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Anomalous Magnetic Moment at Finite Chemical Potential and External Field Effects

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Abstract

We calculate the anomalous magnetic moment of a massive Dirac fermion at non-zero temperature and chemical potential. In the course of our work we also discuss the non-linear electromagnetic field dependence. Finally we derive the effective magnetic moment in curved space and show that the recently proposed large effect due to the gravitational field is not measured in present experiments.

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1 Introduction

Perturbative quantum field theory at finite temperature and density is of current interest in many areas of elementary particle physics (see e.g. Refs.[1,2,3,4,5,6]). At the one-loop level there is a rather straightforward procedure to obtain the temperature correction to conventional elementary processes in the vacuum. The finite-temperature one-loop correction to the anomalous magnetic moment of the electron, $\delta a_e \equiv \delta a_e^\beta$, at low T and zero density has for instance been computed by several groups [7,8,9,10] with the result

$$\delta a_e^\beta = -\frac{2\pi\alpha}{9} \left(\frac{T}{m}\right)^2, \quad (1)$$

where α is the fine-structure constant and m the fermion mass. δa_e^β is expressed in terms of a finite-temperature renormalized Bohr-magneton unit, i.e. $e/2(m + \delta m_\beta)$ [9,10]. Here δm_β is the finite-temperature correction to the mass of the electron which, for T/m sufficiently small, is given by

$$\frac{\delta m_\beta}{m} = \frac{\alpha\pi}{3} \left(\frac{T}{m}\right)^2. \quad (2)$$

δm_β can physically be understood in terms of a straightforward finite-temperature extension of Kibble's [11] analysis of the effective mass of a charged particle moving in an electromagnetic field (see e.g. Refs.[12,13]). Strictly speaking the finite temperature mass is an effective momentum-dependent mass [14,15,9,10,7]. In our case we will consider small momenta, so this complication can be neglected. It is interesting to notice that the measurement of certain highly excited Rydberg levels at finite temperature is consistent with the notion of a finite temperature-dependent effective electron mass [16].

The primary purpose of the present paper is to calculate how the anomalous magnetic moment depends on the chemical potential (μ) and the charge density (ρ) to first order in α . This will be done in the next Section. A computational technique, which is based on conventional methods, enables us to find a closed analytical expression for δa_e , at least for $T=0$ but a non-zero chemical potential. For sufficiently small densities, $n = \rho/e$, we find a correction $\delta a_e \equiv \delta a_e^\beta = 16\alpha(3\pi^2 n)^{1/2}/3\pi m$. Here δa_e^β is expressed in units of

$e/2(m + \delta m_{\beta,\mu})$, where for small T/m [17]

$$\delta m_{\beta,\mu} \approx \delta m_\beta + \frac{\alpha\pi}{2\pi} \left(\left(\frac{\mu}{m} - 2 \right) \sqrt{\left(\frac{\mu}{m} \right)^2 - 1} - 3 \log \left(\frac{\mu}{m} + \sqrt{\left(\frac{\mu}{m} \right)^2 - 1} \right) \right). \quad (3)$$

In Section 3 we will discuss the non-linear magnetic field dependence of δa_e . The Dirac equation in curved space and the recently discussed effect due to the earth's gravitational field, are treated in Section 4. In Section 5 we conclude by comparing the various corrections and give some final comments.

2 Finite temperature and chemical potential

In this Section we evaluate the anomalous magnetic moment using a Gibbs ensemble to describe the electron(s) interacting with a heat and charge bath. The parameters governing the average energy and charge density are the temperature (β^{-1}) and the chemical potential (μ). The technique we use is based on the standard Feynman diagrams of thermo-field dynamics [3]. This real-time formalism contains physical fields and thermal ghost fields, with only physical fields on the external legs. The effective 3-point vertex in QED has therefore, apart from the Lorentz index ν , also additional indices $a, b, c = 1, 2$ corresponding to the physical and thermal ghost fields, respectively. In order to obtain the physical 3-point vertex, one only considers the $(1,1,1)$ -component of Λ_{abc}^ν : (Evans [18] and Kobes [19] have recently addressed the question when this gives the same result as the imaginary-time formalism). The bare vertices are diagonal in the thermal indices and at the one-loop level we only need the $(1,1)$ -component of the propagators to calculate Λ_{111}^ν [8]. We can therefore suppress the abc-indices and write the vertex correction at finite T and μ as

$$\Lambda_\nu(p, p') = (-ie)^2 \int \frac{d^4 k}{(2\pi)^4} T_{\alpha\beta}(p', p, k) D^{\alpha\beta}(k) S(p' - k) S(p - k), \quad (4)$$

where all the γ -matrices from the vertices and the fermion propagators have been absorbed in $T_{\alpha\beta}(p', p, k) = \gamma_\alpha(p' - \not{k} + m) \gamma_\nu(\not{p} - \not{k} + m) \gamma_\beta$. Here $D_{\alpha\beta}(k)$ is the photon propagator at finite T and μ which, to lowest order in perturbation

theory (see e.g. Ref.[20]), has the form

$$D_{\alpha\beta}(k) = g_{\alpha\beta} \left(\frac{-i}{k^2 + i\epsilon} - 2\pi f_B(k_0) \delta(k^2) \right). \quad (5)$$

Similarly, $S(k)$ is obtained from the fermion thermal propagator and has, to lowest order in perturbation theory, the form (see e.g. Ref.[20])

$$S(k) = \frac{i}{k^2 + m^2 + i\epsilon} - 2\pi [\theta(k_0) f_F^\dagger(k_0) + \theta(-k_0) f_F^-(k_0)] \delta(k^2 - m^2). \quad (6)$$

The thermal distribution functions, $f_B(k_0)$ and $f_F^\pm(k_0)$, are given by

$$f_B(k_0) = \frac{1}{e^{\beta|k_0|} - 1}, \quad f_F^\pm(k_0) = \frac{1}{e^{\beta(k_0 \mp \mu)} + 1}. \quad (7)$$

Physically, one can interpret (at least to one-loop order) the second terms in Eqs.(5) and (6) as an exchange of the incoming (or outgoing) particle with an on-shell particle in the heat-bath (see e.g. Refs.[21,12]). We represent the processes diagrammatically in Figs.(1) and (2).

