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ON THE EXTRACTION OF $\alpha_s(M_{Z^0})$ FROM LEP JET DATA

by

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Abstract

We propose a method for summing the dominant contributions to the perturbative higher-order effects on jet shape distributions in e^+e^- annihilation which are due to exponentiation of the leading infrared divergences. We find improved results for the strong interaction coupling constant extracted from various event shape variables by the different groups at LEP, which are in better agreement with each other and with values obtained from other experiments.

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Since it offers large numbers of relatively clean high energy events, LEP currently offers the most precise arena for testing the Standard Model and measuring its parameters. As well as the electroweak sector, this is the case for tests of QCD and measuring the strong coupling constant $\alpha_s(M_{Z^0})$. The latter can in principle be measured via the ratio of hadronic and leptonic final states in Z^0 decay. However, so far the best LEP determinations of $\alpha_s(M_{Z^0})$ are those obtained from measurements of final state event shapes. The interpretation of these measurements is largely based on the cross-section for 3-jet final states which has been evaluated through one loop level [1, 2], and on the tree-level cross-section for 4-jet final states [1, 3]. Certain higher-order collinear singularities have also been resummed to estimate better the 2-jet cross section [4].

It is well known that the coefficient of the $\mathcal{O}(\alpha_s^2)$ one-loop correction to the 3-jet cross section is relatively large. This has created considerable uncertainty in the interpretation of the LEP data [5], and led to calls for the two loop $\mathcal{O}(\alpha_s^3)$ contribution to the 3-jet cross section and the one loop contribution to the 4-jet cross section to be calculated. While clearly important, these calculations are technically very challenging, and one should be convinced before embarking on them that all the useful information obtainable from existing perturbative QCD calculations has been fully used, and that the non-perturbative hadronization uncertainties are controlled sufficiently to make further calculations worthwhile. This letter is directed towards the more efficient exploitation of our present knowledge of perturbative QCD.

Our interest in this question has been stimulated by the ongoing attempts [6, 7, 8] to use precision LEP data to test proposed extensions of the Standard Model such as Grand Unified Theories (GUTs). Precise knowledge of $\alpha_s(M_{Z^0})$ is a key input into these tests, which disfavour minimal non-supersymmetric GUTs [6, 7, 8]. The discussion now centres around different types of supersymmetric GUTs [7], and a more precise measurement of $\alpha_s(M_{Z^0})$ would be an invaluable contribution to this debate. Although motivated by this question, we do not enter into it here, but limit ourselves to arguing that an important class of large higher order perturbative QCD effects can be resummed to give a more reliable (and lower) value of $\alpha_s(M_{Z^0})$ than would be obtained from the one-loop 3-jet cross-section alone.

We argue below that although the $\mathcal{O}(\alpha_s^2)$ corrections to jet cross sections are very large, the dominant part of these corrections is a term proportional to π^2 which is associated with the leading infrared divergence, and consequently can be summed by exponentiation. We claim that this accounts for most of the as yet uncalculated higher-order terms, and justify this by demonstrating that at the order calculated so far these π^2 terms dominate the corrections over a large range of most event shape parameters. With the perturbative

higher-order corrections now under better control the theoretical error on the measurement of $\alpha_s(M_{Z^0})$ is of the same order as the uncertainties in the hadronization procedure. The lower values of $\alpha_s(M_{Z^0})$ obtained by this exponentiation are in better agreement with the values obtained in other experiments.

