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Semi-free Field Representation and String Field Theory in 2D - Gravity.

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ABSTRACT

String field theory for 2D gravity coupled to a $c = 1$ matter field is described. We use the free field representation and take into account the semi-boundedness of the Liouville mode in the last step of calculations. We call such a representation a semi-free field representation. It is shown that the Witten string field theory leads to the Das-Jevicki-Polchinski action for the tachyon field.

Starting from string field theory with a semi-free field representation we calculate string amplitudes. These amplitudes contain, instead of energy conservation δ -functions, energy denominators, residues of which coincide with the Veneziano (Shapiro-Virasoro) amplitudes for open (closed) strings and exhibit a very interesting property: the s , t and u poles become the poles with respect to individual external momentum. These poles are related with higher levels excitations living only with fixed momentum.

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1 Introduction

An outstanding problem of string theory is to understand non-perturbative effects. Let us recall in this context that many hopes were placed in string field theory. String field theory is supposed to be background independent and, therefore, to give a framework for discussing non-perturbative effects.

Recently there has been considerable progress in the problem of quantized 2-dimensional gravity coupled to a $c = 1$ matter [1]-[9]. There are two approaches to this problem. In the first approach one deals with a matrix model [1] (in some sense a discretized string field theory [10]) and in the second one with the continuum Liouville theory [2, 3, 4]. Solvable matrix models have also an interesting interpretation as topological field theories [11, 12].

The recent excitement about matrix models stems from the fact that they give an example of the understanding of non-perturbative effects. It is thus of interest to construct a string field theory in the context of matrix models as well as in the context of the continuum Liouville theory. A reformulation of the matrix models in the form of a string field theory was achieved by Das and Jevicki [5] and is discussed in more recent papers [7]. The fermionic representation of this theory has been considered in [8].

The goal of this letter is to present a string field theory associated with the continuum Liouville theory. We will use the free field representation [13]- [17] and the Witten string field vertex [18]. Following the Polchinski suggestion [4] we will consider the integral over the Liouville field ϕ as an integral, which runs along the real ϕ axis. This inclines one to make a restriction on possible values of the zero mode. However, such a restriction destroys the conformal invariance. To deal with conformal invariant theory one can think about target space as a space with a non-trivial metric which preserves the conformal invariance, and for which the domain with positive zero mode is forbidden. In the first approximation one can perform calculations in the flat background and use the zero-mode restriction only in the last step of calculations. We will refer to the free field representation with the zero-mode restriction as the semi-free representation.

We investigate the free string field theory equation $Q\Psi = 0$ and observe the effect of the jumping of the BRST cohomologies. This is an analogue of the discontinuity of the number of degrees of freedom observed in [2]. Then, starting from the string field theory action we derive the tachyon field action, which in the first order up to some typical factors coincides with the action proposed in [4, 5]. String field theory provides a systematic method of calculation for the higher order tachyon interaction.

String field theory also gives a regular method of calculation of amplitudes. The on-shell amplitudes obtained by the semi-free representation for the closed string reproduce those computed by the Polyakov self-tuning conjecture [2]. This amplitude, being the Veneziano (Shapiro-Virasoro) amplitude for open (closed) strings, exhibits a very interesting property due to the specific kinematics: the s, t and u poles become the poles with respect to the individual external momentum. These poles correspond to the on-shell states with energy-momentum (p, ϵ) equal to $p = \pm n/\sqrt{2}$, $\epsilon = -\sqrt{2} \pm (n + 2)/\sqrt{2}$ (for the open string). They

are related with higher level fields and can be associated with a discrete set of states in the matrix model [5, 3].

The paper is organized as follows. We first present the semi-free field representation and the corresponding string field theory. We then derive general formulae for N-point (on)off-shell tachyon amplitudes and discuss in detail the properties of the 4-point on-shell amplitude. The last section contains our conclusions and a discussion of a possible connection with the σ -model approach [19].

2 String field theory

2.1 Semi-free field representation

Due to the pioneering works of Gervais and Neveu [13] and Curtright and Thorn [14] there are indications that the Liouville mode can be tried by the free field with shifted energy-momentum tensor (for a more recent review, see also [15]). In the $D = 1$ case we have the matter field t combined with the Liouville field ϕ in such a way that the total central charge is equal to $c = 26$.

