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Bounds on f_B from Lattice QCD*

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Abstract

We present a study of *finite* a effects in the leptonic decay constant f_P of heavy pseudoscalar mesons. Exploiting results on three lattices - with inverse lattice spacings $a^{-1} = 3.35, 2.3, 1.53 \text{ GeV}$ - to extrapolate f_P to $a = 0$, we can bound the B-meson decay constant f_B , by $188 \text{ MeV} < f_B < 246 \text{ MeV}$. This range of f_B allows for sizeable CP violation effects in the B-meson decay $B_d(\bar{B}_d) \rightarrow J/\psi K_s$ and puts an experimental determination of f_B in $B \rightarrow \tau \nu$ at LEP within reach.

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1. Introduction

Weak decays of heavy hadrons play a key role in probing the hadronic sector of the successful Standard Model of electroweak interactions with the *three quark-lepton family* scenario [1]. The big challenge is to establish direct CP-violation effects in B decays, as they are predicted in the Cabibbo-Kobayashi-Maskawa (CKM) mechanism [2, 3, 4]. Within this scheme, by virtue of the unitarity of the CKM matrix, the CP-violation effects, in principle, can be predicted entirely in terms of CP-conserving experimental quantities. At present the most important constraints for the CP-violating CKM phase δ come from the observed CP-violation in the $K^0 - \bar{K}^0$ system and from the ARGUS and CLEO results on $B^0 - \bar{B}^0$ mixing and semileptonic B-decays [5, 6, 7]. A prediction of δ is not possible, though, because we are still lacking experimental knowledge of the top mass m_t , the leptonic B-meson decay constant f_B and the B_B factor. Moreover, the determination of CKM matrix elements is hampered by our rather limited ability to predict hadronic matrix elements (decay constants, B-factors and other form-factors).

Theoretical predictions are particularly difficult for heavy-light bound states like the D- and B-mesons. Therefore previous theoretical estimates for f_B differ widely. They are based on QCD sum rules [8, 9], lattice calculations and various potential models [10]. For large mass m_P the scaling law [11]

$$f_P(m_P) \sqrt{m_P} = \text{constant}$$

is expected to be valid. A value $f_B \sim 110$ MeV is then obtained by extrapolating with the scaling law from the value for $f_D \sim 180$ MeV which comes out from both QCD sum rules [8, 9] and lattice calculations [12, 13, 14]. Usually a range $100 \text{ MeV} \leq f_B \leq 200 \text{ MeV}$ has been anticipated in phenomenological discussions [3, 4, 7]. Values for f_B in the lower part of this range would imply rather pessimistic prospects for CP violation experiments, even at B-factories with luminosity as high as $\sim 10^{33} \text{ cm}^{-2} \text{ sec}^{-1}$ [3].

More recent developments in lattice simulations, based on a static treatment of the heavy b-quark [15], led to lattice estimates for f_B which are by a factor *two to three* higher than the quoted conventional values [16, 17, 13]. We should mention that large values for f_B have been obtained in the QCD sum rule and in the potential model approaches as well [8, 10]. In Ref. [18] large values for $f_B \sim 250$ MeV and $\sin \delta \sim 0.89$ were found as a result of a “global fit” to present data.¹ It has been pointed out recently [19] that a value f_B in the range 200 to 300 MeV would drastically change the outlook of the proposed CP violation experiments. For example, CP violating asymmetries in the pilot reaction $B_d(\bar{B}_d) \rightarrow J/\psi K_s$ could be enhanced by an order of

¹The result of this analysis crucially depends on the value $\epsilon'/\epsilon = (2.0 \pm 1.8) \times 10^{-3}$ used as a constraint. This value is controversial, however.

magnitude in rate and could thus safely be measured at storage rings with technically achievable luminosities.

Needless to say, a full lattice determination of f_B , being an *ab initio* calculation from the QCD Lagrangian, would be superior to any other way of predicting this quantity which is so badly needed. A number of pioneering lattice simulations have been carried out previously to study beauty - physics in the quenched approximation [12, 17, 13, 14].

