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Confinement and monopoles in lattice QCD[†]

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Abstract

We investigate monopoles defined by different Abelian projections, in $SU(2)$ lattice gauge theory. We point out that the usual definition of the monopole density is plagued by lattice artifacts, and that the density is not related to confinement. A modification of the monopole density with respect to the vacuum does exist inside the chromoelectric flux tubes connecting heavy $q\bar{q}$ pairs. However this modification depends on the specific Abelian projection used to define monopoles. We argue that the monopole density is not an order parameter for confinement.

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1. A possible mechanism for confinement of colour in QCD is dual superconductivity of type II [1, 2]. The idea is that the QCD vacuum acts as a perfect dielectric medium which expels the chromoelectric field, if it is weaker than some critical value, in the same way as a superconductor expels the magnetic field (dual Meissner effect). For larger values of the field strength, the field penetrates partially in the medium, by formation of Abrikosov flux tubes, up to a second critical value, at which superconductivity is destroyed. The field configuration created by a $q\bar{q}$ pair is a flux tube joining them, or a Nambu string [3, 4]. The static energy is then proportional to the length of the string, $V \sim \sigma R$, which means confinement; σ is known as the string tension.

Dual superconductivity should be produced by vacuum condensation of monopoles or of complexes of monopoles, in the same way as usual superconductivity is produced by condensation of Cooper pairs.

Numerical simulation of the theory on the lattice can provide a test of this mechanism. It has already been shown on a lattice that field configurations produced by a $q\bar{q}$ pair indeed look like chromoelectric flux tubes, with a transverse size of the order of a fraction of a fermi [5]. These configurations have been exposed by use of a cooling technique, which freezes local quantum fluctuations, but preserves long-range correlations and the string tension [5, 6, 7].

A more ambitious program is to identify the monopoles which are supposed to produce the effect, to demonstrate their condensation and to understand the details of the phenomenon.

Monopoles do exist as classical configurations in a gauge theory coupled to a scalar field ϕ^a transforming in the adjoint representation [8, 9]. An Abelian gauge field can be defined in that model [8], which has the monopoles as sources of the magnetic field:

$$F_{\mu\nu} = \hat{\phi}^a G_{\mu\nu}^a - \frac{1}{g} f_{abc} \hat{\phi}^a D_\mu \hat{\phi}^b D_\nu \hat{\phi}^c . \quad (1)$$

Here $G_{\mu\nu}^a$ are the colour components of the field strength tensor, $\hat{\phi}^a \equiv \phi^a / \sqrt{\phi^a \phi^a}$, f_{abc} are the structure constants of the gauge group and D_μ is the covariant derivative. $F_{\mu\nu}$ can also be written in the form [10]:

$$F_{\mu\nu} = M_{\mu\nu} + H_{\mu\nu} , \quad (2)$$

where

$$M_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu , \quad (3)$$

$$B_\mu = \hat{\phi}^a A_\mu^a , \quad (4)$$

and

$$H_{\mu\nu} = \frac{1}{g} f_{abc} \hat{\phi}^a \partial_\mu \hat{\phi}^b \partial_\nu \hat{\phi}^c . \quad (5)$$

In a gauge in which $\hat{\phi}^a$ is oriented, for all x , in some fixed direction of colour space, $H_{\mu\nu} = 0$ and $F_{\mu\nu} = M_{\mu\nu}$. Transforming to such a gauge is called *Abelian projection*.

In QCD there is no fundamental scalar field; a Higgs field could exist at the level of an effective Lagrangian, but there is no known recipe to identify it. 't Hooft [11] suggested that the Abelian projection could be done by a gauge transformation which diagonalizes, for all x , any local operator transforming in the adjoint representation. From purely geometrical arguments the lines where two eigenvalues of the operator are equal can be identified as world lines of monopoles. Of course the resulting monopoles do depend on the choice of

the operator used to perform the Abelian projection. The guess of 't Hooft is that physics should not depend on it.