The free field representation becomes more trustworthy after the papers of David [16] and Distler and Kawai [17] in which the critical exponents [20] were reproduced by using the free field representation.

According to the free field hypothesis, the first quantized action in the conformal gauge for $D = 1$ string is given by

$$S = \frac{1}{8\pi} \int d^2\xi \sqrt{\hat{g}} (\hat{g}^{\alpha\beta} G^{\mu\nu} \partial_\alpha X_\mu \partial_\beta X_\nu - i \hat{R} G^{\mu\nu} b_\mu X_\nu + \text{ghosts}). \quad (1)$$

Here $X_\mu = (t, \phi)$, where t is the embedding dimension and ϕ is the Liouville mode², $G_{\mu\nu} = \eta_{\mu\nu} = \text{diag}(++)$; $\hat{g}$ is a background metric on the worldsheet with curvature $\hat{R}$; $b_\mu = (0, -iQ)$. Conformal invariance relates the background charge Q to the central charge c^ϕ of the Liouville theory

$$c^\phi = 1 + 3Q^2.$$

The requirement of vanishing of the central charge for the system of fields $X = (t, \phi)$ and ghosts (b, c) leads to

$$1 + c^\phi = 26, \quad Q = 2\sqrt{2}.$$

The energy-momentum tensor is given by

$$T(z) = T^X(z) + T^{bc}(z), \quad (2)$$

where

$$T^X(z) = -\frac{1}{2} \partial X_\mu \partial X_\mu(z) + \frac{1}{2} i b_\mu \partial^2 X_\mu(z), \quad (3)$$

²Two-dimensional interpretation was intensively used in [4, 21]

$$T^{bc}(z) = -2b\partial c(z) - \partial bc(z) \quad (4)$$

and the expression for the BRST-charge casts into the form

$$Q_{BRST} = \oint \frac{dz}{2\pi i} j(z), \quad j(z) = c(T^X + \frac{1}{2}T^{bc})(z). \quad (5)$$

Let us note that in the action (1) we do not take into account the contribution of the cosmological term

$$S_{c.t} = \frac{\mu}{2\pi} \int d^2\xi \sqrt{\hat{g}} e^{\alpha\phi} \quad (6)$$

Exactly this term provides the suppression of the zero mode φ of the field $\phi(z)$ in the functional integral when $\varphi \rightarrow \infty$ [4]. When $\varphi \rightarrow -\infty$ the terms containing the background charge ensure the convergence of the functional integral. We are going to use a rather rough approximation in which the contribution of the cosmological term is replaced by the requirement that the region of positive φ is forbidden. In other words one assumes a non-trivial metric in 2-dimensional target space, which is flat for $\varphi < -\delta$ and which ensures that the region $\varphi > \delta$ is forbidden. The consistency of such types of metrics with conformal invariance (i.e. the existence of such types of solutions in the two-dimensional σ -model approach [9]) is an interesting and important question.

Therefore in all our further calculations we will use the following prescription

$$\langle \dots \rangle^\varphi = \int_{-\infty}^0 d\varphi \dots e^{Q\varphi} \quad (7)$$

Through half-infinity configuration space we will lose the energy conservation law (that also takes place in the presence of asymptotically non-flat metrics). Such a convention we call the semi-free representation.

2.2 Vertex operators

We shall use the standard string field theory conformal methods [22, 23]. The usual connection between vector states and conformal vertex operators is

$$\Phi \rightarrow |\Phi\rangle = \lim_{z \rightarrow 0} \Phi(z)|0\rangle,$$

where $\Phi(z)$ is a conformal field and $|0\rangle$ is a conformal vacuum $|0\rangle = |0\rangle^X |0\rangle^{bc}$. The string field $|\Phi\rangle$ has the following mode decomposition

$$|\Phi\rangle = \int d^2f [T(f) + A(f)\alpha_{-1} + D(f)\phi_{-1} + B(f)c_0b_{-1} + \text{higher level terms}] e^{if \cdot x} |0\rangle^X |1\rangle^{bc}, \quad (8)$$

where $|1\rangle^{bc} = c_1|0\rangle^{bc}$; $T(f)$, $A(f)$, etc. are the local fields in the energy-momentum representation, $f_\mu = (k, -i\epsilon)$; $x_\mu = (t, \varphi)$, where t and φ are zero modes of conformal fields

$t(z)$ and $\phi(z)$ respectively. $T(f)$ is a "tachyon" field, $F_\mu = (A, D)$ is a (0)-level "vector" field, B is an auxiliary field.

