + \hat{\Delta}_{22} \sin^2 \beta + 2\hat{\Delta}_{12} \sin \beta \cos \beta). \quad (11)$$

The consistency condition $m_A^2 \geq 0$ then implies: (i) $m_h \leq m_h^{max}$, and (ii) $m_h^- \leq m_h \leq m_h^+$, where

$$(m_h^\pm)^2 = \frac{1}{2}\{a \pm \sqrt{a^2 - 4k}\}M_Z^2. \quad (12)$$

In an analogous way we can determine that: (i) $m_H \geq m_h^{max}$, and (ii) $m_H \geq m_h^+$. Note that for $\gamma = 0$, $m_h^+ = M_Z$, $m_h^- = 0$, and $m_h^{max} = M_Z|\cos 2\beta|$; thus we recover the tree-level results. For comparison with earlier calculations [13,14,15], we also give the one-loop corrected Higgs boson mass sum rule

$$m_h^2 + m_H^2 = m_A^2 + aM_Z^2, \quad (13)$$

which follows from taking the trace of the (h, H) squared mass matrix in (7).

To proceed we need to specify the quantities in Eqs. (9)–(11). For the upper bound on m_h we obtain

$$\begin{aligned} \frac{1}{M_Z^2}(m_h^{max})^2 = \cos^2 2\beta + \gamma \left(\frac{m_t}{M_Z} \right)^4 & \left\{ \ln \frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{m_t^4} + (m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2) \frac{1}{2} \frac{\sin^2 2\theta_t}{m_t^2} \ln \frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2} \right. \\ & \left. + (m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2 \left(\frac{1}{4} \frac{\sin^2 2\theta_t}{m_t^2} \right)^2 g\left(\frac{m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2}\right) \right\} + (t \rightarrow b), \end{aligned} \quad (14)$$

where $\theta_{t(b)}$ is the angle rotating from the $\tilde{t}_{L,R}(\tilde{b}_{L,R})$ eigenstates to the $\tilde{t}_{1,2}(\tilde{b}_{1,2})$ mass eigenstates ($m_{\tilde{t}_1(\tilde{b}_1)} \leq m_{\tilde{t}_2(\tilde{b}_2)}$), and

$$g(r) = 2 - \frac{r+1}{r-1} \ln r. \quad (15)$$

The above formula has been obtained using the expressions for $\Delta_{11,12,22}$ in [11], but the angles $\theta_{t,b}$ have been introduced to simplify the resulting expression and to clarify the limit of degenerate squark masses. Expressions for a and k in Eqs. (9) and (10) above can be derived similarly, but they do not reduce to a simple form as in Eq. (14), and will not be presented here.

A full analysis of the above one-loop bounds on the Higgs boson masses requires studying a nine-dimensional parameter space (*i.e.*, $m_{\tilde{t}_{1,2}}$, A_t , $m_{\tilde{b}_{1,2}}$, A_b , μ , $\tan \beta$, and m_t). This task can be simplified in SUSY GUTs models where a few high energy soft supersymmetry breaking parameters, together with the requirement of radiative electroweak symmetry breaking, determine the low-energy theory [7]. For simplicity and to gauge the size of the various effects, here we reduce drastically the parameter space of the model by assuming a common squark mass (*i.e.*, $m_{\tilde{t}_1} = m_{\tilde{t}_2} = m_{\tilde{b}_1} = m_{\tilde{b}_2} = m_{\tilde{q}}$). This approximation may not be very realistic, but it may not be that inaccurate either, since in this limit small *e.g.*, $\tilde{t}_{1,2}$ mass splittings contribute a $\mathcal{O}[(m_{\tilde{t}_1}^2 - m_{\tilde{t}_2}^2)^2]$ deviation from the degenerate case result.* In this limit the relevant quantities reduce to

$$a = 1 + \epsilon_t / \sin^2 \beta + \epsilon_b / \cos^2 \beta, \quad (16)$$

* The case of large left-right squark mixing, as it affects the Higgs boson mass sum rules, has been considered in [14,15].

$$k = \epsilon_t / \tan^2 \beta + \epsilon_b \tan^2 \beta + \epsilon_t \epsilon_b / \sin^2 \beta \cos^2 \beta, \quad (17)$$

$$\frac{1}{M_Z^2} (m_h^{max})^2 = \cos^2 2\beta + \epsilon_t + \epsilon_b, \quad (18)$$

where $\epsilon_i \equiv \gamma(m_i/M_Z)^4 \ln(m_{\tilde{q}}^4/m_i^4)$, $i = t, b$. One can readily verify that for all values of the parameters involved (and at least for the case of degenerate squark masses), $m_h^+ \geq m_h^{max}$, and hence the bounds derived above reduce to

$$m_h^{min} \equiv m_h^- \leq m_h \leq m_h^{max} \leq m_h^+ \leq m_H. \quad (19)$$