Practical lattice simulations of QCD are faced, however, with a number of problems that must be carefully dealt with before one can claim reliable results from this theoretical approach. For B-mesons, due to their large mass, the limited lattice resolution (*finite lattice constant* a !) is potentially the most dangerous source of a systematic error: with quarks treated as Wilson fermions, as we do throughout this paper, such scaling violations are first-order effects in a . In the instance of *heavy-light* quark systems on the lattice, $\mathcal{O}(a)$ terms translate into $\mathcal{O}(M_i)$ effects, since masses M_i , given in lattice units, are related to the physical masses m_i by $M_i = am_i$. Therefore, with present lattice resolution limited to lattice spacings $a \geq (4\text{GeV})^{-1}$, we have to cope with the problem that $M_B > 1$!

In the following we will present new results for f_B , obtained on a $18^3 \times 48$ lattice at $\beta = 6.26$, which corresponds to a lattice spacing $a = (3.35\text{GeV})^{-1}$. We will combine them with our previous high precision data, obtained at inverse lattice spacings $a^{-1} = 1.53$ and 2.3 GeV [13]. An elaborate analysis of the whole data set will enable us to estimate the continuum value for f_B , *via* extrapolation to lattice spacing $a = 0$. As a result, we will end up with a fairly coherent picture, where the estimate for the pseudoscalar decay constant in the static approximation [16] can be reached by extrapolation from the standard, non-static results as presented here. Moreover, we can bound the leptonic B-meson decay constant to the range $188\text{ MeV} < f_B < 246\text{ MeV}$. We shall discuss the phenomenological implications of this result in Section 3.

2. The Lattice Calculation of f_B

Definitions. In this section, we will briefly define the quantities used in our present analysis². The computation of the pseudoscalar decay constant starts from the correlation functions

$$C_{ij}(T) = \sum_{\vec{X}} \langle O_i(\vec{X}, T) (O_j(\vec{0}, 0))^{\dagger} \rangle \quad i, j = P, A \quad (1)$$

²For any details of the simulation we refer the reader to Ref.[13] and a forthcoming longer publication on our results for heavy-light mesons [20].

of the operators

$$O_A(\vec{X}, T) = \bar{h}(\vec{X}, T) \gamma_0 \gamma_5 l(\vec{X}, T) \quad , \quad O_P(\vec{X}, T) = \bar{h}(\vec{X}, T) \gamma_5 l(\vec{X}, T) \quad . \quad (2)$$

Throughout this section, *capital letters* will be used for quantities in lattice units, and the corresponding *lower case* letters designate the same quantities in physical units. For sufficiently large time T , the correlation functions Eq. (1) are dominated by the lowest pseudoscalar state $|P\rangle$, with normalisation $\langle P|P\rangle = 1$

$$C_{ij}(T) \simeq (\langle 0|O_i(0)|P\rangle \langle P|O_j(0)^\dagger|0\rangle \exp(-M_P T)) \pm (T \rightarrow L_T - T) \quad . \quad (3)$$

The decay constant F_P , using a normalisation where $f_\pi = 132 \text{ MeV}$, is given by ³

$$Z_A \langle 0|O_A(0)|P\rangle = F_P(m_l, m_h) \sqrt{M_P(m_l, m_h)}/2 \quad (4)$$

with Z_A being the perturbative renormalisation constant of the axial vector current [21] and m_l (m_h) the light (heavy) quark mass.

Finite a -effects. In order to convert F_P into physical units, we determine the lattice constant a_ρ by matching to the ρ meson mass: $a_\rho m_\rho = M_\rho$. In this context, we prefer the ρ mass to any other physical observable, since it can be determined most precisely in a lattice calculation. In particular, it allows for a smooth extrapolation to the physical light quark mass. Of course, any other choice, like the proton mass, for example, would be as legitimate. Apart from effects due to the quenched approximation, this would only change the size of the scaling violations. With the present choice of a , the decay constant f_P is given by

$$a_\rho^{-1} F_P(m_l, m_h) = f_P(m_l, m_h) [1 + \mathcal{O}(a_\rho)], \quad (5)$$

a form suggested by a perturbative analysis [22]. Since the scaling violations in this expression are actually found to be rather small (see Ref.[13] as well as Fig. 1), we attempt to describe them by the simple *ansatz*

$$a_\rho^{-1} F_P(m_l, m_h) = f_P(m_P(m_l, m_h)) [1 + b a_\rho] [1 + d M_P(m_l, m_h)] \quad . \quad (6)$$

This form is suggested by the simulation data: the first correction is motivated by the observation that *finite a effects* appear to be still sizeable even at small quark masses. The second correction factor accounts for the expected growth of lattice artifacts with increasing mass of the pseudoscalar state M_P . For fixed m_P , this second term induces a *finite a* correction, which dominates the overall first-order correction (and thus constitutes the full correction term of Eq. (5)), in the case where all masses

³Note that in Ref.[13] a form has been used that assumes that the pseudoscalar state is described effectively as a lattice boson with nearest neighbour kinetic energy. We do not do this here, since lattice artifacts are treated explicitly.

