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TESTS OF THE INFINITE MASS EFFECTIVE THEORY AND PROPERTIES OF THE CHARMING AND BEAUTIFUL MESONS

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ABSTRACT

Using an operator product expansion (OPE) in the inverse of the heavy quark mass M_Q and a dispersive approach, we recover the expressions of the pseudoscalar decay constant f_P , the vector coupling γ_V and some of the form factors of the semi-leptonic B decays predicted by the infinite mass effective Theory (IMET). We find that these observables are governed by the universal $\langle \bar{q}q \rangle$ light quark condensate. We evaluate the corrections induced by finite values of M_c and M_b and conclude that the IMET $M_Q^{-1/2}$ scaling law for f_P is affected by large $1/M_Q$ corrections. Using the updated values of the QCD input parameters, we reestimate f_B and critically review the QCD spectral sum rules (QSSR) results for f_B and f_D . We conclude that the optimum QSSR estimates lead to the average : $f_B = (1.69 \pm .20)f_\pi$ and $f_D = (1.31 \pm .12)f_\pi$. The couplings and masses of some other mesons of the B family are reviewed. We also find that γ_V satisfies the IMET $M_Q^{3/2}$ scaling law, while f_P obeys the modified formula in units of GeV : $f_P \simeq .57 M_Q^{-1/2} (1 - 2.22/M_Q + 1.67/M_Q^2)$, for $M_Q \geq M_c$. We predict the flavour independence : $M_P - M_Q \simeq .7$ GeV and $M_V - M_Q \simeq .8$ GeV of the mass differences between the mesons and the corresponding quark. Finally, we find that the $B \rightarrow D e \nu_e$ form factor $f_+(0) \simeq (.55 \pm .10)$ deviates from the IMET expectation.

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1. INTRODUCTION

For many years, there has been some considerable interest in the theory of weak matrix elements involving one light q and one heavy Q quark. However, for the simplest matrix element :

$$\langle 0 | \bar{Q} \gamma^\mu \gamma^5 q | P \rangle = \sqrt{2} f_P q^\mu, \quad (1.1)$$

where P is a pseudoscalar meson, there is no consensus in the theoretical determinations of the decay constant f_B of the B meson from some different non-perturbative aspects of QCD, while various methods seem to agree that for the D meson¹⁾⁻²⁴⁾ :

$$f_D \simeq (1.2 - 1.5) f_\pi. \quad (1.2)$$

More recently, the new lattice results^{5),25)-27)} on f_B give the value :

$$f_B(lattice) \simeq (1.6 - 3.2) f_\pi. \quad (1.3)$$

This result obtained in the static approximation ($M_b \rightarrow \infty$) is in reasonable agreement with the previous QCD spectral sum rules (QSSR) estimate obtained from different types of sum rules (exponential, moments, non-relativistic) based on the sum rule variables and continuum threshold stability criteria¹⁵⁾⁻¹⁸⁾. The QSSR prediction obtained under this condition is independent on the form of the sum rules and is uniquely determined to be¹⁷⁾ :

$$f_B(QSSR) = (1.6 \pm .1) f_\pi, \quad (1.4)$$

using the updated value of the pole mass¹⁷⁾ :

$$M_b = (4.59 \pm .10) GeV, \quad (1.5)$$

obtained by simultaneous fits of the B, B^* and Υ masses. We have multiplied the error in Ref.17) by a factor two in order to include the systematic uncertainty as the weighted average done in Ref.17) has decreased the error. Allowing larger errors for M_b are unlikely as with the values of M_b outside this range, the method loses its predictive power on the spectra (see section 4). Let me also mention that the estimate of the matrix elements of the $B^0 - \bar{B}^0$ mixing using QSSR within the stability criteria¹⁸⁾ leads to a similar value of f_B once one assumes the value of the B_B weak factor to be 1 as expected from the

vacuum saturation hypothesis. Moreover, other values of f_B ⁹⁾⁻¹³⁾ from QSSR smaller than the results in Eq.1.4 have been obtained outside the region of stability of the QCD continuum threshold (see section 5). Therefore, these results cannot be reliable as they will be strongly affected by the continuum model. One can see that the previous results in Eqs.1.2 to 1.4 do not satisfy the scaling behaviour :

$$f_B = \sqrt{\frac{M_c}{M_b}} f_D, \quad (1.6)$$

expected by extrapolating the infinite mass effective theory (IMET)²⁸⁾⁻³¹⁾ results down to the charm mass. An attempt to understand the origin of this discrepancy is the first motivation of this paper. Our second motivation particularly consists of a re-estimate of the decay constant f_B and of a critical review of the existing results from QSSR. Our last motivation is the estimate of the semi-leptonic B decay form factors.

In section 2 of this paper, we derive some of the IMET results within an OPE in $1/M_Q$. In section 3, we evaluate the corrections to IMET induced by finite quark masses and the role of the splitting between the ground state and its radial excitation. In section 4, we re-estimate the decay constant f_B and the coupling γ_{B^*} by using QSSR moments within a $1/M_Q$ expansion. In section 5, we make a critical review of the existing QSSR estimates of the B, D and B-like mesons couplings. In section 6, we study the mass dependence of these couplings and of the splitting between the quark and the meson masses. In section 7, we estimate the form factor of the $B \rightarrow D e \nu_e$ semi-leptonic decay and the associated CKM mixing angle. Section 8 contains our summary and conclusions.

2. OPE DERIVATION OF THE IMET RESULTS

a) Decay constants of pseudoscalar mesons :

Let us first remind that in IMET²⁸⁾⁻³¹⁾, the heavy quark has an infinite mass and the long-distance physics can be described by an effective theory with a static colour source. In this limit, we have a new spin-flavour global symmetry and a selection rule for the heavy quark velocity³²⁾⁻⁴⁰⁾. In particular, the expressions of the hadronic form factors simplify considerably as they can be expressed in terms of one universal function (the so-called ³⁹⁾Wisgur function). The IMET expression of the Green's function is obtained by

performing the functional over the heavy quark. For instance, in IMET, the axial-vector matrix element defined in Eq.1.1:

$$G = \langle 0 | T Q_\alpha(x) (\bar{Q} \gamma^\nu \gamma^5 q)(0) q_\beta(z) | 0 \rangle, \quad (2.1a)$$

reads in the heavy quark rest frame³¹⁾ :

$$G'(\mu) = \exp(-i M_Q x^0) \delta^3(\vec{x}) \frac{1}{2} (1 + \gamma^0) \gamma^\nu \gamma^5 C(\mu) \hat{G}(\mu), \quad (2.1b)$$

where $C(\mu)$ is a Wilson coefficient and $\hat{G}(\mu)$ is the Green's function of the light quark and external gluon fields :

$$\hat{G}(\mu) = Z_G \langle 0 | T \exp \left(-ig \int d^n w J^\nu(w) A_\nu^a(w) \frac{\lambda_a}{2} \right)_{\alpha\gamma} q_\gamma(0) \bar{q}_\beta(z) | 0 \rangle. \quad (2.2)$$

Z_G is an external renormalization constant. $J^\nu(x)$ is the source function and $\hat{G}(\mu)$ is independent of M_Q . By noting that $\gamma^\nu \gamma^5$ factors out in the analysis, the results in IMET are universal for $\bar{Q}q$ bilinears. Eq.2.2 shows the importance of the light quark contribution which can be reduced to the $\langle \bar{q}q \rangle$ condensate for $z=0$.