Let us examine the free field equation

$$Q|\Phi\rangle = 0 \quad \text{or} \quad \{Q, \Phi(z)\} = 0. \quad (9)$$

Taking into account the decomposition (8) and the expression (5) for the BRST charge Q , we get the following equation for the tachyon field $T(x)$ ((-1)-level)

$$(f^2 + bf - 2)T(f) = 0, \quad (10)$$

or in the coordinate representation

$$\left(\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial \varphi^2} + 2\sqrt{2}\frac{\partial}{\partial \varphi} + 2\right)T(y) = 0 \quad (11)$$

The field redefinition $T(t, \varphi) = e^{-\sqrt{2}\varphi}\tilde{T}(t, \varphi)$ renders the tachyon equation (11) into

$$\left(\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial \varphi^2}\right)\tilde{T}(t, \varphi) = 0 \quad (12)$$

For the (0)-level vector field $F_\mu = (A, D)$, equation (9) gives

$$\frac{1}{2}(f^2 + b \cdot f)F_\mu - if_\mu B = 0, \quad (13)$$

$$\frac{1}{2}(f + b) \cdot F - iB = 0 \quad (14)$$

Equations (13) and (14) are invariant under the gauge transformations generated by $\delta|\Phi\rangle = Q|\Lambda\rangle$ where

$$|\Lambda\rangle = \int d^2f [\lambda(f)e^{ifx} + \text{higher levels terms}]|0\rangle \quad (15)$$

i.e.

$$\delta\Phi = 0, \quad \delta F = if\lambda, \quad \delta B = \frac{1}{2}(f^2 + b \cdot f)\lambda. \quad (16)$$

Eliminating the auxiliary field B we get the following equation

$$f \cdot (f + b)F_\mu - f_\mu(f + b) \cdot F = 0. \quad (17)$$

The F -field can be made equal to zero by the corresponding gauge transformations (16) excluding the case when one of the following conditions 1) $f_\mu = 0$, 2) $f_\mu + b_\mu = 0$ is satisfied. The first condition gives $k = 0$, $\epsilon = 0$, and the second $k = 0$, $\epsilon = -2\sqrt{2}$.

A similar situation takes place for higher-level fields, all of them live only with some fixed discrete momentum $p = \pm \frac{n}{\sqrt{2}}$ and energies $\epsilon = -\sqrt{2} \pm \frac{1}{\sqrt{2}}(n + 2)$.

2.3 String field theory action

The Witten string field theory action for the open string has the form

$$S = \langle (I\Phi)(\infty) Q \Phi(0) \rangle + \frac{2}{3}g \langle (h\Phi)(z_1)(h\Phi)(z_2)(h\Phi)(z_3) \rangle \quad (18)$$

where I is the map from the inside of a unit disk to the outside, and h is the map from the interaction three-string configuration to the upper half-plane. One can also keep in mind the string field theory in some background. From this expression one can extract the following action for the tachyon

$$S_0(T) = \frac{1}{2} \int d^2 f_1 \int d^2 f_2 T(f_1) T(f_2) (f_2^2 - b \cdot f_2 - 2) \delta(k_1 + k_2) \frac{1}{\epsilon_1 + \epsilon_2 + 2\sqrt{2}} = \quad (19)$$

$$= -\frac{1}{2} \int dt \int_{-\infty}^0 d\varphi e^{2\sqrt{2}\varphi} T(t, \varphi) \left[\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial \varphi^2} + 2\sqrt{2} \frac{\partial}{\partial \varphi} + 2 \right] T(t, \varphi). \quad (20)$$

