CERN-TH.6116/91

SPONTANEOUS SYMMETRY BREAKING IN THE RELATIVISTIC MODEL OF ANYON SUPERCONDUCTIVITY

Dmitri Kazakov*

CERN – Geneva

and

Laboratory of Theoretical Physics,
Joint Institute for Nuclear Research, Dubna, USSR[†]

Abstract

We consider the renormalizable abelian Chern-Simons field theory with matter in 2+1 dimensions which is claimed to be an effective model for anyon High- T_c superconductivity. It is shown that a second-order phase transition occurs to a phase with nonzero anomalous v.e.v $\langle \psi\psi \rangle \neq 0$. This phenomenon can be interpreted as a formation of a condensate of Cooper pairs and leads to spontaneous breaking of the gauge invariance accompanied by the appearance of massless fermion excitations.

*E-mail: KAZAKOVD@THEOR.JINRC.DUBNA.SU

[†]Permanent address

1 Introduction

The fact that the high-temperature superconducting materials based on copper oxide planes are, in a reasonable approximation, two-dimensional, suggests an attempt to describe them within 2+1 dimensional models, which have some distinguishing properties. Recently theoretical studies have indicated the existence of a new mechanism for superconductivity in two spatial dimensions. The novel feature of this mechanism is the appearance of anyons, particles with fractional statistics and a violation of the discrete symmetries of time reversal, T , and two-dimensional reflection symmetry, P [1].

An important step towards the understanding of the mechanism of High- T_c superconductivity in the anyon model was the construction of the effective long-range Lagrangian. It is written in terms of the non-linear σ - model for the spin excitations with finite doping incorporated by adding to the Lagrangian the fermionic degrees of freedom describing electrons or holes [2]. Fractional statistics is represented by the Hopf term [3], having a topological nature. It has been shown [4] that the theory of anyon superconductivity can be rewritten in terms of an abelian Chern-Simons gauge model containing the matter fields. The dynamics of the abelian gauge field responsible for the "statistical" interaction is governed by the Chern-Simons term. In the presence of a Chern-Simons field the original bosons and fermions obtain additional phase factors and behave like anyons. This Lagrangian has been the subject of intensive theoretical study, mainly within the mean-field approximation, indicating various attractive features of the anyon model [5].

Recent attempts to go beyond the mean-field approximation in the description of critical properties of the anyon model have been performed within a relativistic field theory approach [6]. The first step in this direction was the construction of a renormalizable effective Lagrangian, which enables one to use the renormalization group technique to study the critical behaviour. It has the form

$$\begin{aligned} \mathcal{L} = & \frac{1}{2}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda + |D_\mu\phi_j|^2 + i\bar{\psi}_j\hat{D}\psi_j - m^2\phi_j^*\phi_j - M\bar{\psi}_j\psi_j - q(\phi_j^*\phi_j)^2 \\ & + \alpha\bar{\psi}_j\psi_j\phi_k^*\phi_k + \beta\bar{\psi}_j\psi_k\phi_j^*\phi_j + \frac{\gamma}{4}[\bar{\psi}_j\psi_k^c\phi_j\phi_k + \bar{\psi}_j^c\psi_k\phi_j^*\phi_k^*] - h(\phi_j^*\phi_j)^3, \end{aligned} \quad (1)$$

where the covariant derivative is, as usual, $D_\mu = \partial_\mu - ieA_\mu$, and we have introduced a global $SU(2)$ symmetry, so that the matter fields are the isospin doublets carrying indices of the fundamental representation of $SU(2)$ ($j, k = 1, 2$). Isospin in relativistic field theory plays the role of a spin in a condensed-matter system. The charge conjugate of a spinor ψ is denoted by ψ^c . In the Majorana basis it is obtained by complex conjugation.

It has been argued in a number of papers [1, 2, 5, 7] that in the anyon model a phase transition could occur which leads to superfluidity and, when a Maxwell electromagnetic field is introduced, to superconductivity. We analyze this possibility in terms of a field-theoretical description of phase transitions based on the renormalization group fixed points.