We try an alternative derivation of the IMET results from the analysis of the two-point correlator :

$$\Psi_5(q^2) \equiv i \int d^4 x e^{iqx} \langle 0 | T J_5^Q(x) (J_5^Q)^\dagger(0) | 0 \rangle, \quad (2.3)$$

associated with the pseudoscalar current :

$$J_5^Q(x) = (m_q + M_Q) : \bar{q} \gamma^5 Q :, \quad (2.4)$$

built from a light quark q and a heavy quark Q . It is known that the correlator obeys the Källen-Lehmann representation :

$$\Psi_5(q^2, M_Q^2) = \int_{(M_Q+m_q)^2}^{\infty} \frac{dt}{t - q^2 - i\epsilon} \frac{1}{\pi} \text{Im} \Psi_5(t). \quad (2.5)$$

On the other hand, for $M_Q^2 \gg \Lambda^2, -q^2, m_q^2$ where Λ is the perturbative QCD scale , one can write the OPE à la SVZ⁴¹⁾ :

$$\Psi_5(M_Q^2, \nu^2) = \int_{(M_Q+m_q)^2}^{\nu^2 \rightarrow \infty} \frac{dt}{t} \frac{1}{\pi} \text{Im} \Psi_5^{\text{pert}}(t) + \sum_n \frac{1}{(M_Q^2)^n} C_{2n}(\mu) \langle O_{2n}(\mu) \rangle, \quad (2.6a)$$

where the first term is the usual perturbative contribution (unit operator) from the bare quark loop written in a dispersive form. The remaining contributions are due to the non perturbative operators $\langle O_{2n} \rangle$ of dimension $2n$. For the case considered here, these operators are the quarks and gluon vacuum condensates ($\langle \bar{q}q \rangle$, $\langle \alpha_s G^2 \rangle$, ...). C_{2n} are their Wilson coefficients which can be evaluated perturbatively in QCD. μ is the subtraction scale which separates the short-and long-distance dynamics and which should disappear in the physical quantities. For $M_Q = \nu$ going to infinity, one can notice in this dispersive representation, that the unit perturbative operator will not contribute. This observation can also be seen by evaluating $\Psi_5^{pert}(M_Q^2, \nu^2)$ using Feynmann rules techniques. Using dimensional regularization, one finds that:

$$\Psi_5^{pert}(M_Q^2, \nu^2) = -\frac{1}{8\pi^2} M_Q^4 \log\left(\frac{M_Q^2}{\nu^2}\right), \quad (2.6b)$$

which indeed shows that at the subtraction point $M_Q^2 = \nu^2$, used also in IMET for avoiding large log contributions, the unit operator does not contribute (we shall come back to this point later). Therefore, we are left with only the lowest dimension non-perturbative contributions. In this case, one obtains :

$$\Psi_5(\infty) = -M_Q \langle \bar{q}q \rangle + \frac{\langle \alpha_s G^2 \rangle}{12\pi}, \quad (2.7)$$

where the quark mass can be understood as the pole mass which has no anomalous dimension. If we saturate the spectral function by a narrow resonance, one obtains by neglecting the mass difference between the quark and the meson :

$$2f_P^2 = -\frac{1}{M_Q} \langle \bar{q}q \rangle \left(1 - \frac{\langle \alpha_s G^2 \rangle}{12\pi \langle M_Q \bar{q}q \rangle}\right). \quad (2.8)$$

We introduce the quark condensate anomalous dimension via its Renormalization Group Improved (RGI) expression⁴²⁾ :

$$\langle \bar{q}q \rangle (M_Q^2) = -\hat{\mu}^3 \left(\log \frac{M_Q}{\Lambda}\right)^{2/-\beta_1}, \quad (2.9)$$

where for the u and d quarks¹⁷⁾ $\hat{\mu} = 189 \text{ MeV}$ and $\beta_1 = -\frac{1}{2}(11 - \frac{2}{3}n_f)$ for n_f flavours. By assuming that IMET works at the c mass, one then deduces the well-known IMET ratio including anomalous dimensions^{30),31)} :

$$f_B = f_D \left(\frac{\alpha_s(M_b)}{\alpha_s(M_c)}\right)^{1/-\beta_1} \sqrt{\frac{M_c}{M_b}}. \quad (2.10)$$

Eq.2.8 would lead to the low value :

$$f_B = 30 MeV, \quad (2.11)$$

which regarding the positivity of the perturbative spectral function in Eq.2.6 can be safely considered as a lower bound. Moreover, it is difficult to do a clean comparison with the IMET prediction, as the absolute value of f_B in the existing wide literature on the subject is obtained through Eq.2.10 with an input value of f_D . Let us now compare the relation in Eq.2.8 with the non-relativistic result of the early days^{19),20),43)}. Then one would have obtained the value of the B meson wave function at the origin :

$$|\phi(0)|^2 = -\frac{1}{12} < \bar{q}q >, \quad (2.12)$$

which is flavour-independent. This is in agreement with potential approach expectations⁴⁴⁾ where the wave function is governed by the reduced mass which is here the light quark constituent mass. A more valuable test of the IMET and non-relativistic predictions requires a reliable value of the wave function controlling f_B . A recent attempt in this direction with the inclusion of the one gluon exchange effect in the B meson wave function is in Ref.21). However, the results there and in some other potential-like approaches¹⁹⁾⁻²³⁾ suffer from its high sensitivity on the QCD α_s value and from the high value ($\alpha_s \simeq 3$) deduced from the fitting procedure.

b) Couplings of vector mesons :

In this case, we shall be concerned with the two-point correlator :

$$\Pi^{\mu\nu}(q^2) \equiv i \int d^4x e^{iqx} < 0 | T J_Q^\mu(x) J_Q^{\nu\dagger}(0) | 0 > = -(g^{\mu\nu} q^2 - q^\mu q^\nu) \Pi(q^2), \quad (2.13)$$

associated to the vector current :

$$J_Q^\mu(x) =: \bar{q} \gamma^\mu Q :. \quad (2.14)$$

An analysis similar to the one in the pseudoscalar channel is also done here. The coupling of the vector meson is introduced as :

$$< 0 | J_Q^\mu | V > = \sqrt{2} \frac{M_V^2}{2\gamma_V} \epsilon^\mu. \quad (2.15)$$

For the infinite quark mass value, one obtains by retaining the lowest dimension condensate contributions :

$$\frac{1}{\gamma_V^2} = -\frac{2}{M_Q^3} \langle \bar{q}q \rangle \left(1 + \frac{\langle \alpha_s G^2 \rangle}{12\pi \langle M_Q \bar{q}q \rangle}\right), \quad (2.16)$$

which well satisfies the IMET scaling behaviour.

c) The form factors of $B \rightarrow D^{(*)} e \nu_e$ semi-leptonic decays :

Let us first discuss the $B \rightarrow D e \nu_e$ case. We shall be concerned with the vertex function:

$$V^\mu(k_1, k_2, q) = - \int d^4x \int d^4y e^{ik_1x - ik_2y} \langle 0 | T J_5^c(x) J^\mu(0) J_5^b(y) | 0 \rangle, \quad (2.17)$$

where $J^\mu = \bar{c}\gamma^\mu b$ and $J_5^Q = (m_q + M_Q)\bar{q}\gamma_5 Q$. V^μ has the general Lorentz decomposition :

$$V^\mu(k_1, k_2, q) = V_+(q)(k_1 + k_2)^\mu + V_-(q)(k_1 - k_2)^\mu \quad (2.18)$$

and obeys the double dispersion relation :

$$V_\pm(k_1, k_2, q) = -\frac{1}{4\pi^2} \int_{(M_b+m_q)^2}^{\infty} \frac{dt'}{t' - k_2^2} \int_{(M_c+m_q)^2}^{\infty} \frac{dt}{t - k_1^2} \text{Im} V_\pm(t', t, -q^2) + \text{subtractions}. \quad (2.19)$$