The anomalous contribution to the magnetic moment is extracted from the real part of $\Lambda_\nu(p', p)$ with on-shell electrons and vanishing external 3-momenta. Writing out Eq.(4) explicitly, we find several products of δ -functions. Using the mass shell condition, only terms with one δ -function contribute to the real part of the thermal correction to Λ_ν . This corresponds to the second, third and fourth terms on the right-hand side in Fig.(2). We write $\text{Re}\Lambda_\nu = \text{Re}\Lambda_\nu^{(0,0)} + \text{Re}\Lambda_\nu^{(\beta,\omega)}$, where $\text{Re}\Lambda_\nu^{(0,0)}$ is the zero temperature and chemical potential part, and $\text{Re}\Lambda_\nu^{(\beta,\omega)}$ contains three terms, one with thermal photons and two with thermal electrons. The photon contribution (the fourth term in Fig.(2)) has been calculated elsewhere [7,8,9,10], and gives the correction in Eq.(1). Vacuum polarization processes will not contribute to the anomalous magnetic moment to the order in perturbation theory that we consider.

Without loss of generality, we will henceforth consider the case with $\mu > 0$, i.e. an excess of electrons. Then, only on-shell thermal states with positive energy ($p_0 - k_0 > 0$ in the δ -functions in Fig.(1) and correspondingly $p'_0 - k_0 > 0$ when the outgoing electron is exchanged with the heat-bath) will contribute

significantly to the integral of the electron part of $\Lambda_\nu^{(\beta,\omega)}(p', p)$, at low T. The Lorentz invariance is broken by the preferred frame of reference, where the heat-bath is assumed to be in thermal equilibrium. However, the rotational invariance remains and we can extract the coupling between electrons and an external magnetic field from the space part, Λ_i , of the vertex function. We are therefore left with (using the on-shell condition and neglecting terms of order $|p|^2$ in the numerator):

$$\text{Re}\Lambda_i^{(\beta,\mu)} = \frac{2\alpha}{\pi^2} \int \frac{d^4k}{2\omega(k)} \frac{(p_i + k_i)(m - \gamma_0 k_0) - m(p'_0 - k_i)}{(q^2 - 2qk)(p'^2 + m^2 - 2pk)} \times \delta(k_0 - \omega(k)) f_F^+(k_0) + p \leftrightarrow p', \quad (8)$$

where $\omega(k) = \sqrt{|k|^2 + m^2}$.

As was mentioned before, the vertex-function should be calculated for on-shell external particles and small external momenta, p_i and p'_i , with $q_i = p'_i - p_i = 0$. We cannot impose this condition from the start because of the infra-red divergence that would arise from the δ -function and the electron propagator with coincident arguments. This problem can, however, be circumvented by performing the integrals with slightly off-shell external particles. At the end one takes either the time-like ($q^2 > 0$) or the space-like ($q^2 < 0$) limit when $q_\mu \rightarrow 0$. In the time-like limit, we keep $q_0 > 0$ to the end of the calculation but let $q_i \rightarrow 0$ and the finite result below is obtained. Even at zero chemical potential it is wellknown that different results for different limits can occur when computing the self-energy or a vertex function at finite temperature (see e.g. Refs.[6,22,18]).

The space-like (static) limit ($q_0 = 0$) is more difficult to evaluate. If we use the lowest order approximation for the photon propagator $D_{\alpha\beta}$, we encounter an infra-red divergence. This is very similar to what happens, for example, when one calculates the partition function in quantum electrodynamics [23, 24]. We therefore believe that it is necessary to use a modified higher order photon propagator, e.g. by making use of a random phase approximation, in order to take into account the effects of screening [25]. This is physically due to the finite background charge density. In analogy with the calculation of the self-energy of a fermion at finite density, we expect that the static limit will give a larger value for the anomalous magnetic moment as compared to the "dynamical", time-like limit [17]. We will, however, restrict ourselves to the

time-like prescription since this preserves the time-ordering underlying the physical picture of the processes under consideration as exhibited in Figs.(1) and (2).

Under these assumptions the calculation becomes straightforward, and we end up with

$$\Lambda_1^{(\beta,\mu)} = \frac{p_1 2\alpha}{m 3\pi} \int_0^\infty ds \sqrt{\frac{s+2}{s}} \frac{s^3 - 3s^2 - 9s - 4}{(s+1)^2} \frac{1}{e^{\beta m(s-\delta)} + 1}, \quad (9)$$

where $\delta = \mu/m - 1$ is a dimensionless parameter related to the charge density. From $\Lambda_1^{(\beta,\mu)}$ we now extract the contribution $\delta a_e^{\beta,\mu} \equiv \delta a_e^{\beta,\mu}$ to the anomalous magnetic moment which, after a change of integration variable, takes the following form

$$\delta a_e^{\beta,\mu} = -\frac{2\alpha}{3\pi} \int_1^\infty \frac{dx}{x^2} \sqrt{\frac{x+1}{x-1}} \frac{1}{(1-6x^2+x^3)} \frac{1}{e^{\beta m(x-\mu/m)} + 1}. \quad (10)$$

At high T , states with $k_0 < 0$ will contribute in a similar way to Eq.(8), which results in an overall multiplicative factor 2 in the limit $T \gg \mu, m$. We can then confirm the results of Refs.[8,9,10], i.e.

$$\delta a_e^{\beta,\mu} = -\frac{\alpha\pi}{9} \left(\frac{T}{m}\right)^2, \quad (11)$$

which should be added to the photon contribution δa_e^β as given by Eq.(1).