(in lattice units) of hadrons made out of "light" quarks only are well below 1 ! In the following we will make use of Eq. (6) as our phenomenological parametrization of the lattice artifacts.

Mass Dependence. Next, we need to parametrize the m_P -dependence of the decay constant at fixed light quark mass. We will achieve this by expanding around the static ($m_P \rightarrow \infty$) limit [16]:

$$f_P(m_P) = \left(\frac{\alpha_s(e^{-2/3}m_B)}{\alpha_s(e^{-2/3}m_P)} \right)^{6/25} m_P^{-1/2} \sum_{n=0}^N c_n m_P^{-n} . \quad (7)$$

At a light quark mass, fixed by $m_P(m_l, m_l) = a_\rho^{-1} M_P(m_l, m_l) = 1.05 \text{ GeV}$, our fit (on a total of 20 data points, at 3 different values of the inverse lattice spacing, $a_\rho^{-1} = 1.53 \text{ GeV}, 2.30 \text{ GeV}, 3.35 \text{ GeV}$) renders a $\chi^2 = 10.0$. This signals a reasonable fit, considering the fact that the data points within each of the 3 sets are correlated. Indeed, Fig. 1 shows that we have found an adequate effective description of the data that will serve as the basis for a reliable prediction of the leptonic B-meson decay constant.

The best fit parameters, with $N = 3$ preset, are $b = 0.532$, $d = -0.129$ and $c_0, c_1, c_2, c_3 = 0.716(65), -1.04, 0.83, -0.28$ measured in powers of GeV.

Since these values are strongly correlated, we quote the errors only for the interesting leading term c_0 and the overall function $f_P(m_P)$. The statistical uncertainty of the latter varies from 8 MeV at $m_P = 3 \text{ GeV}$ to 10 MeV at the B-meson mass. We should mention that both c_1 and c_2 are significantly different from zero, which implies violation of mass scaling $f_P \sqrt{m_P} = \text{const}$ even beyond the B-meson mass range. Violation of this scaling law has been obtained previously in a version of the QCD sum rule approach [9] as well.

Consistency checks. It is tempting to make a consistency check, in order to test whether Eq. (7) remains a valid description in the mass range beyond the B-meson mass: the question is whether c_0 coincides with the static result

$$\hat{f}_P^{\text{stat}} \equiv f_P \sqrt{m_P} (\alpha_s(e^{-2/3}m_P) / \alpha_s(e^{-2/3}m_B))^{6/25} ,$$

that can be computed independently, in the static limit[16]. In Ref.[13], we have determined $\hat{f}_P^{\text{stat}} = 1.4 \text{ GeV}^{3/2}$ at $a_\rho = (1.53 \text{ GeV})^{-1}$ and $\hat{f}_P^{\text{stat}} = 1.1 \text{ GeV}^{3/2}$ at $a_\rho = (2.3 \text{ GeV})^{-1}$. These values decrease further to $\hat{f}_P^{\text{stat}} = 0.94 \text{ GeV}^{3/2}$ at $a_\rho = (3.35 \text{ GeV})^{-1}$. The statistical uncertainty on these numbers is small (about 5%), while the systematic error, due to the perturbative renormalisation of this quantity, may be as large as 20%. It is reassuring to see $\hat{f}_P^{\text{stat}}$ approach c_0 ! In fact, despite the large size of scaling violations in $\hat{f}_P^{\text{stat}}$, these values follow surprisingly well a linear behaviour in a , giving $\hat{f}_P^{\text{stat}} = 0.57 \text{ GeV}^{3/2}$ at $a = 0$. Within the systematic uncertainty of $\hat{f}_P^{\text{stat}}$ this value is in agreement with $c_0 = 0.716(65)$!

The fitted $f_P(m_P)$ and c_0 turn out to be quite stable with respect to variants of Eq. (7), like modifying N to 2 or 4 or switching from polynomial to Padé forms.

