Decay Rates and Average Luminosity in a B-Factory

H. Braun, W. Joho , PSI (Paul Scherrer Institut), 5232 Villigen, Switzerland
K. Hübner, CERN, Geneva, Switzerland

ABSTRACT

The different effects contributing to the decay of the electron and positron beam are discussed and the coupled differential equations describing the decay in an asymmetric B-factory are given. The effect of the vacuum pressure rise by gas desorption owing to synchrotron radiation is taken into account. The equations can be solved numerically and the average luminosity can be calculated as function of the running time \( T \) for data taking with the filling time \( F \) as parameter. The proper choice of \( T \) for a given \( F \) can optimize the average luminosity. Examples relevant for a B-factory in the ISR tunnel at CERN (BFI) are given, taking into account the constraints of the LEP injector chain, which is proposed to be used also for this collider.

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Abstract
The different effects contributing to the decay of the electron and positron beam are discussed and the coupled differential equations describing this decay in an asymmetric B-factory are given. The effect of the vacuum pressure rise by gas desorption owing to synchrotron radiation is taken into account. These equations can be solved numerically and the average luminosity can be calculated as function of the running time \( T \) for data taking with the filling time \( F \) as parameter. The proper choice of \( T \) for a given \( F \) can optimize the average luminosity. Examples relevant for a B-factory in the ISR tunnel at CERN are given, taking into account the constraints of the LEP injector chain, which is proposed to be used also for this collider.

I. Introduction
CERN and PSI have investigated the possibility of building a B-Factory in the ISR tunnel (BFI) [1,2]. This collider facility with two separate rings could operate in an asymmetric mode (3.5 GeV \( e^+ \) vs. 8 GeV \( e^- \)). The beam currents decay after a fill due to particle losses. The luminosity is proportional to the product of the intensities in the two beams and has thus an even stronger decay rate. For the experimentalist the key number is the average luminosity \( \langle L \rangle \), which depends on the useful running time \( T \) between two fillings and the filling time \( F \), which cannot be used for physics. A schematic curve for the time dependent luminosity is shown in fig. 1. The preferred filling method is topping up: After each running period \( T \) the circulating beams are supplemented by injecting new particles to bring the luminosity back to its peak value.

For given fill parameters one can optimize the ratio \( \eta \) of average to peak luminosity by an appropriate choice \( T_{opt} \) of the running time \( T \). The optimization of the average luminosity has been treated in several reports [4,5,6,7,8]. In our approach we take into account that the currents and the beam energies can be very different in the two rings. Therefore we have written a computer code named LUMIFILL solving the general case of the beam decays and calculating the average luminosity. We assumed that the beam cross-section at the interaction point would be constant during a physics run.

For the calculations two cases of operation for the main rings were taken into account (see table 1). The first case is the performance of the machine which should be reached fairly early, while the second case corresponds to a machine upgraded for ultimate luminosity. In both cases two interaction points and a circumference of 963 m were assumed. More numerical examples have been worked out for a variety of injector and main ring scenarios and compiled in a report [3].

<table>
<thead>
<tr>
<th>Case</th>
<th>( \mathcal{L} = 10^{23} \text{cm}^{-2}\text{s}^{-1} )</th>
<th>( \mathcal{L} = 10^{24} \text{cm}^{-2}\text{s}^{-1} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( e^+ ) e^-</td>
<td>( e^+ ) e^-</td>
</tr>
<tr>
<td>E [GeV]</td>
<td>3.5</td>
<td>8.0</td>
</tr>
<tr>
<td>I [A]</td>
<td>1.28</td>
<td>0.56</td>
</tr>
</tbody>
</table>

Table 1: Main ring parameters at \( t = 0 \).