At low temperatures we have several cases which have to be treated separately. To distinguish the physical properties in the various situations we calculate the charge density ρ as a function of μ (or δ) and β to lowest order in α , i.e.

$$\rho = \frac{e}{\pi^2} \int_0^\infty dk k^2 \left\{ \frac{1}{e^{\beta(\omega-\mu)} + 1} - \frac{1}{e^{\beta(\omega+\mu)} + 1} \right\}, \quad (12)$$

where $\omega(k) \geq m$. When $\mu > 0$ the last term will always give an exponentially small contribution. In such a case we can neglect the last term in the expression for ρ . After a change of variable ($s = \sqrt{k^2/m^2 + 1} - 1$) in Eq.(12), we then have the following expression for the charge density :

$$\rho = \frac{em^3}{\pi^2} \int_0^\infty ds (s+1) \sqrt{s(s+2)} \frac{1}{e^{\beta m(s-\delta)} + 1}. \quad (13)$$

We now discuss ρ and $\delta a_e^{\beta,\mu}$ in three different cases.

Case I ($0 < \mu < m, T < m$)

We write the approximation as $\delta\beta m < -1, \beta m \gg 1$. To lowest order in $e^{\beta m}$ and T/m we find

$$\begin{aligned} \rho &= \frac{em^3}{\pi^2} e^{\beta m} \int_0^\infty ds (s+1) \sqrt{s(s+2)} e^{-\beta m s} \\ &= 2e \left(\frac{Tm}{2\pi}\right)^{3/2} e^{\beta m} (1 + \mathcal{O}(T/m)), \end{aligned} \quad (14)$$

which is exponentially small for $m - \mu \gg T$ and is zero at $T = 0$. The anomalous magnetic moment for these small densities is given by

$$\delta a_e^{\beta,\mu} = \frac{16\alpha\pi}{3} \frac{n}{Tm^2} (1 + \mathcal{O}(T/m)), \quad (15)$$

where the particle density is given by $n = \rho/e$. We notice that $\delta a_e^{\beta,\mu}$ is not an exponential function of n . However, one should remember that this approximation is only valid when the density n is actually exponentially small in T .

Case II ($|\mu - m| < T < m$)

This implies in terms of β and δ : $|\delta\beta m| < 1, \beta m \gg 1$. We now expand in $\delta\beta m$, i.e.

$$\frac{1}{e^{\beta m(s-\delta)} + 1} = \frac{1}{e^{\beta m s} + 1} (1 + \delta\beta m \left(\frac{e^{\beta m s}}{e^{\beta m s} + 1}\right) + \dots) \quad (16)$$

As only small s will contribute in the limit, and keeping only the first two terms in Eq.(16), we obtain

$$\rho = \frac{2\sqrt{2}em^3}{\pi^2} \left(\frac{T}{m}\right)^{3/2} \left(\gamma + \frac{\gamma'}{2} \frac{\mu - m}{T}\right), \quad (17)$$

where

$$\gamma = \int_0^\infty \frac{dt t^2}{e^{t^2} + 1} = 0.3390 \dots, \quad \gamma' = \int_0^\infty \frac{dt}{e^{t^2} + 1} = 0.5360 \dots \quad (18)$$

Thus, when $\mu = m$ the charge density is determined in terms of T . The correction to the anomalous magnetic moment, obtained in a similar way, can be written in the following form:

$$\delta a_e^{\beta,\mu} = \frac{16\sqrt{2}\alpha}{3\pi} \sqrt{\frac{T}{m}} \left(\gamma' + \gamma'' \frac{\mu - m}{T} \right), \quad (19)$$

where

$$\gamma'' = \int_0^\infty \frac{dt e^{t^2}}{(e^{t^2} + 1)^2} = 0.3368 \dots \quad (20)$$

If $\delta a_e^{\beta,\mu}$ is expressed in terms of the particle number density n we obtain

$$\delta a_e^{\beta,\mu} \approx \frac{\alpha}{2\pi} \sqrt{\frac{T}{m}} (1.66 + 66.15 \frac{n}{m^3} (\frac{m}{T})^{3/2}). \quad (21)$$

If $\mu = m$ then $\delta a_e^{\beta,\mu}$ takes the form

$$\delta a_e^{\beta,\mu} \approx 0.539 \frac{16\alpha}{3\pi} \frac{(3\pi^2 n)^{1/3}}{m}. \quad (22)$$

Depending on what is experimentally measured, one may fix T , and either n or μ , and express $\delta a_e^{\beta,\mu}$ in terms of the convenient parameters. The non-analytic behaviour at small T is a potential source of fairly large corrections to $\delta a_e^{\beta,\mu}$. However, at 300 K, a much higher density than is presently accessible in $(g-2)$ experiments is required for this approximation to be valid, as can be seen from Fig.(3).

Case III ($T \ll \mu - m < m$)

This case resembles electrons in a metal with a filled Fermi sea. The Fermi energy is, in this approximation, far below the rest mass of the electrons, and the finite temperature smooths the otherwise well-defined Fermi surface.