$q = k_2 - k_1$ is the momentum transfer. We shall evaluate the LHS in the limit where both M_c^2, M_b^2 are infinite and $M_c^2 = M_b^2$, i.e $q_{max}^2 = 0$. In this way, one obtains :

$$V_+(0) = \frac{1}{M_b} \langle \bar{q}q \rangle, \quad (2.20)$$

where, as in the previous case, the perturbative loop graph does not contribute at $M_Q^2 = \nu^2$. Saturating the vertex with the B and D mesons, we can deduce for $-k_i^2 \ll M_Q^2$:

$$V_+(0) = -2f_B f_D f_+(0). \quad (2.21)$$

Using the expression of the decay constants in IMET, one obtains :

$$f_+(0) = 1, \quad (2.22)$$

in agreement with the Ademollo-Gatto⁴⁵⁾ theorem and the IMET result in this limit^{32),33)}. In order to go further, we can analyze in the same way the parity violating part of the $B \rightarrow D^* e \nu_e$ amplitude defined as :

$$\langle D^*(k_1) | V_{\mu\nu} | B(k_2) \rangle = 2i\epsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta V(q^2). \quad (2.23)$$

By retaining the lowest non-perturbative contribution, one gets :

$$V(0) = \frac{1}{2} \langle \bar{q}q \rangle \frac{1}{M_b^3}, \quad (2.24)$$

while a saturation of the spectral function by the D^* and B poles leads to :

$$V(0) = -\frac{f_B}{\gamma_{D^*}} \frac{F(0)}{2M_B}, \quad (2.25)$$

for $M_B = M_{D^*}$, where we have introduced the dimensionless form factor F for convenient comparison with the IMET result. Using the previous expressions of the couplings, one can deduce the relation :

$$F(0) = 1, \quad (2.26)$$

for $M_B = M_{D^*}$. The expressions of the other form factors entering in the $B \rightarrow D^* e \nu_e$ decay can be derived in a similar way where their QCD expressions in the infinite mass limit are also governed by the light quark condensate . This can be a further indication of the universality of the form factors for infinite quark mass values. The agreement of the previous results of decay constants and form factors with those predicted by IMET is certainly encouraging for a better control of the approximations used in IMET when the results are applied to the real world.

3. FINITE QUARK MASS CORRECTIONS TO f_B AND γ_{B^*}

For a finite value of the quark mass, the effect of the perturbative dispersive integral in Eq.2.4 cannot be ignored, whereas the effects of high dimensions non-perturbative condensates must be included. However, life is not so simple, as the perturbative QCD expression of the correlator is subtraction-scale dependent. Its spectral function reads for $m_q = 0$ (hereafter, we shall neglect the light quark mass)¹⁷⁾ :

$$Im\Psi_5^{pert}(t) = \frac{3}{8\pi} M_Q^2 t(1-x)^2 \left(1 + \frac{4}{3} \left(\frac{\alpha_s}{\pi} \left(\frac{9}{4} + 2Li_2(x) + \log x \log(1-x) \right. \right. \right. \\ \left. \left. \left. 2Li_2(x) - \frac{3}{2} \log\left(\frac{1}{x} - 1\right) - \log(1-x) + x \log\left(\frac{1}{x} - 1\right) - \frac{x}{1-x} \log x \right) \right) \right), \quad (3.1)$$

where $x = \frac{M_Q^2}{t}$ and $Li_2(x) = -\int_0^x \frac{dx}{x} \log(1-x)$. This scale ambiguity can be avoided by working with high q^2 -derivatives of the correlator. The lowest superconvergent quantity is :

$$M_P^{(1)} \equiv \frac{1}{2!} \left(\frac{d}{dq^2} \right)^2 \Psi_5(q^2)|_{q^2=0} = \int_{M_Q^2}^{\infty} \frac{dt}{t^3} \frac{1}{\pi} \text{Im} \Psi_5(t). \quad (3.2)$$

Using the QCD expression of the correlator known in the literature, saturating the spectral function by the B meson and using the positivity of the higher states contributions to the spectral function, we can deduce the bound^(7),8):

$$f_B^2 \leq M_B^2 \left(\frac{1}{4\pi^2} \left(1 + \left(\frac{\alpha_s}{\pi} \right) \frac{2}{3} \left(\frac{1}{4} + \frac{\pi^2}{3} \right) \right) + NPT_P^{(1)} \right), \quad (3.3)$$

with :

$$NPT_P^{(1)} = -\frac{1}{M_b^3} \langle \bar{q}q \rangle \left(1 - \frac{\langle \alpha_s G^2 \rangle}{12\pi M_b \langle \bar{q}q \rangle} - \frac{3 M_0^2}{2 M_b^2} - \frac{60\pi}{4} \rho \frac{\alpha_s \langle \bar{q}q \rangle}{M_b^3} \right), \quad (3.4)$$

where¹⁷⁾ : $M_0^2 = .8 GeV^2$ is the scale of the mixed condensate defined as $g \langle \bar{q} G_a^{\mu\nu} \sigma_{\mu\nu} \frac{\lambda^a}{2} q \rangle = M_0^2 \langle \bar{q}q \rangle$, $\langle \alpha_s G^2 \rangle = (.06 \pm .02) GeV^4$ and $\rho = 3 \pm 1$ indicates the deviation from the vacuum saturation of the four-quark condensate. Eq.3.3 indicates that the role of the perturbative term is important for finite values of the quark mass, while in the extrapolation of the IMET result down to M_c , the non-perturbative series blows up. In fact, the contribution of the mixed (four-quark) condensates are -44% (70%) of the $\langle \bar{q}q \rangle$ condensate. One can improve the result in Eq.3.3 by including the continuum contribution to the spectral integral which we parametrize using the QCD expression from a threshold t_c^{min} , where t_c^{min} is approximately the mass squared of the radial excitation. We introduce the parameter Δ_b as :

$$t_c^{min} = (M_b + \Delta_b)^2. \quad (3.5)$$

Using the lowest order perturbative expression given in Eq.3.1, integrating the spectral integral from M_Q^2 to t_c and using a Δ_b/M_b expansion, we obtain the constraint from Eq.3.2:

$$f_B^2 = \frac{\Delta_b^3}{M_b} \left(\frac{M_B}{M_b} \right)^2 \frac{1}{\pi^2} \left(1 - \frac{9}{2} \left(\frac{\Delta_b}{M_b} \right) + \frac{51}{4} \left(\frac{\Delta_b}{M_b} \right)^2 \right) + (NPT_P^{(1)}) M_B^2, \quad (3.6)$$

where $NPT_P^{(1)}$ are the non-perturbative contributions in Eq.3.4. A generalization of this result for higher numbers of derivatives has been also discussed in Ref.46). Analogous constraint can be derived for the vector coupling by studying the first superconvergent quantity :

$$M_V^{(1)} \equiv \frac{1}{2!} \left(\frac{d}{dq^2} \right)^2 q^2 \Pi(q^2) |_{q^2=0} = \int_{M_Q^2}^{\infty} \frac{dt}{t^2} \frac{1}{\pi} \text{Im} \Pi(t). \quad (3.7)$$