II. Beam Decay
The following effects which can lead to beam decay have been considered: Beam-Beam Bremsstrahlung (BBB), which is sometimes also called radiative Bhabha scattering, Beam-Gas Bremsstrahlung (BGB), Quantum life time and Touschek effect [9]. The cross section \( \alpha_{4\pi} \) for particle losses due to BBB \( e^+ + e^- \rightarrow e^+ e^- + \gamma \) was computed with the formula given in [10]. It has the highest cross section of all beam-beam effects. For a B-factory running on the \( \Upsilon(4S) \) resonance and having a momentum acceptance of \( \pm 0.5\% \) it is \( 0.3 \cdot 10^{-24} \text{cm}^2 \). The dependence of the cross section on the bucket height is very weak. The initial beam lifetime due to BBB is given by

\[
\tau_i = \frac{N_i}{n_i \sigma_{4\pi} \mathcal{L}}
\]  

(1)

\( N_i \) is the total number of particles in ring \( i \) and \( n_i \) is the number of interaction points and \( \mathcal{L} \) is the initial luminosity. For the parameters above the numerical values are

\[
\tau_i = 20h \frac{I_i [A] C [\text{km}]}{n_i \mathcal{L} [10^{20} \text{cm}^{-2}\text{s}^{-1}]}
\]  

(2)

\( I_i \) is the beam current in ring \( i \) and \( C \) the circumference. This formula shows that:
1) The lower current beam decays faster.
2) It is advantageous to have a large circumference.
3) Increasing the luminosity by reducing \( \beta^* \) alone reduces the lifetime (\( \beta^* = \beta \)-function at interaction point).

The effect of the residual gas due to beam-gas interaction can be described by three parameters, the static pressure \( P_s \) without beam, the dynamic pressure \( \frac{dP}{dt} \cdot I \) due to gas desorption.

induced by synchrotron radiation and the $k_{vac}$ value, which is the product of total pressure and lifetime. The energy dependence of $\frac{dP}{dt}$ and $k_{vac}$ is neglected, since it is rather weak in the region we considered. We assumed $P_0 = 1\text{ Torr}$, $\frac{dP}{dt} = 1\text{ Torr} \cdot A^{-1}$ and $k_{vac} = 17\text{ Torr} \cdot h$ based on the experience from LEP [11], taking into account the effects of BGB and inelastic scattering. Since BGB and BGB dominate, we restricted our calculations to these effects.

### III. Equations for the beam decay

The beam decay in the two separate rings is given by the two differential equations for $i = 1, 2$:

$$\frac{dN_i}{dt} = \frac{dN_i}{dt} \bigg|_{BBB} + \frac{dN_i}{dt} \bigg|_{BGB}$$

with $N_i$ the number of particles in ring $i$. The decay due to BGB can immediately be derived from the definition of the luminosity

$$\frac{dN_i}{dt} \bigg|_{BBB} = \frac{dN_i}{dt} \bigg|_{BGB} = -n_e \sigma_{\text{el}} L(0) N_i(t) N_j(t) N_1(0) N_2(0)$$

where $n_e$ is the number of interaction points, while the decay due to BGB is given by

$$\frac{dN_i}{dt} \bigg|_{BGB} = \frac{-1}{k_{vac}} \left( \frac{e}{\tau_{\text{rev}}} \frac{dP}{dt} N_i^2 + P_0 N_i \right)$$

with $e$ = elementary electric charge and $\tau_{\text{rev}}$ = revolution time. Substituting for $N_i$ the normalized currents

$$Y_i \equiv \frac{N_i(t)}{N_i(0)} = \frac{I_i(t)}{I_i(0)}$$

in (3) gives together with (4) and (5) the two coupled differential equations

$$-\dot{Y}_1 = A_{12} Y_1 Y_2 + A_{11} Y_1^2 + B_G Y_1$$
$$-\dot{Y}_2 = A_{21} Y_1 Y_2 + A_{12} Y_2^2 + B_G Y_2$$

with

$$A_{12} = \frac{n_e \sigma_{\text{el}} L(0)}{N_1(0)}$$
$$A_{11} = \frac{1}{k_{vac}} \frac{dP}{dt} I_1(0)$$
$$A_{21} = \frac{n_e \sigma_{\text{el}} L(0)}{N_2(0)}$$
$$A_{22} = \frac{1}{k_{vac}} \frac{dP}{dt} I_2(0)$$
$$B_G \equiv \frac{P_0}{k_{vac}}$$