According to the usual correspondence between field theory models and statistical mechanics [8], the infrared stable fixed points of the renormalization group transformation are associated with second-order phase transitions. In the vicinity of a stable fixed point the Green functions exhibit a power-like behaviour with the powers equal to the proper

anomalous dimensions. The same kind of behaviour is typical of second-order phase transitions where the power-like scaling behaviour is governed by the critical exponents.

The renormalization group study of the model eq.(1) has been performed in refs.[6, 9]. All the fixed points were found and their stability properties were examined. The characteristic feature of the Chern-Simons model is that the gauge coupling e is not renormalized, the β function is identically zero [10]. Hence the coupling e is considered as a free parameter of the model being inversely proportional to the coupling of the Hopf term. This is the origin of the fractional statistics with the phase factor $\theta \propto e^2$. On the other hand, the matter β functions $\beta_\alpha, \beta_\beta, \beta_\gamma$ and β_h are nontrivial and posses a set of the fixed points of the form [9]

$$\alpha^* = \alpha_i e^2, \beta^* = \beta_i e^2, \gamma^* = \gamma_i e^2, h^* = h_i e^4,$$

where $i = 1, \dots, 18$. With respect to the infrared behaviour there are 4 stable fixed points and 14 saddle points. For our purposes we are interested in the stable fixed points which are associated with the second order phase transitions. The saddle points are usually associated with the tricritical points, though the interpretation here is not so clear.

One of the most important problems in the description of a phase transition is to find a proper order parameter. In the field-theoretical approach the order parameter is usually associated with the expectation value of the field or of the products of fields. In our case when we have both scalar and spinor fields with nonzero charge there could be several order parameters. The most natural then is to take the Lorentz invariant products and try to minimize the potential. The different phases will then be distinguished by the values of the order parameters.

2 Minimization of the Potential and Condensates

Consider the tree-level potential corresponding to the Lagrangian eq.(1). It can be rewritten as follows

$$\begin{aligned} V = & m^2 \phi_j^* \phi_j + q(\phi_j^* \phi_j)^2 + h(\phi_j^* \phi_j)^3 + M C^{\alpha\beta} \psi_{\beta j}^* \psi_{\alpha j} - \alpha C^{\alpha\beta} \psi_{\beta j}^* \psi_{\alpha j} \phi_k^* \phi_k \\ & - \beta C^{\alpha\beta} \psi_{\beta j}^* \psi_{\alpha k} \phi_k^* \phi_j - \frac{\gamma}{4} [C^{\alpha\beta} \psi_{\beta j}^* \psi_{\alpha k} \phi_k \phi_j + C^{\alpha\beta} \psi_{\beta j} \psi_{\alpha k} \phi_k^* \phi_j^*], \end{aligned} \quad (2)$$

where we have introduced explicit spinor indices denoted by $\alpha, \beta = 1, 2$. Our conventions follow ref.[11]. The spinor indices can be raised and lowered

$$\psi^\alpha = C^{\alpha\beta} \psi_\beta, \quad \psi^\beta C_{\beta\alpha} = \psi_\alpha$$

with the help of the antisymmetric tensor

$$C_{\alpha\beta} = -C_{\beta\alpha}, \quad C^{\alpha\beta} = -C^{\beta\alpha}, \quad C^{\alpha\beta} C_{\beta\gamma} = -\delta_\gamma^\alpha.$$

In contrast the Dirac γ matrices are symmetric. Thus, for the anticommuting spinors the scalar combination is symmetric

$$\{\psi_\alpha, \chi_\beta\} = 0 \Rightarrow \psi^\alpha \chi_\alpha = \chi^\alpha \psi_\alpha.$$

To find the minimum of the potential we differentiate it with respect to the fields, giving

$$\begin{aligned} \frac{\delta V}{\delta \phi_i} &= m^2 \phi_i^* + 2q(\phi_k^* \phi_k) \phi_i^* + 3h(\phi_k^* \phi_k)^2 \phi_i^* \\ &\quad - \alpha C^{\alpha\beta} \psi_{\beta k}^* \psi_{\alpha k} \phi_i^* - \beta C^{\alpha\beta} \psi_{\beta i}^* \psi_{\alpha k} \phi_k^* - \frac{\gamma}{2} C^{\alpha\beta} \psi_{\beta i}^* \psi_{\alpha k}^* \phi_k = 0, \end{aligned} \quad (3)$$