The perturbative expression of the spectral function reads :

$$\text{Im} \Pi^{\text{pert}}(t) = \frac{\text{Im} \Psi_5^{\text{pert}}(t)}{3M_Q^2 t} (2+x) - \frac{\alpha_s}{6\pi^2} \left((3+x)(1-x)^3 \log \frac{x}{1-x} + 2x \log x + (3-x^2)(1-x) \right). \quad (3.8)$$

Using the non-perturbative contributions of the moment in Ref.46), one can also deduce the sum rule :

$$\frac{1}{\gamma_{B^*}^2} = \left(\frac{\Delta_b}{M_b} \right)^3 \left(\frac{M_{B^*}}{M_b} \right)^2 \left(\frac{1}{\pi^2} \left(1 - \frac{5}{2} \left(\frac{\Delta_b}{M_b} \right) + \frac{3}{4} \left(\frac{\Delta_b}{M_b} \right)^2 \right) + NPT_V^{(1)} \right), \quad (3.9)$$

with :

$$NPT_V^{(1)} = -2 \frac{1}{M_b^3} \langle \bar{q}q \rangle \left(1 + \frac{\langle \alpha_s G^2 \rangle}{12\pi M_Q \langle \bar{q}q \rangle} - \frac{3 M_0^2}{2 M_b^2} - \frac{90\pi}{4} \rho \frac{\alpha_s \langle \bar{q}q \rangle}{M_b^3} \right). \quad (3.10)$$

One can notice from Eqs.3.6 and 3.9 that the IMET scaling law can be realized if the splitting Δ_b is constant or decreases for increasing values of M_b . There is no good argument for the behaviour of Δ_b . So we will study this point numerically. One can also notice that the Δ_b corrections are smaller for the vector than for the pseudoscalar channels. Therefore, we might expect that IMET will be better realized in the case of vector meson.

4. QSSR RE-ESTIMATE OF f_B , γ_{B^*} AND M_b

Let us now estimate the decay constant and coupling using QSSR. All of these quantities have already been estimated in the past⁽⁶⁾⁻¹⁸⁾ using different forms (Laplace, Moments, Non-relativistic) of QSSR. As has been repeatedly mentioned^(16),17), the apparent disagreements between various previous estimates come from the different criteria used for extracting the optimized results and from the uses of different values of the quark mass. In particular, low values of f_B quoted in the literature⁽⁹⁾⁻¹³⁾ correspond to high values of the quark mass and/or mainly do not satisfy the t_c stability criterion which is necessary

for the result to be insensitive to the form of the continuum model. Here, we shall revise the analysis based on the moments of the type :

$$M_P^{(n)} \equiv \frac{1}{(n+1)!} \left(\frac{d}{dq^2} \right)^{n+1} \Psi_5(q^2)|_{q^2=0} = \int_{M_Q^2}^{\infty} \frac{dt}{t^{n+2}} \frac{1}{\pi} \text{Im} \Psi_5(t). \quad (4.1)$$

and

$$M_V^{(n)} \equiv \frac{1}{(n+1)!} \left(\frac{d}{dq^2} \right)^{n+1} q^2 \Pi(q^2)|_{q^2=0} = \int_{M_Q^2}^{\infty} \frac{dt}{t^{n+1}} \frac{1}{\pi} \text{Im} \Pi(t), \quad (4.2)$$

which are close to the IMET spirit, as an expansion in the inverse of the heavy-quark mass is used in their QCD expression. The moments read :

$$M_P^{(n)} = \int_{M_Q^2}^{\infty} \frac{dt}{t^{n+2}} \frac{1}{\pi} \text{Im} \Psi_5^{pert}(t) + NPT_P^{(n)} \quad (4.3)$$

and :

$$M_V^{(n)} = \int_{M_Q^2}^{\infty} \frac{dt}{t^{n+1}} \frac{1}{\pi} \text{Im} \Pi^{pert}(t) + NPT_V^{(n)}, \quad (4.4)$$

where :

$$\begin{aligned} NPT_P^{(n)} = & - \frac{M_b}{M_b^{2(n+1)}} \langle \bar{q}q \rangle \left(1 - \frac{\langle \alpha_s G^2 \rangle}{12\pi M_Q \langle \bar{q}q \rangle} \right. \\ & \left. - \frac{(n+1)(n+2)}{4} \frac{M_0^2}{M_b^2} - \frac{(n+2)(n^2+10n+9)\pi}{4} \rho \frac{\alpha_s \langle \bar{q}q \rangle}{M_b^3} \right) \end{aligned} \quad (4.5)$$

and :

$$\begin{aligned} NPT_V^{(n)} = & - \frac{M_b}{M_b^{2(n+1)}} \langle \bar{q}q \rangle \left(1 + \frac{\langle \alpha_s G^2 \rangle}{12\pi M_Q \langle \bar{q}q \rangle} \right. \\ & \left. - \frac{(n+1)(n+2)}{4} \frac{M_0^2}{M_b^2} - \frac{(n+2)(n+4)(n+5)\pi}{4} \rho \frac{\alpha_s \langle \bar{q}q \rangle}{M_b^3} \right). \end{aligned} \quad (4.6)$$

The QSSR approach becomes familiar. One is looking for the values of n (number of derivatives) and t_c (continuum threshold) where the results present stability (minimum or inflexion point). In the stability in n , we are in a region where there is an equilibrium

between the perturbative and non-perturbative contributions, where the non-perturbative physics can already be felt but the OPE still makes sense. In the stability region in t_c , we are less sensitive to the continuum effects and so to the detailed structure of the higher mass states. For this reason, the simple QCD parametrization of the spectral function which is an average is largely sufficient for the analysis. We shall use the following QCD input parameters obtained from an updated QSSR analysis of the hadron systems¹⁷⁾ :

$$M_b = (4.59 \pm .1)GeV, \quad M_c = (1.47 \pm .1)GeV, \quad \rho = 3 \pm 1, \\ M_0^2 = (.80 \pm .02)GeV^2, \quad < \alpha_s G^2 > = (.06 \pm .02)GeV^4, \quad (4.7)$$

where we have multiplied by two the errors quoted in Ref.17) in order to take into account the systematic error related to the method. The behaviour of our results versus n and t_c is given in Figs 1 and 2, from which, we deduce the optimum results:

$$f_B = (1.72 \pm .16)f_\pi, \quad \gamma_{B^*} = (15.5 \pm 2.0) \quad (4.8)$$

and the corresponding values of n and t_c :

$$n \simeq 7 - 8, \quad \sqrt{t_c} \simeq (7.1 - 7.7)GeV. \quad (4.9)$$

We have tested the accuracy of the method by estimating the known B and B^* masses from the ratio of finite energy sum rules (FESR)-like moments :

$$R_P^{(n)} \equiv M_P^2 = \frac{\int_0^{t_c} \frac{dt}{t^{n+2}} \frac{1}{\pi} \text{Im} \Psi_5^{pert}(t) + NPT_P^{(n)}}{\int_0^{t_c} \frac{dt}{t^{n+2}} \frac{1}{\pi} \text{Im} \Psi_5^{pert}(t) + NPT_P^{(n+1)}}, \quad (4.10)$$

which is the meson mass squared in our duality ansatz parametrization of the spectral function. Analogous ratio can be also constructed for the vector channel. Then, one obtains with the same values of n and t_c as in Eq.4.5, where the ratio $R_{P,V}^{(n)}$ also stabilizes, the values :

$$M_B = (5.23 \pm .09)GeV, \quad M_{B^*} = (5.45 \pm .15)GeV \quad (4.11)$$

which are in good agreement with the data and which confirm previous QSSR estimates obtained from different forms of the sum rules based on stability criteria¹⁶⁾⁻¹⁸⁾.