We write $\delta < < 1$, $\delta\beta m \gg 1$, $\beta m \gg 1$. The Sommerfeldt-expansion is well suited for a perturbation series in δ and $\delta\beta m$ of the integral $I = \int ds h(s)f(s)$. Here $h(s)$ is a smooth function and $f(s)$ is almost a step function

at $s = \delta$, like the Fermi distribution function. We now follow a well-known technique (see e.g. Ref.[26]) in the derivation of the expansion series:

$$\begin{aligned} I &= f(0) \int_0^\delta ds h(s) \\ &+ \sum_{n=1}^\infty \frac{d^{2n-1}h}{ds^{2n-1}} \Big|_{s=\delta} \frac{1}{(2n)!} (\beta m)^{-2n} \int_{-\delta\beta m}^\infty dx x^{2n} \left(-\frac{d}{dx} \frac{1}{e^x + 1} \right) \\ &= \int_0^\delta ds h(s) + (\beta m)^{-2} \frac{dh}{ds} \Big|_{s=\delta} \frac{\pi^2}{6} + O((\beta m)^{-4}, e^{-\delta\beta m}). \end{aligned} \quad (23)$$

The charge density in this situation, keeping only the first two terms, is given by

$$\rho = 2 \frac{em^3}{\pi^2} \sqrt{2} \left(\frac{\mu}{m} - 1 \right)^{3/2} \left(1 + \frac{\pi^2}{8} \left(\frac{T}{\mu - m} \right)^2 \right), \quad (24)$$

where the $T = 0$ term corresponds to the ordinary filled Fermi sea. Within the same set of approximations we now find that

$$\delta a_e^{\beta,\mu} = \frac{16\sqrt{2}\alpha}{3\pi} \sqrt{\frac{\mu}{m} - 1} \left(1 - \frac{\pi^2}{24} \left(\frac{T}{\mu - m} \right)^2 \right). \quad (25)$$

For $T = 0$ we also note from Eq.(10) that $\delta a_e \equiv \delta a_e^\mu$ takes the form

$$\delta a_e^\mu = -\frac{2\alpha}{3\pi} \int_1^{\mu/m} \frac{dx}{x^2} \sqrt{\frac{x+1}{x-1}} (1 - 6x^2 + x^3). \quad (26)$$

If we use the ideal gas approximation in calculating the relation between the particle density n and the chemical potential μ , we can more accurately write

$$\frac{\mu}{m} = \left(1 + \left(\frac{3\pi^2 n}{m^3} \right)^{2/3} \right)^{1/2}. \quad (27)$$

For small enough densities n we obtain

$$\delta a_e^\mu = \frac{16\alpha}{3\pi} \frac{(3\pi^2 n)^{1/3}}{m}. \quad (28)$$

Thus, the contribution to δa_e^μ can be significant even when n is small. In Fig.(3) we exhibit δa_e^μ for small densities. Eq.(28) has the same form as Eq.(22) even though they apply to different regimes.

3 External magnetic fields

As was already shown by Schwinger a long time ago [27], an external electromagnetic field, slowly varying in space and time, can be treated to all orders in the field at least to first order in α . Here we will make use of Schwinger's technique in order to evaluate the "effective" magnetic moment in the presence of a constant external magnetic field B .

The generating functional $Z[\bar{\eta}, \eta, A_\mu]$ defined by

$$Z[\bar{\eta}, \eta, A_\mu] = \int d[\psi] d[\bar{\psi}] \exp[i \int d^4x (-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi}(i \not{D} - m)\psi - \bar{\eta}\psi + \bar{\psi}\eta)] , \quad (29)$$

where $D = \partial - eA$ is a covariant derivative, describes second-quantized electrons and positrons interacting with a classical electromagnetic field expressed in terms of the vector potential A_μ . The expectation value of ψ (and $\bar{\psi}$) can be fixed by choosing appropriate $\bar{\eta}$ (and η), i.e. $\varphi(x) \equiv \langle \psi(x) \rangle = -i\delta/\delta\bar{\eta}(x) \log Z$ (and similarly for $\bar{\psi}$). The equation of motion for $\varphi(x)$ tells us how the electrons interact with the electromagnetic field which includes effects due to virtual e^+e^- -pairs.

The integral in Eq.(29) can formally be performed:

$$Z[\bar{\eta}, \eta, A_\mu] = \text{Det}(i \not{D} - m) \exp[i \int d^4x (-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - i \int d^4x \int d^4y \bar{\eta}(x) S(x; y) \eta(y))] , \quad (30)$$

where $S(x; y)$ is the external field propagator, i.e.

$$(i \not{D} - m) S(x; y) = \delta^4(x - y) . \quad (31)$$

It follows that $\varphi(x)$ satisfies the Dirac equation in the external field, i.e. $(i \not{D} - m)\varphi(x) = 0$.

The functional determinant $\text{Det}(i \not{D} - m)$ gives rise to an effective action S_{eff} , i.e.

$$S_{eff} = -i \log \text{Det}(i \not{D} - m) = \int d^4x \mathcal{L}_{eff} , \quad (32)$$

where for a constant magnetic field B and after a suitable renormalization which makes $\mathcal{L}_{eff}$ finite,

$$\mathcal{L}_{eff} = -\frac{1}{8\pi^2} \int_0^\infty \frac{ds}{s^3} \exp(-ms) (esB \coth(esB) - 1 - \frac{1}{3}(esB)^2) . \quad (33)$$

The other factors in $Z[\bar{\eta}, \eta, A_\mu]$ describe the classical background field and the fermions which interact with this field. In the non-relativistic limit these lead to an interaction energy $\Delta E_\pm = \mp g e B / 2m$ for the spin parallel or anti-parallel to the direction of the magnetic field, where g is the gyromagnetic factor. $\mathcal{L}_{eff}$ can now physically be interpreted as modifying $\Delta E_\pm$ due to vacuum polarization effects in terms of a field-dependent fine-structure constant $\alpha(B)$ [28], i.e.