We mention another check on the systematics of our treatment of lattice artifacts: for mesons composed of two light quarks the second square bracket in Eq. (6) is negligible. We can therefore use the experimental values of f_K , f_π and Eq. (6) to approximately ‘predict’ the expected lattice numbers (that include the finite a effects), for both F_K and F_π . These numbers agree with our direct Monte-Carlo results as well as with the ones of the APE group [23] within the $\simeq 10\%$ statistical errors. In other words, one would obtain compatible results if the scale were introduced into Eq. (5) through f_K rather than through m_ρ .⁴ In order to further substantiate our results, we have repeated the above fitting procedures, after replacing a_ρ by a_σ , the latter being defined as the lattice spacing obtained from matching to the string tension. Doing so, we arrive at compatible results, though within larger overall uncertainties, which are characteristic of string tension measurements [24].

At the B-meson mass, our fit yields $f_P(m_P = 5.28\text{GeV}) = 239(10)\text{MeV}$, which is only a mild extrapolation with respect to the last data point. In the following we shall use, in addition to the statistical error, half of the difference between the latter two values, namely 8 MeV, as our estimate of the systematic error due to the phenomenological *ansatz* for treating lattice artifacts. This difference is small because b and d have opposite signs, which leads to a partial compensation of finite a effects.

There is another way to look at the problem, which will help us to get a better insight into the *sign* of the lattice artifacts, for $m_P > 4$. We may proceed as follows:

1. Interpolate the data, at a given value of a_ρ , using a smooth fitting-function $F_P = F_P(M_P)$:

$$F_P(M_P) = \frac{\hat{f}^{stat}}{Z^{stat}} \sqrt{\frac{2K}{M_P}} \left(1 + \sum_{n=1}^3 e_n(\beta) M_P^{-n}\right). \quad (8)$$

The asymptotic form $F_P(M_P \rightarrow \infty)$ was constrained by the hopping parameter expansion [25]. In doing so, we expressed the hopping parameter K in terms of M_P by the relation $1/(2K) = 1/(2K_c) + \exp(M_P - M_1^{stat}) - 1$ for $M_P > 2$. Z^{stat} , M_1^{stat} and K_c are defined in [13]. Eq. (8) enables one to formally extrapolate the data at the two larger a -values up to $m_P \simeq m_B$.

2. Determine lattice artifacts by fitting a linear *ansatz* in a , $a_\rho^{-1} F_P = f_P(m_P) (1 + \delta M_P)$, for each m_P , making use of the previous inter-(and extra-)polator.

Following this strategy, we again find that the lattice artifacts in Eq. (5) are negative

⁴This has not been done here, since it would introduce larger statistical uncertainties than the ones obtained with a_ρ .

for large enough values of m_P , *i.e.* $m_P > 4\text{GeV}$.⁵ Though we are reluctant to believe the actual size of finite a corrections in a situation where $M_P > 3$, we hold their sign to be trustworthy. It allows us to establish, from our data at $a_\rho^{-1} = 3.35\text{GeV}$ alone (*i.e.* without any further reference to the form of the lattice artifacts), the lower bound $f_P(m_l, m_b) > 213\text{MeV}$. As explained below, this value has to be extrapolated to the physical B-meson, *i.e.* to really light quark masses, yielding the result

$$180\text{MeV} < f_B. \quad (9)$$

Extrapolation to light quarks. In our calculation the “light” quark mass has been kept fixed at a value corresponding to $m_P(m_l, m_l) = 1.05\text{GeV}$. In order to find the “true” value for the leptonic decay constant of the physical B-meson we have to correct for the unphysical value of m_l used so far. For the B-mesons B_d and B_u this means extrapolation to zero m_l , the corresponding decay constant will be denoted by f_B . In Ref.[13] the calculations were performed down to light quark masses of the size of the strange quark mass m_s . From this data we estimate

$$0.010\text{GeV}^{-1} < \frac{\partial f(m_l, m_h)}{\partial m_p(m_l, m_l)^2} < 0.030\text{GeV}^{-1}.$$

Using a Taylor expansion, this translates into the final result for the B-decay constant:

$$188\text{MeV} < f_B < 246\text{MeV}. \quad (10)$$

These bounds can evidently be tightened by extending the computations to lower values of m_l . For the B_s decay constant we do not need to extrapolate as far in the light quark mass and get

$$204\text{MeV} < f_{B_s} < 241\text{MeV}$$

with a lower systematic error.

Finite size effects. In Ref.[13] we found no volume dependence in varying L from 0.7 fm to 1.05 fm. In the meantime, we have lattices of physical length $L = 1.5\text{fm}$ [20] and granularity $a_\rho = (1.53\text{GeV})^{-1}$. Again no finite size effects are visible within the 10% statistical errors. So we conclude that finite size effects are under control.