$$S_{int} = \frac{2}{3}g \prod_{r=1}^3 \left[\int d^2 f_r \exp\left(\frac{1}{2} \ln \frac{4}{3\sqrt{3}} (f_r^2 + b \cdot f_r - 2)\right) T(f_r) \right] \delta\left(\sum_r k_r\right) \frac{1}{\sum_r \epsilon_r + 2\sqrt{2}} = \quad (21)$$

$$= \frac{2}{3}g \int dt \int_{-\infty}^0 d\varphi e^{2\sqrt{2}\varphi} \prod_{r=1}^3 \left[\exp\left(-\frac{1}{2} \ln \frac{4}{3\sqrt{3}} \left(\frac{\partial^2}{\partial t^2} + \frac{\partial^2}{\partial \varphi^2} + 2\sqrt{2} \frac{\partial}{\partial \varphi} + 2\right)\right) T(t, \varphi) \right]. \quad (22)$$

The free part coincides with the action of Ref.[4, 5] and the interaction part contains the additional exponential factors. If one deals with the closed string, then it is necessary to take into account the non-polynomial string field action [24], which gives a non-polynomial interaction for the tachyon field. The string field action (18) contains also the interaction between tachyon and higher level fields. Since the quadratic term in (18) does not contain the terms mixing the different levels, there are no corrections to the T^3 term coming from the interaction between tachyon and other fields. Contributions to higher power terms $T^4, \dots$ can be calculated using off-shell conformal methods or level truncation approximation [25].

There is no doubt that boundary effects are important. The discussion of boundary effects and the applicability and limitations of the free field theory can be found in the recent Polyakov paper [2]. According to the Polyakov conjecture the true action has the form

$$S = \int_{-\infty}^0 d\varphi \int_{-\infty}^{\infty} dt \mathcal{L}[T(t, \varphi)] + \int_{-\infty}^{+\infty} dt \mathcal{L}_B[T(t, 0), \frac{\partial T}{\partial \varphi}|_{\varphi=0}, \dots] \quad (23)$$

The free field representation can reproduce in principle the first term; as to the second one, its appearance in the string field theory language seems to be connected with a non-trivial background in the target space and is necessary to provide the hermiticity of the full action. The hermiticity is not only a mathematical trinket, but it is a necessary attribute of the theory which is supposed to satisfy the unitarity conditions.

3 N -tachyon amplitudes

We are going to calculate the N -tachyon off-shell string amplitude for the theory (18) following off-shell conformal theory methods developed in [22, 23]. Let us sketch the main steps of this calculation. The amplitudes are computed using perturbation theory. To invert the propagator we must fix a gauge. A convenient choice is the Siegel gauge $b_0\Phi = 0$, where b_0 is the zero mode of $b(z)$. The propagator in this gauge is $\frac{b_0}{L} = b_0 \int_0^\infty d\tau e^{-\tau L}$, where L is the zero-mode Virasoro generator. The effect of $e^{-\tau L}$ is to introduce a world-sheet strip of width π and length τ . To each Feynman graph is associated a string configuration. External strings are semi-infinite rectangular strips of width π . The i -th string propagator is a strip of length τ_i and width π . The interaction glues the strip ends together in a pairwise manner. The contribution to a particular Feynman graph is

$$A = \left(\prod_{i=1}^{N-3} \int_0^\infty d\tau_i \right) \langle O_1(w_1) O_2(w_2) \oint dw'_1 b(w'_1) \dots O_N(w_N) \rangle_{R_\tau}, \quad (24)$$

where the correlation function is considered on the string configuration R_τ described above. The off-shell tachyon states are represented by the operators

$$O(z) = \int d^2 f T(f) c e^{i f X}(z) \quad (25)$$

Substituting (25) in (24) we get the N -tachyon off-shell amplitude

$$A = \left(\prod_{r=1}^n \int d^2 f_r T_r(f_r) \right) G(\{f_r\}), \quad (26)$$

where

$$G(\{f_r\}) = \left(\prod_{i=1}^{N-3} \int_0^\infty d\tau_i \right) \langle c e^{i f_1 X}(w_1) c e^{i f_2 X}(w_2) \oint dw'_1 b(w'_1) \dots c e^{i f_N X}(w_N) \rangle_{R_\tau}. \quad (27)$$