$$\alpha(B) = \frac{\alpha}{1 - \alpha X(B)} , \quad (34)$$

where

$$X(B) = \frac{1}{2\pi} \int_0^\infty \frac{dx}{x} \exp(-x \frac{m^2}{B}) (\frac{1}{\sinh^2(x)} - \frac{2}{x} \coth(x) + \frac{2}{3}) . \quad (35)$$

For $B/m^2 \ll 1$, we obtain, to lowest order in B/m^2 , the effective fine-structure constant

$$\alpha(B) = \alpha + \frac{2}{45\pi} \alpha^2 (\frac{eB}{m^2})^2 . \quad (36)$$

We therefore obtain the result that $|\Delta E_\pm| \rightarrow g \sqrt{4\pi\alpha(B)} B / 2m$, i.e. since $\delta a = (g - 2)/2$,

$$\delta a = \frac{4}{45} \alpha^2 (\frac{B}{m^2})^2 . \quad (37)$$

If B_0 is measured in units of Gauss, this corresponds to $\delta a \approx B_0^2 \times 8.6 \times 10^{-32}$, i.e. $\delta a \approx 10^{-11}$ for $B_0 = \mathcal{O}(10^{10})$. Even for the large magnetic fields in neutron stars [29] where $B_0 = \mathcal{O}(10^{12})$, or in white magnetic dwarfs [30] where $B_0 = \mathcal{O}(10^7 - 10^8)$ the corrections to the anomalous magnetic moment are extremely small.

4 Magnetic moment in gravitational field

The Dirac equation will be modified in the presence of a gravitational field. It has recently been claimed [31] that the effective magnetic moment of the muon (and thus also of the electron) is shifted because the energy scales with g_{00} . This correction should then be of the order of the electroweak correction for the muon. However, the effect of scaling the energy and time will affect all frequencies measured in a given frame of reference. Therefore no such effect

will be observed in the experiments where one compares different frequencies [32]. This observation was made in Ref.[33]. To show that this intuitive argument holds we derive the relevant equations of motion in the limit of small gravitational fields.

We sketch the appropriate formalism to deal with a generally covariant Dirac equation. Then we perform a Foldy-Wouthuysen (FW) transformation to extract the first relativistic correction to the equation of motion for the spin and the velocity in a weak gravitational field. One can use this method to calculate higher order correction terms but we shall only consider the lowest order.

4.1 Dirac fermion in curved space

It is wellknown how to generalize the Dirac equation to a curved background space-time [34]. The background Riemannian manifold $\mathcal{M}$, is described by the metric tensor $g_{\mu\nu}$ and the vierbeins $e_A{}^\mu$ which transform locally to an inertial frame, and permits us to go between flat and curved indices. The vierbeins are defined by $g^{\mu\nu} = e_A{}^\mu e_B{}^\nu \eta^{AB}$ where η^{AB} is the flat Minkowski metric. We will use Greek letters for curved space-time indices, capital Roman letters for flat space-time indices and small Roman letters starting with i for curved space indices. Curved γ -matrices are obtained from the flat ones by $\tilde{\gamma}^\mu(x) = e_A{}^\mu(x)\gamma^A$. We have three types of indices - curved Lorentz, flat Lorentz and spinor - and we therefore need three connections to write covariant derivatives of objects with these indices. We call the connections $\Gamma^\alpha{}_{\mu\nu}$, $\Omega^A{}_{\mu B}$, ω_μ respectively (see Appendix A).

The Lagrangian is given by [34]

$$\mathcal{L} = \sqrt{-g} \left\{ \frac{i}{2} [\tilde{\psi} \tilde{\gamma}^\mu (\nabla_\mu \psi) - (\nabla_\mu \tilde{\psi}) \tilde{\gamma}^\mu \psi] - m \tilde{\psi} \psi \right\} . \quad (38)$$

This $\mathcal{L}$ is manifestly real as one sees when taking the complex conjugate and using (in the Dirac representation)

$$(\tilde{\gamma}^\mu)^*{}^T = \gamma^0 \tilde{\gamma}^\mu \gamma^0, \quad (\omega_\mu)^*{}^T = -\gamma^0 \omega_\mu \gamma^0, \quad \tilde{\psi} = \psi^{*T} \gamma^0. \quad (39)$$

Here, $()^*{}^T$ means complex conjugation and transposition. We reserve the notation $()^t$ for Hermitian conjugation which is related to the inner product that will be defined below.

To get the Dirac equation one integrates the second term in Eq.(38) by parts (neglecting surface terms) and uses $\partial_\mu \sqrt{-g} = \sqrt{-g} \Gamma^\nu{}_{\mu\sigma}$ and Eq.(64). This gives

$$\frac{1}{\sqrt{-g}} \frac{\delta \mathcal{L}}{\delta \psi} = (i \tilde{\gamma}^\mu \nabla_\mu - m) \psi = 0 . \quad (40)$$

Electromagnetism is, as usual, included by adding the gauge potential, A_μ , to the derivative. The method we shall use in the next section is based on a Hamiltonian formulation, so we rewrite Eq.(40) as

$$i \partial_0 \psi = \{ -i(\tilde{\gamma}^0)^{-1} \tilde{\gamma}^i (\partial_i - iq A_i + \omega_i) - i \omega_0 + m(\tilde{\gamma}^0)^{-1} \} \psi \equiv \mathcal{H} \psi . \quad (41)$$