Pioneering work on $U(1)$ lattice gauge theory [12] has shown that confinement is indeed correlated to monopoles: the response of the ground state in the confined phase to an external field is consistent with that of a superconductor of the second kind, and the density of monopole world lines drops down sharply at the deconfining phase transition. Increasing interest has been devoted to the problem also for $SU(2)$ and $SU(3)$ lattice gauge theories. By analogy with the $U(1)$ case attention has been mostly focused on the study of the density of monopoles and on its behaviour across the phase transition [13–18].

There exist also some investigations into the response to an external field [19, 20]. The response to external colour fields would provide a clear indication of a superconducting behaviour. However the external fields which can be put on a lattice with periodic boundary conditions have strength [21]:

$$a^2 B = \frac{2\pi}{L^2} n, \quad (6)$$

with n integer. The critical field B_c is expected to be $B_c \sim T_c^2$. Since for $SU(2)$

$$\Lambda_L a(\beta) = f(\beta) \simeq \left(\frac{6\pi^2 \beta}{11} \right)^{51/121} \exp \left(-\frac{3\pi^2 \beta}{11} \right), \quad (7)$$

and $T_c/\Lambda_L \simeq 40$, we obtain for the minimal field B which can be put on a lattice:

$$\frac{B}{B_c} \geq \frac{2\pi}{L^2} \frac{1}{(40)^2} f^{-2}(\beta). \quad (8)$$

For $SU(2)$ at $\beta = 2.50$, $f(\beta) \simeq 0.0036$. Therefore, to have $B/B_c \leq 1$ one needs $L \geq 18$: we see that producing a field lower than the critical field requires very large lattices.

The density of monopoles can be defined in different Abelian projections. There is no a priori reason to expect that the density of monopoles is the order parameter of the deconfining phase transition: it is analogous to the density of electrons in a superconductor and it is invariant with respect to $U(1)$ gauge transformations remaining after Abelian projection (see below). We know that the order parameter of superconductivity measures spontaneous breaking of the $U(1)$ gauge invariance and is the vacuum expectation value of some non-gauge-invariant operator like the Cooper pair condensate. However it is interesting to investigate the density of monopoles also in view of testing mechanisms of confinement different from dual superconductivity.

In this paper we study the density of monopoles for different Abelian projections. We find that it is plagued by lattice artifacts, which are less severe in the maximal Abelian gauge. The density has no significant jump at the deconfining phase transition and has no plateau along cooling. A correlation of the monopole density with the flux tubes linking a $q\bar{q}$ pair exists, but it depends on the specific Abelian projection used. Our main conclusion is that the monopole density is not an order parameter for the deconfining phase transition.

2. To perform the Abelian projection, let us choose an operator $\mathcal{R}$ which transforms in the adjoint representation: $\mathcal{R}(x) \rightarrow \Omega(x)\mathcal{R}(x)\Omega^\dagger(x)$, and fix the gauge by requiring $\mathcal{R}$ to be diagonal at each point. Different choices can be made for $\mathcal{R}$. We can use, for example, a plaquette with a definite orientation (corresponding to one of the components of

$F_{\mu\nu}$ in the continuum), or the Polyakov line [13]. It is interesting to note that, if we write $\mathcal{R} = \exp(i\phi^a T^a)$, then ϕ^a can be interpreted as an effective Higgs field.

Gauge fixings of this sort leave a residual $U(1)$ gauge invariance, which, writing the gauge-transformation matrix in terms of Euler angles

$$\Omega(x) = \exp(i\psi\sigma_3/2) \exp(i\theta\sigma_2/2) \exp(i\phi\sigma_3/2) \quad (9)$$

can be easily seen to be parametrized by the ψ angle. The diagonalization leaves also a permutation symmetry which is eliminated by choosing a convention for the ordering of the eigenvalues. The monopoles of this residual $U(1)$ should condense to produce confinement.