$$\begin{aligned} \frac{\delta V}{\delta \phi_j^*} &= m^2 \phi_j + 2q(\phi_k^* \phi_k) \phi_j + 3h(\phi_k^* \phi_k)^2 \phi_j \\ &\quad - \alpha C^{\alpha\beta} \psi_{\beta k}^* \psi_{\alpha k} \phi_j - \beta C^{\alpha\beta} \psi_{\beta k}^* \psi_{\alpha j} \phi_k - \frac{\gamma}{2} C^{\alpha\beta} \psi_{\beta j}^* \psi_{\alpha k}^* \phi_k^* = 0, \end{aligned} \quad (4)$$

$$\frac{\delta V}{\delta \psi_{\alpha i}} = -M C^{\alpha\beta} \psi_{\beta i}^* + \alpha C^{\alpha\beta} \psi_{\beta i}^* \phi_k^* \phi_k + \beta C^{\alpha\beta} \psi_{\beta k}^* \phi_k^* \phi_k + \frac{\gamma}{2} C^{\alpha\beta} \psi_{\beta k}^* \phi_i^* \phi_k^* = 0, \quad (5)$$

$$\frac{\delta V}{\delta \psi_{\beta j}^*} = M C^{\alpha\beta} \psi_{\alpha j} - \alpha C^{\alpha\beta} \psi_{\alpha j} \phi_k^* \phi_k - \beta C^{\alpha\beta} \psi_{\alpha k} \phi_k^* \phi_j - \frac{\gamma}{2} C^{\alpha\beta} \psi_{\alpha k}^* \phi_k \phi_j^* = 0, \quad (6)$$

These classical equations being imposed on quantum operators have to be satisfied for the expectation values. Thus, we have to put them between some quantum states. However, if we put them between the vacuum states, we shall get a trivial zero when $\langle 0|\phi|0 \rangle = \langle 0|\psi|0 \rangle = 0$. Looking for nontrivial v.e.v.s preserving the Lorentz invariance we introduce the following bilinear combinations of quantum operators which could have nonzero v.e.v.s minimizing the potential¹

$$\begin{aligned} \langle \phi_i^* \phi_j \rangle &\equiv a_{ij}, & \langle C^{\alpha\beta} \psi_{\beta i}^* \psi_{\alpha j} \rangle &\equiv c_{ij}, & a^+ &= a, & c^+ &= c, \\ \langle \phi_i \phi_j \rangle &\equiv b_{ij}, & \langle C^{\alpha\beta} \psi_{\beta i} \psi_{\alpha j} \rangle &\equiv d_{ij}, & b^T &= b, \\ \langle \phi_i^* \phi_j^* \rangle &\equiv b_{ij}^*, & \langle C^{\alpha\beta} \psi_{\beta i}^* \psi_{\alpha j}^* \rangle &\equiv d_{ij}^*, & d^T &= d. \end{aligned}$$

Multiplying now eqs.(3,4) by ϕ and ϕ^* and eqs.(5,6) by ψ and ψ^* , respectively, and taking the expectation values between the vacuum states, we get the following matrix equations for a, b, c and d :

$$m^2 a + 2q \text{tra } a + 3h (\text{tra})^2 a - \alpha \text{trc } a - \beta c a - \frac{\gamma}{2} d^* b = 0, \quad (7)$$

$$m^2 b + 2q \text{tra } b + 3h (\text{tra})^2 b - \alpha \text{trc } b - \beta c b - \frac{\gamma}{2} a^T d = 0, \quad (8)$$

$$M c - \alpha \text{tra } c - \beta a c - \frac{\gamma}{2} b^* d = 0, \quad (9)$$

$$M d - \alpha \text{tra } d - \beta d a - \frac{\gamma}{2} c^T b = 0 \quad (10)$$

plus the complex conjugated ones.