Z-factors. One may further worry about the perturbative renormalisation of the lattice current. We would like to stress that $\mathcal{O}(a)$ effects in the current renormalisation are included in our treatment of lattice artifacts. Since the g^2 term in the renormalisation is only 14%, the g^4 effects are not expected to be large. They should therefore not have a noticeable effect on Eqs. (9) and (10).

⁵The lowest order term of the hopping parameter expansion alone implies that lattice artifacts are negative at very large M_P .

In the above discussion we have taken into account all but one of the systematic errors. The impact of the quenched approximation on the results has not been estimated so far. One has to wait for future unquenched calculations for any real understanding of this latter uncertainty. However we do know that the quenched lattice results for f_K and f_π (after using Eq. (6)) are in good agreement with the experimental values.

3. Phenomenological Consequences of a Large f_B

In this section we will discuss for completeness the phenomenological implications of our result, as given in Eq. (10), on the expected size[7, 18, 19] of CP violation effects in B-decays.

As usual, we start from the unitarity triangle

$$\sum_{i=u,c,t} V_{id} V_{ib}^* = 0$$

associated with the $b \rightarrow d$ transitions. Using the standard Wolfenstein parametrization we have $V_{ub}^* - |V_{cd} V_{cb}^*| + V_{td} = 0$ to a very good approximation [5, 7]. After rescaling by $|V_{cd} V_{cb}^*|^{-1}$ this defines a triangle in the complex plane with vertices (ρ, η) , $(1, 0)$ and $(0, 0)$ where

$$\rho = (\text{Re } V_{ub})/|V_{cd} V_{cb}^*|, \quad \eta = -(\text{Im } V_{ub})/|V_{cd} V_{cb}^*|. \quad (11)$$

We use the following phenomenological constraints [5]: ⁶

$$|V_{ub}/V_{cb}| = |V_{us}|\sqrt{\rho^2 + \eta^2} = 0.11 \pm 0.05 \quad (12)$$

$$\text{with } |V_{cb}| = 0.044 \pm 0.009 \text{ and } |V_{us}| = 0.2205 \pm 0.0018,$$

the $K^0 - \bar{K}^0$ CP-violating parameter, ⁷

$$|\epsilon| = 4.00 \times 10^4 B_K |V_{cb}|^2 |V_{us}|^2 \eta \left[F(x_c, x_t) + |V_{cb}|^2 (1 - \rho) F(x_t) \right] = (2.272 \pm 0.022) \times 10^{-3} \quad (13)$$

and the $B^0 - \bar{B}^0$ mixing parameter

$$x_d = 4.15 \times 10^{-7} B_B \tau_B m_B^{-1} f_B^2 |V_{cb}|^2 |V_{us}|^2 \left((1 - \rho)^2 + \eta^2 \right) F(x_t) = 0.71 \pm 0.11. \quad (14)$$

⁶We remind the reader that these numbers depend on theoretical inputs.

⁷Assuming the CPT invariance constraint $\phi_{00} = \phi_{+-}$, we use

$$|\epsilon| = \frac{|\eta_{+-}|}{3} (2 + |\eta_{00}/\eta_{+-}|)$$

and the averages $|\eta_{+-}| = (2.279 \pm 0.022) \times 10^{-3}$ and $|\eta_{00}/\eta_{+-}| = 0.991 \pm 0.003$ [5].

The value of Eq. (12) is given in Ref. [6] but with a smaller systematic error which according to Ref. [1] is unknown. Our value agrees with the one used in Ref. [19]. The value for x_d is obtained from the weighted average $\chi_d = 0.166 \pm 0.033$ of the ARGUS (updated) and CLEO results [5, 6].

We follow Ref.[19] and assume $B_K = 0.94 \pm 0.11$ and take $B_B = 1$. The functions F are the QCD corrected versions of the Inami-Lim functions depending on the charm mass $x_c = m_c^2/M_W^2$ and the top mass $x_t = m_t^2/M_W^2$ [26]. It is obvious from the expression for x_d that a large (small) value for f_B favours positive (negative) ρ .