Since $\epsilon_{t,b} > 0$ (we take $m_{\tilde{q}} > m_t$), $m_h^{max} \geq M_Z |\cos 2\beta|$, and the tree-level upper bound on m_h is relaxed at one-loop. Also, $m_h^+ > M_Z$ and the tree-level lower bound on m_H is strengthened. Furthermore, when $m_h = m_h^{max}$, $m_H, m_A \rightarrow \infty$, and for $m_h = m_h^{min}$, $m_A = 0$ and $m_H = m_h^+$. This last result indicates that the pseudoscalar field A can actually be *lighter* than the h field, which is always the lightest Higgs boson at tree-level. For a given value of $\tan \beta$, m_A is smaller than m_h for the following range of values of m_h

$$(m_h^{min})^2 \leq m_h^2 \leq k M_Z^4 / [a M_Z^2 - (m_h^{max})^2] \approx \epsilon_t M_Z^2 / (4 \sin^4 \beta + \epsilon_t), \quad (20)$$

where in the last relation the small effects of ϵ_b have been neglected. At the lower end of the m_h range in (20) $m_A = 0$, it then grows until it equals m_h at the upper end of the interval.

3. Results

In Figs. 1 and 2 we plot the one-loop bounds on m_h and m_H as they are given by Eq. (19). We do this for $m_t = 120$ GeV and 160 GeV, and $m_{\tilde{q}} = 200$ GeV and 1 TeV. These are the expected ranges for m_t and $m_{\tilde{q}}$ based on one-loop fits to electroweak data [16] and naturalness [17] respectively. We also show the tree-level upper bound on m_h ($m_h^{max, tree}$). The figures clearly show the violation of the tree-level bounds as discussed above. The radiative corrections are enhanced for large m_t ($m_t = 120$ (160) GeV gives $\epsilon_t = 0.064$ (0.089) for $m_{\tilde{q}} = 200$ GeV), although a large value of $m_{\tilde{q}}$ enhances them further ($\epsilon_t = 0.267$ (0.729) for $m_{\tilde{q}} = 1$ TeV). This is because in the unbroken supersymmetry limit ($m_{\tilde{q}} = m_t$) all radiative corrections of this nature vanish. The bottom quark contribution is always small ($\epsilon_b \lesssim 10^{-6}$).

From the figures it is clear that $\tan\beta < 1$ gives the strongest constraints on the parameter space of the model. In fact m_h^{max} (solid line) and m_h^{min} (dashed line) are very close to each other for not so large values of the radiative corrections (*i.e.*, ϵ_t small), thus leaving only a very narrow range of allowed values of m_h for $\tan\beta < 1$. The rather strong one-loop lower bound on m_h (compared with $m_h^{min} = 0$ at tree level) does indeed go to zero for $\epsilon_t = \epsilon_b = 0$ (*i.e.*, $k \rightarrow 0 \Rightarrow m_h^- \rightarrow 0$) and is an indication of how large the top-quark mass and/or the supersymmetry breaking are. For $\tan\beta = 1$ we get $(m_h^{max})^2 \approx \epsilon_t M_Z^2$ and $(m_h^{min})^2 = (m_h^-)^2 = \frac{1}{2}\{1 + \epsilon_t/2 - \sqrt{(1 + \epsilon_t/2)^2 - 4\epsilon_t}\}M_Z^2 \approx \epsilon_t M_Z^2$ for $\epsilon_t \ll 1$. Hence, $\tan\beta = 1$ is basically excluded theoretically for small values of the radiative corrections (see Fig. 1). This one-loop asymmetry constraining m_h is the result of the large m_t/m_b ratio (*i.e.*, $\epsilon_t \gg \epsilon_b$) and thus favors models with $\tan\beta > 1$. This result fits nicely with the embedding of the model in a SUSY GUTs scenario with radiative electroweak symmetry breaking driven by a large top-quark Yukawa coupling (see *e.g.*, [5]). Indeed, a large top-quark Yukawa coupling (λ_t) and a large top quark mass ($m_t = \lambda_t \sin\beta(173 \text{ GeV})$) are only compatible for $\tan\beta \gtrsim 1$ (*e.g.*, for $\lambda_t = 1$, $m_t = 120(160) \text{ GeV}$ requires $\tan\beta = 0.96(2.4)$).