Eqs.(7-10) have the following solutions:

¹To understand how the product of operators can have nonzero v.e.v when a single operator has zero one, we substitute a complete set of states between the two operators and get

$$\langle 0|\phi\phi|0 \rangle = \langle 0|\phi|n \rangle \langle n|\phi|0 \rangle = \langle 0|\phi|1 \rangle \langle 1|\phi|0 \rangle \neq 0,$$

where $|1 \rangle$ is some state (or states) giving nonzero e.v. of the field. This state does not correspond to a fixed number of fermions or bosons since these numbers are not conserved.

1. $a = b = c = d = 0$,
2. $c = b = d = 0$, $tra = \frac{-q \pm \sqrt{q^2 - 3hm^2}}{3h}$
3. $b = d = 0$, $a = \frac{M}{2\alpha + \beta}I$, $c = \frac{m^2 + 2qtra + 3h(tra)^2}{2\alpha + \beta}I$,
4. $a = \frac{M}{2\alpha + \beta + \gamma/2}I$, $c = \frac{m^2 + 2qtra + 3h(tra)^2}{2\alpha + \beta + \gamma/2}I$, $b = a U$, $d = c U$,

where I is the unit matrix and U is some global unitary symmetrical matrix. Performing an $SU(2)$ rotation of the fields one can diagonalize the matrix U up to some overall phase factor which in its turn can be canceled by the gauge transformation. Hence the vacuum is degenerate with respect to the $SU(2)$ and $U(1)$ and choosing the particular state one can take $U = I$.

All the solutions realize local minima of the potential, which have the universal form

$$V = m^2 tra + q(tra)^2 + h(tra)^3$$

with the values of tra given above.

It should be stressed that the solution # 4, which happens to be the most interesting one, exists only when the coupling $\gamma \neq 0$. It leads to spontaneous violation of fermion number and also gauge invariance and corresponds to condensates of bosonic and fermionic pairs.

3 The Spectrum of Excitations in Spontaneously Broken Phase

Now we consider the expansion around the nontrivial minima and find the spectrum of excitations. In what follows we concentrate on the the minimum # 4 which opens up an interesting possibility to get a phase with a condensate of fermion pairs to be identified with the condensate of Cooper pairs in a superconductor.

To perform an expansion of the potential around a nontrivial minimum one has first to define the OPE for the composite operators having nonzero v.e.v.s. Extracting the classical values we have for the quantum operators

$$\begin{aligned}
\phi_i^* \phi_j &= \langle \phi_i^* \phi_j \rangle + : \phi_i^* \phi_j : \\
\phi_i \phi_j &= \langle \phi_i \phi_j \rangle + : \phi_i \phi_j : \\
\phi_i^* \phi_j^* &= \langle \phi_i^* \phi_j^* \rangle + : \phi_i^* \phi_j^* : \\
C^{\alpha\beta} \psi_{\beta i}^* \psi_{\alpha j} &= \langle C^{\alpha\beta} \psi_{\beta i}^* \psi_{\alpha j} \rangle + : C^{\alpha\beta} \psi_{\beta i}^* \psi_{\alpha j} : \\
C^{\alpha\beta} \psi_{\beta i} \psi_{\alpha j} &= \langle C^{\alpha\beta} \psi_{\beta i} \psi_{\alpha j} \rangle + : C^{\alpha\beta} \psi_{\beta i} \psi_{\alpha j} : \\
C^{\alpha\beta} \psi_{\beta i}^* \psi_{\alpha j}^* &= \langle C^{\alpha\beta} \psi_{\beta i}^* \psi_{\alpha j}^* \rangle + : C^{\alpha\beta} \psi_{\beta i}^* \psi_{\alpha j}^* :
\end{aligned}$$

where the quantum operators are now taken in the normal-ordered form and have zero v.e.v.

Substituting these operator products into the Lagrangian eq.(1) we get

$$\begin{aligned}\mathcal{L} = & \frac{1}{2}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda + |D_\mu\phi_j|^2 + i\bar{\psi}_j\hat{D}\psi_j - \frac{\gamma}{4}\text{trc}\phi_j^*\phi_j + \frac{\gamma}{4}\text{trc}\frac{1}{2}(\phi_j^*\phi_j^* + \phi_j\phi_j) \\ & - \frac{\gamma}{4}\text{tra}\bar{\psi}_j\psi_j + \frac{\gamma}{4}\text{tra}\frac{1}{2}(\psi_j^*\psi_j^* + \psi_j\psi_j) - (q + 3h\text{tra})(\phi_j^*\phi_j)^2 - h(\phi_j^*\phi_j)^3 + e^2\text{tra}A_\mu^2 \\ & + \alpha\bar{\psi}_j\psi_j\phi_k^*\phi_k + \beta\bar{\psi}_j\psi_k\phi_k^*\phi_j + \frac{\gamma}{4}[\bar{\psi}_j\psi_k^c\phi_j\phi_k + \bar{\psi}_j^c\psi_k\phi_j^*\phi_k^*].\end{aligned}\quad (11)$$