The virtual gluon correction to the amplitude for the production of three partons in e^+e^- annihilation contains a double infrared divergence which occurs when the virtual gluon is both soft and collinear to one of the final state partons. Using dimensional regularization (in $4 - 2\epsilon$ dimensions) to expose these divergences, the amplitude may be written

$$\mathcal{A}_{3-jet}^{1loop} = \mathcal{A}_{3-jet}^0 \left[\frac{\alpha_s}{\pi} \left(N_c - \frac{1}{2N_c} \right) \frac{1}{2\epsilon^2} + \mathcal{O}\left(\frac{1}{\epsilon}\right) + \text{regular terms} \right] \quad (1)$$

where $N_c (= 3)$ is the number of colours. For a time-like external squared momentum s , the double pole is always accompanied by a factor $(-s)^{-\epsilon}$, and it is this factor which when multiplied by the pole term gives rise to the imaginary part of the amplitude. These leading infrared divergences are known to exponentiate [11]. Thus we can write the 3-jet amplitude as

$$\mathcal{A}_{3-jet} = \mathcal{A}_{3-jet}^0 \exp \left(\frac{\alpha_s}{\pi} \left(N_c - \frac{1}{2N_c} \right) \frac{1}{2\epsilon^2} \left(\frac{1}{-s} \right)^\epsilon \right) \left[1 + \sum_{n=1}^{\infty} d_n \left(\frac{\alpha_s}{\pi} \right)^n \right] \quad (2)$$

where the coefficients d_n contain subleading infrared divergences up to $\left(\frac{1}{\epsilon}\right)^{(2n-1)}$. The cross-section is therefore proportional to

$$|\mathcal{A}_{3-jet}|^2 = |\mathcal{A}_{3-jet}^0|^2 \exp \left\{ \frac{\alpha_s}{\pi} \left(N_c - \frac{1}{2N_c} \right) \left(\frac{1}{\epsilon^2} + \pi^2 + \ln^2 s \right) \right\} \left| \left[1 + \sum_{n=1}^{\infty} d_n \left(\frac{\alpha_s}{\pi} \right)^n \right] \right|^2 \quad (3)$$

The soft and collinear infrared divergences cancel against similar terms in cross sections with more than three final-state partons. To the order that has been calculated so far, this means the four-parton cross-section. Here the infrared divergences arise in the integration of the phase space of one of the four partons when it becomes soft or collinear (or both) with one of the other partons. In this case the double pole is accompanied by a factor $s^{-\epsilon}$. Thus we see that when the pole terms are cancelled we are left with a remnant “ π^2 ” correction, $\alpha_s/\pi (N_c - 1/2N_c) \pi^2/2$, which is large even with the values of $\alpha_s(M_{Z^0})$ found at LEP, namely over 50%. This means that unless we incorporate the exponentiation of this large π^2 term the perturbation expansion is unreliable. This has been done in the case of the Drell-Yan cross-section [9], and it was first suggested that this could be applied to other processes in ref.[10].

There are some event shape parameters for which the higher-order corrections appear to be somewhat smaller than this. However, this is due to an accidental partial cancellation

between these large terms associated with the infrared divergences and other correction terms that have a different origin. Thus an event shape distribution for which the higher-order corrections appear to be modest is in fact doubly unreliable, since we cannot rely on the persistence of this accidental cancellation in higher orders of perturbation theory. We suggest here that one can minimize the inevitable uncertainties by relying rather on distributions for which the higher-order corrections are dominated by the π^2 terms which are known to exponentiate [11]

We realize that without conducting a complete next order calculation one cannot of course be sure how large the other higher-order corrections are. Nevertheless, we think that a very good indication is given by examining the extent to which the exponentiating π^2 corrections dominate the $\mathcal{O}(\alpha_s^2)$ corrections. To this end we define the differential jet cross-section for a generic event shape parameter x by

$$\frac{d\Sigma}{dx} = \frac{\alpha_s}{\pi} A^0(x) \left[1 + \frac{\alpha_s N_c}{\pi} \left(\frac{\pi^2}{2} \left(1 - \frac{1}{2N_c^2} \right) + \eta(x) \right) \right] \quad (4)$$