Let us now re-analysis the errors on M_b . In so doing, we move the value of M_b outside the range in Eq.(1.5). Giving, for instance, $\Lambda = .15\text{GeV}$ and taking $M_b = 5\text{GeV}$, the predictions on M_B and M_{B^*} shift above the experimental values by an amount of 390 and 500 MeV. Inversely, if one uses a lower value of $M_b = 4.2\text{GeV}$ and we still fix Λ at the previous value, the predictions are lower than the data by 290 and 229 MeV for the B and B^* mesons. This analysis indicates that allowing 10% errors in the value of M_b will lead to very inaccurate predictions for the meson masses. We have also tried to see the effects of Λ on the value of M_b by taking the range $\Lambda = .05 - .30\text{GeV}$. We see that lower values of Λ are correlated to lower values of M_b . Fixing $\Lambda = .05\text{GeV}$, we find that we cannot fit simultaneously the B and B^* masses. The B mass tends to be underestimated while the B^* one tends to be overestimated. The lowest value of $M_b = 4.49\text{GeV}$ given by Eq.1.5 corresponds to a departure of 70 MeV from the two experimental masses. Pushing this quark mass for instance until 4.42 GeV, where the B^* mass is reproduced, one underestimates the B mass by 210 MeV, which is too low. Fixing now $\Lambda = .30\text{GeV}$, and taking $M_b = 4.69\text{ GeV}$ from the extremal value given by Eq.1.5, we reproduce the B mass but the B^* one is overestimated by 229 MeV. Better estimate can be obtained by decreasing the value of M_b and this explains the smallness of the errors given in Refs.16b) and 17). Therefore, our analysis indicates that the errors given in Eq.1.5 are reasonable. The effects of the condensate values given in Eq.4.7 on the errors of M_b are less important. Moreover, we cannot play too much with these values as they have already been fixed from an overall fit of the light and heavy quarks systems¹⁷⁾. A re-analysis of the Υ system by Ref.15b) leads to a value of the pole mass of 4.55 GeV where the error of 50 MeV given there is possibly underestimated compared with the results from previous fits of the Υ system reviewed in Ref.17) which lead to an analogous value of M_b . Then, we conclude that the errors given in Eq.1.5 are not underestimated and the b-quark pole mass value from QSSR is :

$$M_b = (4.59 \pm .10)\text{GeV} \quad \text{for} \quad \Lambda = (50 - 300\text{MeV}). \quad (4.12)$$

5. CRITICAL REVIEW OF THE QSSR RESULTS

a) The decay constant f_B

Let me repeat again that the low values obtained in Refs.9)-13) correspond to the choice of the continuum threshold in the region where the t_c stability is not yet reached. This has been proven in Ref.16) where indeed all forms of the sum rules lead to a unique prediction once a stability criterion is used for t_c in addition to the stability required for the sum rule variables. The result of Ref.16) has been improved in Ref.17) with the use of the value of the b quark mass quoted in Eq.1.5 and updated values of the QCD input parameters in Eq.4.7. In this way, one obtains from the Laplace exponential sum rules¹⁷⁾ :

$$\begin{aligned} f_B/f_\pi &= (1.34 \pm .30) \text{ (non - relativistic Laplace SR)} \\ & (1.61 \pm .16) \text{ (relativistic Laplace SR)}. \end{aligned} \quad (5.1)$$

The non-relativistic Laplace SR corresponds to the case of an infinite quark mass value, which, as we have previously seen, is a very crude approximation in the case of the b quark. Therefore, we shall discard this result in the final average.

The authors in Ref.15) using a slightly different form of the moments ($q^2 \neq 0$) obtains their optimum results at the same values of the t_c stability region as in Refs.16),17) and in the present paper which starts from values of t_c of about 50 GeV^2 . Rescaling the result of Ref.15) obtained with the value of the pole mass of 4.7 GeV , one obtains for the mass in Eq.1.5 :

$$f_B = (1.7 \pm .27)f_\pi. \quad (5.2)$$

We also use the previous value of M_b in the estimate of the four-quark matrix element based on stability criteria¹⁸⁾. Using $B_B = (1 \pm .1)$ by assuming that the factorization hypothesis is satisfied within 10% for the heavy quark sector, we obtain :

$$f_B = (1.71 \pm .20)f_\pi. \quad (5.3)$$

Excluding the non-relativistic result due to the crude approximation for obtaining it and averaging the previous results with the one in Eq.4.8 obtained under optimum conditions and corresponding to $M_b = 4.59 \text{ GeV}$, one obtains :

$$f_B = (1.69 \pm .20)f_\pi, \quad (5.4).$$

where we have added the error quadratically and multiplied it by a factor two for an educated guess of the uncertainty of the method. Let us now compare this result obtained

within stability criteria with the existing ones in the QSSR literature and comment on these values.

Using Laplace (non)-relativistic Laplace sum rules, Refs.9) to 11) found very low values:

$$f_B = (.99 - 1.06)f_\pi. \quad (5.5)$$

The disagreement between Refs 9) and 10) is due to the non-inclusion of the α_s correction in Ref.10). However, these two sum rules correspond to the infinite mass limit which, according to our discussion in the previous section, is a very crude approximation. In this limit, the decay constant tends to be underestimated. Moreover, as shown in Refs 16) and 17), the result in Eq.5.5 is obtained for too low a value of the continuum threshold. Therefore, the result on f_B is twice underestimated.

The result of Ref.11) is obtained with the relativistic Laplace SR. In addition to the criticisms raised in Ref.15) on the inconsistencies of the set of parameters used by the authors, we also notice that the authors use $t_c \simeq 33 GeV^2$ which is outside the t_c stability region starting from $50-60 GeV^2$, a value which is independent of the form of the sum rules¹⁵⁾⁻¹⁸⁾. One should also notice that the apparent stability in the Laplace sum rule variable shown in Fig.4 of Ref.11) for f_B , which motivated their choice of t_c and which tends to disappear for increasing t_c , is misleading. In fact, in this high-energy region the continuum effect is already important. The true minima of f_B where the resonance contribution is maximum occur at lower values of the sum rule scale of about $1 - 2 GeV^2$ (see e.g Fig.3 of Ref.11) for f_D) and becomes more pronounced for increasing values of t_c . This point has been explicitly analyzed in Refs.16) and 17) and invalidates the result of Ref.11).

The second class of QSSR results come from Refs.12) to 14). Their results are :

$$\begin{aligned} f_B/f_\pi &= (1.35 \pm .25) && \text{central value of Ref.12} \\ & (1.44 \pm .23) && \text{Ref.13} \\ & 1.74 && \text{Ref.14.} \end{aligned} \quad (5.6)$$

The result of Ref.12) is obtained from the first two derivatives of the two-point correlator where both the stability in n (number of derivatives) and in t_c ($t_c \simeq (1.1 - 1.3) M_B^2$ in Ref.12)) are not yet reached. Therefore the result of Ref.12) must be largely affected by

the continuum. The authors also use a low value of the mixed condensate which plays an important role in this heavy-light quarks channel. All these uncertainties affect their result and explain the difference with the one obtained in this paper from higher moments.