So far we have not said anything about the inner product in the Hilbert space, which, of course, is essential to define unitarity. It is obtained by a straightforward generalization of the flat space inner product:

$$\langle \psi | \phi \rangle = \int d\Sigma_\mu \tilde{\psi} \tilde{\gamma}^\mu \phi = \int_{t=const} d^3x \sqrt{-g} \tilde{\psi} \tilde{\gamma}^0 \phi . \quad (42)$$

This is natural because $\langle \psi | \psi \rangle$ is then the integral of the fourth component of the covariantly conserved current: $j_\mu = \tilde{\psi} \tilde{\gamma}^\mu \psi$. Furthermore, it is conserved in time (neglecting surface terms)

$$\begin{aligned} \partial_0 \langle \psi | \psi \rangle &= \int d^3x \{ \partial_0 (\sqrt{-g} j^0) - \partial_i (\sqrt{-g} j^i) \} \\ &= \int d^3x \{ \partial_\mu j^\mu + \Gamma^\mu{}_{\mu\alpha} j^\alpha \} = 0 . \end{aligned} \quad (43)$$

We shall limit ourselves to a Hermitian Hamiltonian, which we have if $\partial_0 g_{\mu\nu} = 0$. Doing so, the operators O_1, O_2, O_3 become Hermitian in the scalar product of Eq.(42):

$$\begin{aligned} O_1 &= i \partial_0, \\ O_2 &= -i(\tilde{\gamma}^0)^{-1} \tilde{\gamma}^i (\partial_i - iq A_i + \omega_i) - i \omega_0, \\ O_3 &= m(\tilde{\gamma}^0)^{-1}. \end{aligned} \quad (44)$$

Before going on, we shall specify in which small parameters we expand. The FW-transformation will naturally give a series in $(1/m)$. Another physically relevant approximation is to consider $(g_{\mu\nu} - \eta_{\mu\nu})$ and $\partial_\alpha g_{\mu\nu}$ as small.

We will also neglect higher derivative terms. These approximations yield a somewhat simpler Hamiltonian,

$$\begin{aligned}\mathcal{H} &= \mathcal{O} + \mathcal{E} \\ &= -i\frac{e_a}{e_0}\alpha^a(\partial_t - iqA_t) - \frac{1}{2}\alpha^a\sigma^{bc}\partial_c e_{ab} - \frac{1}{2}\sigma^{0a}\partial_a g_{00} \\ &\quad + \frac{m}{e_0}\beta\end{aligned}\quad (45)$$

where $\beta = \gamma^0$, $\alpha^a = \gamma^0\gamma^a$, and $\sigma^{AB} = \frac{i}{2}[\gamma^A, \gamma^B]$. $\mathcal{O}$ and $\mathcal{E}$ are the odd and the even parts of $\mathcal{H}$ to be discussed below. We now use small Roman letters starting with a for flat space indices. Terms containing derivatives of vierbeins can be treated as flat in this approximation.

4.2 Foldy-Wouthuysen transformation

The FW-transformation [36] is a unitary transformation which decouples the upper and lower components of the Dirac equation to a given order in $(1/m)$. One can decompose any 4×4 -matrix in one odd part that mixes the upper and lower component, and one even which does not:

$$M = \frac{1}{2}(M + \gamma^0 M \gamma^0) + \frac{1}{2}(M - \gamma^0 M \gamma^0) \equiv \mathcal{E} + \mathcal{O}, \quad (46)$$

where γ^0 is in the Dirac representation. $\mathcal{E}$ and $\mathcal{O}$ can also be defined by $\mathcal{E} = \gamma^0 \mathcal{E} \gamma^0$, $\mathcal{O} = -\gamma^0 \mathcal{O} \gamma^0$.

We want this decomposition to be Hermitian, but that is in general not the case, if γ^0 is not Hermitian (in the curved scalar product). However, requiring the metric to satisfy $g_{0i} = e_{0i} = 0$, we show that $\langle \psi | \gamma^0 \phi \rangle = \langle \gamma^0 \psi | \phi \rangle$, and $\mathcal{E}$ and $\mathcal{O}$ are indeed Hermitian.

The next step is to perform the FW-transformation, $\mathcal{H}_{FW} = U \mathcal{H} U^\dagger$, so that the odd part becomes of order $(1/m)^2$. We are guided by the standard procedure in flat space [35, 36] where one chooses $U = \exp(\beta \mathcal{O}/2m)$. This will, however, not eliminate the lowest order odd part in $\mathcal{H}$. Instead we choose $U = \exp(\sqrt{e_0} \beta (\beta \mathcal{O}/2m) \sqrt{e_0})$, which also is unitary because $\sqrt{e_0} \beta (\beta \mathcal{O}/2m) \sqrt{e_0}$ is anti-Hermitian.

After transforming $\mathcal{H}$ in Eq.(45) to $\mathcal{H}_{FW}$ we get

$$\mathcal{H}_{FW} \simeq \frac{m}{e_0}\beta + \frac{\beta}{4m}(\sqrt{e_0}\mathcal{O}\sqrt{e_0}^0\mathcal{O} + \mathcal{O}\sqrt{e_0}^0\mathcal{O}\sqrt{e_0}^0). \quad (47)$$

This reduces to the ordinary non-relativistic Hamiltonian for spin $1/2$ particles in flat space.

4.3 Spin and velocity precession

In the actual experiments described in e.g. Ref.[32], one measures the relative difference in precession frequency between the spin and the velocity. To see how this is affected by the presence of a gravitational field, one can compute the time evolution of the spin and velocity operators, as commutators with $\mathcal{H}_{FW}$, and identify the new precession frequencies. This procedure will then give an explicitly gauge invariant expression where different terms should be easy to interpret.