Fig. 2 shows the allowed (ρ, η) range for different values of m_t , using the phenomenological constraints Eqs. (12), (13) and (14). This figure should be compared with Figure 2.1 of Ref.[3], which illustrates previous theoretical prejudices. In Fig. 3 we compare the allowed regions for the vertex (ρ, η) of the unitarity triangle when f_B is taken in the range of Eq. (10) or in the more conventional range $f_B = 140 \pm 40$ MeV. The top mass is allowed to vary in the range

$$90 \text{ GeV} \leq m_t \leq 200 \text{ GeV} . \quad (15)$$

This is the phenomenological bound from LEP, $p\bar{p}$ -collider and neutral current data [27].

The CP-violating asymmetries for B-decays into CP eigenstates are proportional to $\sin(2\phi)$, where ϕ is one of the angles of the unitarity triangle. Of particular interest is the decay $B \rightarrow J/\psi K_s$ for which the CP asymmetry is proportional to $\sin(2\beta) = 2\eta(1-\rho)/((1-\rho)^2 + \eta^2)$. The allowed range is depicted in Fig. 4. The phenomenological bounds allow for substantially larger values of this quantity, in particular if the top mass is close to the present “central” value $m_t \sim 130$ GeV. For example, from inspection of Fig. 2b, one finds that large values of the asymmetry come out for positive values of ρ , which corresponds to the solution found in the phenomenological analysis of Ref. [18]:

$$0.59 \leq \sin(2\beta) \leq 0.95; \quad 0.09 \leq \rho \leq 0.38 \text{ for } m_t \simeq 130 \text{ GeV} . \quad (16)$$

This is to be compared to the situation with conventional f_B , where ρ is definitely negative: $0.12 \leq \sin(2\beta) \leq 0.46$; $-0.63 \leq \rho \leq -0.22$ for $f_B = 140 \pm 40$ MeV at the same value for m_t . Thus, for m_t around 130 GeV and assuming the $\rho > 0$ -solution to be realised, one would gain some factor between 1.3 and 7.9 (!) in amplitude.

Up to now a direct experimental measurement of f_B in the leptonic decay $B \rightarrow \tau \nu_\tau$ is lacking. Theoretically, the branching ratio into this channel is given by

$$\text{Br}(B \rightarrow \tau \nu_\tau) = 5.41 \times 10^{-12} \text{ GeV}^{-4} |V_{ub}|^2 \tau_B f_B^2 m_B m_\tau^2 \left(1 - \frac{m_\tau^2}{m_B^2}\right)^2 (1 + \delta_{QED}) . \quad (17)$$

Using parameters from Ref. [5] and our lattice estimate for f_B (neglecting QED corrections) we get $\text{Br}(B \rightarrow \tau \nu_\tau) = 1.4 \times 10^{-4}$ for the central value and a range $(0.18 \text{ to } 6.1) \times 10^{-4}$ if we add uncertainties linearly. This result looks very promising and a direct measurement of f_B appears to be feasible at LEP.

4. Summary and Conclusions

Within the quenched approximation we have been able, for the first time, to study the limiting behaviour of $f_P \sqrt{m_P}$ near the static limit, $m_P \rightarrow \infty$. This has been achieved by a conventional, non-static calculation, by varying the heavy quark mass m_h and the lattice spacings a in suitable ranges.

Let us summarize the main results:

1. The static limit *in itself* appears to be a poor estimate for the B-meson decay constant: the corrections of $\mathcal{O}(1/m_P)$, as derived from the non-static computations, amount to $\simeq -30\%$ in the vicinity of the b-quark mass and $\simeq -60\%$ at the c-quark mass. This implies that we cannot retrieve the scaling law $f_P \sqrt{m_P} = \text{constant}$.
2. In our approach of treating lattice artifacts, we find the data - for all a -values under study - to be consistent with each other, even when increasing m_h as high as $m_h \simeq 1.7a^{-1}$. We find large scaling violations within the static approximation, which can however still be described by a first order ansatz.
3. The non-static data for f_P have been shown, by means of extrapolation $m_h \rightarrow \infty$, to be in reasonable accord with the static approach[16].

As an upshot of this investigation, we can for the moment (before the advent of Teraflops computers) envisage two viable schemes to compute, by means of extrapolation, the decay constant of the B-meson, f_B , in the limit $a \rightarrow 0$: (a) Either one attempts to fit (and then unfold lattice artifacts by means of extrapolations from) non-static results taken beyond the D-range, or (b) one relies on non-static results collected exclusively in the mass-range below the D-meson (where, with presently accessible lattices, heavy quarks can safely be handled) and subsequently performs extrapolations from a fit constrained to pass through the static limit, with finite a effects taken into account in both cases.