The result of Ref.13) is obtained from two different Laplace sum rules where the author fixes the value of t_c from the matching of the two sum rules at high values of the sum rule scale. The same criticisms as those done for Ref.11), which uses a Laplace sum rule, applies also here. In fact, the true minima are missed by the author, where the matching condition can make sense only in these minima. As a result, the author obtained a coincidence of the two sum rules at too low values of t_c outside the stability region.

Finally, the result in Ref.14) is very crude as it is obtained to the lowest order of perturbation theory and without the contributions of the mixed and four-quark condensates. In fact, it is known that the inclusion of these contributions in the analysis is essential for the estimate of the decay constant. The large value of f_B obtained there is mainly due to the too low value 4.2 GeV of M_b , but this definition is ill-defined without the inclusion of radiative corrections.

In conclusion, there is strong evidence and well founded justification for not doing an average of all existing QSSR estimates naively. In fact, some of these results are not optimum and are even incorrect. The average value of f_B given in Eq.5.4 is the final and optimum result from QSSR and reads :

$$f_B = (1.69 \pm .20)f_\pi. \quad (5.7)$$

f_B agrees with the recent lattice results of Refs.5),25)-27) and challenges the IMET results applied at the b mass scale.

b) The decay constant f_D

Let us first quote the values of f_D from different estimates :

$$\begin{aligned}
f_D/f_\pi &= (1.30 \pm .10) && \text{Ref.8b)} \\
&1.67 && \text{Ref.9)} \\
&1.29 && \text{Ref.11)} \\
&(1.70 \pm .20) && \text{Ref.12)} \\
&1.52 && \text{Ref.13)} \\
&1.45 && \text{Ref.14)} \\
&(1.31 \pm .12) && \text{Ref.16)}.
\end{aligned} \tag{5.8}$$

Let us first notice that the agreement between various estimates of f_D is much better than for the case of f_B . The main reason is that the $\bar{t}_c$ values of about $6-7\text{GeV}^2$ used by different authors belong to the t_c stability region ^{16),17)}.

We have only a few comments to make. The non-relativistic sum rule used in Ref.9) corresponding to the infinite mass limit must be a very bad approximation for the charm. The use of moment sum rules based on $1/M_c$ expansion as in Ref.12) is very dangerous here as the c quark mass is still too low to justify this expansion. However, limiting the analysis to the first two moments is not sufficient, as the role of the continuum is too important in these moments. In fact, for given values of t_c , the lowest moments tend to overestimate the true result (see e.g. the behaviour of these results for different values of the derivatives in Ref.16) and 17)). Moreover, the authors also use a value of the c quark pole mass lower than the one in Eq.4.7. This also implies an overestimate of their result. The result of Ref.14) is a lowest order result, as in the earlier works in Refs.6) to 8), which cannot be optimum and must be inaccurate.

Finally, we conclude that the average value of f_D obtained in Ref.16) remains unchanged. This result comes from the exponential Laplace SR which is more appropriate in this channel as the $1/M_c$ expansion is not used in this analysis. Another reason for the invariance of the result is that the value of the charm quark mass obtained from the sum rule is stable in time¹⁷⁾. The average value from QSSR is therefore^{16),17)} :

$$f_D = (1.31 \pm .12)f_\pi. \tag{5.9}$$

corresponding to the QCD input values in Eq.4.7.

c) SU(3) and SU(4) flavour breakings : the B_s , D_s and B_c mesons

Finally, let us discuss the inclusion of the SU(3)-breaking effects. Following Refs.7) and 16), we estimate it by taking the ratio of the lowest moment $M_P^{(1)}$ in Eq.3.7 in the d and s quark channels obtained within a $1/M_Q$ expansion. Though each separate moment can be highly affected by the continuum, we have verified that this effect as well as of the leading radiative and of the flavour-independent condensates compensate in the ratio. Then, we obtain^{7),16)} :

$$f_{B_s} \simeq f_B \left(\frac{M_{B_s}}{M_B} \right) \left(1 + \frac{3}{2} \frac{m_s}{M_b} \left(1 - \frac{M_b^2}{t_c} \right) - \frac{4\pi^2}{M_b^3} \langle \bar{s}s - \bar{u}u \rangle \right) \simeq 1.08 \left(\frac{M_{B_s}}{M_B} \right) f_B, \quad (5.10)$$

for¹⁷⁾ $m_s/M_b \simeq .04$ and $\langle \bar{s}s \rangle / \langle \bar{u}u \rangle \simeq .6 \pm 0.1$, where B_s is the $\bar{s}b$ pseudoscalar meson. Using the value :

$$M_{B_s} \simeq (5.44 \pm .16) \text{GeV}, \quad (5.11)$$

deduced from averaging the result in Refs.16) and 17) with the phenomenological expectation : $M_{B_s} - M_B \simeq M_{D_s} - M_D$, one obtains :

$$f_{B_s} \simeq (1.11 \pm .03) f_B \simeq (1.88 \pm .24) f_\pi. \quad (5.12)$$

For a crude approximation, one may extrapolate this expression to the D meson. Therefore, we may deduce the value :

$$f_{D_s} \simeq 1.13 f_D, \quad (5.13)$$

where the mass and condensate corrections tend to compensate, explaining the almost constancy of the corrections for the b and c cases.

The decay constant and the mass of the pseudoscalar $B_c(\bar{c}b)$ meson have also been estimated in Ref.46), using moments sum rules. Contrary to the case of the light quarks, the condensates of the heavy quark can be derived using a $1/M_Q$ expansion techniques. In this case, they can be expressed in terms of the gluon condensate as¹⁷⁾ :

$$\begin{aligned} \langle \bar{c}c \rangle &\simeq -\frac{1}{12\pi M_c} \langle \alpha_s G^2 \rangle + \dots, \\ g \langle \bar{c} \sigma^{\mu\nu} \lambda_a G_{\mu\nu}^a c \rangle &\simeq -\frac{M_c}{\pi} \langle \alpha_s G^2 \rangle \left(2 \log \frac{\nu}{M_c} - 1 \right) + \dots, \end{aligned} \quad (5.14)$$

where ν is an arbitrary subtraction scale. In Ref. 46), the choice $\nu = M_b$ has been used and the value $M_{B_c} \simeq 6.9 GeV$ has been obtained. We have checked that the prediction for the mass is quite sensitive to the value of ν contrary to that of the decay constant. Taking $M_c \leq \nu \leq M_b$, we obtain :

$$M_{B_c} \simeq (6.5 \pm .4) GeV, \quad f_{B_c}/f_\pi \simeq (3.7 \pm .5), \quad (5.15)$$

where the maximum value is the result in Ref.46) associated with $\nu = M_b$. This result is in the range given by different potential models^{44),47),23a)}. A search for B_c in LHC experiment⁴⁸⁾, which looks feasible, can help for an accurate test of the different approaches.

d) Couplings and masses of vector, axial and scalar mesons

The coupling of the D^* can be more reliably estimated from the exponential Laplace SR which does not use the $1/M_c$ expansion as the charm mass is still too low for justifying this expansion. In this case, the form of the sum rule is :

$$L_\tau = \int_{M_Q^2}^{\infty} dt \exp(-t\tau) \frac{1}{\pi} \text{Im} \Pi^{pert}(t) + N P T_\tau, \quad (5.16)$$

where the τ variable replaces n .