Using the calculation methods [23] we write A in the following form

$$G(\{f_r\}) = (z_1 - z_2)(z_1 - z_N)(z_2 - z_N) \left(\prod_{i=3}^{N-1} \int dz_i \right) \left(\prod_{r < s} (z_r - z_s)^{f_r f_s} \right) \\ \left(\prod_{r=1}^N \exp \left[\frac{1}{2} N_{00}^{rr} (f_r^2 + b \cdot f_r - 2) \right] \right) \delta \left(\sum_r k_r \right) \left(\sum_r \epsilon_r + 2\sqrt{2} \right)^{-1}. \quad (28)$$

Here z_r are the asymptotic positions (on the real axis) of the N external states; N_{00}^{rr} are zero-zero components of the Neumann function. Note that in eq. (28) we lose the energy conservation law (in contrast to the momentum conservation). The amplitude G can be represented as

$$G = \frac{\tilde{A}_1}{\sum_r \epsilon_r + 2\sqrt{2}} + A_2, \quad (29)$$

where for $\tilde{A}_1$ the "energy conservation" takes place. In the case when external energy-momenta are on-shell, the residue $\tilde{A}_1$ is nothing but the Koba-Nielsen amplitude

$$\tilde{A}_1(\{f_r\}) = \left(\prod_{i=3}^{N-1} \int dz_i \right) (z_1 - z_2)(z_1 - z_N)(z_2 - z_N) \prod_{r < s} (z_r - z_s)^{f_r f_s} \delta\left(\sum_r k_r\right), \quad (30)$$

where $\sum_r \epsilon_r = -2\sqrt{2}$.

Let us consider the particular example when $N = 4$. In this case we choose $z_1 = \infty$, $z_2 = 1$, $z_3 = x$, $z_4 = 0$. The on-shell condition and energy-momentum conservation have the form $f_r^2 + b \cdot f_r - 2 = 0$, $\sum_r f_r + b = 0$. The mass-shell conditions give

$$f_1 \cdot f_3 = f_2 \cdot f_4, \quad f_1 \cdot f_2 = f_3 \cdot f_4, \quad f_1 \cdot f_2 + f_2 \cdot f_4 + f_1 \cdot f_4 = -2, \quad (31)$$

and the s-channel 4-tachyon on-shell amplitude can be written as

$$A_1^{4T} = \int_0^1 dx x^{f_1 \cdot f_2} (1-x)^{f_2 \cdot f_3} = \frac{\Gamma(f_1 \cdot f_2 + 1) \Gamma(f_2 f_3 + 1)}{\Gamma(f_1 \cdot f_2 + f_2 \cdot f_3 + 2)}, \quad (32)$$

This amplitude being written in s, t variables,

$$s = (f_1 + f_2)^2 + b \cdot (f_1 + f_2), \quad t = (f_2 + f_3)^2 + b \cdot (f_2 + f_3), \quad (33)$$

is the usual Veneziano amplitude

$$A_1^{4T} = \frac{\Gamma(\frac{1}{2}s - 1) \Gamma(\frac{1}{2}t - 1)}{\Gamma(\frac{1}{2}s + \frac{1}{2}t - 2)}. \quad (34)$$

Due to the special kinematical relations this amplitude can be presented as a function of the individual external momentum. Indeed, mass-shell conditions for external states ,

$$\epsilon_i = -\sqrt{2} + e_i k_i, \quad i = 1, 2, 3, 4; \quad e_i = \pm 1, \quad (35)$$

and energy-momentum conservation in the particular case $e_i = 1$, $i = 1, 2, 3$, $e_4 = -1$, give

$$k_4 = -\sqrt{2}, \quad k_1 + k_2 + k_3 = \sqrt{2}$$

and

$$s = 4 - 2\sqrt{2}k_3, \quad t = 4 - 2\sqrt{2}k_1$$

and therefore

$$A_1^{4T} = \frac{\Gamma(1 - \sqrt{2}k_1) \Gamma(1 - \sqrt{2}k_3)}{\Gamma(2\sqrt{2}k_2)} \quad (36)$$

This amplitude has the s -poles, which correspond to the state with the fixed discrete momentum and energy

$$p = \pm \frac{n}{\sqrt{2}}, \quad \epsilon = -\sqrt{2} \pm \frac{1}{\sqrt{2}}(n + 2) \quad (37)$$

The $n = 0$ corresponds to the (0)-level excitation living only with zero momentum.