The leading order differential cross-section and higher order corrections for various different event shape parameters have been obtained from ref.[2], and the different values of η are tabulated in the Table.[†] As can be seen for most of the variables and over most of the ranges shown, η turns out to be a number whose magnitude is smaller than or of the order of 1. The one obvious exception is oblateness for which the higher-order corrections turn out to be negative, reflecting the presence of a huge negative correction not associated with leading infrared divergences, which dominates the π^2 correction. It can also be seen from the Table that the quantity η acquires an increasing negative contribution as one goes to the 2-jet region. This is to be expected, since it is known that the higher order corrections to the total hadronic width do *not* have the large π^2 corrections discussed here, and this is because of a divergent cancelling correction in the two-jet region. These corrections have been resummed in ref.[4]. Further large corrections can be seen in the thrust and C distributions near the edge of the range available for the three-parton cross-section. Here the differential cross-section is dominated by the four-parton cross section, which has only been calculated at tree level.

Our recommendation is that the value of $\alpha_s(M_{Z^0})$ should be extracted from the region of the event shape parameters away from both the two-jet and four-jet regions. The smallness

[†]In the case of energy-energy correlations (EEC) there is some disagreement between the different calculations of the higher-order corrections. Here we have taken the results quoted in ref.[2]. The values of η for the asymmetry in energy-energy correlations (AEEC) [12] can be deduced from the Table and give amplified values between -2.2 and -3.6.

of η over the middle range for all event shape parameters, with the exception of oblateness[†], constitutes *prima facie* evidence that, after taking into account the above-mentioned exponentiation, the higher-order corrections should be under control, and we would expect the next order non-infrared-divergent perturbative correction to be of order $(\alpha_s N_c/\pi)^2$, i.e. around 1%. On the other hand since only the leading infrared divergences (and hence leading π^2) exponentiate there is a genuine ambiguity of order $\alpha_s^2 N_c^2 \eta/2$ which is of order 5%. We consider this to be the largest theoretical uncertainty. Within this uncertainty, our improved estimate of $\alpha_s(M_{Z^0})$ is given by the solution to the equation

$$\alpha_s^{old}(M_{Z^0}) = \alpha_s^{new}(M_{Z^0}) \frac{\exp \left\{ \frac{17\pi\alpha_s^{new}(M_{Z^0})}{12} \right\}}{\left(1 + \frac{17\pi\alpha_s^{new}(M_{Z^0})}{12} \right)} \quad (5)$$

At this stage we mention that some of the LEP groups [13, 14] have attempted to absorb the higher-order correction by extracting the value of α_s at some scale μ which is much smaller than M_{Z^0} . This scale is determined from the optimisation of their fits to various quantities. One could expect to find such a low value of μ if one includes in the fitted region ranges of parameters available only to the four-parton cross-section. This has only been calculated to leading order, and is expected to acquire substantial positive one-loop corrections as is the case for the three-parton cross-section. Hence a larger effective value of the coupling constant would be required in order to fit data using the tree level expression, and consequently a small value of μ . The two-loop renormalisation group is then used to calculate the value of α_s at M_{Z^0} . This is an inconsistent procedure as long as only the first non-leading contributions to the jet cross-sections are known perturbatively, and it is this that leads to their reported scale dependence. As a subsidiary argument, we note that whereas one cannot state for sure that M_{Z^0} is precisely the scale at which α_s should be extracted, renormalisation group arguments tell us that it should be of order M_{Z^0} , whereas refs.[13, 14] favour values of μ as low as 5 GeV. A far better and more systematic way to proceed is to use the exponentiated form for the large perturbative corrections, avoid the four-jet region, and extract α_s at M_{Z^0} .

If we accept that the perturbative corrections are indeed under control, then the theoretical uncertainties in the determination of $\alpha_s(M_{Z^0})$ are of the same order as those arising from the hadronization. Although considerably less important than at PETRA energies, these effects are still significant, leading to a reduction of the value of $\alpha_s(M_{Z^0})$ by about 6%. There is good agreement between the various different hadronization procedures used. Nevertheless, most LEP groups quote an error of around 3-4% from uncertainties in the

[†]We also do not consider AEEC which has large values of η as mentioned above.