M_{D^*} is extracted from the ratio of FESR :

$$R_\tau = \frac{\int_{M_Q^2}^{t_c} t dt \exp(-t\tau) \frac{1}{\pi} \text{Im} \Pi^{pert}(t) + N P T'_\tau}{\int_{M_Q^2}^{t_c} dt \exp(-t\tau) \frac{1}{\pi} \text{Im} \Pi^{pert}(t) + N P T_\tau}. \quad (5.17)$$

The behaviour of the results versus τ and t_c are given in Fig.3. At the region of stability, one deduces :

$$\gamma_{D^*}^{-1} = (.40 \pm .06) , M_{D^*} = (1.97 \pm .07) GeV. \quad (5.18)$$

By comparing this result with the one in Eq.4.8, one can verify that the couplings of vector mesons approximately satisfy the IMET scaling law in Eq.2.16. This result is comparable with the one in Ref.12b) but disagrees with that of Ref.49). The authors in Ref.49) obtain

their result from a lowest order perturbative graph and at a value 1GeV of the sum rule scale where the stability is not yet reached. The stability at $\tau \simeq 1.6GeV^{-2}$ is lost in their analysis as they use too low a value of the charm quark mass. Their alternative result obtained from the longitudinal part of the correlator must be affected by the D meson.

For the B-meson families, we use the moments within a $1/M_b$ expansion. Then, we obtain with the QCD input values in Eq.4.7^(46),17) :

$$\gamma_{B_A^*} \simeq (14.0 \pm .5) \quad \gamma_{B_c^*} \simeq (14.0 \pm 1.). \quad (5.19)$$

$$f_{B_\delta} \simeq (2.0 \pm .4)f_\pi. \quad (5.20)$$

The values of the meson masses are^(46),17) :

$$M_{B_A^*}/M_{B^*} \simeq 1.05, \quad M_{B_c^*} \simeq M_{B_c}, \quad M_{B_\delta}/M_B \simeq 1.04. \quad (5.21).$$

which indicate a quite good symmetry between parity doublets.

6. QUARK MASS DEPENDENCE OF f_P , γ_V AND $M_{P,V}$

We study the quark mass dependence of f_P and γ_V from the moments in Eqs 4.1 and 4.2. However, the meson mass is not known in these expressions, so we introduce it via the ratio of moments in Eq.4.9. Therefore in our analysis, we do a simultaneous determination of the meson couplings and masses. In order to illustrate our analysis, we show in Figs.4 and 5 the n and t_c behaviours of f_P and M_P as well as of γ_V and M_V for a given value of the quark mass. We can notice that the decay constant and meson mass output stabilize at the same values of n and t_c . This increases the reliability of our results. These stabilities are lost for :

$$M_Q \geq 13GeV, \quad (6.1)$$

which gives a warning on the upper limit for the applicability of the method. Our analysis gives the flavour independent mass differences :

$$M_P - M_Q \simeq .7GeV, \quad M_V - M_Q \simeq .8GeV, \quad \Delta_Q \equiv \sqrt{t_c^{min}} - M_Q \simeq 2.5GeV, \quad (6.2)$$

These results are also expected from IMET non-relativistic approaches. The third result can be interpreted as the splitting between the radial excitation and the quark. Nevertheless, as the QCD continuum gives an average of the higher states contributions, this value should

only be considered as a very crude estimate of the mass of the radial meson. However, its constancy indicates that for increasing M_Q , it becomes difficult to separate the lowest ground state from the continuum and thus explaining the failure of the approach for M_Q above 13 GeV.

The quark mass dependences of f_P and $1/\gamma_V$ are given in Figs.6 and 7, where the predictions from the IMET scaling laws :

$$f_P = \sqrt{\frac{M_b}{M_Q}} f_B, \quad \frac{1}{\gamma_V} = \left(\frac{M_{B^*}}{M_V} \right)^{3/2} \frac{1}{\gamma_{B^*}} \quad (6.3)$$

and the one in Eq.2.8, are also shown. One can notice that f_P shows a net departure from the IMET scaling law while γ_V seems to satisfy approximately it even until M_c . This is however in agreement with our qualitative expectations from Eqs.3.6 and 3.9, where the Δ_b/M_b corrections are smaller for the vector than for the pseudoscalar channels. As the lattice groups²⁷⁾, we search for an interpolating formula of the result in Fig.6 for $M_Q \geq M_c$. Then, we obtain in units of GeV :

$$f_P \simeq .57 M_Q^{-1/2} \left(1 - \frac{2.22}{M_Q} + \frac{1.67}{M_Q^2} \right), \quad (6.4)$$

where the M_Q^{-1} corrections can be considered as effective corrections which take into account the remaining higher order ones. The error bars corresponding to the previous numbers are typically 10 – 15%. Similar strengths of the $1/M_Q$ corrections are also obtained by the lattice groups²⁷⁾. It is interesting and encouraging that the two different non-perturbative QCD approaches lead to similar results (at least within the errors).

7. QSSR ESTIMATE OF THE $B \rightarrow De\nu_e$ FORM FACTOR

We shall be concerned with the vertex function $V_+(0)$ defined in Eq.2.16 associated with the three-point function in Eq.2.15. The perturbative expression of its spectral function reads⁵⁰⁾ :

$$\text{Im} V_+(t', t, 0) = -\frac{3M_c M_b}{2(t' - t)^2} \left((M_b^2 - M_b M_c)t - (M_c^2 - M_c M_b)t' + M_c M_b (M_c^2 - M_b^2) \right), \quad (7.1)$$

while its non-perturbative contributions from the $\langle \bar{q}q \rangle$ and mixed $g \langle \bar{q}G^2 q \rangle$ condensates read for spacelike k_i^2 :

$$V_+^{(3)}(0) = \frac{1}{2} \langle \bar{q}q \rangle \frac{M_c M_b (M_c + M_b)}{(k_1^2 - M_c^2)(k_2^2 - M_b^2)} \quad (7.2)$$

and⁵⁰⁾ :

$$V_+^{(5)}(0) = -\frac{M_0^2}{12} \langle \bar{q}q \rangle M_c M_b \left(\frac{2(2M_c + M_b)}{r_1^2 r_2} + \frac{2(2M_b + M_c)}{r_2^2 r_1} + \frac{3M_c^2(M_b + M_c)}{r_1^3 r_2} + \frac{3M_b^2(M_b + M_c)}{r_2^3 r_1} + \frac{M_b^2(2M_b + M_c) + M_c^2(2M_c + M_b)}{r_2^2 r_1^2} \right), \quad (7.3)$$

with $r_1 = k_1^2 - M_c^2$ and $r_2 = k_2^2 - M_c^2$. The resonance contribution to the vertex reads :

$$V_+^{res}(0) = -2f_D f_B M_D^2 M_B^2 \frac{f_+(0)}{(k_1^2 - M_D^2)(k_2^2 - M_B^2)}, \quad (7.4)$$

where as in section 2, $f_+(0)$ is the dimensionless form factor. The continuum effect is parametrized by the QCD spectral function from thresholds t_c^B and t_c^D for the D and B mesons. Analogously to the IMET approach, we shall treat the b quark as heavy, i.e we shall do an expansion in $1/M_b$, but we treat the charm as a light one, i.e we shall not take the limit $-k_1^2 \ll M_c^2$. Therefore, we shall work with the FESR quantity :

$$V_+^{res}(n, \tau, 0) = -\frac{1}{4\pi^2} \int_{M_b^2}^{t_c^B} \frac{dt'}{t'^n} \int_{M_c^2}^{t_c^D} dt \exp(-t\tau) \text{Im} V_+^{pert}(t', t, 0) + NPT(n, \tau), \quad (7.5)$$

which we have obtained by transferring the QCD continuum contribution in the RHS of the sum rule. $NPT(n, \tau)$ are the non-perturbative contributions which one can deduce from Eqs.7.2 and 7.3. The resonance contribution is :

$$V_+^{res}(n, \tau, 0) = -2f_D f_B M_D^2 M_B^2 \frac{f_+(0)}{M_B^{2n}} \exp(-\tau M_D^2). \quad (7.6)$$