It is interesting to note that the 4-point amplitude corresponding to $e_i = 1$, $i = 1, 2$; $e_j = -1$, $j = 3, 4$ has only one pole in the t -channel,

$$A_1^{4T(++--)} \sim \frac{2}{t-2}, \quad (38)$$

corresponding to the tachyon, which can have an arbitrary momentum.

The "vector particles" (0-level excitations) on-shell amplitudes are trivial since s , t and u variables for them are fixed, i. e. $s = t = u = 4$. The 2-"vector"-2-tachyon scattering amplitudes also have only tachyon poles. Indeed, up to the kinematical factor the amplitudes look like

$$\begin{aligned} A_1^{2T2V(++++)} &\sim A_1^{2T2V(--++)} \sim \frac{1}{s-2}, \\ A_1^{2T2V(+--+)} &\sim A_1^{2T2V(-+--)} \sim A_1^{2T2V(----)} \sim 1 \end{aligned}$$

and other types of amplitudes are forbidden by the energy-momentum conservation. In a similar way, starting from a closed string with string field theory action [24] one can obtain the N -point tachyon amplitudes. For example, for $N = 4$

$$A_{1,(cl)}^{4T(+++-)} = \int d^2z |z|^{2f_1 \cdot f_2} |1-z|^{2f_2 \cdot f_3} = \frac{\Gamma(\frac{1}{2}s-1)\Gamma(\frac{1}{2}t-1)\Gamma(\frac{1}{2}u-1)}{\Gamma(\frac{1}{2}s+\frac{1}{2}t-2)\Gamma(\frac{1}{2}s+\frac{1}{2}u-2)\Gamma(\frac{1}{2}t+\frac{1}{2}u-2)}, \quad (39)$$

which also due to the special kinematics

$$s = 1 - \sqrt{2}k_3, \quad t = 1 - \sqrt{2}k_1, \quad u = 1 - \sqrt{2}k_2$$

contains Γ functions of the individual external momentum [2].

4 Discussion

In this paper Witten's approach to the string field theory was used for the $D=1$ open string theory and it was shown that it reduces in the first approximation to the Das-Jevicki-Polchinski action for the tachyon field. The jumping of the BRST cohomologies was observed and the scattering amplitudes were obtained.

2D gravity coupled to a scalar field can be regarded as the critical string in a 2D target space and corresponding σ -model methods can be applied [9]. Target space equations of motion have non-trivial solutions, one of which was identified as a black hole [26]. This means that string field theory should be considered in a non-trivial background.

The statement that all fields corresponding to higher excitations can be removed on-shell is true only in the special background (flat space and linear dilaton field for closed strings). In more complicated background the situation is more subtle. As is noted by Mandal, Sengupta

and Wadia [19], presumably the non-perturbative regime of the matrix model corresponds to theories more complicated than with flat space and a linear dilaton.

Let us note that the expression (18), being considered in an arbitrary background, presents the interaction of all possible degrees of freedom and one should extract from this expression degrees of freedom suitable for describing some particular background in an appropriate regime. In this sense the formula (18) gives a universal action as can be suspected from string field theory. According to the above mentioned hypothesis, the boundary terms call upon one to provide the hermiticity in non-trivial backgrounds and are background dependent. This hypothesis looks rather attractive and must be tried out very carefully. $D = 1$ string theory gives us the much desired proving ground for testing the capacity of the string field theory. In this context it would be interesting to consider a generalization to superstring models. There is encouraging formulation of non-critical superstrings in terms of Super-Liouville theory [27, 28] as well as an explicit form for the superstring field theory action [29, 30] similar to (18). This will be the subject of future investigations.

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