As discussed above, the pseudoscalar field A can be lighter than h at the one-loop level. From Eq. (20) one can verify that for small ϵ_t (Fig. 1), this happens for a range of m_h only slightly above m_h^{min} . However, for large ϵ_t (Fig. 2) this value can be substantive: for $\tan\beta = 2$, $m_A < m_h$ for $m_h \lesssim 28(43) \text{ GeV}$ for $m_t = 120(160) \text{ GeV}$ and $m_{\tilde{q}} = 1 \text{ TeV}$. These results agree with those in Figs. 1a,1b in [11] where m_A is plotted versus m_h for various values of $\tan\beta$ and the choices in our Fig. 2.

The heavy Higgs scalar H is bounded below by m_h^+ and this is shown as the dot-dashed curves in Figs. 1 and 2. For $\tan\beta < 1$ and large radiative corrections (Fig. 2), the lower bound on m_H is pushed to masses above $\approx 125 \text{ GeV}$, thus beyond the range of difficult detectability at LEP. In any rate, there is an absolute lower bound on this mass (which occurs for $\tan\beta \gg 1$) given by $m_H \gtrsim \sqrt{1 + \epsilon_t} M_Z$.

Finally, let us comment of the one-loop corrections to the Higgs boson mass sum rule $m_h^2 + m_H^2 = m_A^2 + aM_Z^2$ (see Eq. (13)). Since $a \approx 1 + \epsilon_t/\sin^2\beta$, for $\tan\beta > 1$ and not too large radiative corrections (*i.e.*, Fig. 1), the one-loop correction is rather small ($\lesssim 10\%$ relative to

the case $a = 1$). (This result agrees with a similar calculation shown in Fig. 7 of Ref. [15].) In general, even though the one-loop corrections to the sum rule could be large (*i.e.*, Fig. 2), their relevance to observable physics is not very apparent. On the other hand, corrections to the allowed ranges of the Higgs boson masses, as presented in this paper, are much more directly applicable in constraining the parameter space of the model. Let us close this section with a comment regarding the early results of Li and Sher [12]. These authors considered $m_t \lesssim 60$ GeV and $\tan \beta \gg 1$ and concluded correctly that the shift in m_h^{max} was $\lesssim 1$ GeV. However, had they also studied the $\tan \beta = 1$ case, they would have obtained a shift in m_h^{max} from zero at tree-level to $m_h^{max} \approx 14$ GeV at one-loop, which is quite substantive, despite the smallness of the value of m_t used.

4. Conclusions

We have shown that the one-loop radiative corrections to the neutral Higgs bosons masses in the minimal supersymmetric extension of the standard model relax the tree-level bounds on m_h and m_H . Furthermore, a new theoretical lower bound on m_h arises which can be quite restrictive for $\tan \beta < 1$ since only a narrow range of h masses is then allowed. In fact, for $\tan \beta = 1$ and not too large radiative corrections, this range is basically null. For $\tan \beta > 1$ the new bound is rather weak and thus these results favor the picture of the model embedded in the SUSY GUTs scenario with radiative electroweak symmetry breaking. Also, the pseudoscalar Higgs field A can be lighter than the “lightest” Higgs field h in a non-negligible region of parameter space. Finally, the lower bound on the heavy Higgs field H is increased at one-loop.

Our numerical results have been derived assuming degenerate squark masses, although our expressions can be readily generalized to the more general case of non-negligible squark splitting. Clearly, the LEP analyses of the supersymmetric Higgs data have to be re-evaluated in the light of the recent one-loop calculations. Our results in this paper should help delineate the one-loop allowed regions of parameter space.

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FIGURE CAPTIONS

- 1) One-loop mass bounds on the Higgs bosons of the minimal supersymmetric standard model for a degenerate squark mass of $m_{\tilde{q}} = 200 \text{ GeV}$ and (a) $m_t = 120 \text{ GeV}$, (b) $m_t = 160 \text{ GeV}$. The value of m_A varies from $m_A = 0$ at the dashed line ($m_h = m_h^{\min}$) to $m_A \rightarrow \infty$ and the solid line ($m_h = m_h^{\max}$).
- 2) Same as Fig. 1 but for $m_{\tilde{q}} = 1 \text{ TeV}$.

Fig. 1

Fig. 2