To find the spectrum of excitations we need now to diagonalize the quadratic form of the Lagrangian eq.(11). This can be achieved by a Bogoliubov canonical transformation mixing the field with the complex conjugated one. One has

$$\phi_j = \frac{\rho_j + i\tau_j}{\sqrt{2}} \quad \phi_j^* = \frac{\rho_j - i\tau_j}{\sqrt{2}}, \quad (12)$$

$$\psi_j = \frac{\chi_j + i\xi_j}{\sqrt{2}} \quad \psi_j^* = \frac{\chi_j - i\xi_j}{\sqrt{2}}. \quad (13)$$

Substituting eqs.(12,13) into eq.(11) we get for the quadratic part of the Lagrangian

$$\begin{aligned}\mathcal{L}_{free} = & \frac{1}{2}\epsilon^{\mu\nu\lambda}A_\mu\partial_\nu A_\lambda + \frac{1}{2}M_A A_\mu^2 + \frac{1}{2}(\partial_\mu\rho_j)^2 + \frac{1}{2}(\partial_\mu\tau_j)^2 - \frac{m_\rho^2}{2}\rho_j^2 - \frac{m_\tau^2}{2}\tau_j^2 \\ & + \frac{1}{2}\chi_j(i\hat{\partial} - M_\chi)\chi_j + \frac{1}{2}\xi_j(i\hat{\partial} - M_\xi)\xi_j,\end{aligned}\quad (14)$$

where the masses are gauge and $SU(2)$ invariant:

$$M_{\chi_i} = 0, \quad M_{\xi_i} = \frac{M\gamma}{2\alpha + \beta + \gamma/2}, \quad (15)$$

$$m_{\rho_i}^2 = 0, \quad m_{\tau_i}^2 = \frac{\gamma(m^2 + 2q\frac{2M}{2\alpha + \beta + \gamma/2} + 3h\frac{4M^2}{(2\alpha + \beta + \gamma/2)^2})}{2\alpha + \beta + \gamma/2}, \quad (16)$$

$$M_A = \frac{4Me^2}{2\alpha + \beta + \gamma/2}. \quad (17)$$

Eqs.(15-17) define the spectrum of excitations in a spontaneously broken phase. Since the $U(1)$ gauge as well as the global $SU(2)$ symmetry are broken down to the global $U(1)$ by the v.e.v.s, Goldstone bosons appear. These are the fields ρ_i , one of which can be absorbed into the longitudinal component of the massive gauge field A_μ . Hence we are left with one Goldstone boson. Eq.(15) defines the other massless excitations, χ_j , which are fermions.

To find the masses one has to substitute the values of the couplings associated with the fixed point. There exists only one fixed point which is infrared stable and has nonzero value of the coupling γ [9]. One has

$$\alpha^* = 3.0013 e^2, \quad \beta^* = -3.6686 e^2, \quad \gamma^* = 4.8514 e^2, \quad h^* = 0.7379 e^4.$$

Substituting these values in eqs.(15-17) gives ($M \gg m, q$)

$$M_{\xi_i} = 1.0193 M, \quad M_A = 0.8404 M, \quad m_{\tau_i} = 0.6312 M.$$

4 Discussion

Thus, we conclude that in the model described by the effective Lagrangian eq.(1) a second order phase transition occurs to a phase with nonzero condensates of bosonic and fermionic pairs, violating gauge invariance together with fermion number. It happens at some critical temperature, which could be associated with the bare masses. We suggest that

$$M_B^2 \sim m_B^2 \sim T - T_c.$$

Then the correlation length, being inversely proportional to the renormalized mass, goes to ∞ at the transition point. Since the anomalous dimensions of the two masses happen to be equal to each other [9]