It is, however, not completely trivial to find the relevant operators for these equations. We start with the velocity. As, for instance $(-i\partial_t - qA_t)$ is not Hermitian, flat space expressions do not help us much. The safe way is to take the time derivative of the position operator. Both x^i and $\mathcal{H}_{FW}$ are Hermitian and so will their commutator be. But the choice of x^i is not trivial either. If we say that x^i was the position operator before the FW-transformation, then $x_{FW}^i = U x^i U^\dagger$ will be a complicated non-local operator (and the transformation will be useless). The same problem arises, of course, even in flat space and is discussed in Refs.[36, 35]. We follow their resolution and choose x^i to be the position operator *after* the FW-transformation, which has the correct non-relativistic limit.

The same argument applies to the spin operator σ^i , but this time we have to worry about the type index, if it is curved or flat. The index of the coordinates x^i is naturally curved, because it is not the manifold but its tangent space that is transformed by the vierbeins. We will therefore consider the equations of motion of x^i and $\sigma^i = e_a{}^i \sigma^a$.

The external electromagnetic field is chosen to be a constant purely magnetic field in the frame of an observer at rest, i.e. $F_{ij} = \partial_i A_j - \partial_j A_i = \text{const.}$, with curved indices. The calculations that we omit here are somewhat tedious

but straightforward. To the order in $(1/m)$ considered it is only necessary to keep the upper component of $\mathcal{H}_{FW}$ which is rewritten as

$$\mathcal{H}_{FW} = \frac{m}{e_0^0} - \frac{e_a^i e_a^j}{2me_0^0} (\partial_i - iqA_i)(\partial_j - iqA_j) - \frac{1}{2m} \epsilon^{abk} \sigma^k \frac{e_a^i e_a^j F_{ij}}{e_0^0} + \frac{1}{2m} D^i (\partial_i - iqA_i), \quad (48)$$

D^i being a term of order $(\partial_a e_a^i)$ containing σ -matrices. Summation over repeated indices is understood, even when both are downstairs. The velocity becomes

$$\dot{x}^i = i[\mathcal{H}_{FW}, x^i] = \frac{e_a^i e_a^j}{me_0^0} (-i\partial_j - qA_j) + \frac{i}{2m} D^j. \quad (49)$$

We see that in general $\dot{x}^i$ is not parallel to $(-i\partial_i - qA_i)$, which it is in flat space. It also contains the term D^i , mixing spin-up and spin-down components, i.e. the velocity depends on the relative orientation of the spin and the gravitational field.

When computing the acceleration $\ddot{x}^i$, we neglect terms of order $(\partial_a e_a^i \dot{x}^i)$. This approximation is motivated by the smallness of both $\partial_a e_a^i$ and $\dot{x}^i$ in the $(g-2)$ -electron experiments [32]. The velocity in $(g-2)$ -muon experiments is highly relativistic and one must keep all powers of $\dot{x}^i$. We find

$$\ddot{x}^i = \frac{e_a^i e_a^j}{me_0^0} F_{jk} \dot{x}^k + \partial_i e_0^0 + \text{terms of order } (1/m^2). \quad (50)$$

In an analogous way we compute

$$\ddot{\sigma}^i = \frac{e_a^i e_a^j}{me_0^0} F_{jk} \sigma^j + \partial_j e_a^i \sigma^a \dot{x}^j + \frac{1}{2} [\sigma^i, D^j] \dot{x}^j. \quad (51)$$

The second term of Eq.(50) is nothing but the acceleration due to the gravitational field. In the Newtonian approximation we have [37]

$$\partial_i e_0^0 = -2\partial_i g_{00} = -\partial_i \phi \quad (52)$$

where ϕ is the classical gravitational potential. Finite-temperature and relativistic corrections to these equations are discussed in Ref.[10]. For small

enough velocities, Eqs.(50) and (51) look exactly the same. Thus, experiments measuring the difference of spin and velocity precession frequencies will not give an observation of the effect proposed in Ref.[31]. The fact that e_0^0 (or g_{00}) does not equal one, only scales the energy, and thus the time, and does not affect the relative frequencies. We therefore confirm the conclusion of Ref.[33].

There are, however, other correction terms in Eqs.(50) and (51) that are measurable in principle, such as the coupling between spin, velocity and the gravitational field, which are more important in muon experiments where the velocity is relativistic.

5 Final remarks

We have considered three different corrections to the anomalous magnetic moment in Sections 1, 2 and 3, and found that only the effect from finite density in Section 1 is large enough to be experimentally measurable. In order to see to which physical situations the three cases in Section 1 apply, we will consider an experiment by Wesley and Rich [38] and a typical metal at room-temperature. The density in the experiment described in Ref.[38] was of the order of $10^3 - 10^4 \text{ cm}^{-3}$. If we assume the electrons to be thermalized at a temperature about 300 K , then this corresponds to case I and the correction to the anomalous magnetic moment is very small, $\delta a_e^{\mu, \theta} \sim 10^{-21}$. In actual experiments it is not clear to us how to determine the electron temperature and if the electrons are thermalized at all. It may seem strange that the same density, at zero temperature, gives a much larger correction, $\delta a_e^{\mu, \theta} \sim 10^{-11}$, at zero temperature where case III applies. This correction is comparable in size to the recently completed QED α^4 vacuum correction which gives a contribution $\delta a_e = -1.434(138)(\alpha/\pi)^4$ [39]. The reason why the correction is so much larger at zero temperature is that a filled Fermi sea is formed and that happens below the Fermi temperature which is very small in the present case, $T_F = (3\pi^2 n)^{2/3}/(2mk_B) \sim 10^{-8} \text{ K}$. At 300 K , a density of approximately 10^{19} cm^{-3} is needed for case II to be valid and an even larger density for case III. (In the work by Dehmelt et al. [40] one measures the anomalous magnetic moment of a single electron in a Penning trap and in such a case our considerations should not apply.)