We estimate $f_+(0)$ from the sum rule in Eqs.7.5 and 7.6. Our result is given in Fig.8 for different values of n and τ , where we have shown the values of $f_+(0)$ corresponding to the QCD input parameters given in Eqs.4.7 and 4.12. Stability in the change of the continuum threshold is reached for values of the thresholds roughly similar to those in the two-point functions analysis ($t_c^D \simeq 8 - 10 \text{ GeV}^2$, $t_c^B \simeq 60 - 80 \text{ GeV}^2$). The optimum prediction corresponding to these central values of the input parameters is :

$$f_+(0) \simeq .55. \quad (7.7)$$

We do the error analysis by considering the correlations between the values of the decay constants and the associated quark mass (minimum value of the decay constant corresponds to maximum value of the quark mass) from the analysis of the decay constant. Then, we obtain an error of about 7.3%. Multiplying that error by two as an educated guess of the systematic uncertainty, we finally obtain :

$$f_+(0) \simeq (.55 \pm .10). \quad (7.8)$$

We compare this result with the one in Refs.50) obtained from double exponential sum rules. By rescaling their input values for the decay constants and continuum thresholds, we obtain a good agreement of the results from the two different aspects of QSSR. This large deviation from the ideal IMET symmetry limit in this channel signals the large role of the finite mass corrections for the form factor. A similar result is also being derived in Ref.51). One can, for instance, use this result in the semileptonic decay width in order to determine the CKM mixing angle V_{cb} . In so doing, we use the following pole parametrization of the form factor for its time-like q^2 -dependence:

$$f_+(q^2) = \frac{f_+(0)}{1 - q^2/(M_B + M_D)^2}. \quad (7.9)$$

The decay width reads :

$$\Gamma(B \rightarrow D e \nu_e) = \frac{G_F^2 M_B^5}{192\pi^3} |V_{bc}|^2 f_+^2(q_{max}^2) 1.6, \quad (7.10)$$

where $q_{max}^2 = (M_B - M_D)^2$ and 1.6 comes from the phase space. Using the experimental values of the B^- lifetime⁵²⁾ and of the B^- branching ratio from CLEO⁵³⁾ :

$$\tau_B = (1.18 \pm .11) 10^{-12} \text{ sec},$$

$$Br(B^- \rightarrow D e \nu_e) = (1.60 \pm .67), \quad (7.11)$$

we deduce :

$$|V_{cb}| = (.044 \pm .011). \quad (7.12)$$

The error is large (25%) but the result is in amazing agreement with the standard PDG one.

8. SUMMARY AND CONCLUSIONS

1. In this paper, we have tested the accuracy and validity of the IMET approach when the IMET results are extrapolated to finite values of the c and b quark masses. In so doing, we have derived and recovered the IMET results using an OPE of the hadronic correlators and a dispersive approach. We have found that the IMET results are governed by the universal $\langle \bar{q}q \rangle$ light quark condensate which can be related to the quark constituent mass⁵⁴⁾ within a chiral Lagrangian framework and/or to the so-called Wisgur function. We have evaluated the corrections to this ideal result and found that these corrections are larger in the pseudoscalar than in the vector channels. Therefore, we have concluded that IMET cannot be used to estimate f_D and even f_B .

2. We have quantitatively checked this conclusion by estimating the decay constant f_B and coupling γ_{B^*} using QSSR moments within a $1/M_Q$ expansion and using the latest updated values of QCD input parameters from an overall QSSR fit of the hadron systems¹⁷⁾. We have also given a critical review of the existing estimates of the decay constants $f_{B,D}$ and conclude that the average of optimum values given by QSSR are :

$$f_D = (1.31 \pm .12)f_\pi \quad (6.8)$$

$$f_B = (1.69 \pm .20)f_\pi, \quad (6.7)$$

where the value of f_D is still equal to the old average given in Ref.16). These values are in good agreement with lattice estimates in Refs.1)-5) and 25)-27).

3. We have also reviewed the QSSR results for the couplings and masses of the other B meson families and compared the results with those from potential models. This issue is discussed in paragraphs c and d of section 5.

4. We have studied the heavy quark mass dependence of the decay constants and masses of the pseudoscalar and vector mesons. We found that the IMET scaling law in Eq.2.10 is largely affected by the $1/M_Q$ correction in the pseudoscalar channel. Our analysis suggests the modified scaling law in units of GeV :

$$f_P \simeq .57M_Q^{-1/2} \left(1 - \frac{2.22}{M_Q} + \frac{1.67}{M_Q^2}\right), \quad (6.4)$$

for $M_Q > M_c$, while in the case of the vector meson, the IMET scaling law in Eq.2.16 is better satisfied. We also found that the mass splitting between the meson and the corresponding quark Q is almost flavour-independent and reads :

$$M_P - M_Q \simeq .7GeV, \quad M_V - M_Q \simeq .8GeV, \quad \Delta_Q \equiv \sqrt{t_c^{min}} - M_Q \simeq 2.5GeV. \quad (6.2)$$

The first two results agree with the IMET expectations. The third result, which might be approximately (the QCD continuum is an average of all higher states) interpreted as the mass difference between the radial excitation and the quark, indicates that for large M_Q the separation of the ground state from the continuum cannot be done. This fact explains the failure of the method for M_Q larger than 13 GeV.

5. Finally, we have estimated the form factor associated with the $B \rightarrow D e \nu_e$ decay and found :

$$f_+(0) = (.55 \pm .10), \quad (7.8)$$

which indicates a large deviation from the IMET result and from the Ademollo-Gatto theorem⁴⁵⁾. Using this value at zero recoil via a simple pole parametrization of the form factor, we obtain the value of the CKM mixing angle to be :

$$|V_{cb}| \simeq (.044 \pm .011) \quad (7.11)$$

which is in amazing agreement with the PDG world average, though less accurate.

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FIGURE CAPTIONS

Fig.1 : n and t_c behaviours of f_B and M_B^2 .

Fig.2 : n and t_c behaviours of γ_{B^*} and M_{B^*} .

Fig.3 : n and t_c behaviours of γ_{D^*} and M_{D^*} .

Fig.4 : n and t_c behaviours of f_P and M_P^2 for a given value of M_Q .

Fig.5 : n and t_c behaviours of γ_V and M_V for a given value of M_Q .

Fig.6 : M_Q dependence of f_P . The full line is the prediction from QSSR by using the value of the meson mass from the ratio of moments. The two points with the error bars are the predictions using the experimental values of M_D and M_B . The dotted-dashed line is the fit from Eq.6.4. The dashed curves are the IMET predictions extrapolated to the c and b masses.

Fig.7 : M_Q dependence of γ_V . The dashed curves are the IMET predictions extrapolated to the c and b masses.

Fig.8 : n , τ and t_c behaviours of $f_+(0)$.

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