A completely different possibility proposed in the literature [15] consists of fixing the gauge by maximizing, on a lattice, the quantity

$$R = \sum_{x,\mu} \text{tr} (\sigma_3 U_\mu(x) \sigma_3 U_\mu^\dagger(x)), \quad (10)$$

where $U_\mu(x)$ are the link variables. In the continuum limit this corresponds to the gauge $D_\mu^3 A_\mu^\pm = \partial_\mu A_\mu^\pm - ig[A_\mu^3 \frac{\sigma_3}{2}, A_\mu^\pm] = 0$.¹

After fixing a gauge (we shall refer to the choices described above as "field-strength", "Polyakov", and "maximal Abelian" gauges), we perform a coset decomposition of $SU(2)$ with respect to $U(1)$ by writing the gauge-transformed links $\tilde{U}$ in the form $\tilde{U}_\mu(x) = c_\mu(x) u_\mu(x)$ where c is parametrized by the non-diagonal generators of the $SU(2)$ group, while u is diagonal (Abelian projection) [13]. The magnetic flux can be constructed from the Abelian u matrices as in lattice electrodynamics [12]: by writing

$$u_\mu(x) = \exp(i\theta_\mu(x)\sigma_3) \quad (11)$$

with $\theta_\mu(x)$ between $-\pi$ and π , we get for the Abelian plaquette

$$U_P = \exp(i\theta_P\sigma_3) \quad (12)$$

with θ_P between -4π and 4π . Define $\bar{\theta}_P$ between $-\pi$ and π and the integer n_P from

$$\bar{\theta}_P = \theta_P - 2\pi n_P. \quad (13)$$

Then $\bar{\theta}_P$ defines the physical flux and n_P gives the number of Dirac strings passing through the plaquette [12]. The oriented sum of n_P over the plaquettes forming a three-dimensional cube gives the number N of monopoles inside the cube. The density of monopoles plus antimonopoles is proportional to the sum over the lattice of the absolute values of N (the sum of N over the lattice would give zero because of monopole-current conservation and periodic boundary conditions).²

3. The first question that we want to address concerns the very consistency of the above definition of the monopoles. With this definition the number of Dirac strings passing through

¹As remarked by 't Hooft [11], in this gauge massless ghosts propagate, which could obscure the physical meaning of the monopole picture.

²The authors of refs. [12, 13, 15, 17] investigated the behaviour of the length of monopole current loops instead of the monopole density. On a symmetric lattice the two quantities are approximately equal. We have checked that also on an asymmetric lattice both quantities lead to (qualitatively) the same results.

a plaquette is bounded to be smaller than or equal to 2, so it is clear that the definition makes sense only when the monopoles are dilute enough. To check this point, one can count the monopole number as obtained from the magnetic flux through cubes of side larger than one lattice spacing. One can define the number of monopoles from non-elementary cubes in such a way that it is additive by definition [12, 16], but of course this is not interesting if we want a check on the consistency of the definition; rather, we define the density ρ_L of monopoles constructed from cubes of size $L = 1, 2, 3, \dots$ using in eq. (12) an $L \times L$ Wilson loop rather than a plaquette and defining again $\bar{\theta}_{L \times L}$ as lying between $-\pi$ and π [16]. It is clear that, in the continuum limit, this gives a legitimate definition of the monopole density, and the density measured with larger cubes should be equal to the one obtained with elementary cubes. In the spirit of Wilson's renormalization group, we can say that we are performing a "blocking" on the operator measuring the monopole number.

In Table 1 we give the monopole densities ρ_1 , ρ_2 , and ρ_3 measured on a 12^4 lattice in the field-strength, Polyakov and maximal Abelian gauges. In both gauges where an operator is diagonalized the results are qualitatively similar: the density of monopoles ρ_L strongly depends on L . This seems quite natural if we realize that the value of ρ_1 is not small, the monopoles are not dilute and lattice artifacts play an important role. In the maximal Abelian gauge ρ_1 is much smaller and, in agreement with our expectation, the spread in the values of ρ_L is not so large. The fact that the monopole density depends on the size of the cubes signals that it is affected by renormalization, the effect being least in the maximal Abelian gauge.