In a metal, on the other hand, the typical conduction electron density is of the order of 10^{22} cm^{-3} . We then find that case III is applicable at 300 K and that the correction is $\delta a_e^{\mu,\beta} \sim 10^{-5}$, a correction which e.g. is larger than the $\mathcal{O}(\alpha^2)$ QED vacuum correction. One may question whether the one-loop approximation is valid in such a strongly correlated electron gas.

The general behaviour is thus that $\delta a_e^{\mu,\beta}$ is fairly large at zero temperature and decreases above the Fermi temperature. We observe from Eq.(10) that $\delta a_e^{\mu,\beta}$ becomes negative at a sufficiently high temperature. One must then also include contributions from thermal positrons. In addition to $\delta a_e^{\mu,\beta}$ there is also the negative, density-independent, photon contribution as given by Eq.(1). The density and temperature dependence of $\delta a_e^{\mu,\beta}$ is shown in Fig.(3) where one sees that $\delta a_e^{\mu,\beta}$ depends only on the density below T_F and becomes temperature dependent above T_F .

Finally, we observe that δa_e^{μ} becomes negative for a sufficiently large chemical potential. For a degenerate electron gas, e.g. in the formation process of a neutron star [29], in which case one may have $\mu/m = \mathcal{O}(10)$, we can encounter a situation where $g = 2(1 + \delta a_e^{\mu})$ is close to zero.

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Appendix A

The finite μ and $T = 0$ contributions to the anomalous magnetic moment, i.e. δa_e^{μ} , can be written in the following form as a function of the parameter $\xi = \sqrt{\mu/m} - 1$:

$$\delta a_e^{\mu} = -\frac{4\alpha}{3\pi} \int_0^\xi dx \left(\frac{\sqrt{x^2+2}}{(x^2+1)^2} + (x^2-5)\sqrt{x^2+2} \right)$$

$$= -\frac{4\alpha}{3\pi} (I_1(\xi) + I_2(\xi)) , \quad (53)$$

where

$$I_1(\xi) = \int_0^\xi dx \frac{\sqrt{x^2+2}}{(x^2+1)^2} , \quad (54)$$

and

$$I_2(y) = \frac{22 \log(\sqrt{2}) - 22 \log(\xi + \sqrt{2 + \xi^2}) - 9\xi\sqrt{2 + \xi^2} + \xi^3\sqrt{2 + \xi^2}}{4} . \quad (55)$$

Thus $I_1(\xi)$ is the only integral we have to evaluate numerically in this limit.

Appendix B

The connections in Section 3 are as follows

$$\Gamma_{\mu\nu}^{\alpha} = \frac{1}{2} g^{\alpha\beta} (\partial_\mu g_{\nu\beta} + \partial_\nu g_{\mu\beta} - \partial_\beta g_{\mu\nu}) , \quad (56)$$

$$\Omega_{\mu B}^A = e^A_{\lambda} (\partial_\mu e_B^{\lambda} + \Gamma_{\mu\alpha}^{\lambda} e_B^{\alpha}) , \quad (57)$$

$$\omega_{\mu} = \frac{1}{8} [\gamma^A, \gamma^B] e_A^{\nu} (\partial_\mu e_{B\nu} - \Gamma_{\mu\nu}^{\alpha} e_{B\alpha}) . \quad (58)$$

Curved (flat) indices are raised and lowered with $g_{\mu\nu}$ (η_{AB}), and e^A_{λ} is the inverse of e_A^{λ} . The covariant derivatives are defined in the usual way as

$$\nabla_{\mu} V^{\nu} = \partial_{\mu} V^{\nu} + \Gamma_{\mu\alpha}^{\nu} V^{\alpha} , \quad (59)$$

$$\nabla_{\mu} V^A = \partial_{\mu} V^A + \Omega_{\mu B}^A V^B , \quad (60)$$

$$\nabla_{\mu} \psi = \partial_{\mu} \psi + \omega_{\mu} \psi , \quad (61)$$

$$\nabla_{\mu} \bar{\psi} = \partial_{\mu} \bar{\psi} - \bar{\psi} \omega_{\mu} . \quad (62)$$

With these definitions we have that

$$\nabla_{\mu} g_{\alpha\beta} = 0, \quad \nabla_{\mu} e_A^{\nu} = 0, \quad \nabla_{\mu} \gamma^A = 0 , \quad (63)$$

and we use in particular

$$\nabla_{\mu} \tilde{\gamma}^{\nu} = \partial_{\mu} \tilde{\gamma}^{\nu} + \Gamma_{\mu\alpha}^{\nu} \tilde{\gamma}^{\alpha} + [\omega_{\mu}, \tilde{\gamma}^{\nu}] = 0 . \quad (64)$$

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Figure captions

Fig.1 Scattering in the presence of a thermal heat-bath. A $\times$ at the beginning (end) of a line indicates that the particle in question is an ambient on-shell particle emerging from (going into) the heat-bath. Summing over all quantum numbers which preserve the final state leads to the on-shell part of the thermal real-time propagator denoted by $\times$ on a conventional Feynman propagator.

Fig.2 Vacuum corrections to the QED vertex and thermal exchange or absorption processes which, quantum mechanically, can interfere with the vacuum processes.

Fig.3 Density dependence of $\delta\alpha_e^{\mu,\beta}$ at various temperatures. The kink in a curve corresponds to the density at which the temperature equals the Fermi temperature. At higher densities, the gas becomes degenerate and $\delta\alpha_e^{\mu,\beta}$ depends essentially only on the density.

Figure 2

Figure 1

Figure 3