The relative luminosity $l(t)$ is defined as $L(t)/L(0)$ giving

$$l(t) = Y_1(t) \cdot Y_2(t)$$

Hence the ratio $\eta$ of average luminosity to peak luminosity in terms of relative populations is given by

$$\eta(T) = \frac{1}{F+T} \int_0^T Y_1 Y_2 dt$$

An analytic solution of (6) and thereby a closed expression of (8) exists only in the two special cases where either $A_{11} = A_{12} = A_{21} = 0$ (no BGB i.e. perfect vacuum) or $A_{12} = A_{21} = 0$ (no beam decay due to BGB). These solutions are given in [3,4,6]. In all other cases (6) can only be solved by numerical means. This is done in a new Fortran program LUMIFILL with a Runge-Kutta algorithm. Fig. 2 gives an example of $Y_1(t), Y_2(t)$ and $l(t)$. The ratio $\eta$ is also evaluated by this code. The assumed injector performance together with $T$ determines the filling time. The interval $F$ (see fig. 1) is the sum of the latter and the time required to switch on the detector, which was set to 2 min in the examples given later.

![Decay of normalized beam currents and luminosity](image)

**Figure 2**: Decay of normalized beam currents and luminosity of case 2 ($L = 10^{34} \text{cm}^{-2}\text{s}^{-1}$).

### IV. Injector parameters

The LEP injector chain [12] is planned to be used as the BFI injector. It consists of the LEP Injector Linac (LIL) providing either positrons or electrons for the Electron-Positron accumulation ring (EPA). The Proton Synchrotron (PS) and the Super Proton Synchrotron (SPS) form the rest of the injector complex.

The BFI high energy ring will be filled with electrons of $8\text{GeV}$ using the chain LIL-EPA-PS-SPS-PS, while the low energy ring only needs LIL-EPA-PS to bring the positrons to $3.5\text{GeV}$. There are various schemes for the operation of the injection chain, which differ in the number of bunches and the cycling pattern. The most favoured schemes are based on the use of 8 bunches in the PS and SPS.

Also we assume a lepton acceleration interleaved with the proton acceleration, as it is done for LEP, so that the fixed target proton programme is not affected at all by the filling of BFI. Proper scheduling of the fills would avoid any conflict with LEP. The total cycle time is $14.4\text{s}$. If the current lepton cycling time of $1.2\text{s}$ is taken, 4 lepton cycles fit between the proton cycles, but with some changes also 0.6 s are achievable.
leading to 8 lepton cycles per total cycle. More details of the LEP injector chain for BFI can be found in a special note [13]. The present intensity limits are summarized in table 2. The following transfer efficiencies based on LEP experience are used: EPA-PS 80%, PS-SPS 90% and 30% stacking efficiency in the BFI. The corresponding stacking rates of BFI are summarized in table 3. For the 8 GeV electrons the SPS is the bottleneck due to a longitudinal instability. For the positrons the stacking limit would come from the present positron production rate of LIL, determining the EPA stacking rate. An improvement of the LIL performance is possible [14].

V. Results for the BFI collider

Case 1, $L = 10^{33} cm^{-2} s^{-1}$:

With this luminosity long running times are possible. After 2 h we still have 44% of the initial luminosity and the average luminosity is more than 60% (fig. 3). Operation of the injector complex could proceed in the following way: The lepton cycles are left at 1.2 s and the LEP preinjector (=LPI consisting of LIL and EPA) $e^+$-production is improved by a factor of 6.5 in order to have topping up times of less than 15 min.

Case 2, $L = 10^{34} cm^{-2} s^{-1}$:

With this high luminosity only short runs provide a good average luminosity. For example: after 1 h the luminosity decayed already to 27% of its peak value (fig. 2) and the average luminosity dropped to about 40% (fig. 3). To keep the filling times comparable with those in case 1 we assumed that the $e^-$-cycles have to be shortened from 1.2 to 0.6 s and that LPI positron production is improved by a factor of 13.

The effect of various vacuum conditions (better or worse than assumed as nominal on the preceding page) and of other filling schemes can be found in the more detailed account [3]. Fig. 3 shows $\eta(T)$ also for a perfect vacuum when only BBB determines the beam decay.

VI. Conclusions

The CERN injector complex with LIL-EPA-PS-SPS gives acceptable filling rates for the BFI collider rings, provided that LPI is upgraded by an amount which depends on the case considered. The calculations with the computer code LUMIFILL have shown, that for the initial design goal of $L = 10^{33} cm^{-2} s^{-1}$ run times of about 2 h or less give a relative average luminosity $\eta$ of more than 60%. For a 10 times higher luminosity the BBB-effect reduces the useful running times to less than about 1 h in order to keep $\eta$ above 40%.

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References