$$\gamma_M = \gamma_m = \frac{4}{3}[2\alpha^2 + 2\alpha\beta + \beta^2 + \frac{3}{4}\gamma^2 - 14e^4]\frac{1}{64\pi^2}$$

one has essentially one correlation length, which behaves like

$$\xi \sim (T - T_c)^{-\nu},$$

where

$$\nu = \frac{1}{2(1 + \gamma_m)} \approx \frac{1}{2(1 + 5.08(e^2/4\pi)^2)}.$$

is gauge invariant. When $T \rightarrow T_c$ the masses go to zero and the condensates disappear. However, the present approach does not allow us to find the value of T_c .

We suggest that the condensate of fermion pairs should be associated with that of Cooper pairs in a superconductor with massless fermion excitations playing an important role in the mechanism of superconductivity.

To consider electrically charged particles, such as electrons, one has to introduce the Maxwell gauge field. It can be done in a gauge invariant way by adding the photon field to the covariant derivative together with the usual kinetic term for the gauge field. In three dimensions the electromagnetic interaction is superrenormalizable with the electric charge having dimension of mass. It can probably be measured in units of the thickness of the superconducting planes.

The addition of an electromagnetic interaction does not change the renormalizability properties of the model as well as the renormalization group flow. However, if the phonons (the scalar fields) are also charged, which is needed for gauge invariance of the fermion number violating interaction, it will lead to some problems in the spontaneously broken phase. Indeed, in the case of two abelian gauge fields the gauge mass term in eq.(11) will contain a linear combination of them. However, the usual diagonalization does not work due to the different nature of the gauge fields, i.e. the different form of their kinetic terms. Hence, the electromagnetic interaction should probably be introduced at the level of the effective Lagrangian eq.(14) rather than eq.(1) to avoid the undesirable mixing. This means that only linear combinations of the fields of eq.(12,13) will have definite electric charge.

Acknowledgements

The author is grateful to D.A.Ross and P.K.Mitter for valuable discussions and helpful comments. I would like to thank CERN TH Division, where this work was done, for kind hospitality and financial support.

References

- [1] Y.H.Chen, B.I.Halperin, F.Wilczek and E.Witten, *Int. J.Mod.Phys.* **B3** (1989) 1001; B.I.Halperin, J.March-Russel and F.Wilczek, *Phys.Rev.* **B40** (1989) 8726.
- [2] X.G.Wen and A.Zee, *Nucl.Phys.* **B326** (1989) 619; R.Shankar, *Nucl.Phys.* **B330** (1990) 433.
- [3] F.Wilczek and A.Zee, *Phys.Rev.Lett.* **51** (1983) 2258; Y.S.Wu and A.Zee, *Phys.Lett.* **147B** (1984) 325.
- [4] X.G.Wen and A.Zee, *Phys.Rev.Lett.* **63** (1989) 461.
- [5] T.Banks and J.D.Lykken, *Nucl.Phys.* **B336** (1990) 500; J.D.Lykken, J.Sonnenschein and N.Weiss, *Int.J.Mod.Phys.* **A6** (1991) 1335.
- [6] G.V.Grigoryev and D.I.Kazakov, *Phys.Lett.* **253B** (1991) 411.
- [7] D.V.Khveshchenko and Ya.I.Kogan, *Int.J.Mod.Phys.* **B4** (1990) 631.
- [8] J.Zinn-Justin, "*Quantum Field Theory and Critical Phenomena*", Clarendon Press, Oxford, 1989.
- [9] L.V.Avdeev, G.V.Grigoryev and D.I.Kazakov, *Renormalizations in Abelian Chern-Simons Field Theories with Matter*, CERN-TH.6091/91.
- [10] S.Coleman and B.Hill, *Phys.Lett.* **159B** (1985) 184; G.Semenoff, P.Sodano and Yong-Shi Wu, *Phys.Rev.Lett.* **62** (1989) 715.
- [11] S.J.Gates, M.T.Grisaru, M.Rocek and W.Siegel, "*Superspace*", Benjamin & Cummings, Reading MA, 1983.