Another crucial point in testing the consistency of the definition of monopoles is whether the density scales properly in the continuum limit. For this purpose we define the "physical" density $\rho_{\text{phys}}/\Lambda_L^3 = \rho_{\text{latt}}(\beta)/f^3(\beta)$, with $f(\beta)$ in eq. (7). Of course, this defines a true physical quantity only if it turns out to be independent of β .

As seen from Table 2, the density does not scale in the field-strength and Polyakov gauge, but seems instead to diverge as a^{-3} . This can be considered as an indication of additive renormalization. It is well-known that on the lattice any local operator undergoes a finite renormalization, which disappears in the continuum limit but can be important at the finite values of β where the Monte Carlo simulation is performed. In various cases, as for the gluon condensate [22, 23] or the topological charge [24, 25], these renormalizations are known to be numerically relevant.

The scaling of the monopole density defined in the maximal Abelian gauge has been studied in [17] on 8^4 , 10^4 , and 12^4 lattices. On a 12^4 lattice, results not incompatible with scaling have been observed. Since the results depend strongly on the lattice size, after confirming the results of [17] we measured the density also on a 16^4 lattice. Our results are shown in the last column of Table 2 and are not incompatible with scaling, on the same level as in [17]. This is consistent with the indication that renormalization effects are less important in the maximal Abelian gauge. The value of the physical density is

$$\rho_{\text{phys}}/\Lambda_L^3 \simeq 3 \times 10^5, \quad (14)$$

which corresponds to about 10 – 15 monopoles/fm³.

After having summarized our understanding of problems with the lattice definition of the monopole density we now turn to its role in the confinement mechanism. The real test should be to measure the order parameter, the monopole condensate, analogous to the condensate

of Cooper pairs. At present we are not able to identify it. However, if monopoles are related to confinement in the same way as in the $U(1)$ case [12], we could expect the density of monopoles to drop at the deconfinement phase transition.

We studied the problem on an asymmetric $12^3 \times 4$ lattice, where the deconfinement phase transition is known to occur at $\beta \simeq 2.29 \pm 0.01$ and which is usually considered to be large enough to qualitatively reproduce the physics of confinement. In Table 3 we report the behaviour of the monopole density across the phase transition, for the three gauges. No relevant drop is observed. Such a drop could be masked by lattice artifacts in the gauges I and II^3 , but it should be visible in the gauge III , where scaling of the monopole density indicates that renormalization effects are not so large.

Another way of investigating the role of monopoles in the confinement mechanism is by studying their behaviour along the cooling process. The string tension and flux tubes show a plateau vs. cooling and drop at the point where configurations responsible for confinement are destroyed by the cooling process [5]. If monopole density is correlated with string tension and flux tubes, it should behave in a similar way. ⁴

In Fig. 1 we show the typical behaviour of the monopole densities obtained from cubes of different sizes in the maximal Abelian gauge. We see no plateau vs. cooling, no behaviour similar to that of the string tension.

A possible explanation of the behaviour under cooling could again be given in terms of renormalization [17, 18]. Since cooling eliminates these renormalizations, such a process could hide the existence of a plateau. However, in the maximal Abelian gauge, where renormalizations are supposed to be small, a plateau should be expected, which is not observed.

4. The results of the previous section indicate that it is worthwhile to look at quantities in which at least additive renormalization terms are expected to cancel. We looked at the correlation of the number of monopoles with a pair of Polyakov lines:

$$\Gamma(\vec{x}_\perp) = \frac{\langle \text{tr}(P(\vec{x}_1)P(\vec{x}_2)) | N(\vec{x}, t) | \rangle}{\langle \text{tr}(P(\vec{x}_1)P(\vec{x}_2)) \rangle} - \langle | N(\vec{x}, t) | \rangle \quad (15)$$

with $\vec{x} = \frac{\vec{x}_1 + \vec{x}_2}{2} + \vec{x}_\perp$, where $\vec{x}_\perp$ is the displacement in the direction orthogonal to the axis joining the $q\bar{q}$ pair, and the Polyakov line is in the time direction. $\Gamma(\vec{x}_\perp)$ is similar to the correlations which have been used in studying the flux tubes [5]. In physical terms, it monitors how the density of monopoles is influenced by the presence of a $q\bar{q}$ pair.