hadronization effect, with $\aleph$ quoting more conservative error estimates of 7-8%, except in the case of the y_3 distribution for which they claim a lower error of around 2% [13]. The differential cross-sections calculated to order α_s^2 in ref.[1] cannot be used directly in a hadronization programme. They must first be modified in the way proposed in ref.[16]. The part of the four-parton differential cross-section for which the invariant mass-squared of any pair of partons is less than a fraction y of the centre of mass energy squared, s , is included in the three-parton cross section. The three-parton and four-parton cross-sections are then folded with corresponding hadronization algorithms, which determine the probability of a given parton configuration fragmenting into a given hadron jet configuration. The difficulty here is that, for small values of the cut y , terms proportional to $\ln^2 y$ appear (with equal coefficients and opposite sign) in the three-parton and four-parton cross sections, defined in the manner described in ref.[16]. In principle this should not matter, but, since these three- and four-parton cross-sections are folded with different hadronization algorithms, some sensitivity to the value of y can arise. It is important to note that the origin of these double logarithms is precisely the same as the large corrections discussed above, namely they accompany the leading infrared divergence. Following the arguments set out above, it is likely that these leading logarithms in y can also be exponentiated. The result of such an exponentiation in the three-parton cross-section regularizes the behaviour of the cross-section as $y \rightarrow 0$, and as such is likely to have the effect of reducing the sensitivity to the y -cut and thus reducing the hadronization uncertainties. [†]

In the last two columns of the Table we have reproduced the values of $\alpha_s(M_{Z^0})$ extracted from the various event shape parameters from different LEP groups. We have extracted from refs.[13, 14, 17, 18] the values obtained when the scale μ is chosen equal to M_{Z^0} , and the errors quoted refer to an estimate of the systematic hadronization error alone. In the rightmost column of the Table we give our corrected estimate of $\alpha_s(M_{Z^0})$ after taking into account the exponentiation of the π^2 higher order corrections. We notice that the foremost effect of this is to reduce the value by around 10%. It also has the effect of improving further the agreement between the various groups and helping to reconcile the estimates obtained from different event shape parameters. Excluding the value extracted from oblateness, we see that all of these values are consistent with the average value of

$$\alpha_s(M_{Z^0}) = .118 \quad (6)$$

on which we estimate a hadronization error of $\pm .004$, and a maximum theoretical error of $\pm .005$.

[†]It has been pointed out [19] that in higher orders this algorithm for jet clustering gives rise to spurious double logarithms in y which spoil the exponentiation, and an alternative algorithm has been proposed [20] for which the exponentiation does indeed work.

This value is in better agreement with the value obtained from deep inelastic scattering [21]:

$$\alpha_s(M_{Z^0}) = .109 \pm .007 \quad (7)$$

Since this determination comes from space-like momenta, there are no troublesome π^2 terms associated with the infrared divergences, and hence we expect no large higher order corrections.

Both of the values (6,7) are moreover consistent with the value of $\alpha_s(m_b)$ determined from Υ decay [22], which corresponds to

$$\alpha_s(M_{Z^0}) = .109 \pm .005 \quad (8)$$

with a systematic error that is difficult to estimate. Here again there are no exponentiating π^2 terms associated with the double infrared logarithms, since this is the total decay rate of a colour-singlet initial state.

We find gratifying the agreement between the three determinations (6,7,8). They are all somewhat smaller than the central value obtained from the total cross-section in e^+e^- annihilation [23], namely

$$\alpha_s(M_{Z^0}) = .140 \pm .020 \quad (9)$$

but we note that since the total cross-section begins at zeroth order in α_s , the errors on this value are much larger than in eqs.(6,7,8), and in fact the discrepancy is only one standard deviation. Taking the weighted average of all these determinations (6,7,8,9) of $\alpha_s(M_{Z^0})$, we find

$$\alpha_s(M_{Z^0}) = .113 \pm .004 \quad (10)$$

As a final caveat we point out that the dominant errors on this estimate are systematic.