In view of the phenomenological importance of f_B to CP violation in the CKM scenario, it would be highly desirable to tighten the bounds set in Eq. (10). We are currently improving the precision of the $a_\rho^{-1} = 3.35 \text{ GeV}$ lattice results in order to test Eq. (6) more stringently [20]. It will further be possible to improve the bound of Eq. (10) through calculations at smaller (light-) quark masses and a smaller value of the lattice

spacing [28]. Moreover, in this context, it is very important to explore the impact of quark loops within full QCD.

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References

- [1] see e.g.
D. G. Hitlin, in *Proceedings of the Snowmass Summer Study on High Energy Physics*, Snowmass, Colorado 1990; CALTECH preprint, CALT-68-1722 (1991), and references therein.
- [2] N. Cabibbo, Phys. Lett. **10** (1963) 513;
M. Kobayashi and K. Maskawa, Prog. Theor. Phys. **49** (1973) 652.
- [3] see e.g.
T. Nakada, ed., CERN Yellow report, CERN 90-02 (1990); and references therein.
- [4] P. Franzini, Phys. Rep. C173 (1989) 1;
E. A. Paschos and U. Türke, Phys. Rep. C178 (1989) 145;
T. Nakada, PSI report, PSI-PR-91-02.
- [5] Particle Data Group, J. J. Hernández et al., Phys. Lett. B239 (1990) 1.
- [6] B. Gittelman, in *Proceedings of the 25th International Conference on High Energy Physics*, Singapore, 1990, eds. K. Phua and Y. Yamaguchi, World Scientific, Singapore, 1991.
- [7] C. Dib, I. Dunietz, F. Gilman and Y. Nir, Phys. Rev. D41 (1990) 1522.
- [8] T. M. Aliev and V. L. Eletsky, Sov. J. Nucl. Phys. 38 (1983) 936;
V. S. Mathur and M. T. Yamazwaki, Phys. Rev. D29 (1984) 2057;
L. J. Reinders, H. Rubinstein and S. Yazaki, Phys. Lett. B104 (1981) 305;
Phys. Rep. C127 (1985) 1;
L. J. Reinders, Phys. Rev. D38 (1988) 947;
C. A. Dominguez and N. Paver, Phys Lett. B197 (1987) 423; B246 (1990) 493;
S. Narison, Phys. Lett. B198 (1987) 104; and in "QCD Spectral Sum Rules",
World Scientific Lecture Notes in Physics **26**;
World Scientific, Singapore, 1989;
A. Pich, Phys. Lett. B206 (1988) 322.
- [9] S. Narison, Phys. Lett. B218 (1989) 238.
- [10] see e.g.
M. Suzuki, Phys. Lett. B162 (1985) 392;
D. Silverman and H. Yao, Phys. Rev. D38 (1988) 214;
S. Capstick and S. Godfrey, Phys. Rev. D41 (1990) 2856.
- [11] E. V. Shuriak, Nucl. Phys. B198 (1983) 83;
M. A. Shifman and M. B. Voloshin, Sov. J. Nucl. Phys. 45 (1988) 292;
H. D. Politzer and M. B. Wise, Phys. Lett. B208 (1988) 504.