We studied the behaviour of this quantity on 12^4 and $12^3 \times 4$ lattices at a distance between the lines equal to 4. Cooling was vital to reduce the errors. The scaling of $\Gamma(0)$ vs. β (at cooling step 8) on 12^4 lattice is shown in Table 4, for the Polyakov gauge: the correlated density seems to scale better than the density itself. For the maximal Abelian gauge, Γ is of the order of magnitude of the uncorrelated monopole density, which again indicates that additive renormalizations, if present, are small.

Fig. 2 summarizes results for two typical values of β – one in the confined, the other in the deconfined phase (on $12^3 \times 4$ lattice). In the confined phase the behaviour is similar

³The behaviour of the monopole density vs. β in both these gauges has been studied in [13] for $SU(2)$. A small, 5^4 , lattice was used. Our Monte Carlo reproduces accurately the results presented in [13].

⁴Some evidence in favour of this correlation has been reported recently in [16].

to what we have found for flux tubes [5]: a sizeable signal is still observed after 8 – 10 cooling steps, later it vanishes. The dependence on $\vec{x}_\perp$ is similar to the dependence of the parallel chromoelectric field in a flux tube, as shown in Fig. 3. However, though the absolute magnitude of the correlation in all three gauges is larger in the confined than in the deconfined phase, it is still dependent on the Abelian projection used, having e. g. different signs in field-strength and maximal Abelian gauge. Nevertheless, our results (see also Table 5) indicate that the correlated density (measured in physical units) drops through the phase transition. This drop seems not present in the Polyakov gauge, but to convincingly disprove the drop we would need much higher statistics.

5. Summarizing, the study of the role of monopoles in the confinement mechanism proves to be a difficult problem. First of all, the proper order parameter for the underlying physical mechanism is not known. The monopole density, studied in the present work and by other authors, was chosen by analogy with the $U(1)$ case where it turns out to be strongly correlated to confinement. Second, the study of the monopole density leads to difficult technical problems, which have been clearly illustrated in previous sections.

Our main results can be summarized as follows:

- The density of monopoles defined via the Abelian projection shows a strong dependence on the chosen gauge and on the size of cubes used in its definition. The density is affected by lattice artifacts, as appears from the fact that the physical density (independent of β) cannot be defined in the field-strength and Polyakov gauge. The density defined in the maximal Abelian gauge seems to be less affected by this problem, and a physical density can be extracted.
- The density of monopoles neither shows correlation with the string tension along cooling, nor a drop across the deconfinement phase transition.
- The monopole density is correlated with the flux tube in the presence of the $q\bar{q}$ pair. However, for different choices of Abelian projection the correlations differ (even in sign). This means that even after partially eliminating lattice artifacts by using correlations, the monopole densities do not become independent of the gauge chosen for Abelian projection. This repeatedly indicates that the monopole density is not an order parameter for the superconducting state.

Our results show that the monopole density in $SU(2)$ lattice gauge theory is, contrary to the $U(1)$ case, not correlated to confinement. We do not think that our results are inconsistent with the picture of confinement by monopole condensation, but it seems necessary, in order to clarify the problem, to understand the real order parameter of colour superconductivity.

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β	ρ_1	ρ_2	ρ_3
I			
2.40	0.3224(5)	0.0576(1)	0.0173(1)
2.45	0.3223(4)	0.0576(1)	0.0173(1)
2.50	0.3224(6)	0.0575(1)	0.0173(1)
II			
2.40	0.3286(5)	0.0571(1)	0.0173(1)
2.45	0.3246(5)	0.0569(1)	0.0173(1)
2.50	0.3186(6)	0.0567(1)	0.0173(1)
III			
2.40	0.0280(3)	0.0192(2)	0.0123(1)
2.45	0.0197(3)	0.0148(2)	0.0108(1)
2.50	0.0138(3)	0.0110(2)	0.0090(1)

Table 1: The (raw) monopole densities determined from cubes of size 1^3 , 2^3 , and 3^3 on a 12^4 lattice. (I – field-strength gauge, II – Polyakov gauge, III – maximal Abelian gauge.)