Encouraged by the recent experimental hints for supersymmetric GUTs [6, 7, 8], and since exact perturbative two-loop 3-jet cross-sections and one-loop 4-jet cross-sections are unlikely to be available for some time, we have undertaken a careful reanalysis of the extraction of $\alpha_s(M_{Z^0})$ from the LEP jet data, based on the plausible idea that exponentiation of the leading infrared divergences sums the dominant contributions to the perturbative higher-order effects on the event shape distributions in e^+e^- annihilation. We have succeeded in providing a more convergent determination of $\alpha_s(M_{Z^0})$ amongst the different LEP experiments that is in excellent agreement with that of deep inelastic scattering and Υ decay. This lower value of $\alpha_s(M_{Z^0})$ may have far-reaching consequences for the Grand Unification programme.

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Table Caption:

Coefficients of higher-order corrections for various event shape variables, together with the values of $\alpha_s(M_{Z^0})$ extracted by different LEP groups, and the improved values we obtain using exponentiation of the leading infrared divergences. The event shape variables are listed in order of decreasing apparent reliability, as measured by the magnitude of the residual coefficients $\eta(x)$. For each variable, values corresponding to two-jet configurations are listed first, and those dominated by four-jet configurations last.

Table

$\frac{d\Sigma}{dx}$	x	$A^0(x)$	$\eta(x)$	Experimental results	
				$\alpha_s(M_{Z^0})$	$\alpha_s^{corr}(M_{Z^0})$
$\frac{1}{\sigma_0} \frac{d\sigma}{dC}$	.2	33.5	-.72		
	.4	11.9	.22	.137±.003	.123 (OPAL)
	.6	5.88	.44	.123±.008	.112 (N)
	.7	4.46	.41		
$\frac{1}{\sigma_0} \frac{d\sigma}{dT}$	.95	115.0	-.62		
	.9	37.9	.20	.144±.003	.128 (OPAL)
	.85	17.6	.41	.131±.003	.118 (N)
	.8	9.11	.73		
	.75	4.58	1.5		
	.7	1.64	4.5		
$\frac{\epsilon_J(1-\epsilon_J)}{\sigma_0} \frac{d\sigma}{d\epsilon_J}$	.4	3.10	-.64		
	.3	1.58	-.18		
	.2	1.61	-1.3		
	.1	2.36	-.16		
$\frac{N_{3-jet}}{N_0}$	.05	5.75	-1.3	.125±.003	.113 (DELPHI)
	.1	2.46	-.87	.126±.005	.114 (L3)
	.15	1.16	-.84		
$\frac{\sin^2 \chi}{\sigma_0} \frac{d\Sigma^{EEC}}{d\chi}$	-.75	1.92	-1.5		
	-.5	1.51	-1.2		
	-.25	1.32	-1.3	.131±.006	.118 (OPAL)
	0	1.22	-1.1	.130±.004	.117 (L3)
	.25	1.14	-.89	.123±.005	.112 (N)
	.5	1.08	-.81		
$\frac{1}{\sigma_0} \frac{d\sigma}{dy_3}$	.75	1.05	-.47		
	.05	115.0	-2.5		
	.1	37.9	-1.3		
	.15	17.6	-.99	.128±.002	.116 (N)
	.2	9.11	-1.1	.125±.003	.113 (OPAL)
	.25	4.58	-1.2		
$\frac{1}{\sigma_0} \frac{m_H^2}{s} \frac{d\sigma}{dm_H^2/s}$	.3	1.64	-1.5		
	.05	5.8	-1.7		
	.1	3.75	-1.2		
	.15	2.65	-1.4	.144±.007	.128 (N)
$\frac{1}{\sigma_0} \frac{d\sigma}{dO}$	.2	1.82	-2.0		
	.25	1.15	-2.7		
	.3	5.25	-5.6		
$\frac{1}{\sigma_0} \frac{d\sigma}{dO}$	.4	3.39	-7.3	.158	.138 (OPAL)
	.5	1.77	-12.2	.184	.156 (N)

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