- [12] M.B. Gavela et al., Nucl. Phys. B306 (1988) 677;
M.B. Gavela et al., Phys. Lett. 206B (1988) 113;
C. Bernard, T. Draper, G. Hockney and A. Soni, Phys. Rev.D38(1988)3540;
T.A. De Grand and D.R. Loft, Phys. Rev. D38 (1988) 954.
- [13] C. Alexandrou, S. Güsken, F. Jegerlehner, K. Schilling and R. Sommer, Phys. Lett. B256 (1991) 60.
- [14] C. Bernard et al., Talk presented at the International Symposium *Lattice '90* Tallahassee, Fl., to be publ. in Nucl. Phys.
- [15] E. Eichten, in *Field Theory on the Lattice*, Nucl. Phys. B (Proc. Suppl.) 4 (1988) 147.
- [16] E. Eichten, Review presented at the International Symposium *Lattice '90* Tallahassee, Fl., to be publ. in Nucl. Phys., FERMILAB-Conf-90/219-T.
- [17] C.R. Allton et al., Nucl. Phys. B349(1991)598;
C.R. Allton et al., Talk presented at the International Symposium *Lattice '90* Tallahassee, Fl., to be publ. in Nucl. Phys.
- [18] J. Maalampi and M. Roos, Particle World 1 (1990) 148.
- [19] M. Lusignoli, L. Maiani, G. Martinelli and L. Reina, Univ. di Roma, Preprint n.792 (1991).
- [20] C. Alexandrou, S. Güsken, F. Jegerlehner, K. Schilling and R. Sommer, in preparation.
- [21] L. H. Karsten and J. Smit, Nucl.Phys. B183 (1981) 103;
R. Groot, J. Hoek and J. Smit, Nucl. Phys. B237(1984)111;
M. Bochicchio, L. Maiani, G. Martinelli, G. C. Rossi and M. Testa, Nucl. Phys. B262 (1985) 331;
- [22] For a brief discussion in the context of an improved fermion action see G. Heatlie et al., Rome University preprint no. 737 (1990).
- [23] The APE group, Talk presented by M. Guagnelli at the International Symposium *Lattice '90* Tallahassee, Fl., to be publ. in Nucl. Phys.
- [24] R. Sommer, PhD thesis, preprint WU-B-86;
K. Schilling and R. Sommer, Z. Phys. C 29 (1985) 95;
C. Michael and M. Teper, Nucl. Phys. B314(1989)347.
- [25] K. G. Wilson, in *New Phenomena in Subnuclear Physics*, Erice 1975, Plenum, New York (1977);
A. Hasenfratz, Z. Kunst, P. Hasenfratz and C. B. Lang, Phys. Lett. 110B (1982) 289.

- [26] T. Inami and C. S. Lim, Prog. Theor. Phys. 65 (1981) 297; (E) ibid. 1722;
 Y. Nir, Nucl. Phys. B306 (1988) 14;
 W. A. Kaufman, H. Steger and Y. P. Yao, Mod. Phys. Lett, A3 (1988) 1479;
 W. A. Kaufman and Y. P. Yao, Phys. Rev. D39 (1989) 3373;
 A. Datta, J. Fröhlich and E. A. Paschos, Z. Phys. C46 (1990) 63;
 A. J. Buras, M. Janin and P. H. Weisz, Nucl. Phys. B347 (1990) 491.
- [27] F. Dydak, in *Proceedings of the 25th International Conference on High Energy Physics*, Singapore, 1990, eds. K. Phua and Y. Yamaguchi, World Scientific, Singapore, 1991;
 J. Ellis and G. L. Fogli, Phys. Lett. B249 (1990) 543;
 P. Langacker and M. Luo, Univ. of Pennsylvania preprint, UPR-0466T (1991).
- [28] A. Abada, Ph. Boucaud, J. Galand, O. Pene, M. Crisafulli, G. Martinelli, R. Sarno, S. Güsken, K. Schilling and R. Sommer, in preparation

Figure Captions

1. $a_\rho^{-1} F_P(m_l, m_h)$ as a function of $m_P^{-1} = a_\rho M_P^{-1}$. The fit described in the text is given by the dashed line ($a_\rho^{-1} = 3.35 \text{ GeV}$), the dashed-dotted line ($a_\rho^{-1} = 2.3 \text{ GeV}$) and the dotted line ($a_\rho^{-1} = 1.53 \text{ GeV}$). The solid curve represents the extrapolation to $a = 0$.
2. Constraints from $|V_{ub}/V_{cb}|$ (dashed circles), x_d (dotted circles), and $|\epsilon|$ (solid hyperbolas) on the rescaled unitarity triangle for $188 \text{ MeV} \leq f_B \leq 246 \text{ MeV}$ and (a) $m_t = 90 \text{ GeV}$, (b) $m_t = 130 \text{ GeV}$, and (c) $m_t = 200 \text{ GeV}$.
3. Allowed region (68% CL) for (ρ, η) for $90 \text{ GeV} \leq m_t \leq 200 \text{ GeV}$ (a) for large $f_B = 217 \pm 29 \text{ MeV}$, favouring $\rho > 0$, and (b) for low $f_B = 140 \pm 40 \text{ MeV}$, favouring $\rho < 0$.
4. Allowed region (68% CL) for the amplitude $\sin(2\beta)$ relevant for the CP asymmetry in $B \rightarrow J\psi K_s$ decays. The full line is for the high f_B range, and the dotted for the low f_B range.

Figure 1

Figure 2

Figure 3

Figure 4