β	I (12^4 lattice)	II (12^4 lattice)	III (16^4 lattice)
	$\rho_{\text{phys}}/\Lambda_L^3 [10^6]$	$\rho_{\text{phys}}/\Lambda_L^3 [10^6]$	$\rho_{\text{phys}}/\Lambda_L^3 [10^5]$
2.40	3.311(5)	3.375(5)	2.865(16)
2.45	4.829(6)	4.863(7)	2.947(24)
2.50	7.051(13)	6.968(13)	3.047(33)
2.55	–	–	2.993(42)
2.60	–	–	2.776(65)

Table 2: The physical monopole densities vs. β . (I – field-strength gauge, II – Polyakov gauge, III – maximal Abelian gauge.)

β	I	II	III
	$\rho_{\text{phys}}/\Lambda_L^3 [10^5]$	$\rho_{\text{phys}}/\Lambda_L^3 [10^5]$	$\rho_{\text{phys}}/\Lambda_L^3 [10^5]$
2.20	7.51(2)	5.96(3)	1.91(2)
2.25	10.80(2)	7.95(2)	2.23(2)
2.40	33.07(5)	21.44(9)	2.45(3)

Table 3: The physical monopole densities vs. temperature, on a $12^3 \times 4$ lattice. (I – field-strength gauge, II – Polyakov gauge, III – maximal Abelian gauge.)

β	$\Gamma_{\text{phys}}/\Lambda_L^3 [10^5]$
2.40	7.3(1.3)
2.45	8.1(0.9)
2.50	12.0(2.6)
2.52	10.7(1.3)

Table 4: Γ vs. β , $x_\perp = 0$, cooling step 8 (Polyakov gauge).

β	I	II	III
	$\Gamma_{\text{phys}}/\Lambda_L^3 [10^4]$	$\Gamma_{\text{phys}}/\Lambda_L^3 [10^4]$	$\Gamma_{\text{phys}}/\Lambda_L^3 [10^4]$
2.20	2.0(3.1)	8.0(3.9)	-5.2(1.4)
2.25	1.9(1.1)	8.2(1.4)	-3.3(0.5)
2.40	0(0.2)	8.1(3.7)	-0.3(0.1)

Table 5: The physical monopole densities correlated with Polyakov loops vs. temperature, on a $12^3 \times 4$ lattice (cooling step 7). (I – field-strength gauge, II – Polyakov gauge, III – maximal Abelian gauge.)

Figure captions

Figure 1:

The (raw) monopole densities, defined from cubes of size L ($\times$: $L = 1$, $\diamond$: $L = 2$, $\square$: $L = 3$), vs. cooling ($\beta = 2.45$, 12^4 lattice, maximal Abelian gauge).

Figure 2:

(a) Correlation of monopole density with a pair of Polyakov lines, $\Gamma(\vec{x}_\perp)$, at $\beta = 2.25$ on a $12^3 \times 4$ lattice (confined phase).

[$\Gamma(0)$ for Polyakov gauge ($\times$), and for maximal abelian gauge ($\square$); $\Gamma(1)$ for Polyakov gauge ($\diamond$), and for maximal Abelian gauge ($\bullet$).]

(b) The same as (a) at $\beta = 2.40$ (deconfined phase).

Figure 3:

Correlation of monopole density with a pair of Polyakov lines $\Gamma(\vec{x}_\perp)$ vs. $\vec{x}_\perp$, in the Polyakov gauge (12^4 lattice, $\beta = 2.45$).

Fig. 1

Fig. 2a

Fig. 2b

Fig. 3